



HAESE MATHEMATICS

Mathematics

Applications and Interpretation SL



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for use with

IB Diploma Programme

REVISION GUIDE

MATHEMATICS: APPLICATIONS AND INTERPRETATION SL REVISION GUIDE

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FOREWORD

The aim of this Guide is to help you prepare for tests and the final examination for the Mathematics: Applications and Interpretation SL course.

This Guide covers all five Topics in the Mathematics: Applications and Interpretation SL syllabus. All of the relevant material from the Mathematics: Core Topics SL and the Mathematics: Applications and Interpretation SL textbooks is covered.

For each Topic, there is a theory summary and a set of skill builder questions.

- The theory summaries highlight the important facts and concepts. They are intended to complement your textbook and International Baccalaureate booklet. When a formula can be found in the formula booklet, it may not be repeated in this Guide.
- The set of skill builder questions are designed to help consolidate your understanding of each Topic. They should be used to reinforce key ideas, and to identify any areas of weakness. Within each Topic, the questions are logically ordered according to the chapters of the textbook, so they can be used for test preparation.

Following the coverage of all five Topics, the Guide has ten mixed questions sets, each containing 12 questions. Each set contains questions from every Topic, as well as cross-topic questions. It is recommended that you attempt all of the questions in a mixed questions set in one sitting, as this will give you practice in answering questions from a range of topics in a short time frame.

The Guide concludes with four trial examinations, written by IB teachers from around the world. Each trial examination contains two papers: Paper 1, which contains shorter questions, and Paper 2, which contains longer questions. This format is consistent with the Mathematics: Applications and Interpretation SL final examination. Full solutions are provided, but it is recommended that you work through a complete paper before checking the solutions.

We recommend completing each trial examination under exam conditions. You are encouraged to print the formulae summary (see page 5), and have it alongside you as you complete the trial examinations.

- If you are having trouble with a question, it is often a good strategy to move on to other questions, and return to it later. Time management is very important during the examination, and too much time spent on a difficult question may mean that you do not leave yourself sufficient time to complete other questions.
- Set out your work clearly with full explanations. A correct answer with no working will not necessarily receive full marks.
- If you make a mistake, draw a single line through the work you want to replace. Do not cross out work until you have replaced it with something you consider better.
- Diagrams and graphs should be sufficiently large, well labelled, and clearly drawn.

- Remember to leave answers correct to three significant figures unless an exact answer is more appropriate or a different level of accuracy is requested in the question.
- Check for key words. If the word “hence” appears, then you must use the result you have just obtained. “Hence, or otherwise” means that you can use any method you like, although it is likely that the best method uses the previous result.
- It is important to read the question carefully. Rushing into a question may mean that you miss subtle points. Underlining key words may help.
- Remember that questions in the examination are often set so that, even if you cannot complete one part, the question can still be picked up in a later part.

After completing a trial examination, you should identify areas of weakness.

- Return to your notes or textbook and review any material you found challenging.
- Ask your teacher or a friend for help if further explanation is needed.
- Summarise each Topic. Summaries that you make yourself are the most valuable.
- If you have had difficulty with a question, try it again later. Do not just assume that you know how to do it once you have read the solution.

In addition to the formula booklet, your graphics display calculator is an essential aid.

- Make sure you are familiar with the model you will be using.
- In trigonometry questions, remember to check that the graphics calculator is in degrees.
- Important features of graphs may be revealed by zooming in or out.
- When using your graphics calculator, it is always important to reflect on the reasonableness of the results.

We hope this Guide will help you structure your revision program effectively. Remember that good examination techniques will come from good examination preparation.

We welcome your feedback:

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FORMULAE SUMMARY



PRIOR LEARNING

Area of a parallelogram	$A = bh$, where b is the base, h is the height
Area of a triangle	$A = \frac{1}{2}(bh)$, where b is the base, h is the height
Area of a trapezoid	$A = \frac{1}{2}(a + b)h$, where a and b are the parallel sides, h is the height
Area of a circle	$A = \pi r^2$, where r is the radius
Circumference of a circle	$C = 2\pi r$, where r is the radius
Volume of a cuboid	$V = lwh$, where l is the length, w is the width, h is the height
Volume of a cylinder	$V = \pi r^2 h$, where r is the radius, h is the height
Volume of a prism	$V = Ah$, where A is the area of the cross-section, h is the height
Area of the curved surface of a cylinder	$A = 2\pi r h$, where r is the radius, h is the height
Distance between two points (x_1, y_1) and (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

TOPIC 1: NUMBER AND ALGEBRA

ARITHMETIC SEQUENCES

$$u_n = u_1 + (n - 1)d$$

$$S_n = \frac{n}{2}(2u_1 + (n - 1)d) \quad \text{or} \quad S_n = \frac{n}{2}(u_1 + u_n)$$

GEOMETRIC SEQUENCES

$$u_n = u_1 r^{n-1}$$

$$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \quad r \neq 1$$

COMPOUND INTEREST

$$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}, \quad \text{where}$$

FV is the future value
 PV is the present value
 n is the number of years
 k is the number of compounding periods per year
 $r\%$ is the nominal annual rate of interest

EXPONENTS AND LOGARITHMS

$$a^x = b \Leftrightarrow x = \log_a b, \quad \text{where } a > 0, b > 0, a \neq 1$$

PERCENTAGE ERROR

$$\varepsilon = \left| \frac{V_A - V_E}{V_E} \right| \times 100\%, \quad \text{where } V_E \text{ is the exact value and } V_A \text{ is the approximate value.}$$

TOPIC 2: FUNCTIONS

STRAIGHT LINES

$y = mx + c$ or $ax + by + d = 0$ or $y - y_1 = m(x - x_1)$
 $m = \frac{y_2 - y_1}{x_2 - x_1}$

QUADRATIC FUNCTIONS

$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$

TOPIC 3: GEOMETRY AND TRIGONOMETRY

MEASUREMENT

Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) and (x_2, y_2, z_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
Volume of a right-pyramid	$V = \frac{1}{3}Ah$, where A is the area of the base, h is the height
Volume of a right cone	$V = \frac{1}{3}\pi r^2h$, where r is the radius, h is the height
Area of the curved surface of a cone	$A = \pi rl$, where r is the radius, l is the slant height
Volume of a sphere	$V = \frac{4}{3}\pi r^3$, where r is the radius
Surface area of a sphere	$A = 4\pi r^2$, where r is the radius
Length of an arc	$l = \frac{\theta}{360} \times 2\pi r$, where θ is the angle measured in degrees, r is the radius
Area of a sector	$A = \frac{\theta}{360} \times \pi r^2$, where θ is the angle measured in degrees, r is the radius

TRIGONOMETRY

Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Cosine rule	$c^2 = a^2 + b^2 - 2ab \cos C$ and $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
Area of a triangle	$A = \frac{1}{2}ab \sin C$

TOPIC 4: STATISTICS AND PROBABILITY

Interquartile range = $Q_3 - Q_1$

Mean of a set of data $\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n}$, where $n = \sum_{i=1}^k f_i$

PROBABILITY

Probability of an event A $P(A) = \frac{n(A)}{n(U)}$

$$P(A) + P(A') = 1$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) \quad \text{for mutually exclusive events}$$

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A)P(B) \quad \text{for independent events}$$

Expected value of a discrete random variable X , $E(X) = \sum x P(X = x)$

BINOMIAL DISTRIBUTION

For $X \sim B(n, p)$:

- Mean $E(X) = np$
- Variance $\text{Var}(X) = np(1 - p)$

TOPIC 5: CALCULUS

DIFFERENTIATION

$$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$$

INTEGRATION

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

Area between a curve $y = f(x)$, where $f(x) > 0$, and the x -axis $= \int_a^b y dx$

TRAPEZOIDAL RULE

$$\int_a^b y dx \approx \frac{1}{2}h((y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})), \quad \text{where } h = \frac{b-a}{n}$$

TOPIC 1: NUMBER AND ALGEBRA

APPROXIMATION AND ESTIMATION

Rounding numbers:

- If the digit after the one being rounded is less than 5, round down.
- If the digit after the one being rounded is 5 or more, round up.

Significant figures are counted from the first non-zero digit from the left.

For example: $3.413 \approx 3.41$ (to 2 decimal places)

$0.034\ 56 \approx 0.0346$ (to 3 significant figures)

$236.5 \approx 237$ (to the nearest whole number)

You are expected to give answers either exactly or rounded to 3 significant figures unless otherwise specified in the question.

A **measurement** is accurate to $\pm \frac{1}{2}$ of the smallest division on the scale.

An **approximation** is a value given to a number which is close to, but not equal to, its true value.

An **estimation** is a value which is found by judgement or prediction instead of carrying out a more accurate measurement.

If the exact value is V_E and the approximate value is V_A then:

- **absolute error** = $|V_A - V_E|$
- **percentage error** = $\frac{|V_A - V_E|}{V_E} \times 100\%$

SCIENTIFIC NOTATION (STANDARD FORM)

A number is in **scientific notation** if it is written in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

SEQUENCES AND SERIES

A **number sequence** is a set of numbers defined by a rule. Often, the rule is a formula for the **general term** or **n th term** of the sequence.

A sequence which continues forever is called an **infinite sequence**. A sequence which terminates is called a **finite sequence**.

Arithmetic sequences

In an **arithmetic sequence**, each term differs from the previous one by the same fixed number.

$u_{n+1} - u_n = d$ for all $n \in \mathbb{Z}^+$, where d is a constant called the **common difference**.

For an arithmetic sequence with first term u_1 and common difference d , the n th term is $u_n = u_1 + (n - 1)d$.

Geometric sequences

In a **geometric sequence**, each term is obtained from the previous one by multiplying by the same non-zero constant, called the **common ratio** r .

$u_{n+1} = ru_n$, so we can find $r = \frac{u_{n+1}}{u_n}$ for all $n \in \mathbb{Z}^+$.

For a geometric sequence with first term u_1 and common ratio r , the n th term is $u_n = u_1 r^{n-1}$.

Series

A **series** is the sum of the terms of a sequence.

For a finite sequence with n terms, the corresponding series is $S_n = u_1 + u_2 + \dots + u_n$.

Using **sigma notation** or **summation notation** we write $u_1 + u_2 + u_3 + \dots + u_n$ as $\sum_{k=1}^n u_k$.

For a **finite arithmetic series**, $S_n = \frac{n}{2}(u_1 + u_n)$ or $S_n = \frac{n}{2}(2u_1 + (n - 1)d)$.

For a **finite geometric series** with $r \neq 1$, $S_n = \frac{u_1(r^n - 1)}{r - 1}$.

Compound interest

The value of a compound interest investment after n time periods is

$$u_n = u_0(1 + i)^n$$

where u_0 is the initial value of the investment

and i is the interest rate per compounding period.

To find the **real value** of the investment, we divide by the inflation multiplier each year.

You should be able to use the TVM solver on your calculator to solve problems involving compound interest investments and loans.

Depreciation

Depreciation is the loss in value of an item over time.

The value of an item after n years is $u_n = u_0(1 - d)^n$

where u_0 is the initial value of the item

and d is the rate of depreciation per year.

POLYNOMIAL EQUATIONS

The highest power of x in a polynomial equation is called its **degree**.

If a polynomial equation has degree n then it may have up to n real solutions.

You should be able to use your graphics calculator to solve polynomial equations.

EXPONENTIALS AND LOGARITHMS

Laws of exponents	
$a^m \times a^n = a^{m+n}$	$a^0 = 1, a \neq 0$
$\frac{a^m}{a^n} = a^{m-n}$	$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$
$(a^m)^n = a^{mn}$	$(ab)^n = a^n b^n$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	

The **logarithm in base 10** of a positive number is the power that 10 must be raised to in order to obtain that number.

If $10^x = b$ for $b > 0$, we say that x is the logarithm of b in base 10, and write $x = \log b$.

$\log 10^x = x$ and $10^{\log x} = x$ for any $x > 0$.

The **natural logarithm** is the logarithm in base e . The natural logarithm of x is written as $\ln x$ or $\log_e x$.

$\ln e^x = x$ and $e^{\ln x} = x$ for all $x > 0$.

SKILL BUILDER QUESTIONS

- 1 A circle has diameter 6.24 cm. Find the area of the circle, rounding your answer to:
 - a 3 significant figures
 - b 4 decimal places.
- 2 Estimate using a one figure approximation:
 - a 382×21
 - b 6.91×0.875
 - c $38\,107 \div 213$
- 3 The speed of a cricket player's bowl is recorded as 141.6 km h^{-1} , rounded to 1 decimal place. In what range of values does the actual speed s lie?
- 4 The dimensions of a block of land are measured to be $17 \text{ m} \times 22 \text{ m}$, rounded to the nearest metre. What are the boundary values for the actual area A of the block of land?
- 5 Terry measures the dimensions of a box as 15 cm by 12 cm by 8 cm, rounded to the nearest centimetre.
 - a Use Terry's measurements to estimate the volume of the box.
 - b The actual dimensions of the box are 15.3 cm by 11.8 cm by 8.4 cm.
 - i Find the actual volume of the box.
 - ii Find the absolute and percentage error in Terry's estimate.
- 6 The radius of a circle is measured as 7 cm, rounded to the nearest centimetre.
 - a Use this measurement to estimate the area of the circle.
 - b Find the boundary values for the area of the circle.
 - c Hence find the maximum percentage error in the estimate.
- 7 Express in exponent form with a prime number base:
 - a 64
 - b 125×5^k
 - c $\frac{9^m}{81^n}$
- 8 Write without brackets:
 - a $(-3m^3)^4$
 - b $\left(\frac{xy^2}{2}\right)^5$
 - c $7s^2t \times (4st^3)^3$
- 9 Simplify:
 - a $4^0 + 4^{-1}$
 - b $(2\frac{3}{4})^{-2}$
 - c $2^2 + 2^1 + 2^{-1}$
- 10 Expand the brackets and write in simplest form:
 - a $(x^2 + x^{-2})^2$
 - b $(x^4 - x^2)(x^3 + 3)$
- 11 Write without negative exponents:
 - a a^2b^{-3}
 - b $\frac{2m^{-2}n^3}{m^5n^{-5}}$
 - c $\frac{12a^{-3}}{b^{-5}}$
- 12 Write in scientific notation:
 - a 42 000
 - b 0.000 067 8
 - c 526 000 000
- 13 Use your calculator to evaluate the following, giving your answer in scientific notation:
 - a $(3.57 \times 10^6) \times (2.38 \times 10^3)$
 - b $\frac{4.61 \times 10^{-7}}{3.45 \times 10^8}$
 - c $(0.000\,08)^4$
- 14 Use technology to solve:
 - a $8x - 2 = 3x^2$
 - b $3x^3 + 7x^2 - 3x = 2$
- 15 Solve using technology:
 - a $\begin{cases} 2x - 3y = 2 \\ 5x + 3y = 5 \end{cases}$
 - b $\begin{cases} 3x - 7y = -8 \\ 6x + 11y = 12 \end{cases}$
 - c $\begin{cases} 2x + y + 3z = -3 \\ x - y + 2z = 1 \\ 3x - 2y + 5z = 4 \end{cases}$
- 16 A triangle is defined by the lines with equations $y = x + 2$, $x + y = 9$, and $x = 4$.
 - a Find the coordinates of the triangle's vertices.
 - b Find the area of the triangle.

- 17** Consider the sequence 8, 13, 18, 23, 28,
- a** Show that the sequence is arithmetic.
 - b** Find the formula for its general term.
 - c** Find the 42nd term.
 - d** Determine whether each number is a member of the sequence: **i** 153 **ii** 4067
- 18** Find k given the consecutive arithmetic terms:
- a** 3, k , 11
 - b** -2 , $k + 4$, $k^2 + 11$
 - c** $k - 5$, $2k$, $2k^2$
- 19** An empty hamster cage has mass 800 g. When 5 hamsters are placed in the cage, the total mass is 1400 g.
- a** Find the average mass of the hamsters in the cage.
 - b** Hence write an arithmetic sequence for u_n , the approximate total mass when n hamsters are placed in the cage.
- 20** Find the general term u_n of the geometric sequence which has:
- a** $u_5 = 324$ and $u_{10} = 78\,732$
 - b** $u_8 = -10$ and $u_{12} = -160$
- 21** Consider the sequence $2, 2\sqrt{3}, 6, 6\sqrt{3}, \dots$
- a** Show that the sequence is geometric.
 - b** Find the formula for its general term.
 - c** Find the 10th term.
 - d** Find the first term which exceeds 1000.
- 22** An endangered species of bird has population 217. However, with a successful breeding program it is expected to increase by 42% each year.
- a** Find the expected population size after:
 - i** 5 years
 - ii** 10 years.
 - b** How long will it take for the population to reach 30 000?
- 23** Paige invests €500 in an account that pays 7.2% p.a. compounded monthly.
- The amount of money in Paige's account at the end of each month follows a geometric sequence with common ratio r .
- a** Find the value of r .
 - b** Find the value of the account after 3 years.
 - c** Given that inflation averages 2% p.a. over the 3 years, find the real value of the investment after 3 years.
- 24** Stan invests £3500 for 33 months at 8% p.a. interest compounded quarterly. Find its maturing value.
- 25** How much should I invest now to produce \$30 000 in 5 years' time, if the money can be invested at a fixed rate of 4.8% p.a. interest compounded monthly?
- 26** A television was purchased for £2000, and depreciates at 30% p.a. for 3 years.
- a** Find the value of the television at the end of this period.
 - b** By how much has the television depreciated?
- 27** Lauren deposits \$10 000 in an account that compounds interest monthly. 4.5 years later, the account has balance \$12 000.
- a** What annual rate of interest did the account pay?
 - b** How long will it take Lauren to double her deposit?
- 28** The first four terms of an arithmetic sequence are 51, 45, 39, 33.
- a** Write down the common difference d .
 - b** Find the 20th term u_{20} .
 - c** Find the sum of the first 20 terms.
- 29** Find the sum of:
- a** $11 + 15 + 19 + 23 + \dots$ to 20 terms
 - b** $7 + 12.5 + 18 + 23.5 + \dots + 106$
 - c** $1 - 2 + 3 - 4 + 5 - 6 + 7 - \dots$ to 100 terms
 - d** the integers from 1 to 200 not divisible by 3.
- 30** An arithmetic sequence has terms $u_7 = 1$ and $u_{15} = -23$.
- a** Find the first term u_1 and common difference d .
 - b** Find the 27th term u_{27} .
 - c** Find the sum of the first 27 terms of the series.

- 31** The first term of a finite arithmetic series is 18 and the sum of the series is -210 . The common difference is -3 . Suppose there are n terms in the series.
- a** Show that $\frac{n}{2}(39 - 3n) = -210$. **b** Hence find n .
- 32** Consider the arithmetic sequence 7, 10, 13, 16, 19,
- a** Write down an expression for the sum of the first n terms S_n .
b Find n such that $S_n = 140$.
- 33** Find the sum of:
- a** $10 + 5 + 2\frac{1}{2} + 1\frac{1}{4} + \dots$ to 8 terms **b** $2 + 10 + 50 + 250 + \dots$ to 10 terms
c $\sum_{k=1}^{20} 3 \times (-2)^{k+2}$.
- 34** Find the sum of the series:
- a** $10 + 14 + 18 + 22 + \dots + 138$ **b** $6 - 12 + 24 - 48 + 96 - \dots + 1536$
- 35** Emma takes out a home loan of \$120 000 at 7.2% p.a. interest compounded monthly. The loan is to be repaid over 20 years.
- a** Calculate the monthly repayment.
b Calculate the amount of money still owing on the loan after one year.
c **i** Calculate the amount paid in the first year of the loan.
ii By how much has the principal been reduced at the end of the first year?
iii Explain why the loan does not reduce by the full amount of your first year's repayment.
d Suppose that after 1 year, the interest rate falls to 6.95% p.a.
i Calculate the new monthly repayment.
ii If Emma is able to keep paying the original repayments, how much earlier will the loan be paid off?
- 36** Oscar decides to start a new business venture which involves taking out a bank loan. The bank charges an interest rate of 6.55% p.a. compounded quarterly.
- His quarterly repayments are \$933.62, and must be repaid over 8 years.
- a** How much did Oscar borrow?
b How much interest will he pay over the 8 year period?
c **i** Calculate the outstanding balance at the end of the sixth year.
ii At the end of the 6th year, Oscar pays a lump-sum of \$3000 off the loan. Assuming his repayments remain the same, how much sooner will Oscar repay the loan?
- 37** Mary takes out a loan of \$10 000 to purchase a car. The bank charges an interest rate of 8% p.a. compounded monthly. Mary will repay the loan with quarterly repayments over 5 years.
- a** Calculate the quarterly repayment.
b Find the balance of the loan after 3 years.
c After 3 years, Mary decided she would like to have the loan paid off in 1 year's time. What must her quarterly repayment increase to for this to occur?
- 38** Cassie made an initial investment of €2000 into a savings account, and followed it with regular deposits of €500 per quarter. The account pays 1.2% interest per quarter, and inflation is 0.3% per quarter.
- a** Explain why the real interest rate is approximately 0.9% per quarter.
b Find the real value of Cassie's investment after 5 years.
- 39** Bill collects \$81 000 as his share of a lottery win. He decides to retire from work and buy an annuity to provide \$2000 per month, until he gets a pension in four years' time.
- a** What annual interest rate, compounded monthly, is needed for Bill's plan to work?
b How much will Bill actually receive each month over the period of the annuity if he receives 7% p.a. interest compounded monthly?

- 40** Celia and Mike want to provide each of their two children with \$200 per month for the next ten years. Interest on the investment is 5.2% p.a. calculated monthly.
- How much should be invested now to provide such an annuity?
 - Calculate the total interest earned over the term of the annuity.
- 41** Anne places her retirement savings of £400 000 into an annuity fund which returns 4.9% p.a. interest compounded quarterly. She plans to make monthly withdrawals of £3000 from the fund.
- How long will her money last?
 - How much should she withdraw each month if she wants the money to last for 20 years?
- 42** Simplify:
- $\log(10^9 \times 1000^b)$
 - $\log\left(\frac{10^n}{100}\right)$
 - $\log(2^t \times 5^t)$
- 43** Use your calculator to write the following in the form 10^x where x is correct to 4 decimal places:
- 2
 - 200
 - 0.02
- 44** Simplify:
- $\ln(e^k \times e^4)$
 - $\ln\left(\frac{e}{e^m}\right)$
 - $e^{2 \ln 6}$
 - $e^{-\ln 3}$
- 45** Use your calculator to write the following in the form e^k where k is correct to 4 decimal places:
- 47
 - 500
 - 0.023
- 46** In acoustics, the intensity of sound is measured in **decibels** (dB). The **sound intensity level** (SIL) is given by $L = 10 \log\left(\frac{I}{I_0}\right)$ dB, where I is the sound intensity, and $I_0 = 10^{-12} \text{ w/m}^2$ is the reference sound intensity.
- Find, in dB, the SIL of a snare drum with sound intensity $3 \times 10^{-2} \text{ w/m}^2$.
 - A lawn mower has SIL 85 dB. Find its sound intensity in w/m^2 , giving your answer in the form $a \times 10^k$, $1 \leq a < 10$, $k \in \mathbb{Z}$.

TOPIC 2: FUNCTIONS

PROPERTIES OF LINES

The **gradient** of the line passing through $A(x_1, y_1)$ and $B(x_2, y_2)$ is $m = \frac{y\text{-step}}{x\text{-step}} = \frac{y_2 - y_1}{x_2 - x_1}$.

The gradient of any horizontal line is zero. The gradient of any vertical line is undefined.

The **y-intercept** of a line is the value of y where the line cuts the y -axis.

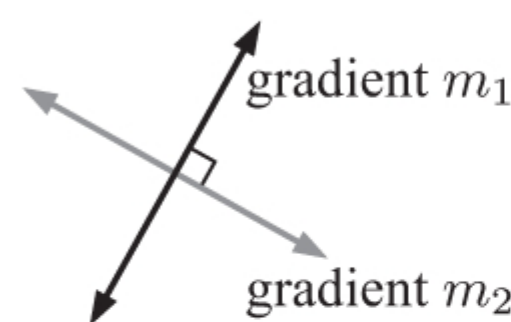
The **x-intercept** of a line is the value of x where the line cuts the x -axis.

PARALLEL AND PERPENDICULAR LINES

The gradients of parallel lines are equal.

The gradients of perpendicular lines are negative reciprocals.

$$m_1 = -\frac{1}{m_2}$$



EQUATION OF A LINE

The equation of a line can be presented in:

- **gradient-intercept form** $y = mx + c$ where m is the gradient and c is the y -intercept.
- **general form** $ax + by = d$
- **point-gradient form** $y - y_1 = m(x - x_1)$

You should be able to find the equation of a line given:

- its gradient and the coordinates of any point on the line
- the coordinates of two distinct points on the line.

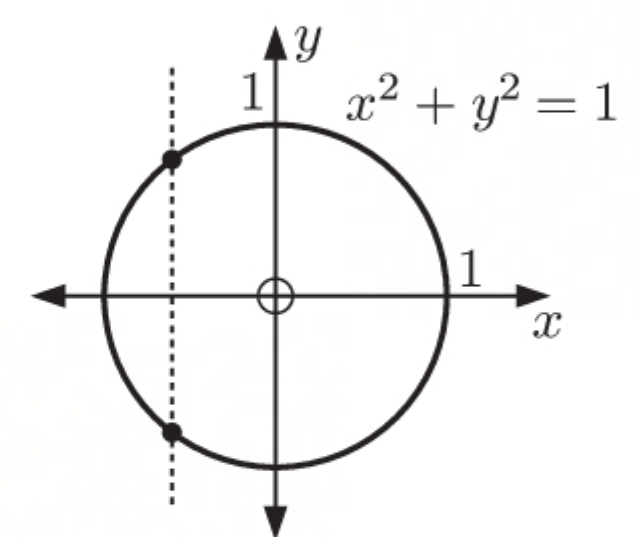
FUNCTIONS $y = f(x)$

A **relation** between variables x and y is any set of points in the (x, y) plane.

A **function** is a relation in which no two different ordered pairs have the same x -coordinate or first component. For each value of x there is at most one value of y or $f(x)$.

We test for functions using the **vertical line test**. A graph is a function if no vertical line intersects the graph more than once.

For example, the graph of the circle $x^2 + y^2 = 1$ shows that this relation is not a function.



The **domain** of a relation is the set of values that x can take.

To find the domain of a function, remember that we cannot:

- divide by zero
- take the square root of a negative number.

The **range** of a relation is the set of values that y or $f(x)$ can take.

INVERSE FUNCTIONS

A function is **one-to-one** if, for each value of y , there is only one value of x . One-to-one functions satisfy the **horizontal line test**.

If a function $f(x)$ is one-to-one, it has an **inverse function** $f^{-1}(x)$. If f maps x to y , then f^{-1} maps y back to x .

$y = f^{-1}(x)$ is a reflection of $y = f(x)$ in the line $y = x$.

The domain of f^{-1} is equal to the range of f .

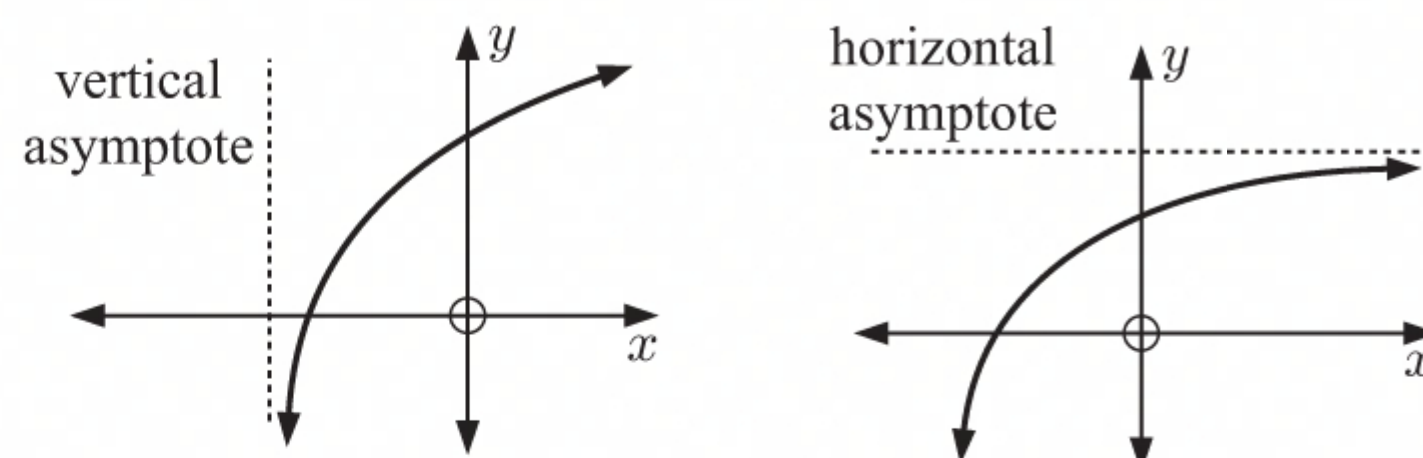
The range of f^{-1} is equal to the domain of f .

GRAPHS OF FUNCTIONS

The **x -intercepts** of a function are the values of x for which $y = 0$. They are the **zeros** of the function.

The **y -intercept** of a function is the value of y when $x = 0$.

An **asymptote** is a line that the graph *approaches* or begins to look like as it tends to infinity in a particular direction.



To find vertical asymptotes, look for values of x for which the function is undefined.

To find horizontal asymptotes, consider the behaviour as $x \rightarrow \pm\infty$.

You should be able to use technology to:

- graph a function
- find the domain and range
- find axes intercepts
- find turning points
- find asymptotes
- find where functions meet.

MODELLING

Mathematical models are developed using a **modelling cycle**:

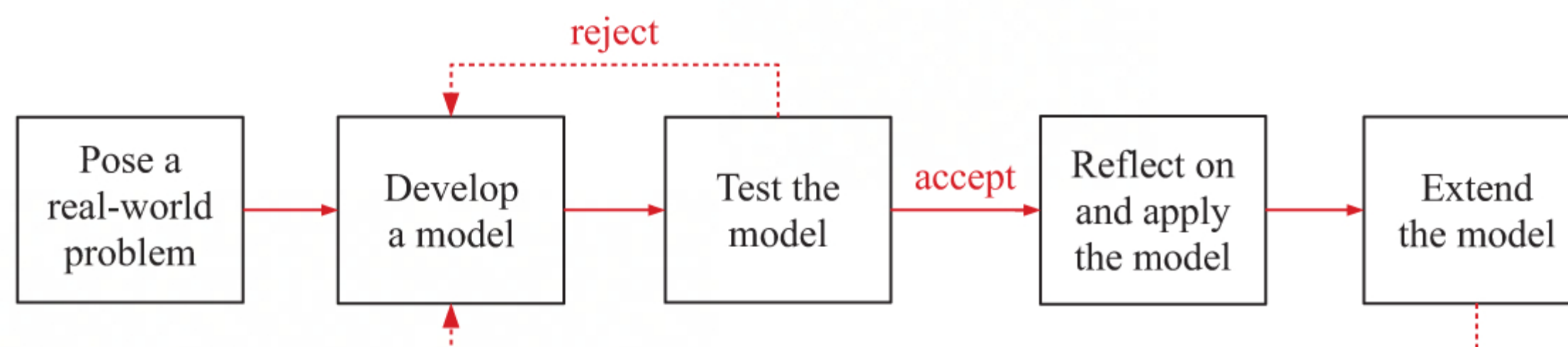
Step 1: **Pose** a real-world problem. Make **assumptions** which simplify the problem without missing key features.

Step 2: **Develop** a model which represents the problem with mathematics. This may involve a formula or an equation.

Step 3: **Test** the model by comparing its predictions with known data. If the model is unsatisfactory, return to **Step 2**.

Step 4: **Reflect** on your model and **apply** it to your original problem, interpreting the solution in its real-world context.

Step 5: If appropriate, **extend** your model to make it more general or accurate as needed.



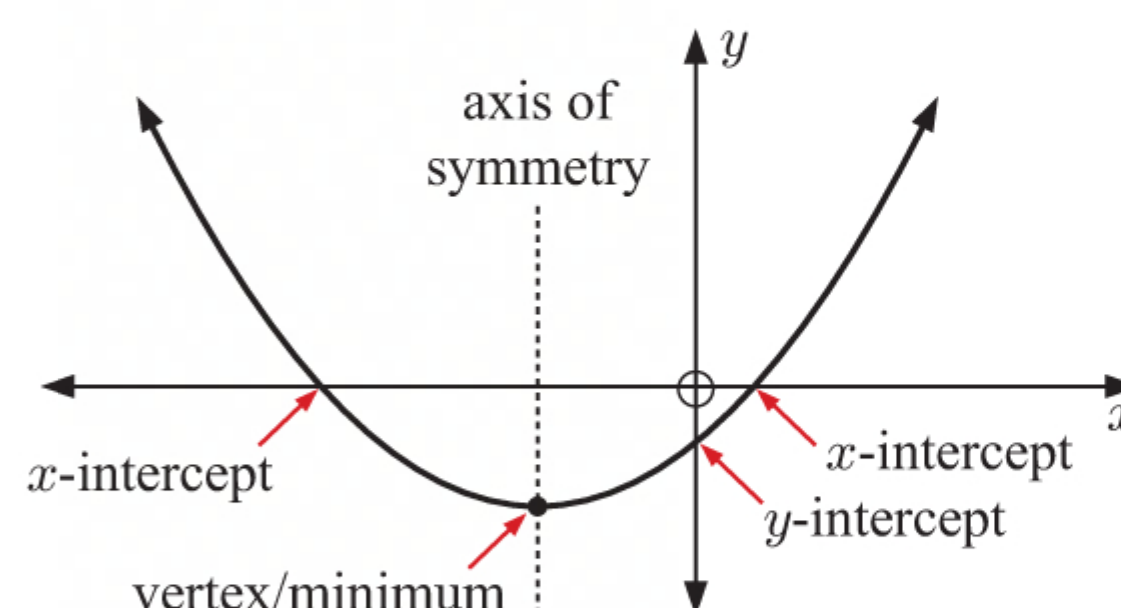
You should be able to solve systems of equations using technology to find unknown parameters in models.

QUADRATIC FUNCTIONS

A **quadratic function** has the form $y = ax^2 + bx + c$, $a \neq 0$.

The graph is a parabola with the following properties:

- It is *concave up* if $a > 0$ and *concave down* if $a < 0$.
- Its axis of symmetry is $x = \frac{-b}{2a}$.
- Its vertex has x -coordinate $\frac{-b}{2a}$. The y -coordinate of its vertex is found by substituting $x = \frac{-b}{2a}$ into the function.
 - If $a > 0$ the vertex is a minimum turning point.
 - If $a < 0$ the vertex is a maximum turning point.



You should be able to use technology to find points at which:

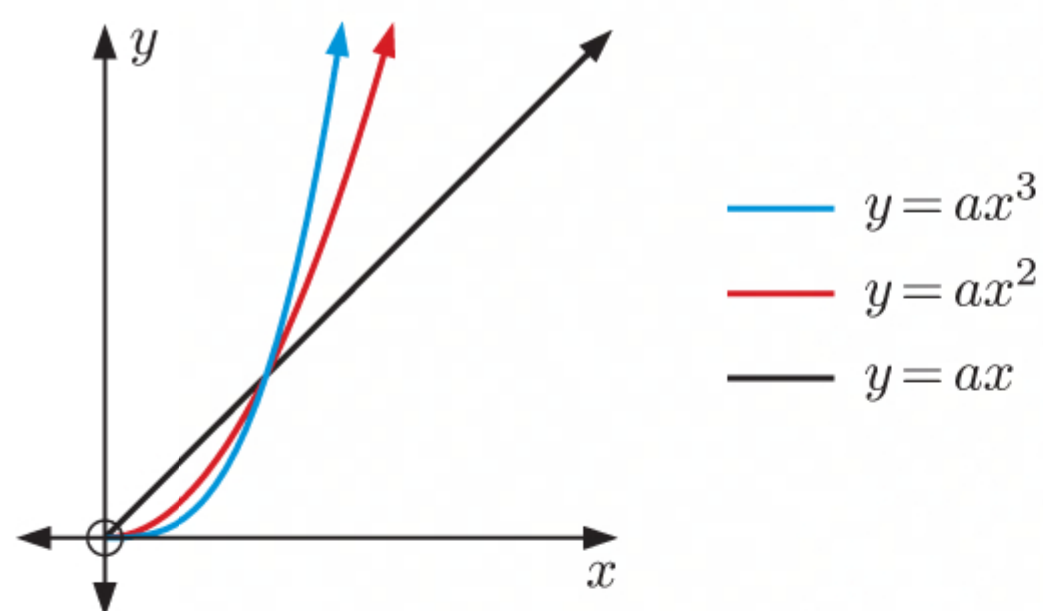
- a linear function meets a quadratic
- two quadratic functions meet.

VARIATION MODELS

Variation models have the form $y = ax^n$, $n \in \mathbb{Z}$, $n \neq 0$.

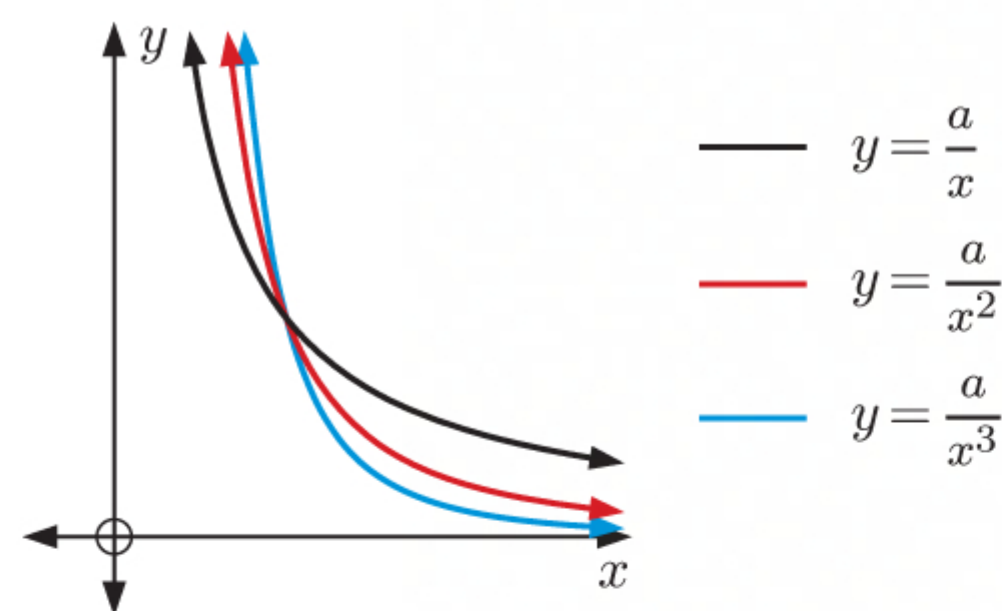
- If $n > 0$ we have **direct variation**.

The graph passes through the origin $(0, 0)$.



- If $n < 0$ we have **inverse variation**.

The graph is asymptotic to both the x and y axes.



You should be able to:

- use a point which lies on the graph of a variation model to find the exact equation of the variation model
- use technology to find the variation model which best fits a set of data.

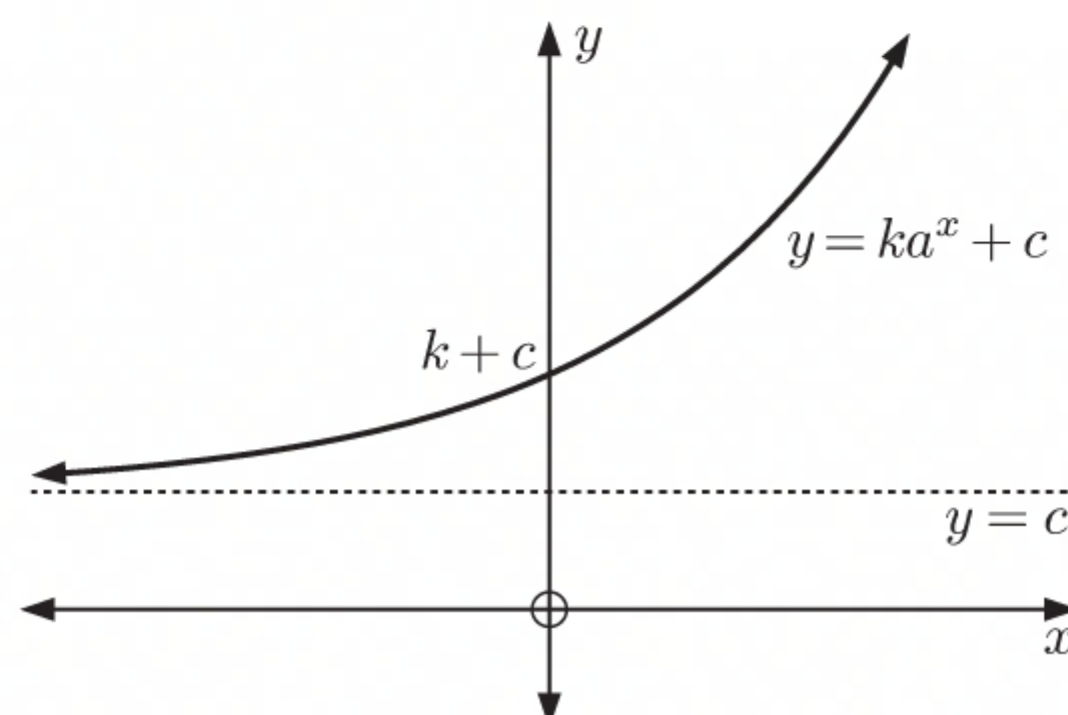
EXPONENTIAL FUNCTIONS

In this course you need to deal with exponential functions of the form:

- $y = ka^x + c$
- $y = ka^{-x} + c$

In each case:

- a and k control the steepness of the curve
- $y = c$ is the equation of the **horizontal asymptote**.



You will also need to deal with natural exponential functions of the form $y = ke^{rx} + c$.

Exponential functions are commonly used to model **growth** and **decay** problems.

Exponential equations are equations where the variable appears in an index or exponent. You should be able to solve exponential equations using technology.

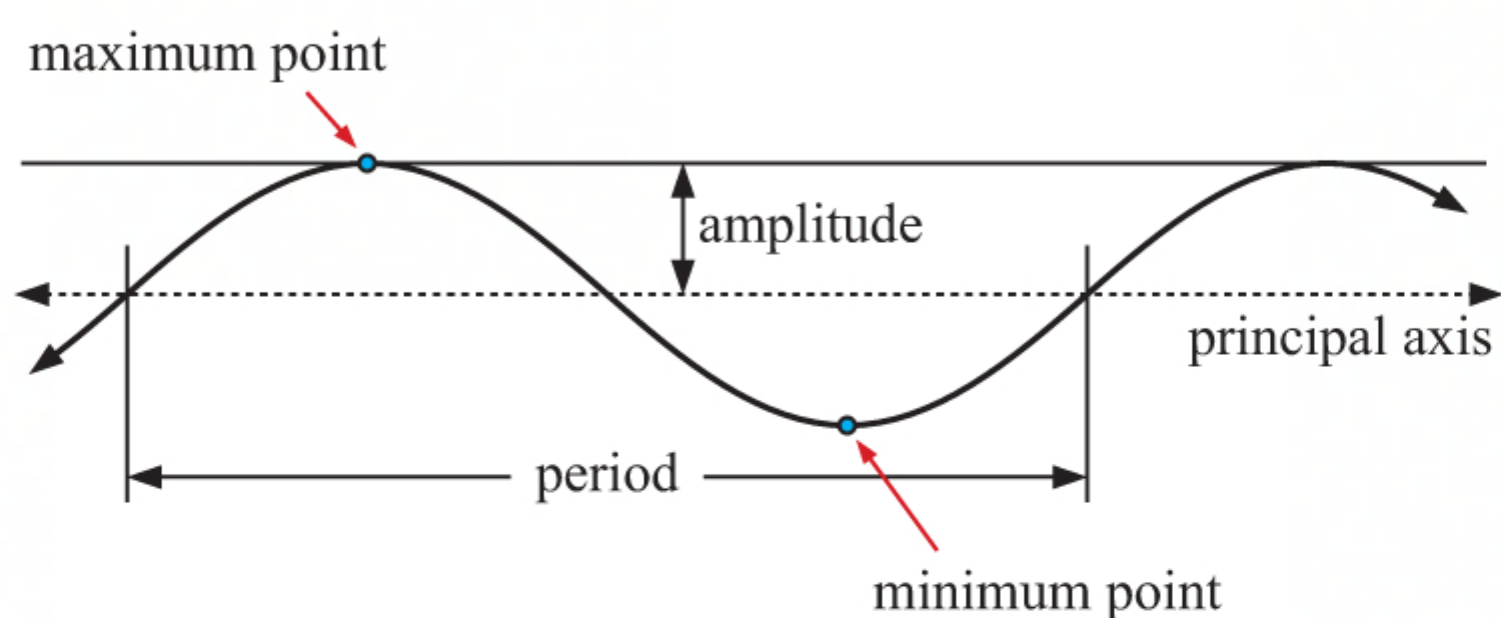
PERIODIC FUNCTIONS

A **periodic function** is one which repeats itself over and over in a horizontal direction.

For example, a **wave** oscillates about a horizontal line called the **principal axis**.

The **period** of a periodic function is the length of one cycle.

The **amplitude** is the distance between a maximum or minimum point and the principal axis.



SINUSOIDAL MODELS

The sinusoidal models considered in this course have the form $y = a \sin(bx) + d$ or $y = a \cos(bx) + d$.

For these models:

- the **amplitude** is $|a|$
- the **principal axis** is $y = d$
- the **period** is $\frac{360^\circ}{b}$.

SKILL BUILDER QUESTIONS

- 1 Find, in gradient-intercept form, the equation of the line which has:
 - a gradient -2 and passes through $(-5, 6)$
 - b gradient $\frac{5}{8}$ and y -intercept 5 .
- 2 Find the equation of the line which is:
 - a parallel to $2x - y = -3$ and passes through $(5, 3)$
 - b perpendicular to $y = -4x + 3$ and passes through $(-1, 5)$.
- 3 Find k given that $(-1, -6)$ lies on the line $7x - y = k$.
- 4 A line passes through the points $(-3, 4)$ and $(-1, 10)$. For this line, find:
 - a the gradient
 - b the equation
 - c the axes intercepts.
- 5 Draw the graph of:
 - a $y = -4x + 8$
 - b $7x + 4y = 14$
- 6 Consider the line with equation $y = 3x + 4$.
 - a Find the gradient and axes intercepts.
 - b Determine whether the following points lie on the line:
 - i $(1, 1)$
 - ii $(2, 10)$
 - c Draw the graph of $y = 3x + 4$, showing your results from **a** and **b**.
- 7 Tammy buys tickets to a stage show. Tickets cost \$30 for adults, and \$15 for children. She spends a total of \$120 buying tickets for x adults and y children.
 - a Explain why $30x + 15y = 120$.
 - b If Tammy bought tickets for 4 children, how many adult tickets did she buy?
 - c Find the x -intercept of the line $30x + 15y = 120$, and interpret your answer.
 - d Draw the graph of $30x + 15y = 120$. Mark two points on your graph to indicate your answers to **b** and **c**.
- 8 The line $ax + by = 20$ is perpendicular to $y = \frac{3}{2}x + 1$, and passes through the point $(2, 2)$. Find a and b .
- 9 Solve for x using the null factor law:
 - a $4x(x + 7) = 0$
 - b $5(x + 6)(3x - 5) = 0$
 - c $-2(x - 3)(4x + 3)^2 = 0$
- 10 Use technology to solve:
 - a $\frac{2}{x} = 5x - 3$
 - b $2^x - x^3 = 0$
- 11 Determine whether the given point satisfies the quadratic function:
 - a $y = 2x^2 + 5x - 1$ $(-1, -4)$
 - b $y = \frac{1}{2}x^2 - 6x - 3$ $(10, -19)$
- 12 Consider the quadratic function $f(x) = -x^2 + 2x + 5$.
 - a Copy and complete the table of values.

x	-3	-2	-1	0	1	2	3
$f(x)$							
 - b Hence sketch the graph of $y = f(x)$.
- 13 Find the zeros of the following functions:
 - a $y = x^2 - x - 12$
 - b $f(x) = 5x - x^2$
 - c $y = 8x^2 - 2x - 3$
- 14 Find the axes intercepts of:
 - a $y = (2x - 1)(x + 3)$
 - b $f(x) = (x + 1)^2$
 - c $y = 3x^2 + 4x - 4$
- 15 Sketch the graph of each function by considering the coefficient of x^2 and the axes intercepts.
 - a $y = x^2 - 2x - 8$
 - b $f(x) = -(2x + 1)(x - 3)$
 - c $y = -\frac{1}{2}(x - 4)^2$
- 16 A quadratic function has axis of symmetry $x = -1$, and one of its x -intercepts is 2 . Find the other x -intercept.
- 17 The quadratic function $f(x) = 2x^2 + bx - 3$ has axis of symmetry $x = 6$.
 - a Find the value of b .
 - b Find the coordinates of the vertex.

18 For each of the following quadratics:

i Find the axes intercepts.

ii Find the axis of symmetry.

iii Find the coordinates of the vertex, and state whether it is a maximum turning point or a minimum turning point.

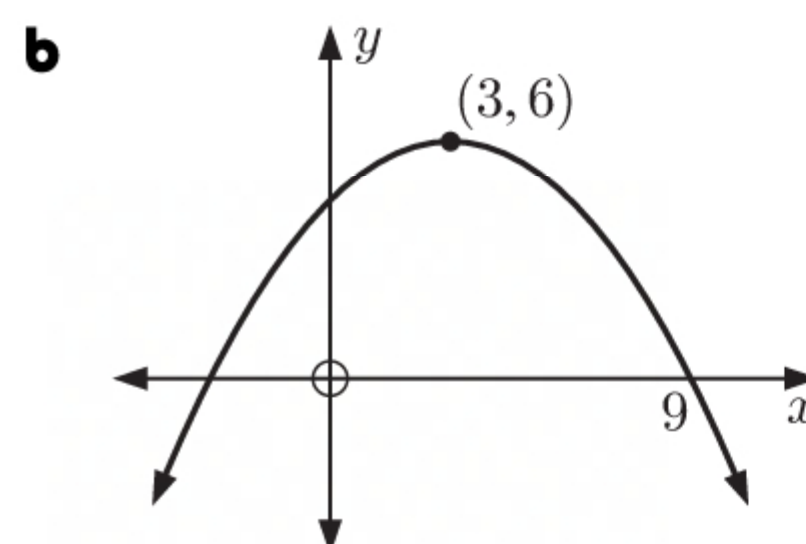
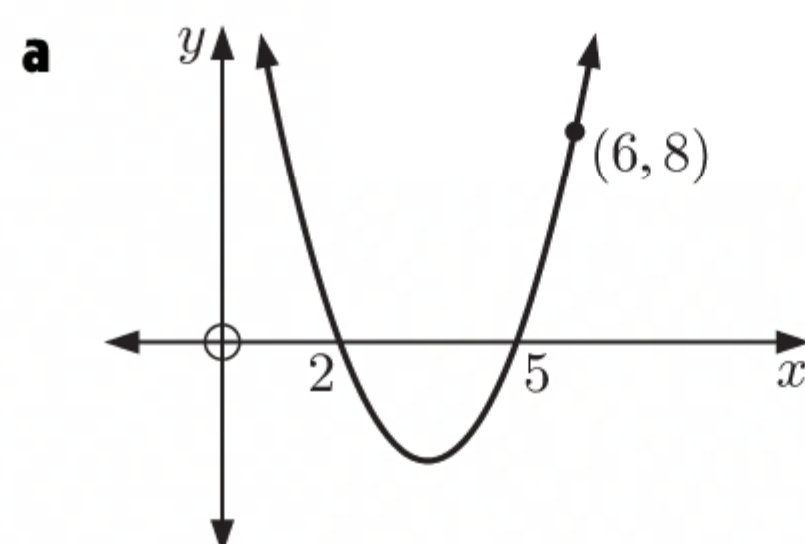
iv Sketch the quadratic.

v State the domain and range.

a $y = -(x - 1)(x + 3)$

b $y = 2(x + 7)(x - 2)$

19 Find the equation of the quadratic with graph:



20 Find the coordinates of the point(s) of intersection of:

a $y = x^2 - 4x - 5$ and $y = 3x - 11$

b $y = -2x^2 + 5x$ and $y = 5 - 2x$

21 Find, to 3 significant figures, the coordinates of the points of intersection of:

a $y = x^2 - 2$ and $y = -x^2 + x + 6$

b $y = 5x^2 - x$ and $y = x^2 - 4x + 4$

22 The daily profit made by a local baker selling x homemade pies is given by $P = -0.05x^2 + 9x - 60$ dollars.

a Copy and complete this table.

x	0	20	40	60	80	100
P		100		300		340

b Use the points in **a** to sketch the graph of P against x .

c Find:

i the number of pies that need to be sold to maximise the profit

ii the maximum possible daily profit

iii the number of pies that need to be sold to make a profit of \$200

iv the amount of money the baker loses if no pies are sold.

23 Jacob's rainwater tank started leaking. The amount of water in the tank after t hours is given by $W(t) = 1000 - 0.5t$ litres.

a Find $W(0)$, and interpret your answer.

b Find t when $W(t) = 700$, and explain what this represents.

c How long will it take for the tank to empty?

24 Consider the graph of $y = f(x)$ alongside.

Decide whether each statement is true or false:

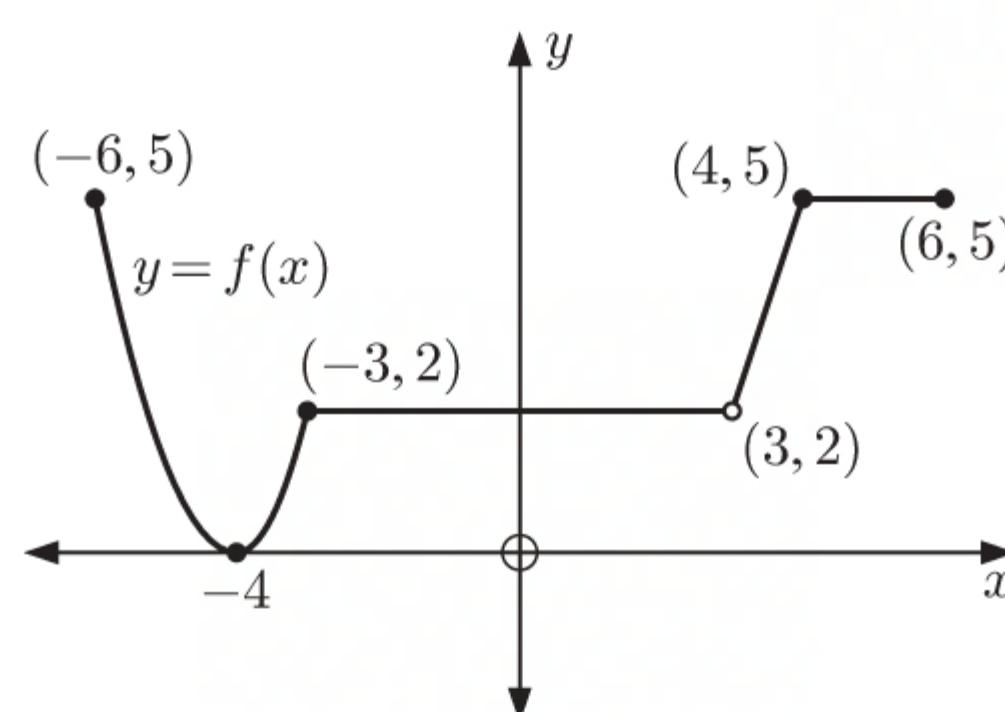
a 0 is in the domain of f .

b 0 is in the range of f .

c 6 is in the range of f .

d 3 is in the domain of f .

e 2 is in the range of f .



25 A function f is defined as $f(x) = \sqrt{x+4}$ for $-4 \leq x \leq 12$, $x \in \mathbb{R}$.

a Find: **i** $f(-4)$ **ii** $f(0)$ **iii** $f(12)$.

b Sketch $y = f(x)$.

c Hence write down the range of $f(x)$.

26 State the domain and range of each function:

a $f(x) = \sqrt{5-x}$

b $f(x) = \frac{2}{x+4}$

c $f(x) = \frac{1}{(x-1)^2}$

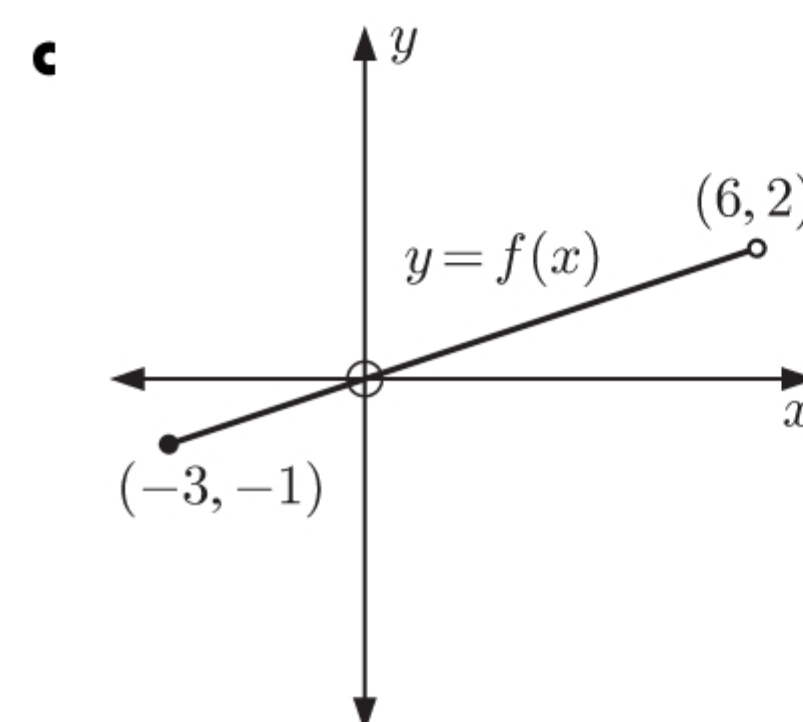
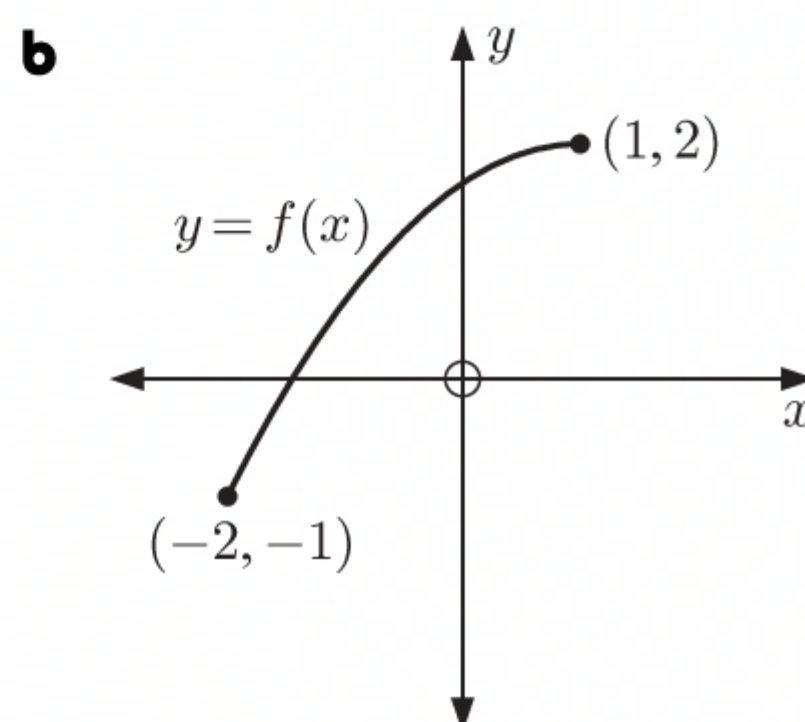
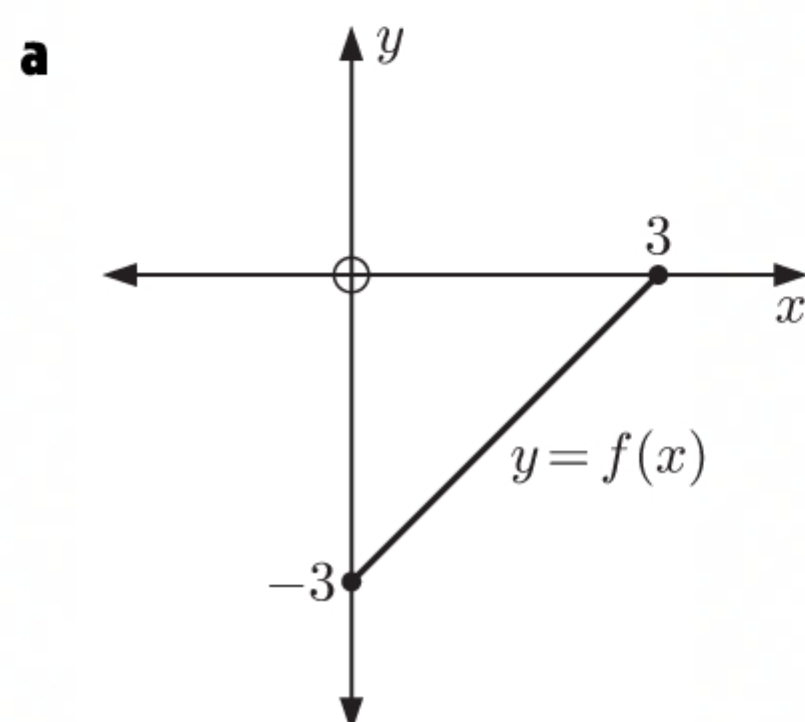
27 Consider $h(x) = x^2 - 2^{-x} + \frac{1}{x}$.

- a** Determine $h(-2)$.
- b** Solve $h(x) = 2$.
- c** Write down the equation of the vertical asymptote.
- d** Sketch $y = h(x)$, illustrating your results from **a**, **b**, and **c**.
- e** Determine the range of $y = h(x)$.

28 Consider $f(x) = 2^{x-1}$.

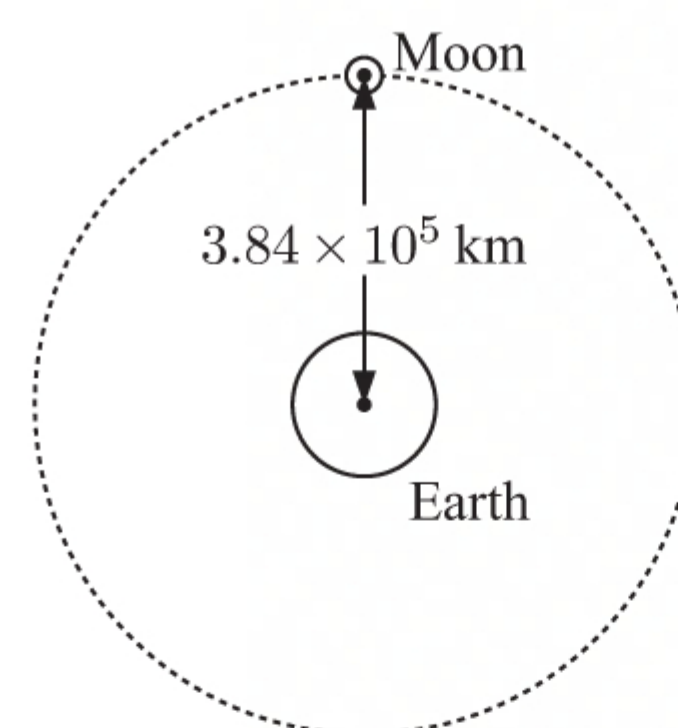
- a** Find $f(1)$ and $f(2)$.
- b** Graph $y = f(x)$ and its inverse function on the same set of axes.
- c** State the domain and range of $f^{-1}(x)$.

29 Copy the graphs of the following functions and draw the graphs of $y = x$ and $y = f^{-1}(x)$ on the same set of axes. In each case, state the domain and range of both f and f^{-1} .



30 The moon is approximately 3.84×10^5 km from the Earth, and completes one full orbit in about 28 days.

- a** Find a model for the distance D km travelled by the moon in t days.
- b** List any assumptions you have made in **a**.
- c** Use your model in **a** to estimate the distance travelled by the moon:
 - i** in one full orbit
 - ii** in 7 days.
- d** The moon actually completes an orbit of the Earth in 27 days, 7 hours, and 43 minutes. Discuss whether your answers in **c** are overestimates or underestimates.



31 In one hour, Isabelita can prepare 120 spring rolls, and Arturo can prepare 100 spring rolls.

- a** How long will it take to prepare 600 spring rolls if:
 - i** Isabelita works by herself
 - ii** Arturo works by himself
 - iii** Isabelita and Arturo work together?
- b** Discuss any assumptions you have made in **a**, and whether or not they are reasonable.

32 An oven technician charges a call-out fee of \$50 and an additional \$20 per hour for the duration of the appointment.

Let $\$C$ be the amount charged for an appointment lasting t hours.

- a** Sketch the graph of C against t for $0 \leq t \leq 3$.
- b** Find the linear model connecting C and t .
- c** Use your model to calculate the amount charged for a $1\frac{1}{2}$ hour appointment.
- d** Mikhal schedules an appointment to repair his oven. The technician arrives at 9:00 am, and the total amount charged is \$118. At what time did the technician complete the repairs?

33 The cost of hiring a car is summarised in the table alongside.

Let $\$C$ be the total cost of hiring a car for t days.

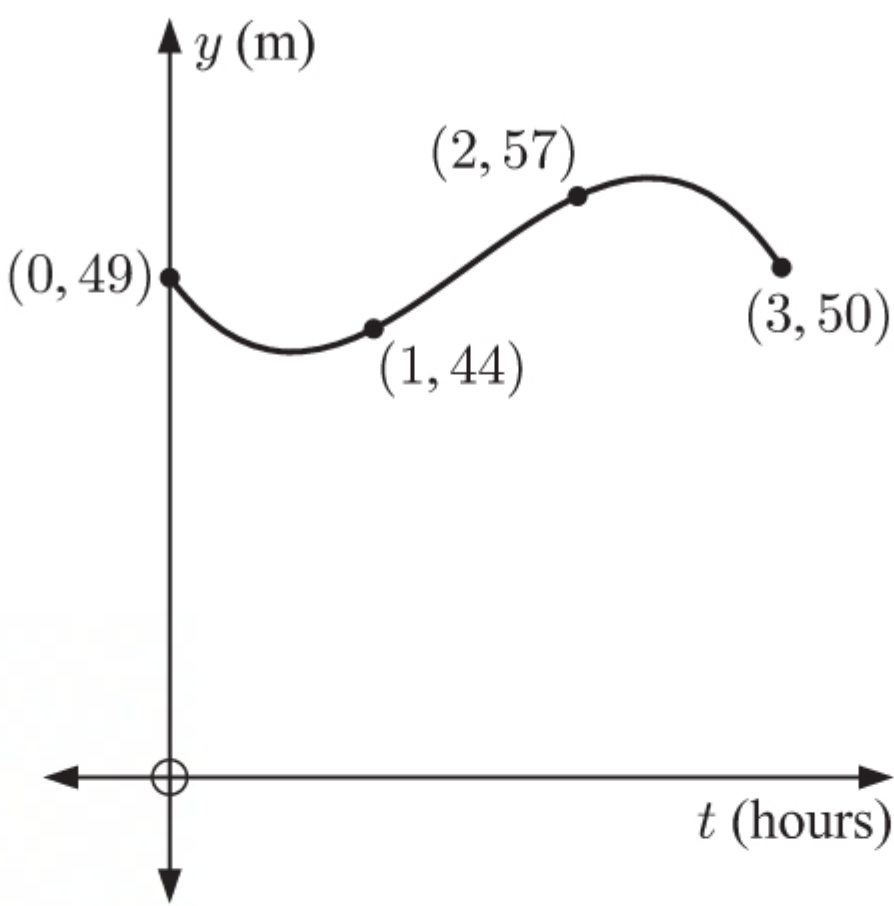
- a** Draw a graph of C against t for $0 \leq t \leq 10$.
- b** Find the cost of hiring a car for:
 - i** 2 days
 - ii** 5 days
 - iii** 9 days.

Hire period (t days)	Cost
1 - 2	\$40 per day
3 - 6	\$33 per day
7+	\$29 per day

- c** Georgia and Tim are planning an 8 day holiday. They require a car for the first 2 days, and the last 3 days.

Is it cheaper for them to hire one car for the first 2 days and a separate car for the last 3 days, or to hire one car for the whole holiday?

34 Stephen and Hugh take a walk in a national park. They measure their elevation in metres above sea level before they start, and every hour throughout their walk. Their elevation after t hours can be modelled by the function $y = at^3 + bt^2 + ct + d$ for $0 \leq t \leq 3$.



- a** State the value of d .
- b** Use technology to find a , b , and c .
- c** Estimate their elevation after $2\frac{1}{2}$ hours.
- d** Their actual elevation after $2\frac{1}{2}$ hours is 60 m.
Calculate the percentage error in your estimate in **c**.

35 Suppose $w \propto z$ and that $w = 27$ when $z = 9$. Find:

- a** w when $z = 2$
- b** z when $w = 45$.

36 The potential difference V across a resistor is proportional to the current I running through it. When the potential difference is 9 volts, the current is 0.01 amps.

- a** Find the proportionality constant.
- b** Find the current running through the resistor if the potential difference is 12 volts.
- c** Find the potential difference if there is 0.018 amps of current running through the resistor.

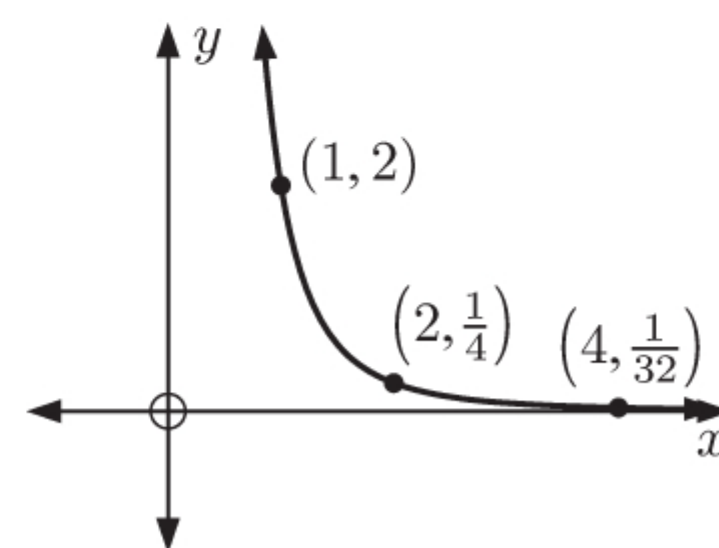
37 The table below shows the cost of purchasing x kg of tomatoes.

Weight (x kg)	0	1	2	3	4	5
Cost ($\$C$)	0	3.5	7	10.5	14	17.5

- a** Draw the graph of C against t .
 - b** Explain why C and x are directly proportional.
 - c**
 - i** Find a formula connecting C and x .
 - ii** Hence find the cost of purchasing 10 kg of tomatoes.
- 38** The mass of an orange is directly proportional to the cube of its diameter.
The diameter of orange A is 14.6% larger than that of orange B. What percentage heavier is orange A than orange B?
- 39** Suppose y is inversely proportional to x . Explain what happens to y if:
- a** x is tripled
 - b** x is divided by 4
 - c** x is multiplied by $\frac{5}{3}$
 - d** x is decreased by 60%.
- 40** When a constant force is applied by an object, the *pressure* applied by the object is inversely proportional to the *area* over which the force is applied.
When applied to an area of 2 m^2 , the object applies 300 Pa of pressure.
- a** Find the pressure when the force is applied over 0.8 m^2 .
 - b** Find the percentage increase in area required to reduce the pressure by 15%.
- 41** Suppose A is inversely proportional to the cube of r , and that $A = 7$ when $r = 2$.
- a** Find A when $r = 5$.
 - b** Find r when $A = 16$.
- 42** An ice cream cone company wants to adjust the existing dimensions of their cones, but maintain the same volume. Their cones currently have radius 2.8 cm and height 14.3 cm.
- a** Explain why the height of the cone is inversely proportional to the square of its radius.
 - b** Find the height of the cone if the radius chosen is 3.2 cm.
 - c** Find the radius of the cone if the height chosen is 10.8 cm.
 - d** It is decided that the possible values for the radius should be between 2.5 cm and 3.5 cm. Can you suggest why this was done?

43 It is suspected that y is inversely proportional to a power of x .

- Calculate x^2y , x^3y , and x^4y for the marked points.
- Hence determine the correct model connecting y and x .
- Find the value of y when $x = 5$.



44 The diameters of circular rugs and their corresponding masses are recorded in the table below.

Diameter (d m)	0.77	1.22	1.69	2.25
Mass (m kg)	0.97	2.44	4.68	8.30

- Do you think there is direct variation or inverse variation between the variables? Explain your answer.
- Use technology to obtain a power model which best fits the data.
- Estimate the mass of a rug with diameter 1.5 m.

45 For each data set, obtain the power model which best fits the data:

a

x	2	3	5	6
y	1.13	8.60	111	275

b

x	1	4	5	7	8
y	72.1	4.6	2.9	1.5	1.1

46 If $f(x) = 2 \times 3^{-x}$, find:

- $f(0)$
- $f(1)$
- $f(-2)$

47 For the function $g(x) = 5^x - 5$, show that:

- the y -intercept is -4
- the x -intercept is 1.

48 Construct a table of values for $x = -3, -2, -1, 0, 1, 2, 3$, then use your table to graph each function:

- $y = 4 \times 2^x$
- $y = 7 + 3^{-x}$

49 Consider $y = -1 + 2^{-x}$.

- Find the y -intercept.
- Find any asymptotes of the function.
- State the domain and range of the function.
- Hence sketch the function.

50 The exponential function $y = a \times 2^x + b$ passes through the points alongside:

- Write down two linear equations which could be used to determine the values of a and b .
- Solve the linear equations simultaneously to find a and b .
- Hence find the values of p and q .

x	0	1	2	3
y	20	p	35	q

51 Consider the exponential function $f(x) = 2 \times \left(\frac{1}{3}\right)^x + 1$.

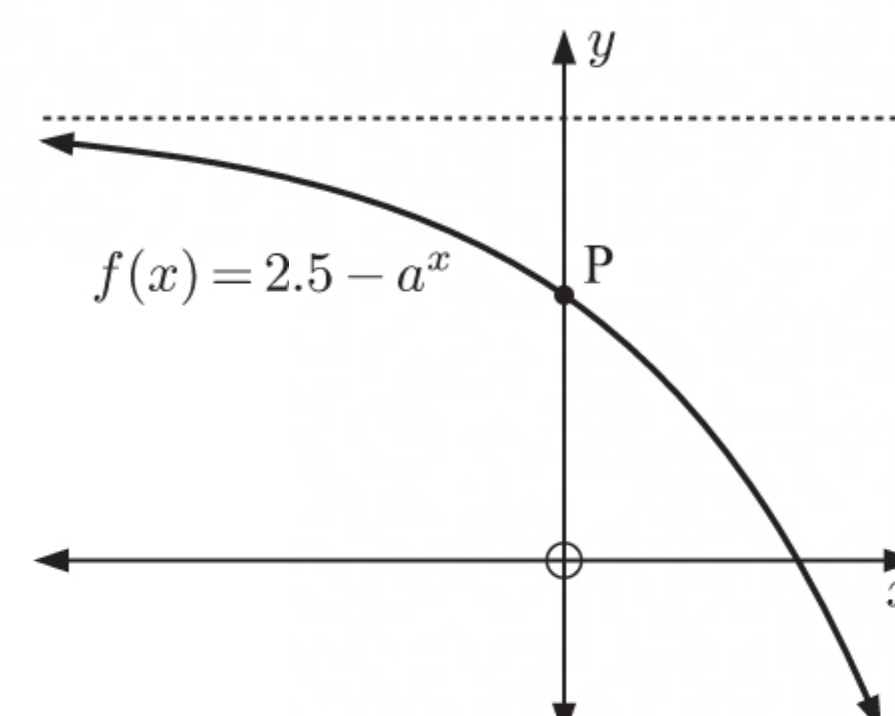
- Find:
 - $f(0)$
 - $f(2)$
 - $f(-1)$
- State the equation of the horizontal asymptote.
- Sketch the graph of the function.
- State the domain and range of the function.

52 Use technology to solve:

- $5^x = 14$
- $3 \times 2^{x-1} = 60$
- $40 \times (0.9)^x = 25$

53 The graph shows the function $f(x) = 2.5 - a^x$ where a is a positive constant. The point $(3, -5.5)$ lies on the graph.

- Write down the coordinates of P.
- Find the value of a .
- Find the equation of the horizontal asymptote.



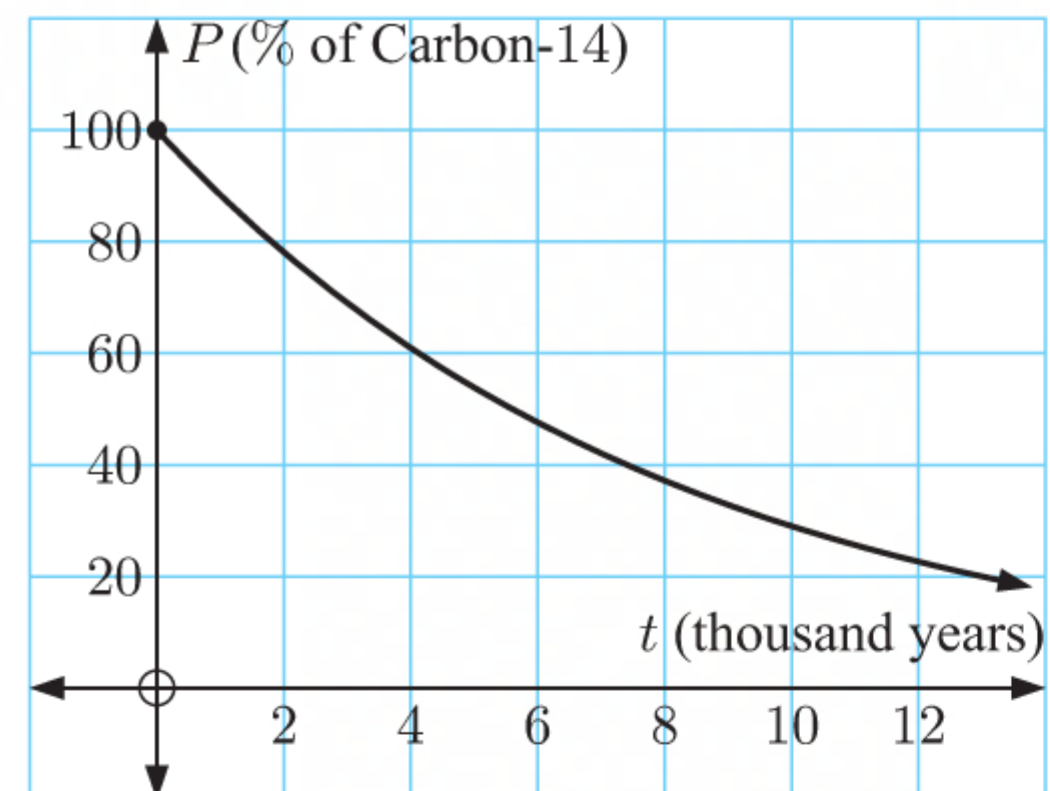
- 54** The population of bees in a hive after t weeks is given by $P(t) = a(0.95)^t + b$.

Initially, there are 2500 bees, and after 2 weeks there are 2383 bees.

- Find the value of:
 - a
 - b .
- Find the population after:
 - 3 weeks
 - 5 weeks.
- State the horizontal asymptote, and interpret this value.

- 55** The graph alongside shows the percentage P of radioactive Carbon-14 remaining in an organism t thousands of years after it dies.

- Use the graph to estimate:
 - the percentage of Carbon-14 remaining after 4000 years
 - the number of years for the percentage of Carbon-14 to fall to 50%.
- The equation of the graph is $P = 100 \times (1.1318)^{-t}$, $t \geq 0$.
 - Calculate the percentage of Carbon-14 remaining after 8000 years.
 - How long will it take for the percentage to fall to 15%?



- 56** The number of people N on a small island t years after settlement, increases according to the formula $N = 120 \times (1.04)^t$.

- Find the number of people who started the settlement.
- Find the number of people on the island after 4 years.
- How many years will it take for the number of people to double?

- 57** A radioactive substance has a half-life of 4 days. The weight of a sample of this substance after t days is $W(t) = 100 \times a^t$ mg, where $a > 0$.

- Find the initial weight of the sample.
- Calculate the value of a , correct to 4 decimal places, and interpret this value.
- Find the weight of the sample after 6 days.
- How long will it take for the weight of the sample to fall to:
 - 60 mg
 - 30 mg?

- 58** Sketch on the same set of axes as the graph of $y = e^x$:

- $y = e^x - 1$
- $y = 2e^x$
- $y = e^{\frac{x}{3}}$

For each graph, state the y -intercept and equation of the horizontal asymptote.

- 59** **a** Use technology to help sketch the graph of $f(x) = 6 - e^{-0.5x}$.

- State the domain and range of $f(x)$.
- Describe the behaviour of $y = f(x)$ as $x \rightarrow \pm\infty$.
- Find k such that $f(x) = k$ has:
 - one solution
 - no solutions.

- 60** Find the amplitude, principal axis, and period of:

- $f(x) = \sin 4x$
- $f(x) = -2 \sin \frac{x}{2} - 1$.

- 61** For each of the following functions:

- State the amplitude.
- State the principal axis.
- State the period.
- Sketch the function.

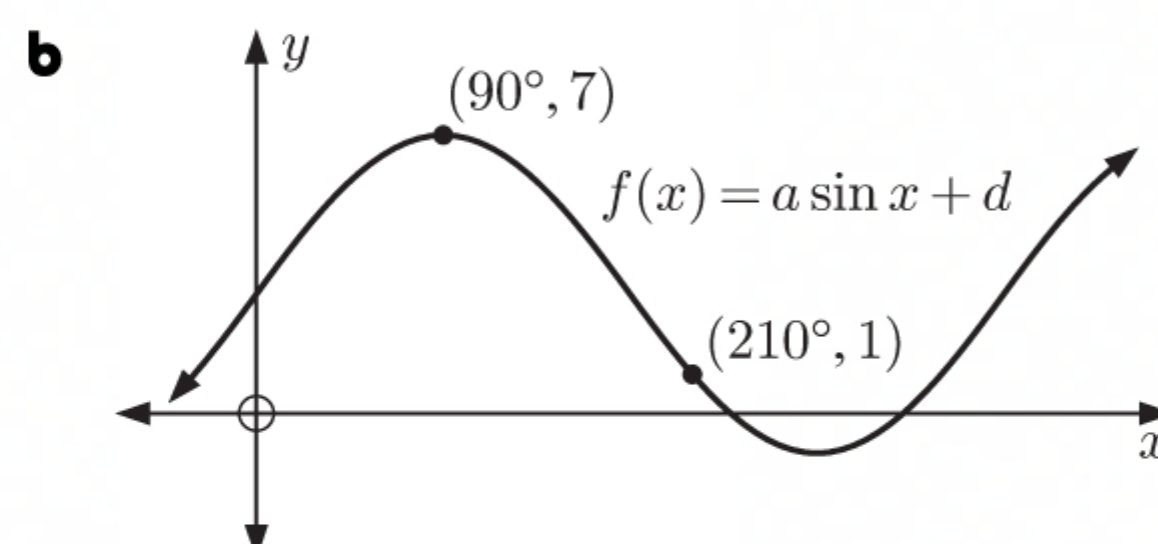
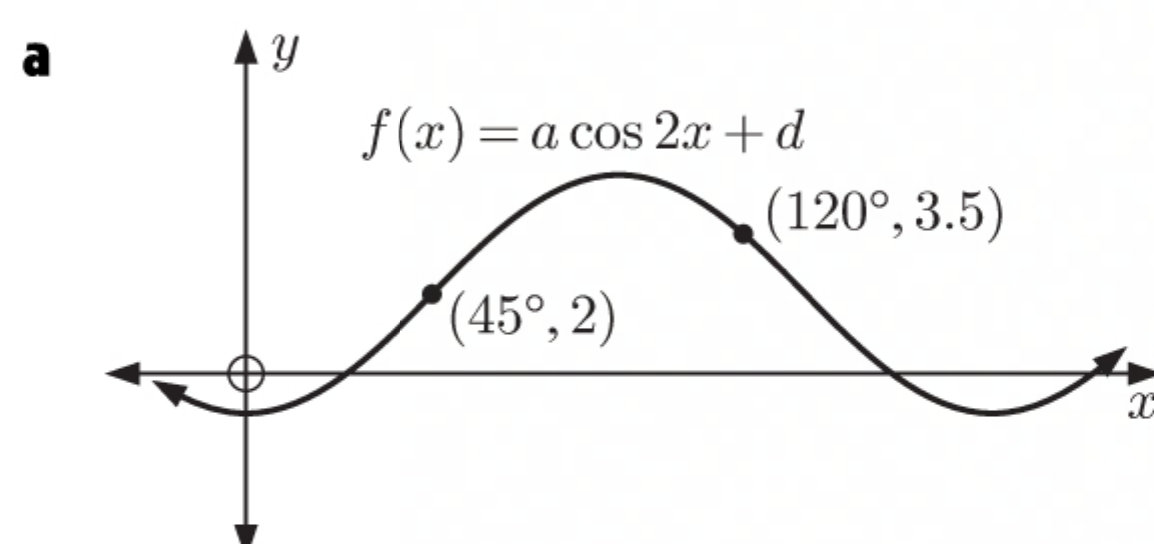
a $y = \sin x + 2$ for $-180^\circ \leq x \leq 180^\circ$

b $y = 3 \cos 2x$ for $0^\circ \leq x \leq 360^\circ$

c $y = \cos \frac{x}{2} - 1$ for $0^\circ \leq x \leq 360^\circ$

d $y = 10 - 6 \sin 3x$ for $0^\circ \leq x \leq 360^\circ$

- 62** Find the unknowns in each function:



- 63** In a busy harbour, the time difference between successive high tides is about 12.3 hours. The water level varies by 2.4 metres between high and low tide. The first high tide is 4.7 metres.

Let H m be the height of the tide t hours after the first high tide.

- a** Find a cosine model for H in the form $H = a \cos(bt)^\circ + d$.
 - b** Sketch a graph of the water level in the harbour for $0 \leq t \leq 24$.
- 64** The temperature inside Pam's caravan t hours after 12 noon is given by the function $T(t) = 5 \sin(15t)^\circ + 24$ °C.
- a** Sketch the graph of T against t for $0 \leq t \leq 24$.
 - b** Find the temperature inside Pam's caravan at:
 - i** 2 pm
 - ii** 9 pm.
 - c** Find the minimum temperature inside Pam's caravan, and the time at which it occurs.

TOPIC 3: GEOMETRY AND TRIGONOMETRY

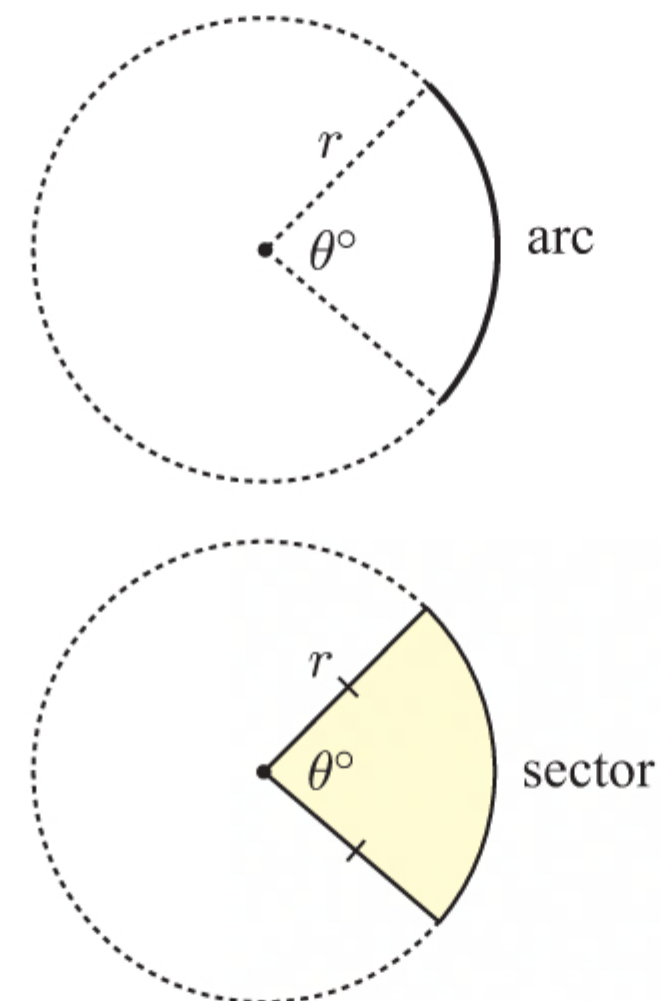
ARCS AND SECTORS

An **arc** is a part of a circle which joins any two different points.

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r$$

A **sector** is the region between two radii of a circle and the arc between them.

$$\text{Area} = \frac{\theta}{360} \times \pi r^2$$



GEOMETRY OF 3-DIMENSIONAL FIGURES

The **surface area** of a three-dimensional figure with plane faces is the sum of the areas of the faces.

The **volume** of a solid is the amount of space it occupies.

The **capacity** of a container is the quantity of fluid it is capable of holding. You should understand how the units of volume and capacity are related.

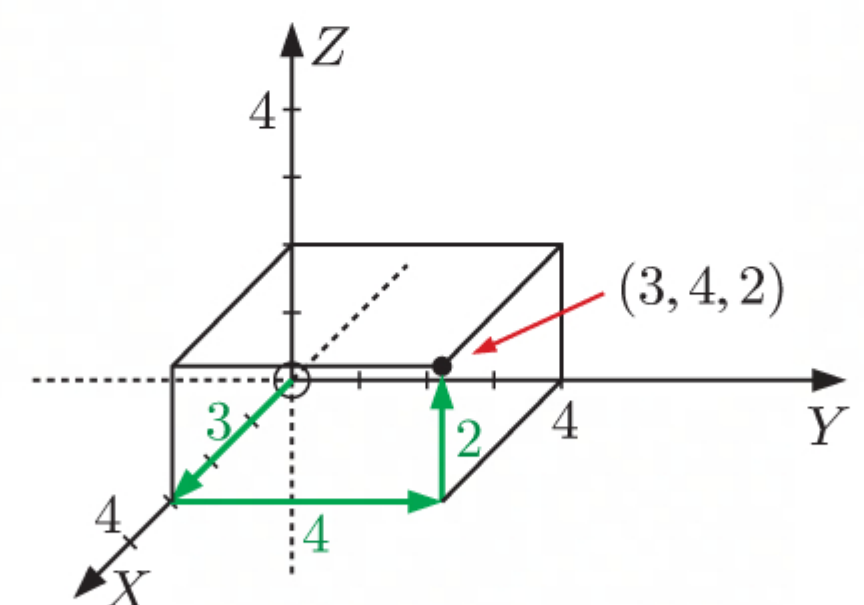
You should be able to calculate the surface area and volume of 3-dimensional figures, including solids of uniform cross-section, pyramids, spheres, and cones.

3-DIMENSIONAL COORDINATE GEOMETRY

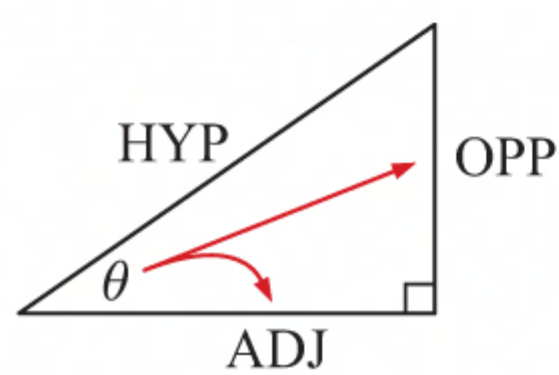
In 3-dimensional coordinate geometry, we specify an origin O, and three mutually perpendicular axes called the *X*-axis, the *Y*-axis, and the *Z*-axis.

For points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$:

- the **distance** $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
- the **midpoint** of $[AB]$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$.



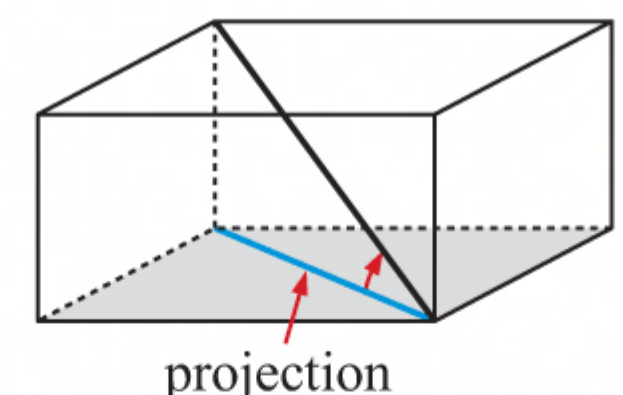
RIGHT ANGLED TRIANGLE TRIGONOMETRY



$$\begin{aligned}\cos \theta &= \frac{\text{ADJ}}{\text{HYP}} \\ \sin \theta &= \frac{\text{OPP}}{\text{HYP}} \\ \tan \theta &= \frac{\text{OPP}}{\text{ADJ}}\end{aligned}$$

Angle between a line and a plane

The **angle between a line and a plane** is the angle between the line and its **projection** on the plane.



True bearings

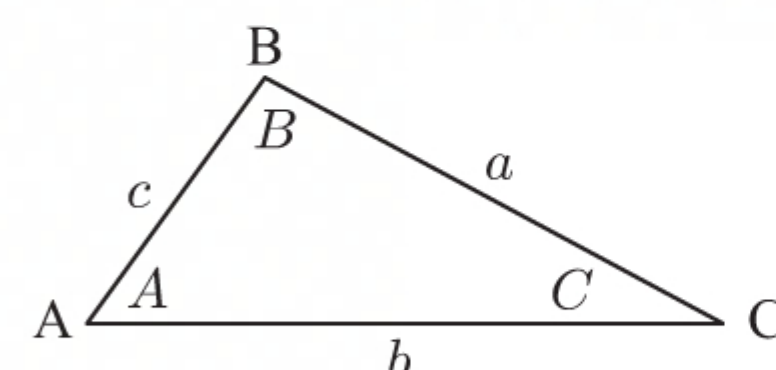
True bearings are used to describe the direction of one object from another. The direction is measured clockwise from true north.

NON-RIGHT ANGLED TRIANGLE TRIGONOMETRY

Area formula: $\text{Area} = \frac{1}{2}ab \sin C$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

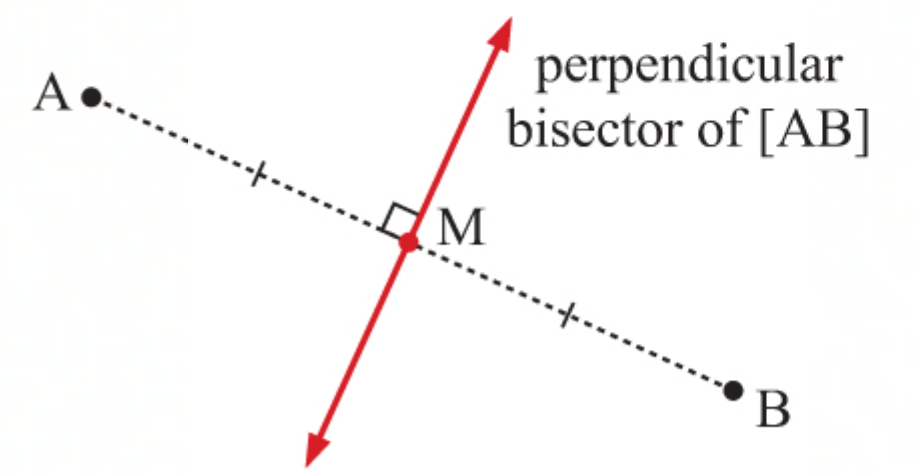
Sine rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ or $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



PERPENDICULAR BISECTORS

The **perpendicular bisector** of a line segment $[AB]$ is the line perpendicular to $[AB]$ which passes through its midpoint.

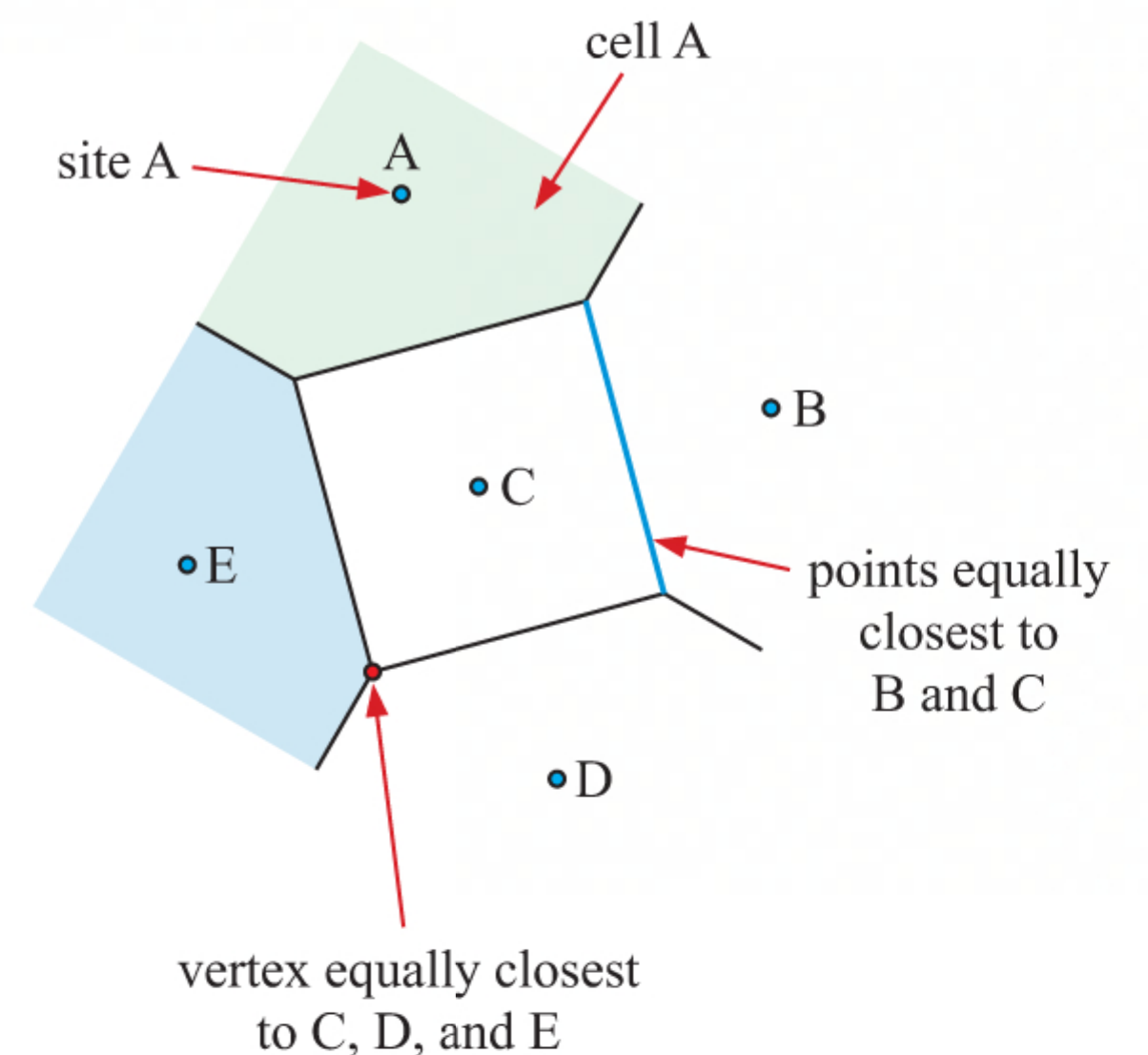
Points on the perpendicular bisector are equidistant from A and B.



VORONOI DIAGRAMS

In a Voronoi diagram:

- Important locations are called **sites**.
- Each site is surrounded by a region or **cell** which contains the points which are closer to that site than to any other site.
- The lines which separate the cells are called **edges**. Each point on an edge is equally closest to the two sites whose cells are adjacent to that edge.
- The points at which the edges meet are called **vertices**. Each vertex is equally closest to the sites whose cells meet at that vertex.



Adding a new site to a Voronoi diagram

To add the cell for a new site X to an existing Voronoi diagram with sites $P_1, P_2, P_3, \dots, P_n$, we follow these steps:

- Step 1:* Identify the site P_i whose cell contains the new site X . Construct the perpendicular bisector of $[P_i X]$, within this cell. At any point where this line meets an existing edge, create a new vertex.
- Step 2:* For each site P_j whose cell is adjacent to a new vertex, construct the perpendicular bisector of $[P_j X]$ within that cell through the vertex. Continue to create new vertices as in *Step 1*. Repeat this process until no more new vertices are created. At this time cell X is complete.
- Step 3:* Remove any segments of edges from the original Voronoi diagram which now lie within cell X .

Nearest neighbour interpolation

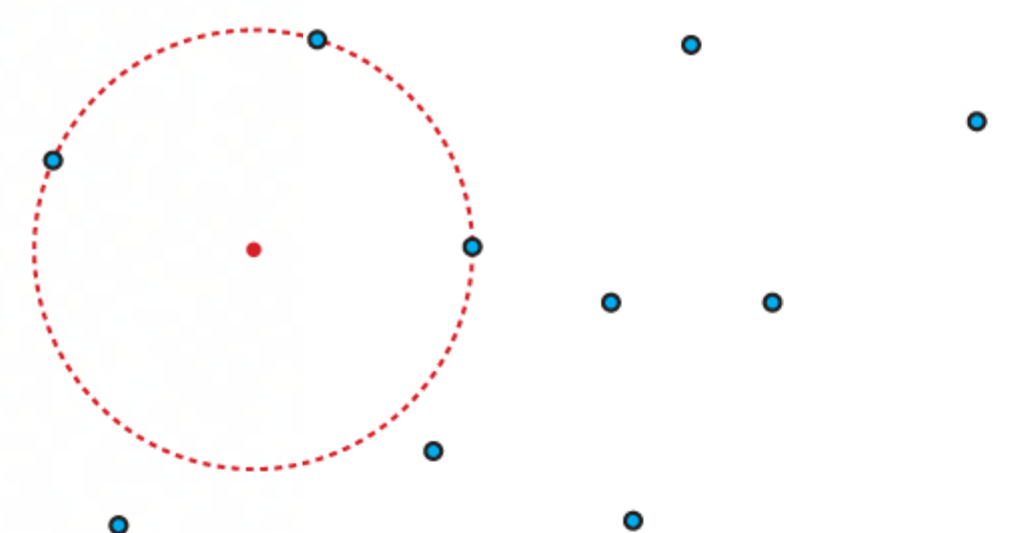
If we are given the values of a variable at a set of known data points, we can *estimate* the value of the variable at some other point. We use the variable's value at the *nearest* known data point.

From a Voronoi diagram with the known data points as sites, we can quickly identify the nearest known data point to any given point.

If the given point lies on an edge or at a vertex, we take the average of the closest known data points.

The Largest Empty Circle problem

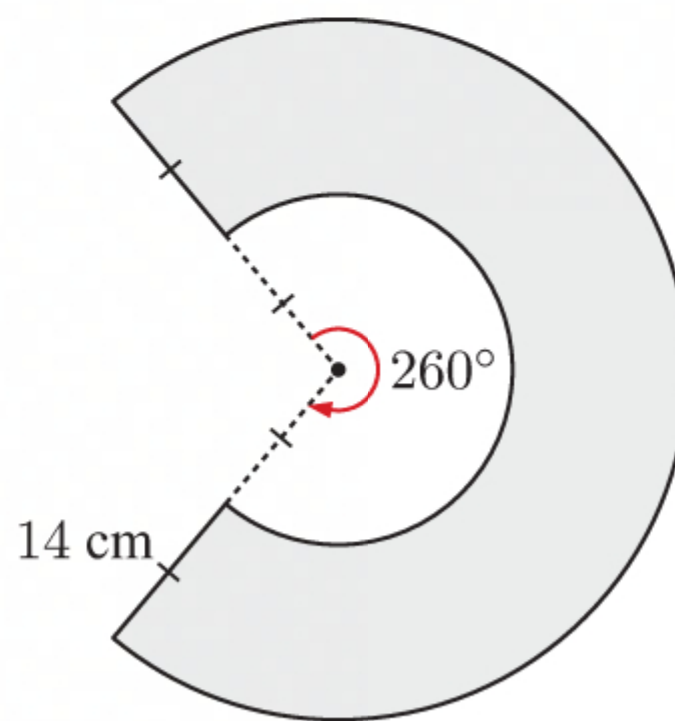
The **Largest Empty Circle problem** is the problem of finding the largest circle whose interior does not contain any sites.



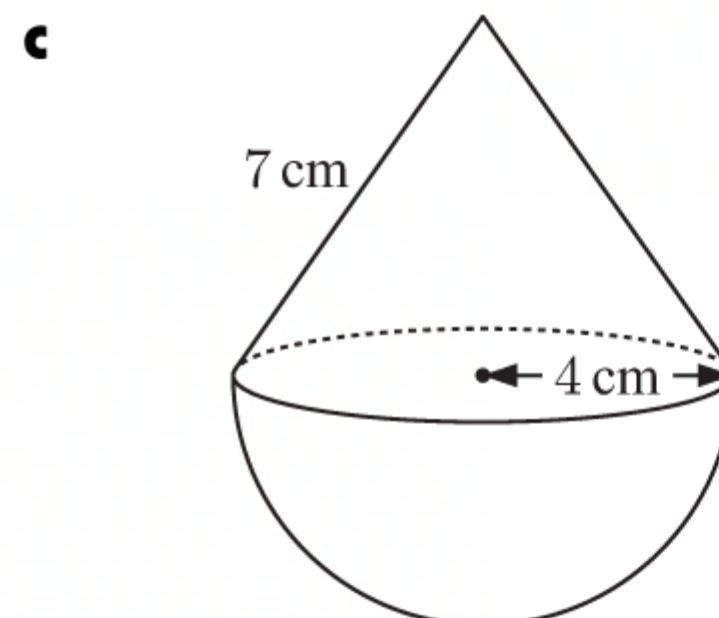
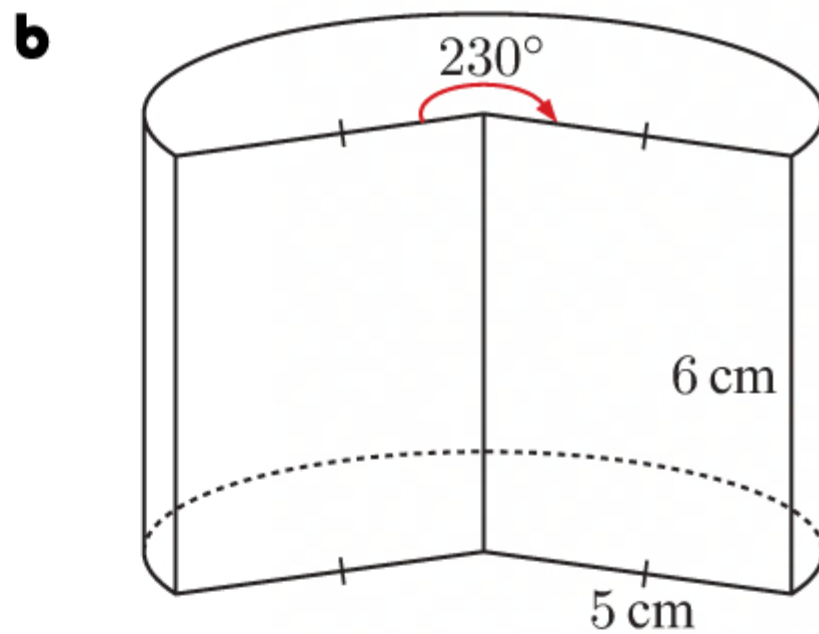
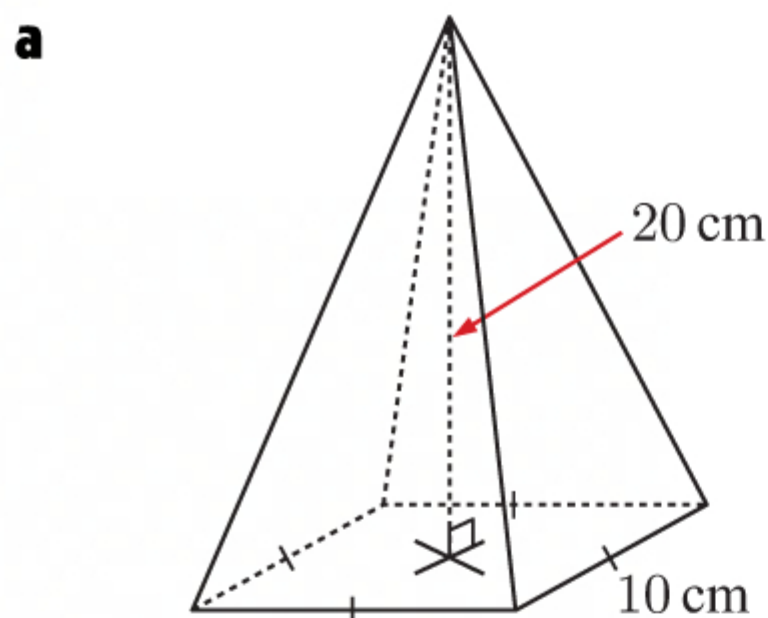
In the problems considered in this course, the optimal position for the circle's centre will occur at one of the vertices of the Voronoi diagram. The vertex with the greatest distance from its nearest site is the optimal position for the circle's centre.

SKILL BUILDER QUESTIONS

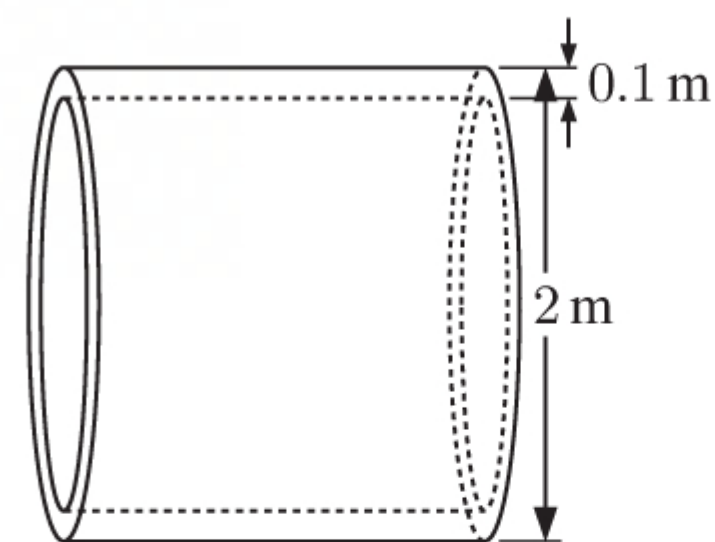
- 1** For the given figure, find to 3 significant figures:
- the perimeter
 - the area.



- 2** Find the surface area of each solid:

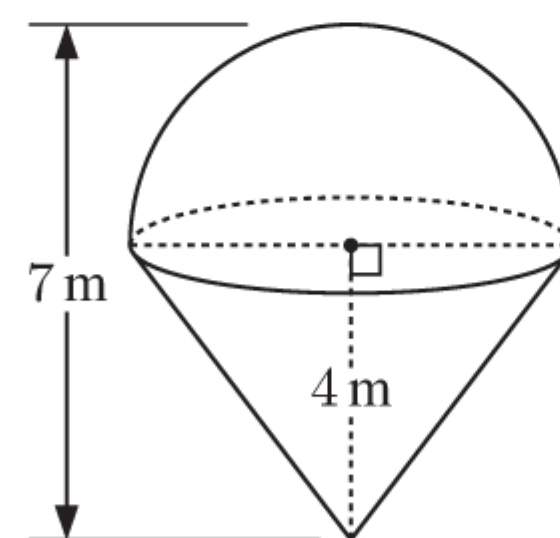



- 3** The surface area of a beach ball is 2800 cm^2 . Find the radius of the beach ball.
- 4** A pipe used to drain stormwater is made from 3 m^3 of concrete. Find the length of the pipe.

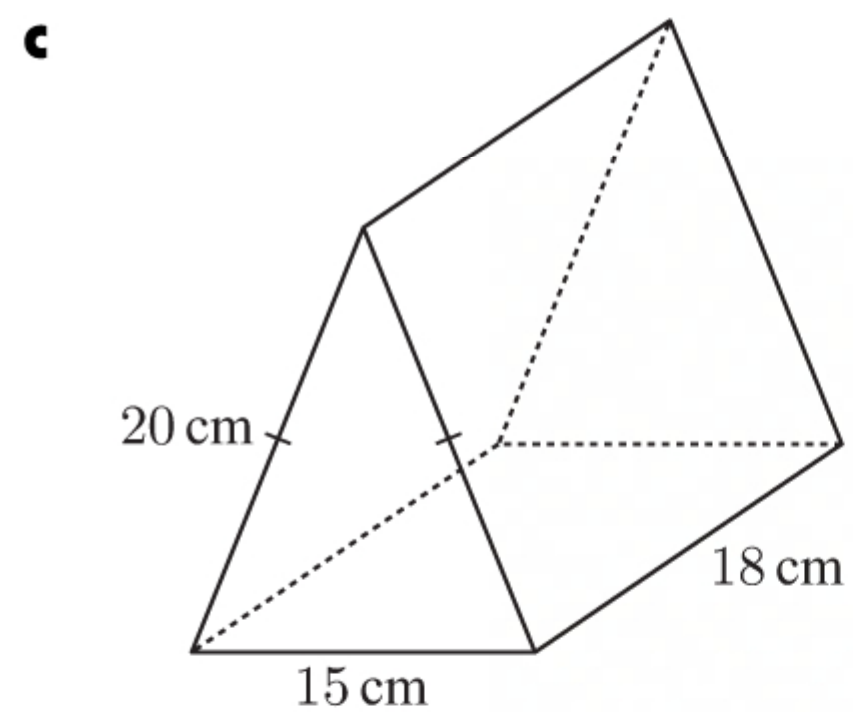
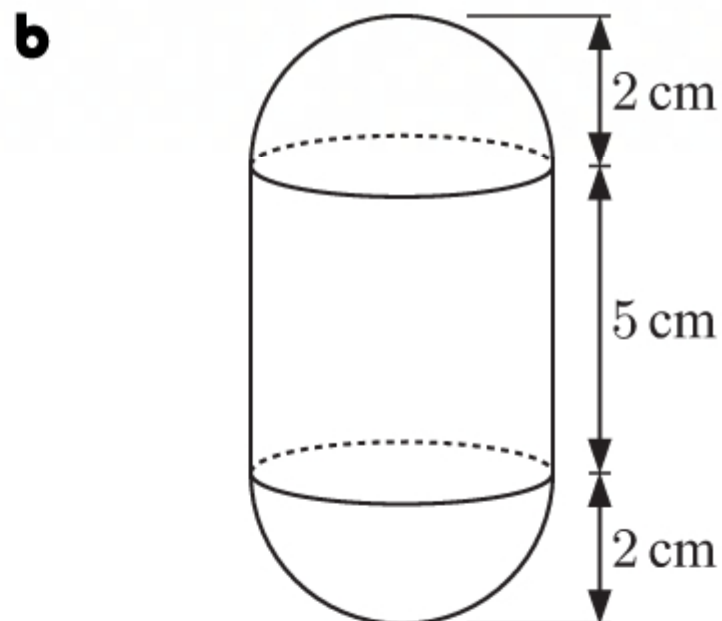
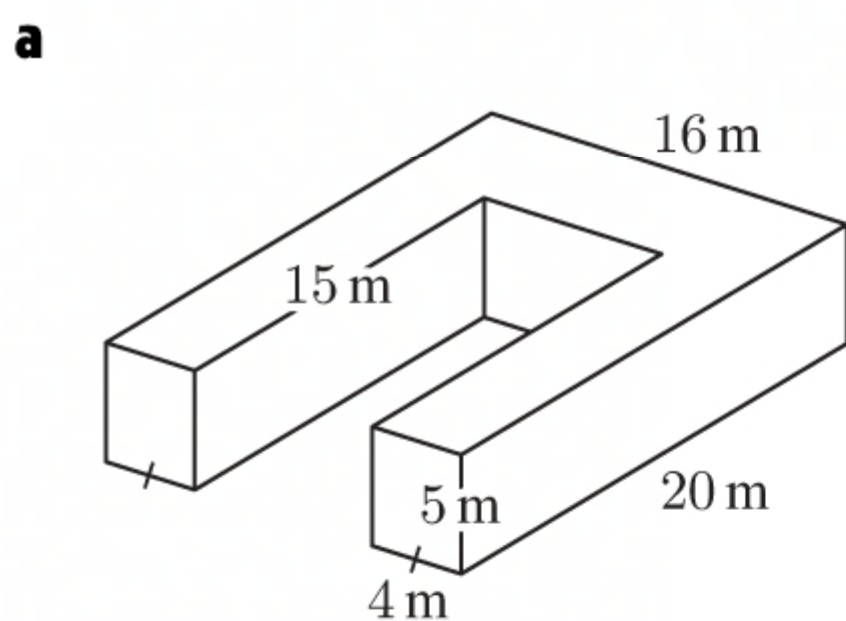


- 5** A sector of a circle of radius 10 cm has perimeter 40 cm. Find:
- a** the arc length of the sector **b** the area of the sector.
- 6** A large artificial ice cream for a shop front display is to be made with a hemisphere on top of an inverted cone.

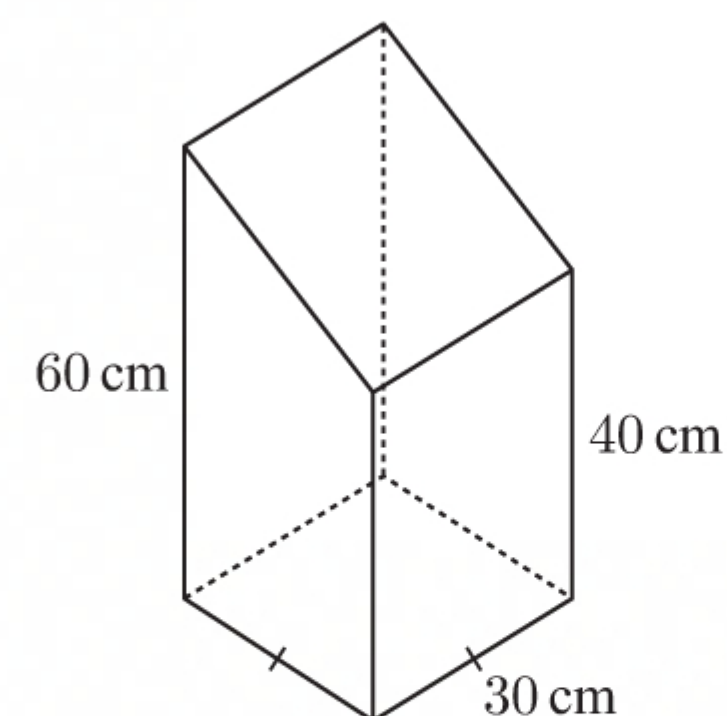
The total height of the structure is 7 m, and the cone is 4 m high.



- a** Show that the radius of the cone is 3 m.
- b** Calculate the total volume of the ice cream.
- c** Find the slant height of the cone.
- d** Find the total surface area of the ice cream.
- e** The ice cream is to be made from a lightweight polymer, weighing 1.23 kg per m^2 . Calculate the total weight of the ice cream.
- 7** Find the volume of:
- 
- The diagram shows an inverted cone representing a container. A horizontal line at the top represents the water surface, with a radius of 7 m indicated by a solid vertical line from the center to the edge. A dashed vertical line from the center to the vertex represents the height, labeled as 4 m.



- 8 How many of these petrol containers can be completely filled with 300 L of petrol?



- 9 For each pair of points, find:

i the distance AB

ii the midpoint of [AB].

a $A(2, 4, 1)$ and $B(4, 0, 7)$

b $A(3, -5, 2)$ and $B(-1, 2, -3)$

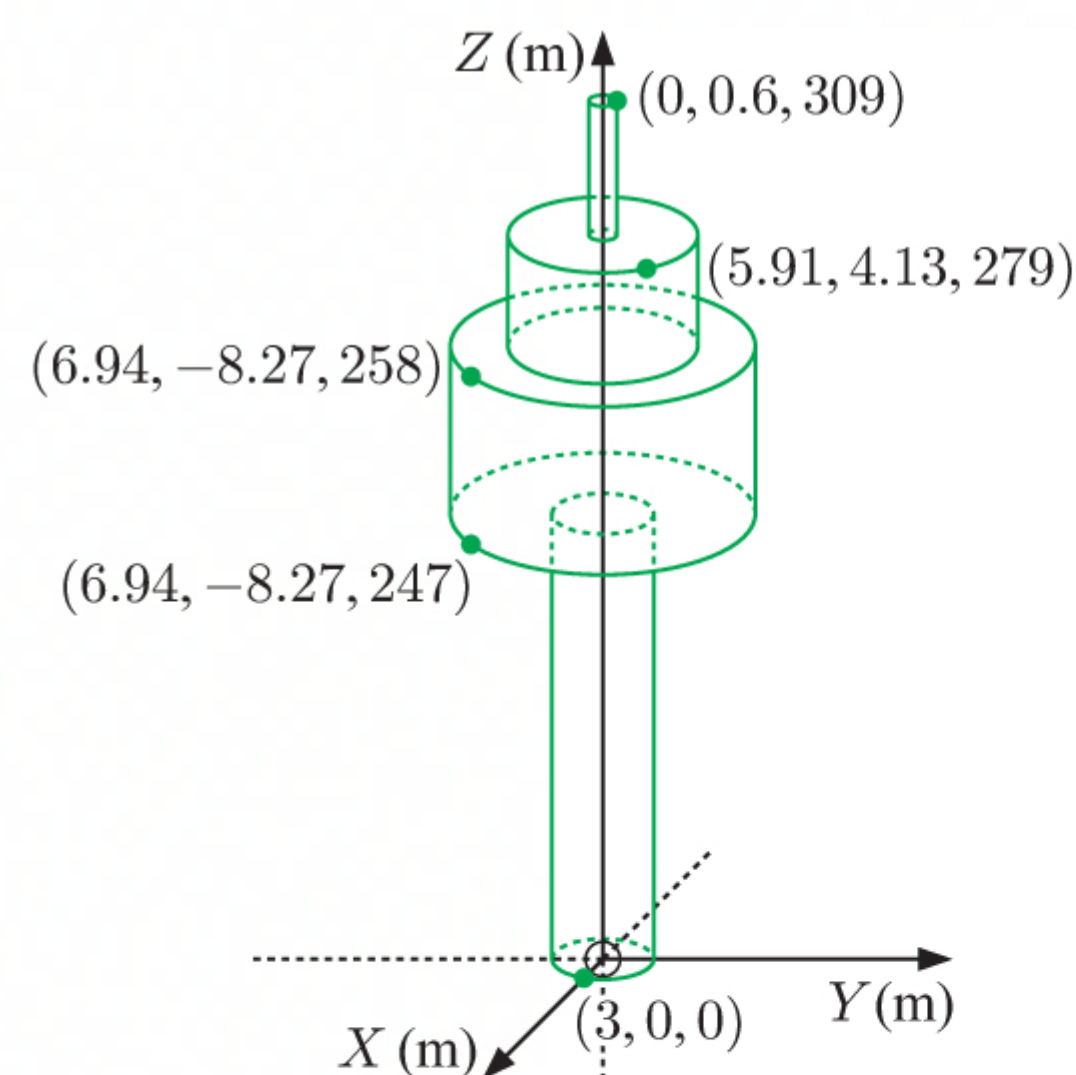
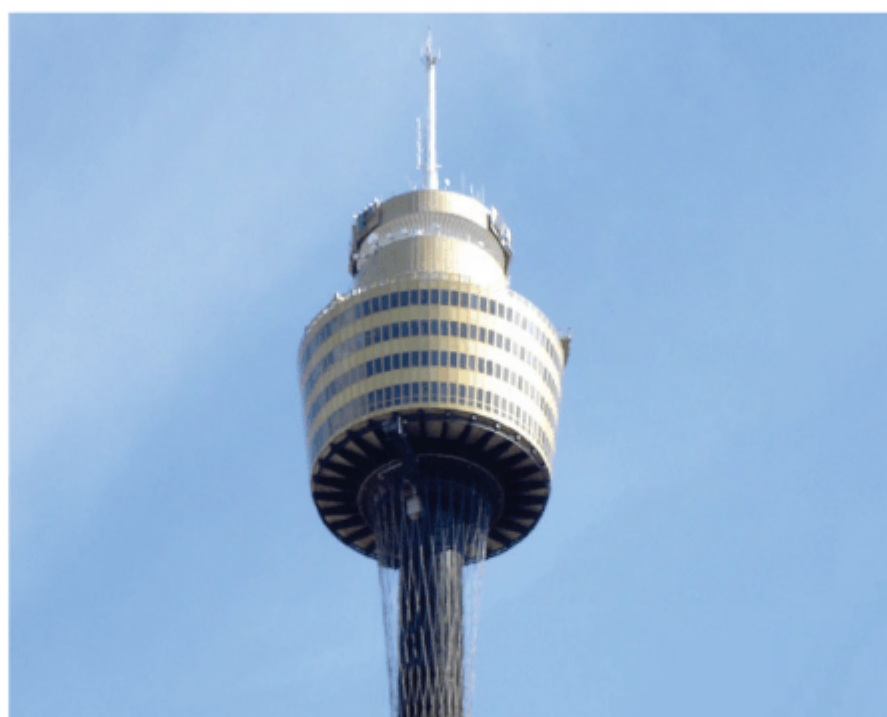
c $A(-6, 0, 5)$ and $B(-3, -3, 1)$

- 10 The distance from $P(k, 6, -5)$ to $Q(2, -1, -8)$ is 9 units. Find the possible values of k .

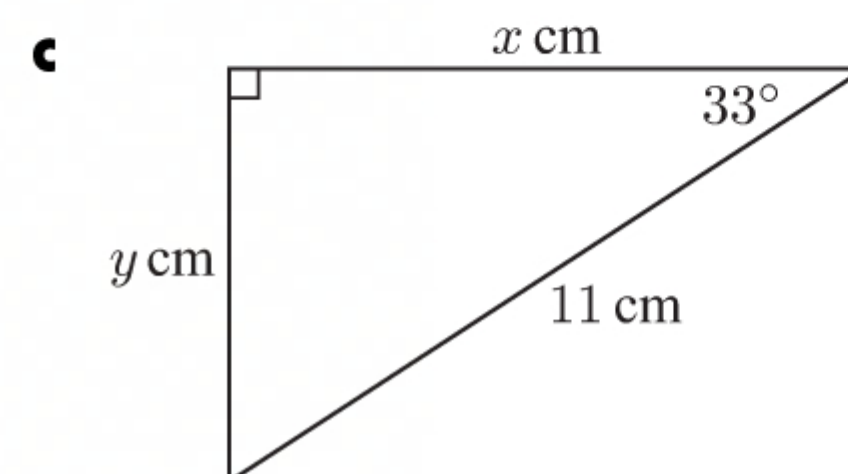
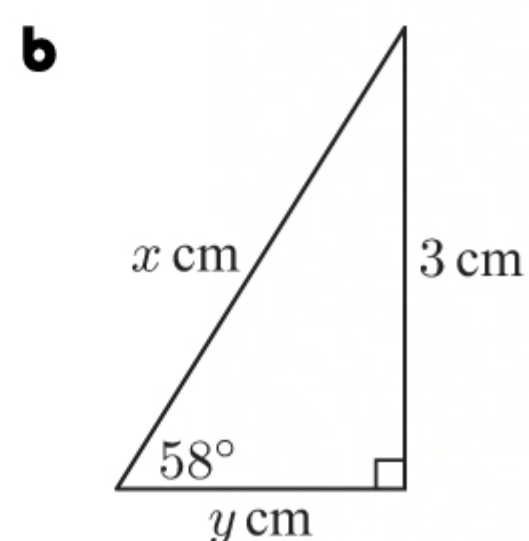
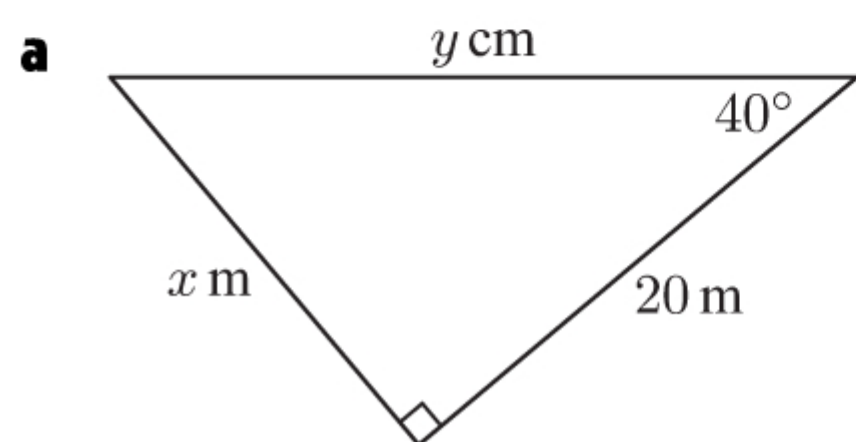
- 11 Suppose A is $(3, 4, -6)$, and $M(-\frac{1}{2}, 9, -7)$ is the midpoint of [AB]. Find the coordinates of B.

- 12 Sydney tower in Australia is the second tallest observation tower in the Southern Hemisphere.

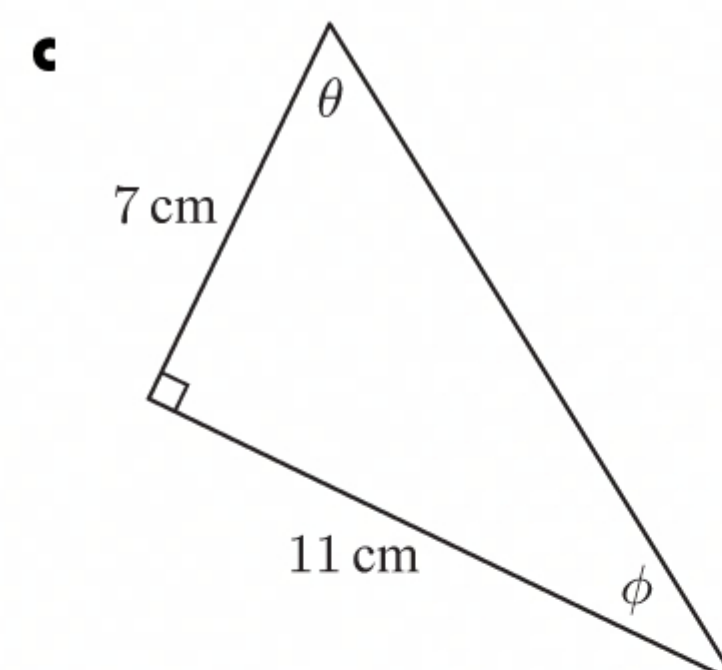
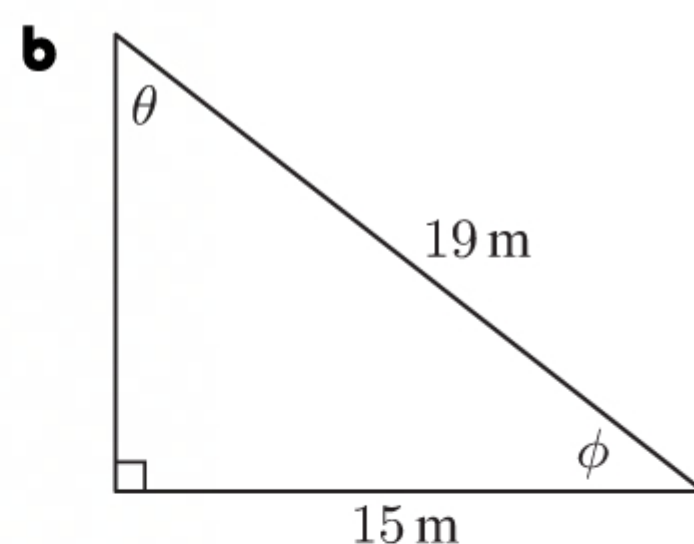
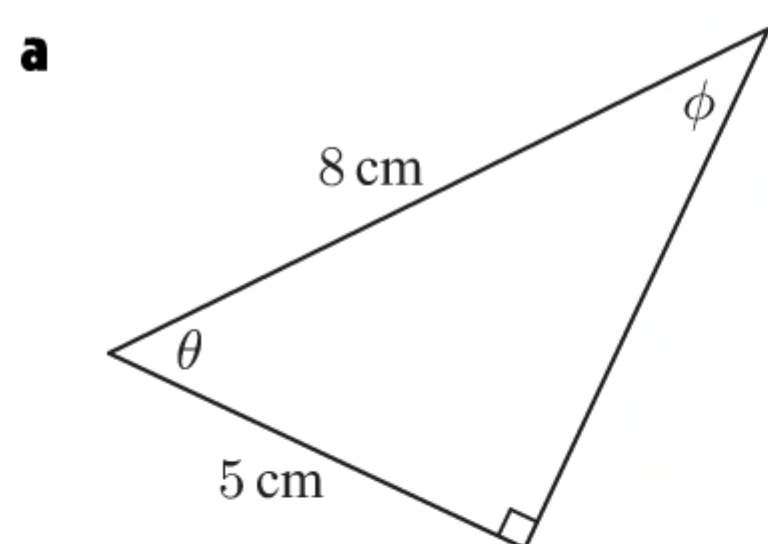
Find the volume of the tower.



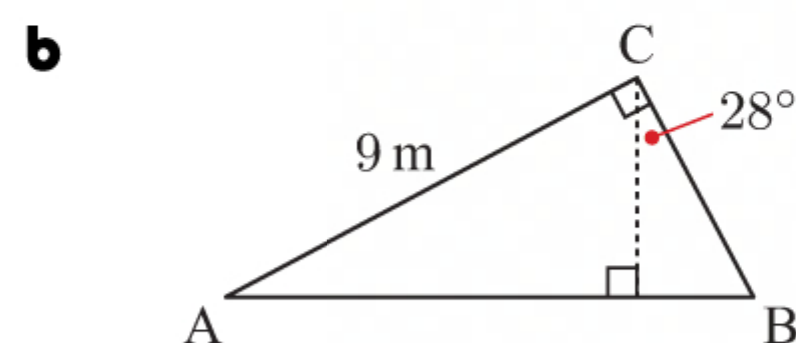
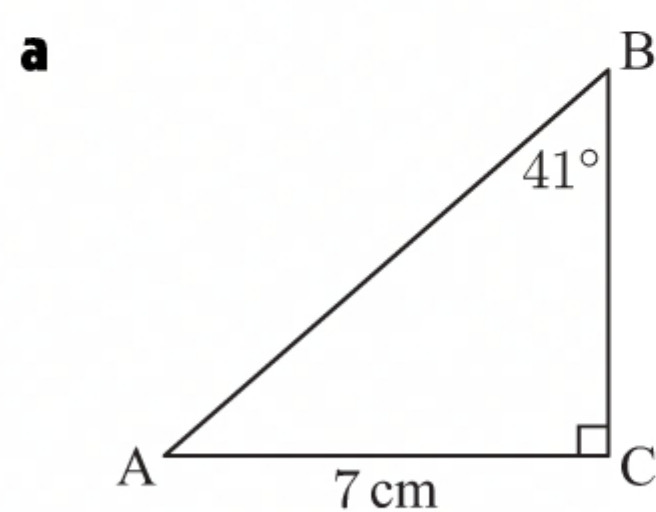
- 13 Find, correct to 3 significant figures, all unknown sides:



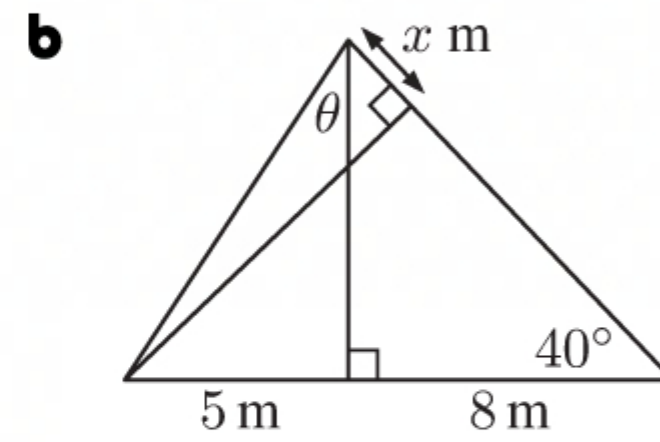
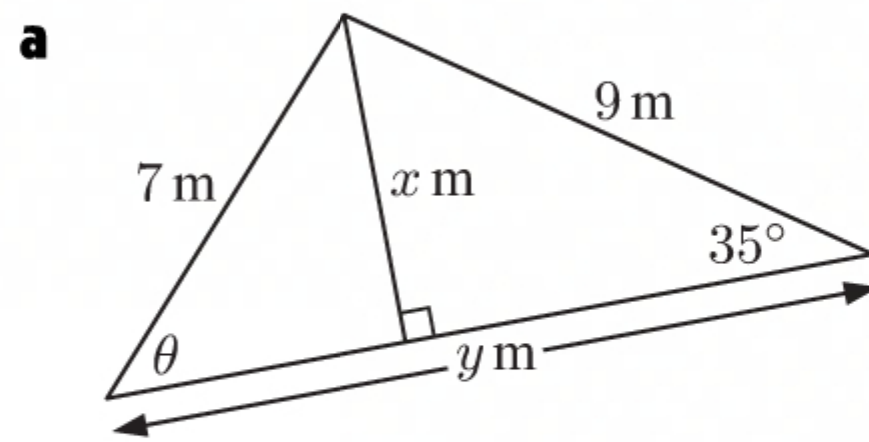
- 14 Find all unknown angles:



- 15 Find the perimeter and area of triangle ABC:



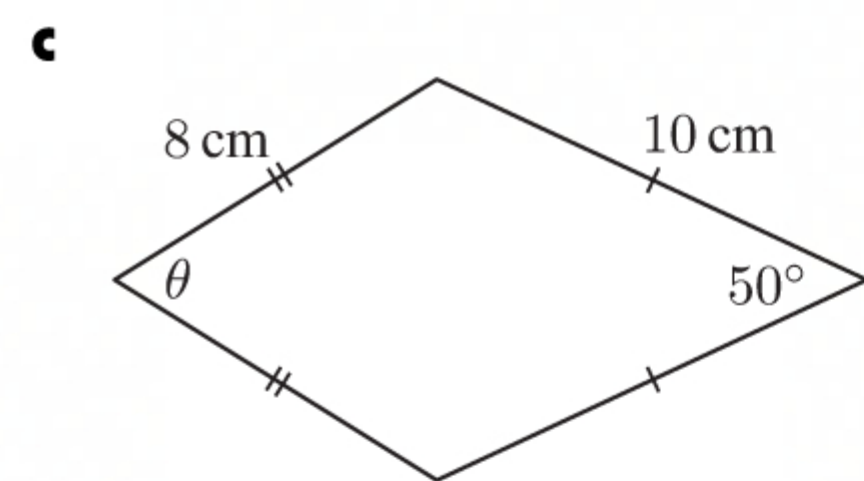
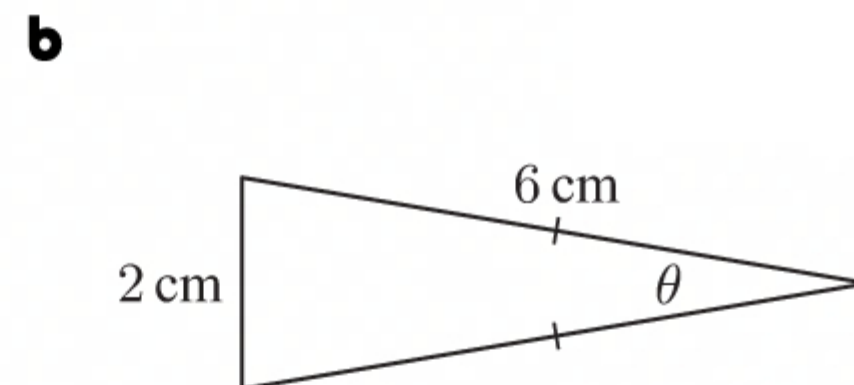
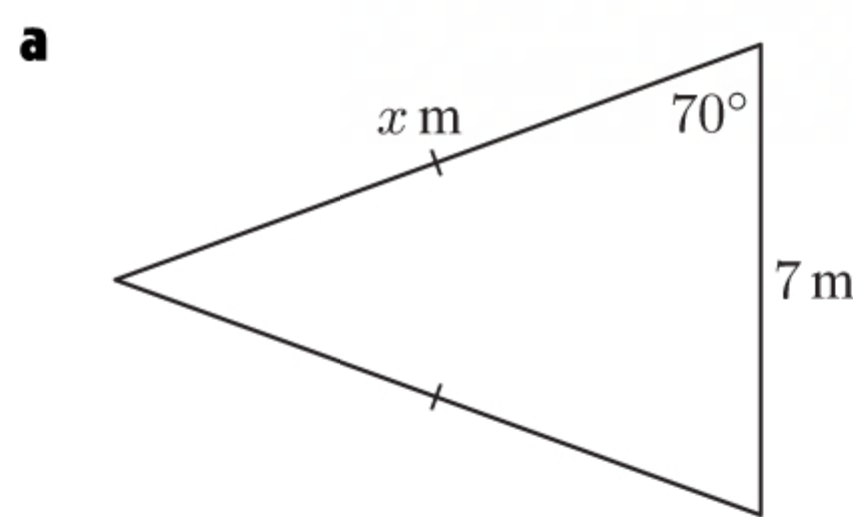
16 Find all unknowns in these figures:



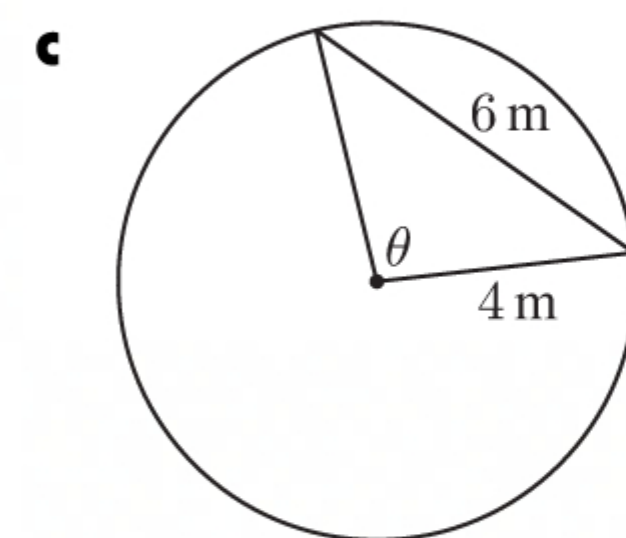
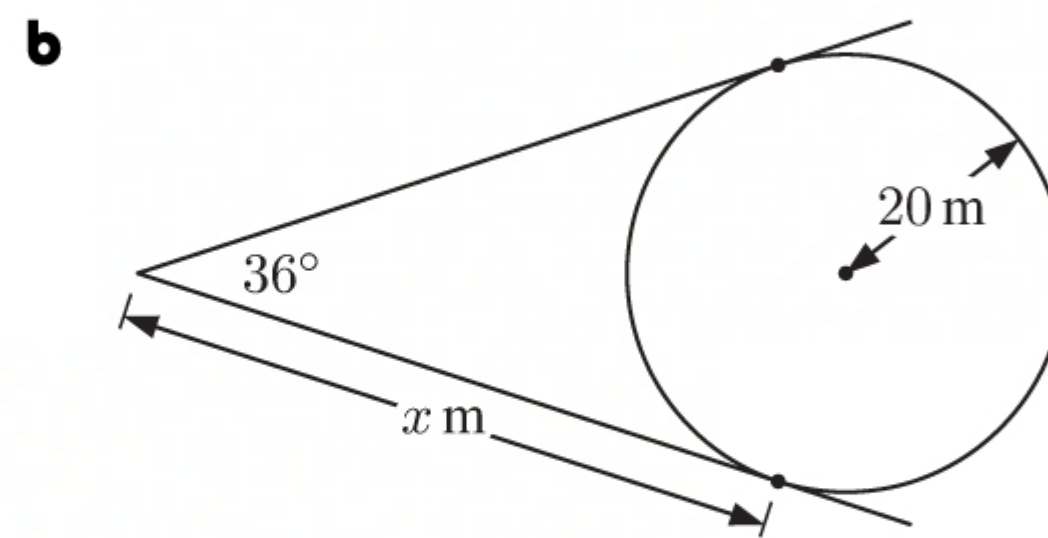
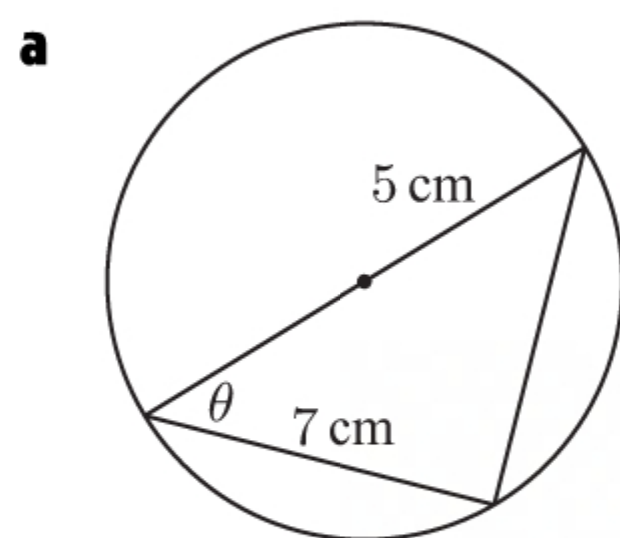
17 A rhombus has diagonals of length 5 cm and 8 cm.

- Draw a diagram and label it with the given information.
- Find the length of the sides of the rhombus.
- Find the measure of the larger angle in the rhombus.

18 Find the unknowns, correct to 3 significant figures:



19 Find the value of the unknown:



20 At 2:35 pm Fari sees an airplane directly overhead. At 2:38 pm he estimates that the angle of elevation to the plane is 15° . The plane is travelling in a straight line at 110 m s^{-1} . Calculate:

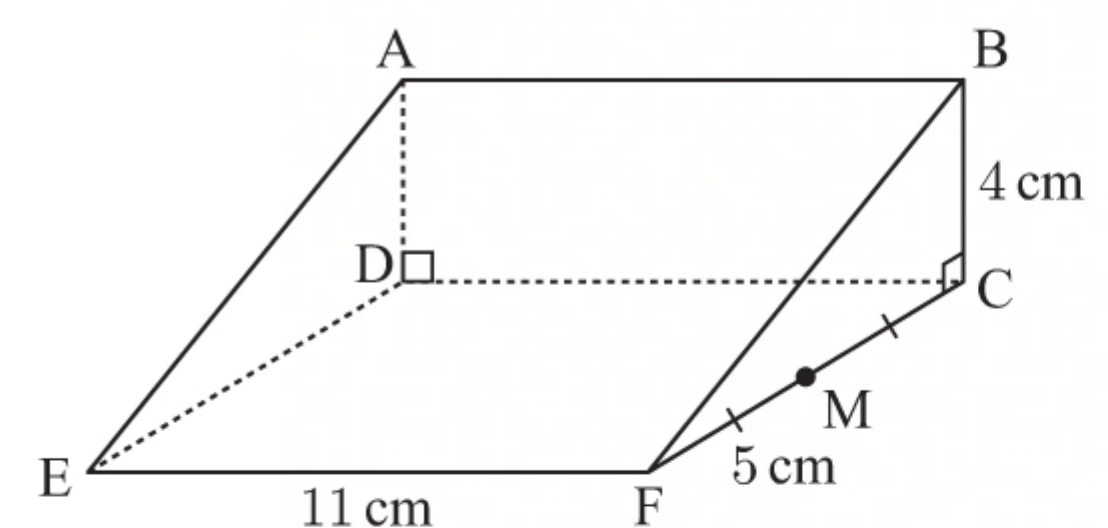
- the height of the plane above the ground
- the angle of elevation to the plane at 2:42 pm.

21 A helicopter lands 5 km east and 7 km south of its starting point.

- Find the helicopter's distance from its starting point.
- Find the helicopter's bearing from its starting point.

22 Find the angle between the following line segments and the base plane of the triangular prism:

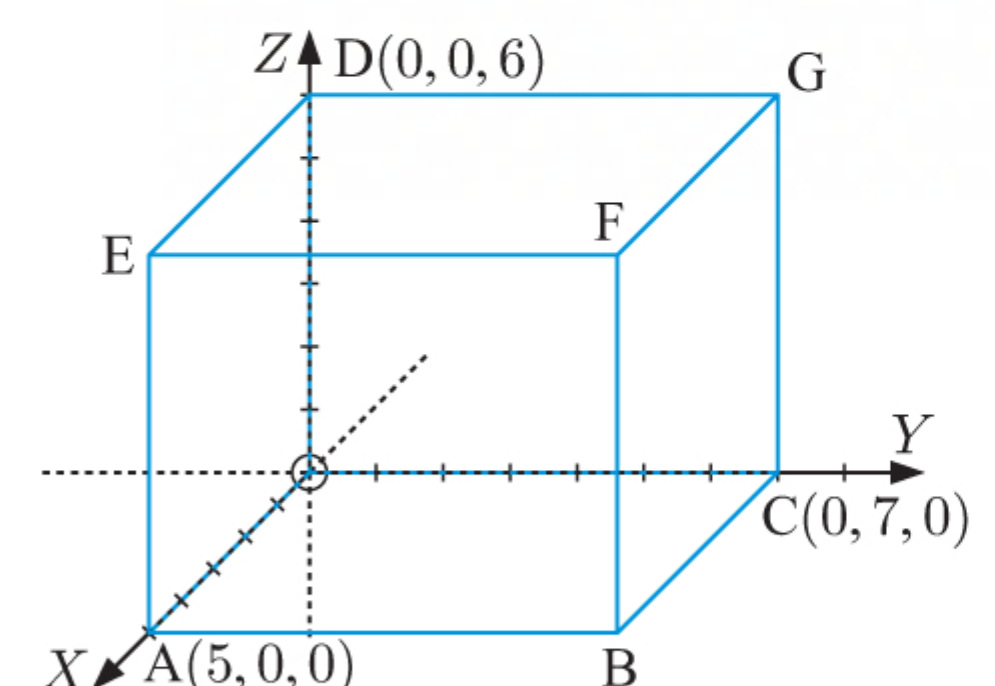
- [AE]
- [BD]
- [BE]
- [AM]

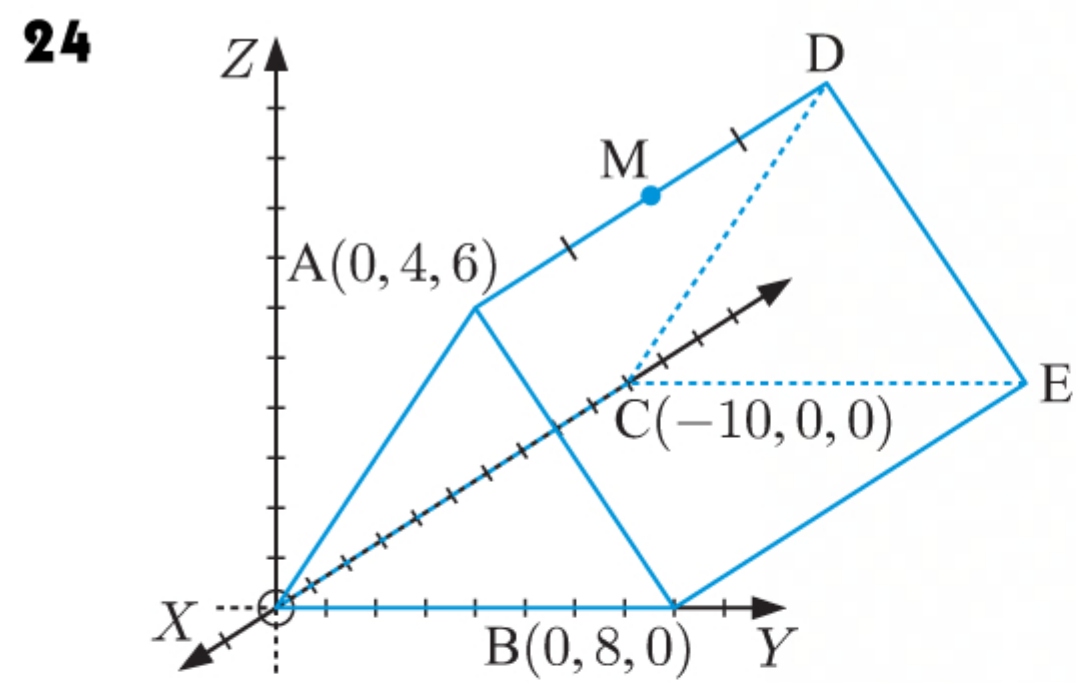


23 Consider the rectangular prism shown.

Find the angle between the following line segments and the base plane ABCO:

- [CD]
- [OF]
- [AG]

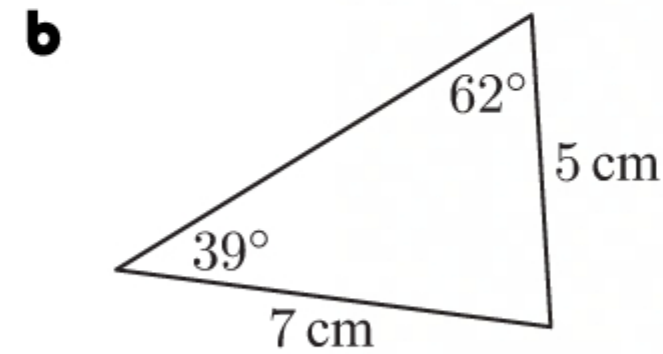
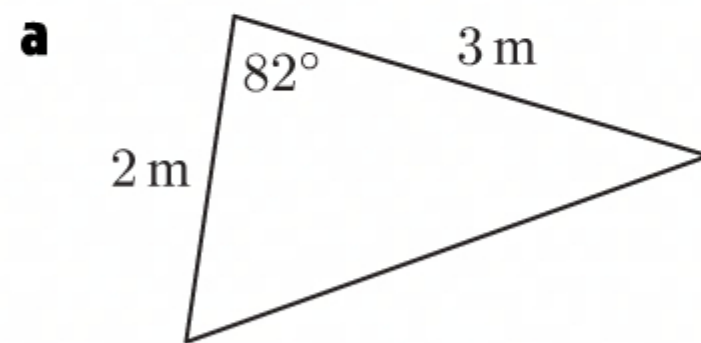




Consider the triangular prism shown.

- State the coordinates of M.
- Find the measure of \widehat{CMD} .
- Find the angle between the following line segments and the base plane BECO:
 - [OD]
 - [EM]

25 Find the area of:

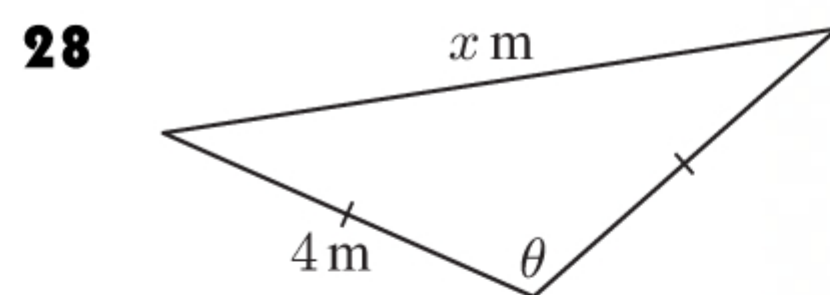


26 Triangle ABC has $AB = 8$ cm, $BC = 10$ cm, and $AC = 12$ cm.

- Draw a diagram clearly showing this information.
- Find the smallest angle in triangle ABC.
- Find the area of triangle ABC.

27 In triangle ABC, $AB = 72$ cm, $BC = 61$ cm, and $\widehat{ABC} = 43^\circ$.

- Calculate the length of AC.
- Find the measure of \widehat{ACB} .

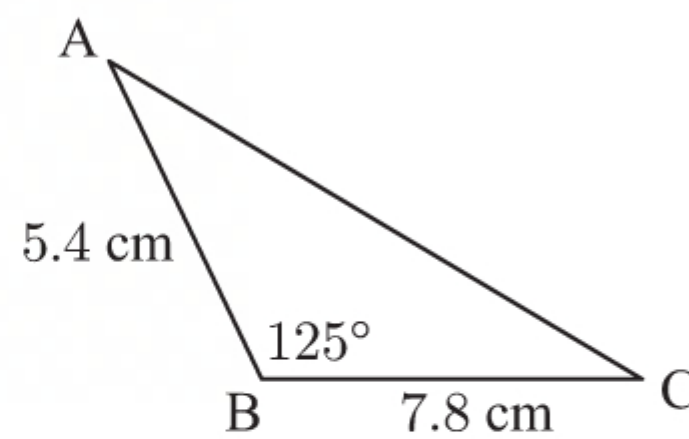


The area of the triangle shown is 4 m^2 .

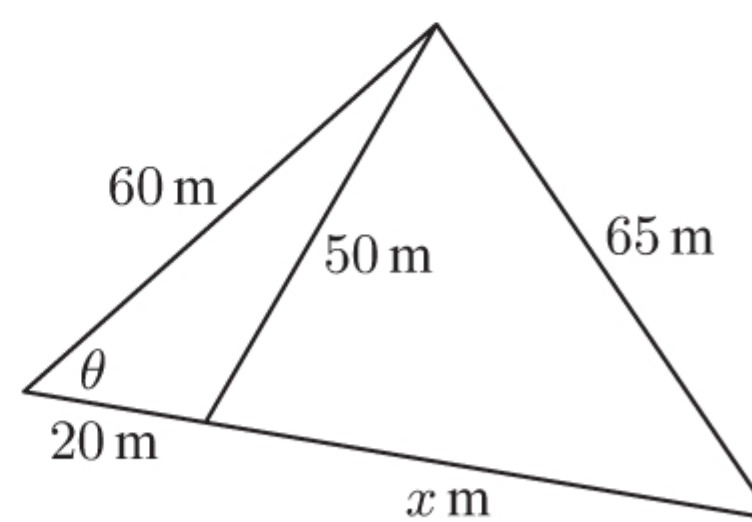
- Given that θ is obtuse, find the value of θ .
- Find x .

29 Find:

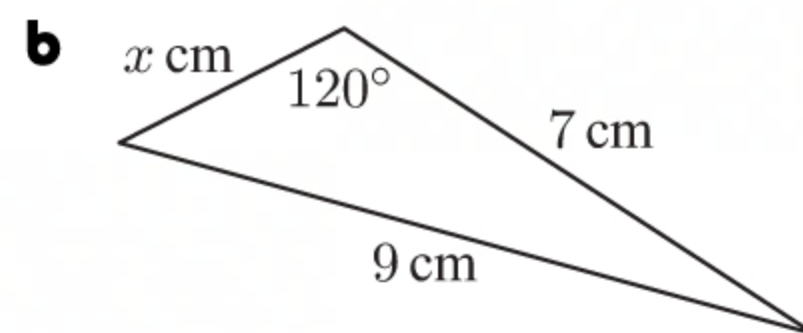
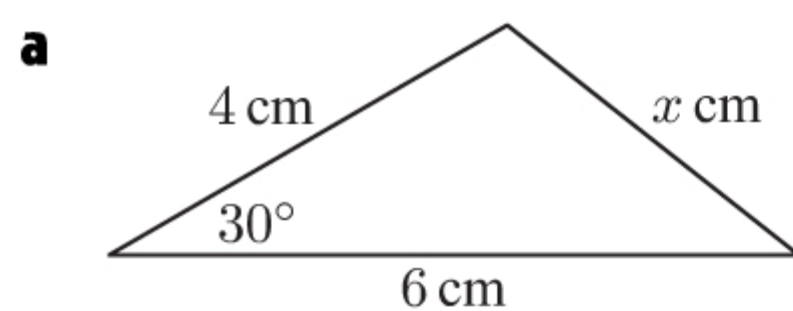
- the area of the triangle
- the length of [AC].



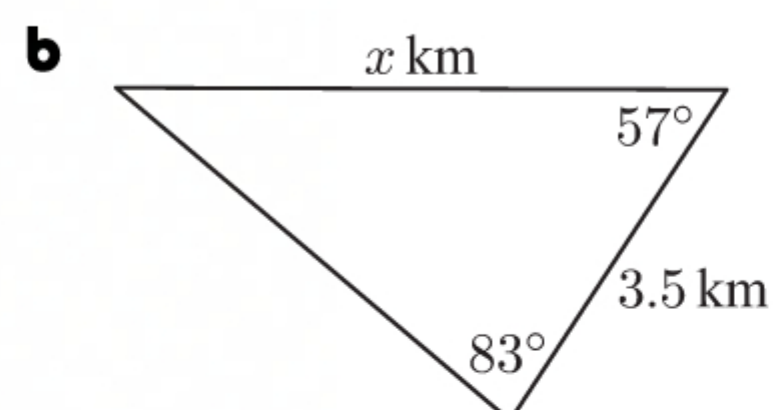
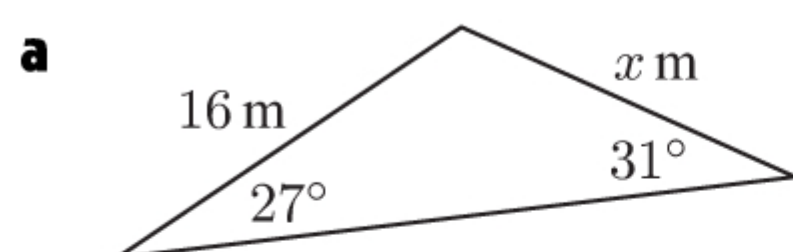
- 30**
- Find $\cos \theta$ but not θ .
 - Hence find the value of x .



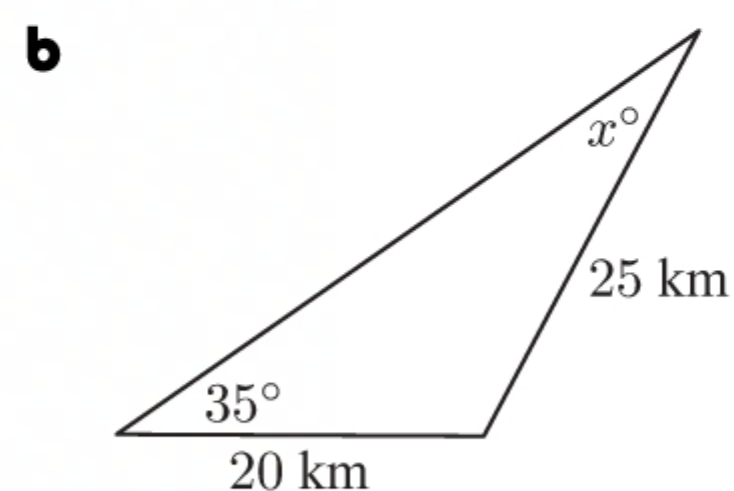
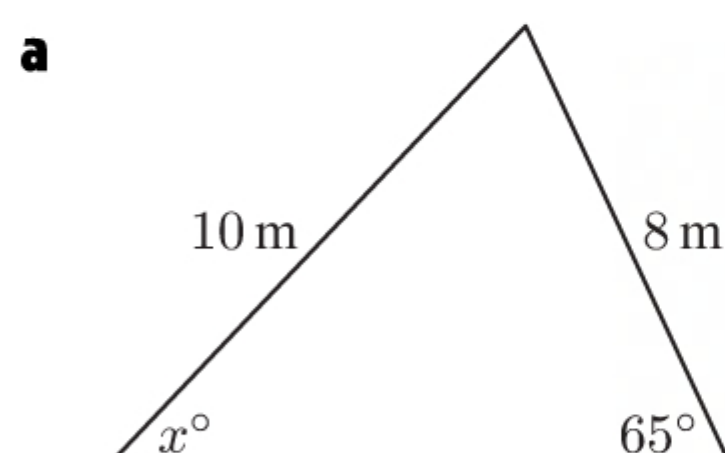
31 Find x :



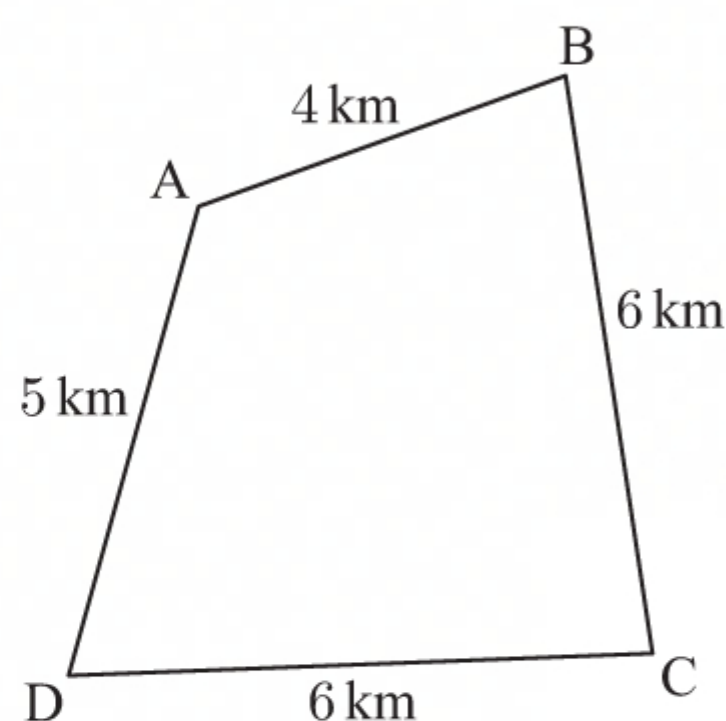
32 Find the value of x :



33 Find the value of x :



34

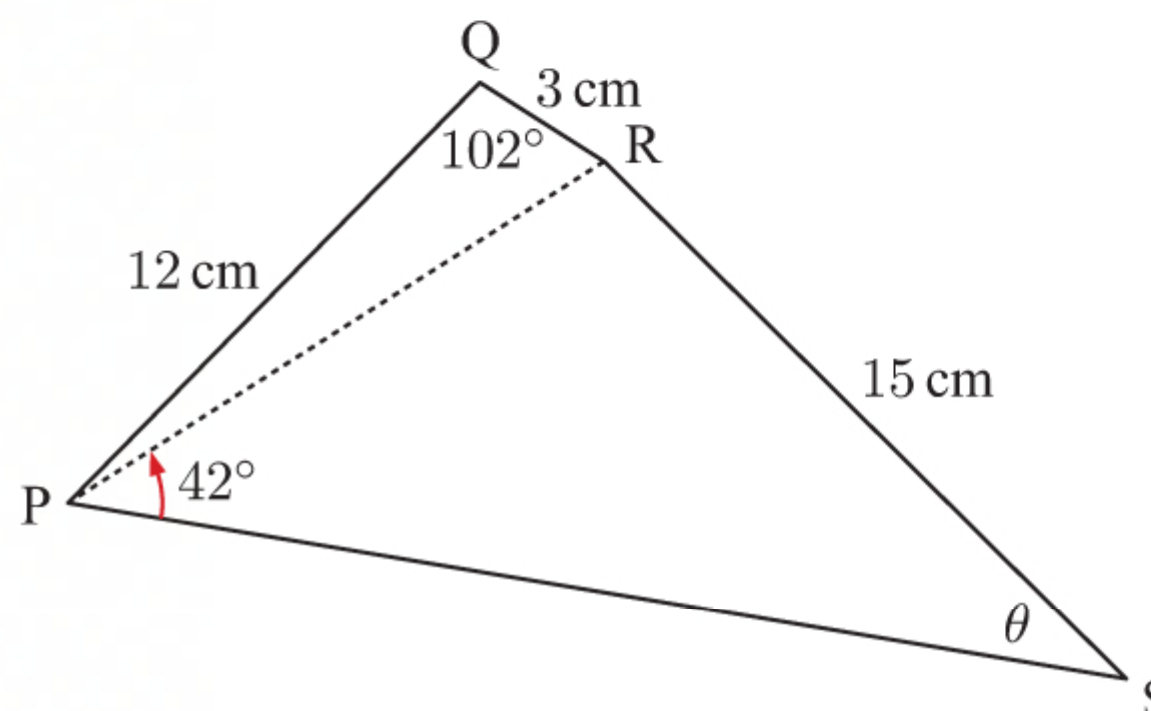


In quadrilateral ABCD, the diagonal [BD] has length 8 km.

Find the length of the diagonal [AC].

35 Quadrilateral PQRS has the measurements shown.

- Find the length of [PR].
- Determine the measure of the angle marked θ .

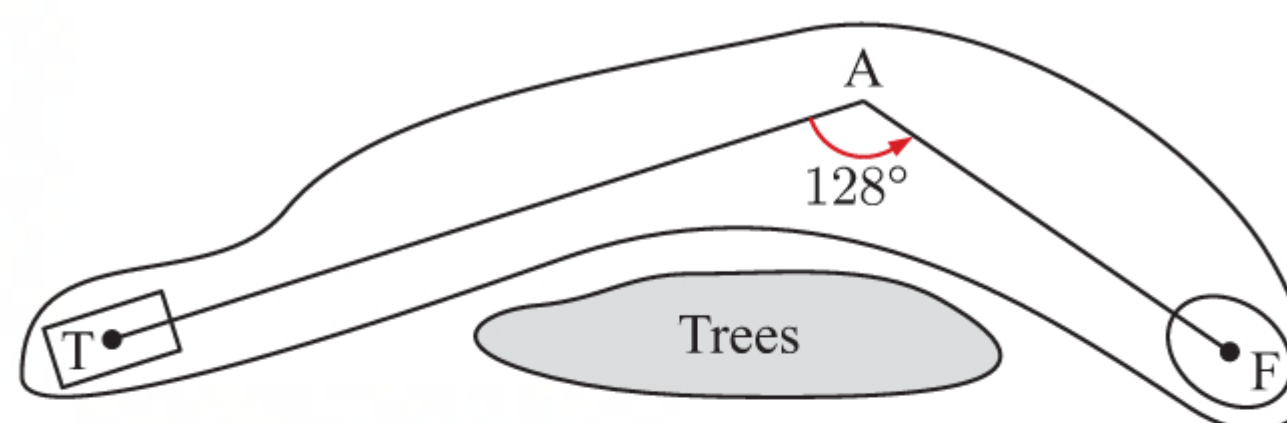


36 Monument X is observed from two points B and C which are 330 m apart. \widehat{XBC} is 63° and \widehat{BCX} is 75° .

- Draw a neat, labelled diagram to illustrate this information.
- Find the distance between the monument and B.

37 The 5th hole at the Flagstaff golf course has the layout shown. From T to A on the fairway, the distance is 240 m, and from A to F the distance is 135 m.

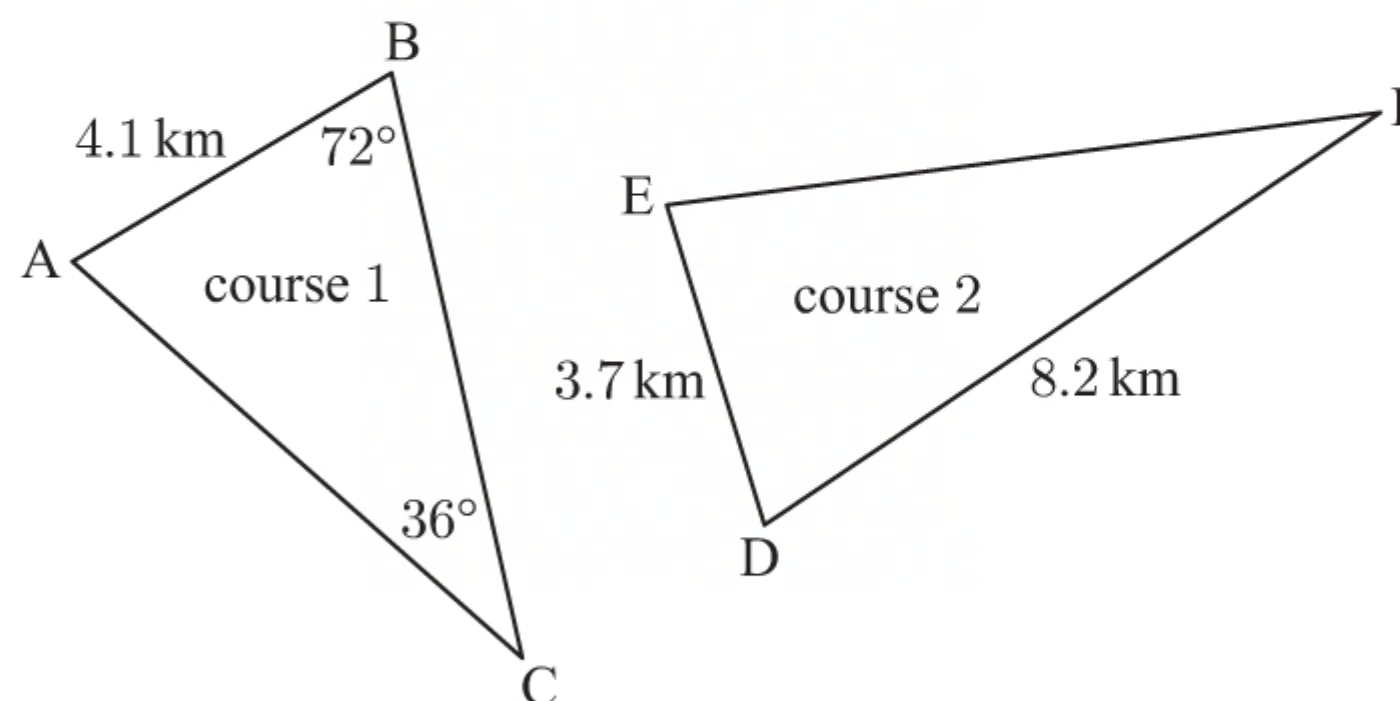
- Find the distance from T to F in a straight line.
- Find the measure of \widehat{ATF} .



38 The diagram below shows the routes of two triangular orienteering courses ABC and DEF. The distance from E to F is 20% longer than the distance from A to C.

Find:

- the length of [EF]
- the measure of \widehat{DEF}
- the total area covered by course 2
- the total length of course 1.



39 Find the equation of the perpendicular bisector of:

- A(3, 5) and B(7, 3)
- M(7, 2) and N(1, -5)

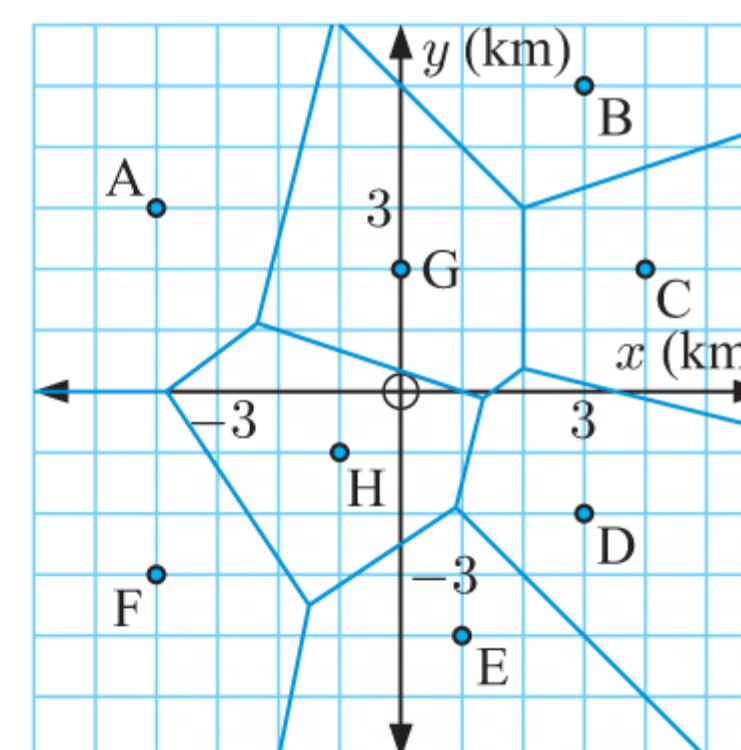
40 A line segment has equation $4x - 3y + 2 = 0$. Its midpoint is (4, 6).

- State the gradient of:
 - the line segment
 - its perpendicular bisector.
- State the equation of the perpendicular bisector. Write your answer in the form $ax + by + d = 0$.

41 This Voronoi diagram shows the bus stops in a particular suburb.

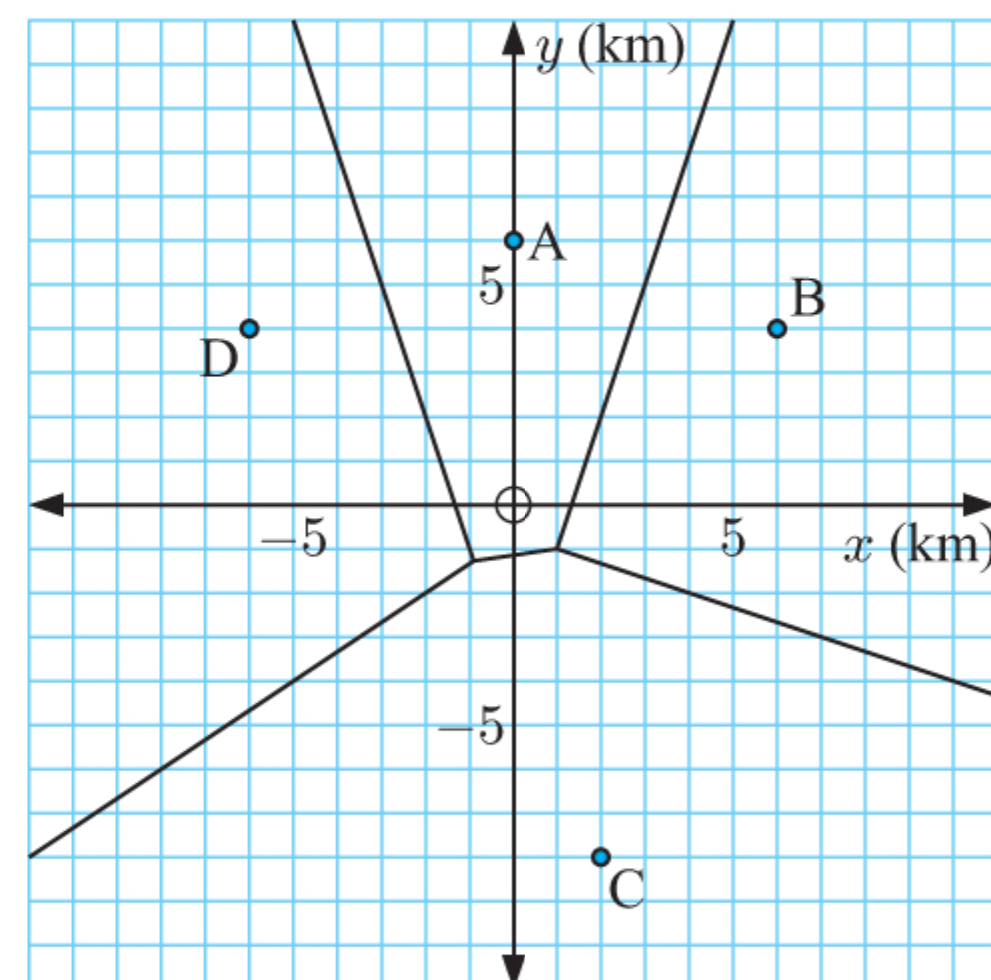
Children living in the area walk to the nearest stop to catch the bus to school.

- Identify the nearest bus stop for a child living at:
 - (0, -2)
 - (3, 2)
 - (-5, 5)
 - (-1, -4)
- Jerome is closest to bus stops B, C, and G.
 - Where does Jerome live?
 - How far does Jerome need to walk to get to any of these bus stops?

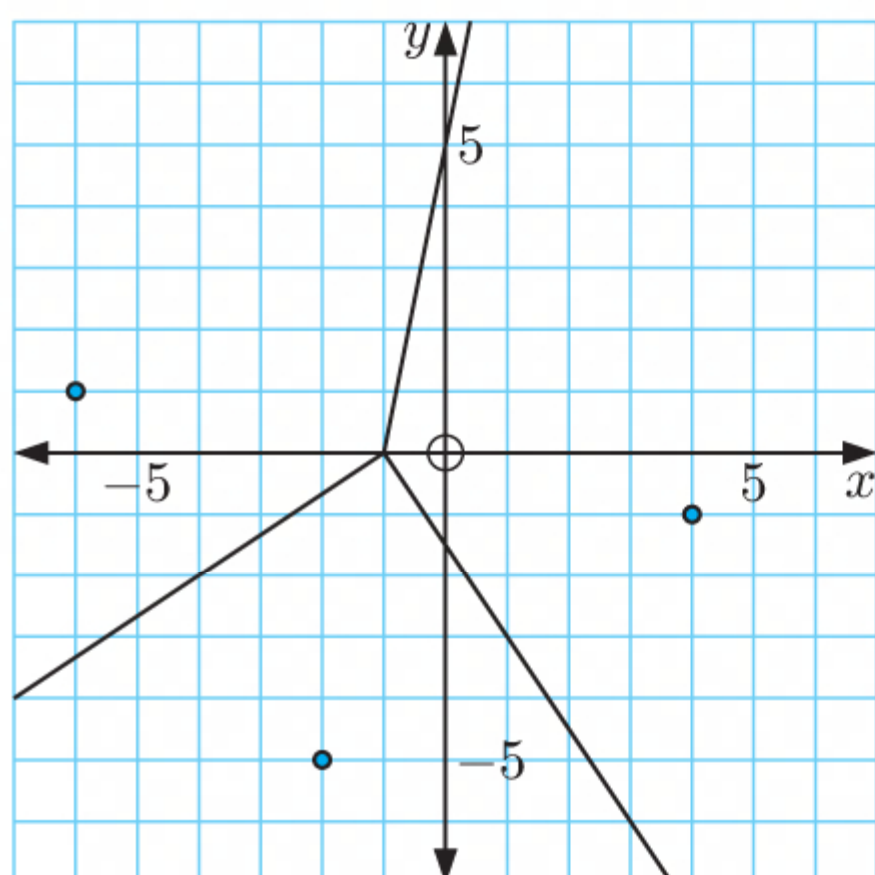


- 42** Terrence owns a fast food chain. The locations of his restaurants in a particular city are shown in the Voronoi diagram alongside.

- a** Find the closest restaurant for someone living at:
- i** $(-2, 8)$ **ii** $(5, -5)$ **iii** $(-9, -6)$
- b** Terrence opens a new restaurant E at $(2, 4)$.
- i** Redraw the diagram to include the new restaurant.
 - ii** Determine the area of the region whose closest restaurant is E.
 - iii** What proportion of residents who are now closest to restaurant E were originally closest to restaurant B?



43



Consider the Voronoi diagram alongside.

Redraw the diagram with a new site added at:

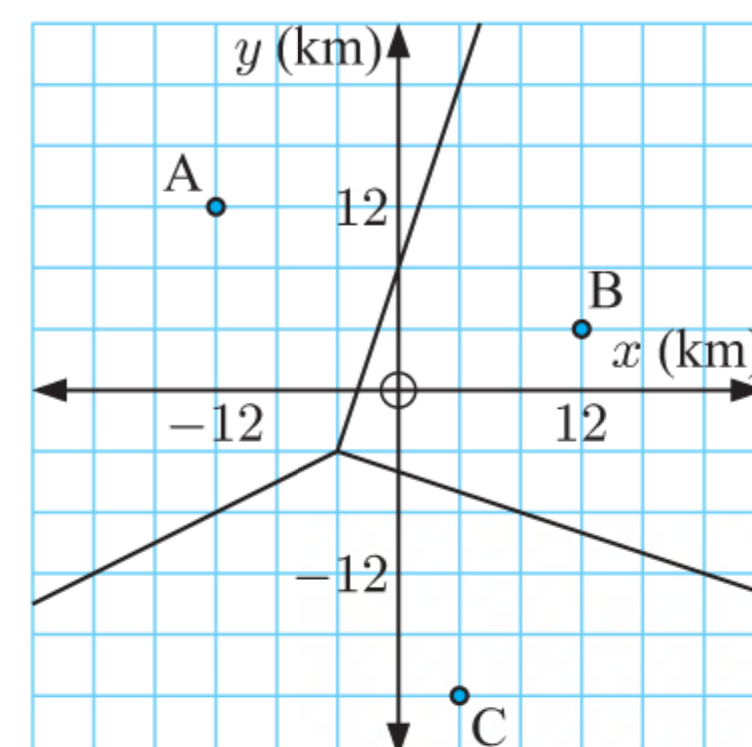
- a** $(2, -5)$ **b** the vertex of the existing diagram.

- 44** The internet speed is measured at 6 pm at three exchanges in a particular area.

Location	Internet speed (Mbps)
A	44.3
B	42.7
C	45.9

Use nearest neighbour interpolation to estimate the internet speed at 6 pm at:

- a** $(12, 16)$ **b** $(-20, -12)$ **c** $(-4, -4)$

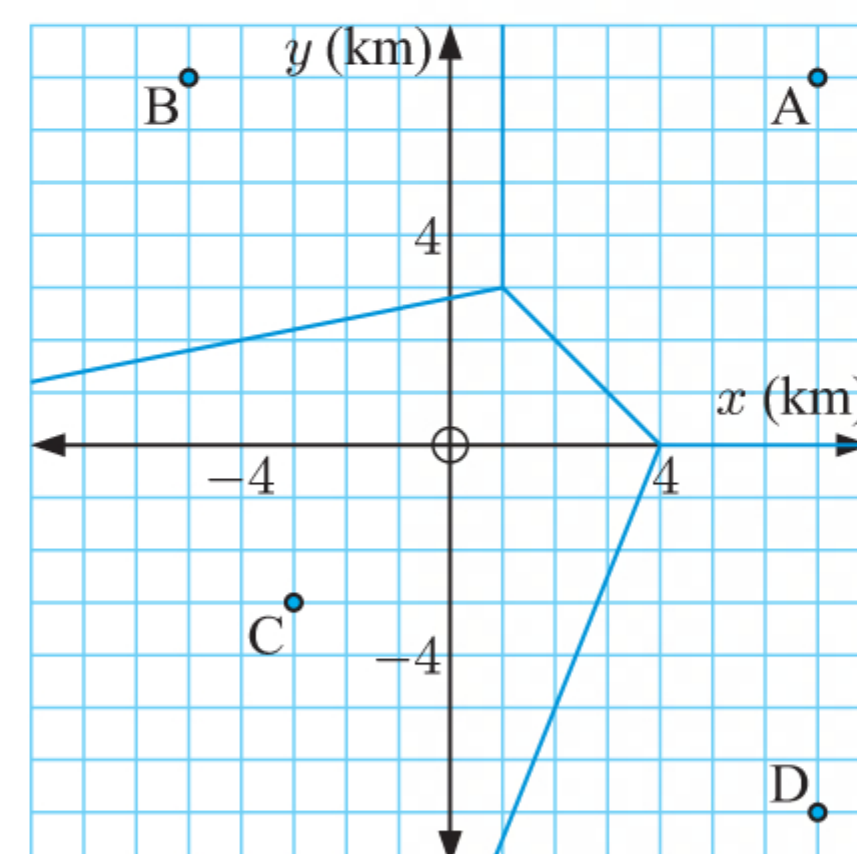


- 45** This Voronoi diagram shows the locations of a particular bank's automatic teller machines (ATMs) around a city.

After receiving customer feedback, the bank has decided to add a new ATM.

The new ATM will be placed so that it is as far as possible from the existing ATMs.

- a** Where should the bank place the new ATM?
- b** Redraw the Voronoi diagram with the new ATM.
- c** Allan is currently at $(-1, 1)$ and needs to withdraw money. How far does Allan need to walk to get to the nearest ATM?
- d** Write down an assumption made in **c**.



TOPIC 4: STATISTICS AND PROBABILITY

SAMPLING

We obtain data from a **sample** of a population when it is impractical to obtain data from the entire population.

You should know the four main categories of **error** that can arise from sampling:

- **Sampling errors** occur when a characteristic of a sample differs from that of the population.
- **Measurement errors** are inaccuracies in measurement during data collection.
- **Coverage errors** occur when a sample does not truly reflect the population.
- **Non-response errors** occur when a large number of people selected for a survey choose not to respond.

SAMPLING METHODS

- In **simple random sampling**:
 - ▶ Each member of the population has the same chance of being selected in the sample.
 - ▶ Each set of n members of the population has the same chance of being selected as any other set of n members.
- In **systematic sampling**, the sample is created by selecting members of the population at regular intervals.
- In **convenience sampling**, members are chosen for the sample because they are easier to select or more likely to respond.
- In **stratified sampling** or **quota sampling**, the population is divided into subgroups, and the number of members sampled from each subgroup is proportional to the fraction of the population represented by that subgroup. If the members of each subgroup are randomly selected, the sample is a **stratified sample**. If the members are specifically chosen, the sample is a **quota sample**.

TYPES OF DATA AND ITS REPRESENTATION

Categorical data refers to data which describes a particular quality or characteristic.

Discrete data can take any of a set of exact number values $\{x_1, x_2, x_3, \dots\}$. It is normally **counted**.

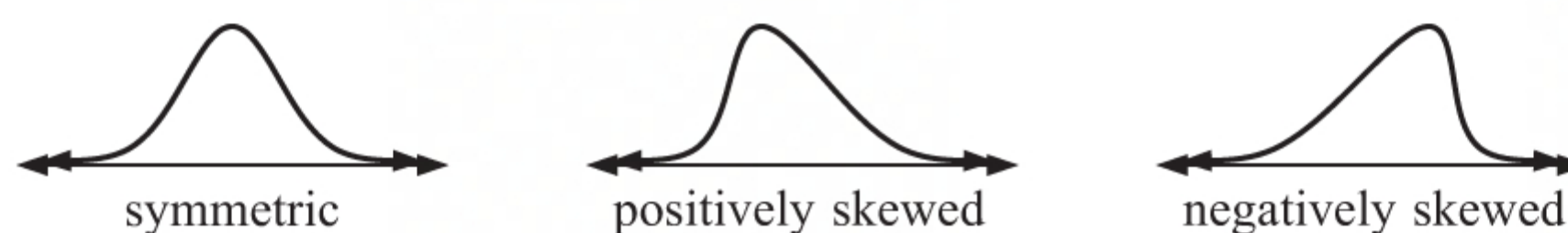
Continuous data can take any numerical value within a certain range. It is normally **measured**.

Grouped data is numerical data which is collected in groups or classes. The **modal class** is the class with the highest frequency.

A **column graph** is used to display discrete data and grouped data. The columns have spaces between them.

A **frequency histogram** is used to display continuous data. The classes are of equal width, and there are no spaces between the columns.

Data may be symmetric, positively skewed, or negatively skewed.



We use a **cumulative frequency graph** to display the cumulative frequency for each data value in a distribution. This enables us to read off the values at each percentile.

MEASURING THE CENTRE OF DATA

The **mean** of a set of scores is their arithmetic average.

For a large population, the **population mean** μ is generally unknown. The **sample mean** \bar{x} is used as an approximation for μ .

For ungrouped data, $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$

For data in a frequency table, $\bar{x} = \frac{\sum xf}{\sum f}$ where f is the frequency of each value.

For grouped data we can only estimate the mean. We use the **mid-interval value** within each group to represent all scores within that group.

The **median** is the middle value of an ordered data set.

- For an **odd number** of data, the median is one of the original data values.
- For an **even number** of data, the median is the average of the two middle values, and may not be in the original data set.

The **mode** is the most frequently occurring score. If there are two modes we say the data is **bimodal**. For continuous data we refer to a **modal class**.

PERCENTILES

The **k th percentile** is the score a such that $k\%$ of the scores are less than a .

The **lower quartile** (Q_1) is the 25th percentile.

The **median** (Q_2) is the 50th percentile.

The **upper quartile** (Q_3) is the 75th percentile.

You should know how to generate a **cumulative frequency graph** and use it to estimate Q_1 , Q_2 , and Q_3 .

MEASURING THE SPREAD OF DATA

The **range** is the difference between the maximum and the minimum data values.

The **interquartile range** $IQR = Q_3 - Q_1$.

The **variance** σ^2 is the average of the squares of the distances from the mean.

The **standard deviation** σ is the square root of the variance.

You should be able to use technology to calculate standard deviation.

OUTLIERS

Outliers are extraordinary data that are separated from the main body of the data. We test for outliers by calculating upper and lower boundaries:

- upper boundary = $Q_3 + 1.5 \times IQR$
- lower boundary = $Q_1 - 1.5 \times IQR$

Any data outside of these boundaries is considered an outlier.

BOX AND WHISKER DIAGRAMS

A **box and whisker diagram** or **box plot** illustrates the **five-number summary** of a data set:

- minimum value
- Q_1
- median
- Q_3
- maximum value

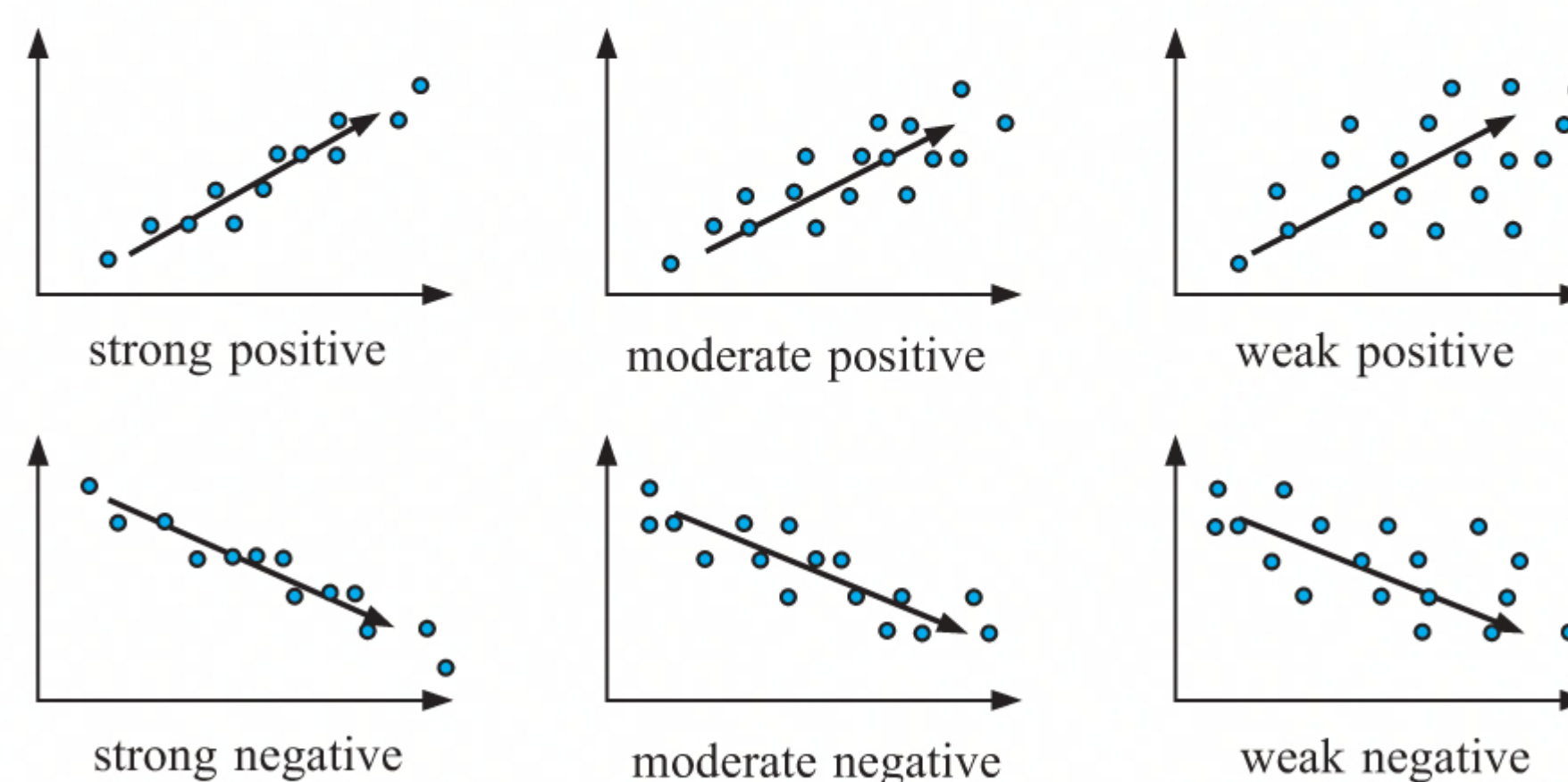


An outlier is indicated by an asterisk *.

BIVARIATE STATISTICS

Correlation refers to the relationship between two numerical variables.

We can use a **scatter diagram** to help identify **outliers** and to describe the correlation between variables. We consider **direction**, **strength**, and **linearity**.



If a change in one variable *causes* a change in the other variable then we say there is a **causal relationship** between them.

To measure the strength of the relationship between two variables, we use **Pearson's product-moment correlation coefficient** r .

The correlation coefficient lies in the range $-1 \leq r \leq 1$.

- The sign of r indicates the direction of correlation.
 - ▶ A positive value for r indicates the variables are positively correlated.
 - ▶ A negative value for r indicates the variables are negatively correlated.
- The size of r indicates the strength of correlation.
 - ▶ A value of r close to $+1$ or -1 indicates strong correlation between the variables.
 - ▶ A value of r close to zero indicates weak correlation between the variables.

Line of best fit

If two variables are linearly correlated, we can draw a line of best fit to illustrate their relationship.

We can draw a **line of best fit by eye**, which passes through the **mean point** (\bar{x}, \bar{y}) , and which fits the trend of the data.

To get a more accurate line of best fit, we use a method called **linear regression**. The line obtained is called the **least squares regression line**. You should be able to find this line using your calculator.

When using a line of best fit to estimate values, **interpolation** is usually reliable, whereas **extrapolation** may not be.

Spearman's rank correlation coefficient of a bivariate data set is defined as the Pearson product-moment correlation coefficient of the variables' **ranks**. It is often used when the data is clearly non-linear, but has an upward or downward trend.

PROBABILITY

A **trial** occurs each time we perform an experiment.

The possible results from each trial of an experiment are called its **outcomes**.

The **sample space** U is the set of all possible outcomes of an experiment.

Experimental probability

In many situations, we can only measure the probability of an event by experimentation.

experimental probability = relative frequency of event

Theoretical probability

If all outcomes are equally likely, the probability of event A is $P(A) = \frac{n(A)}{n(U)}$.

For any event A , $0 \leq P(A) \leq 1$.

For any event A , A' is the event that A does not occur. A and A' are **complementary events**, and $P(A) + P(A') = 1$.

The event that both A **and** B occur is written $A \cap B$.

The event that A **or** B **or both** occur is written $A \cup B$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

For **disjoint** or **mutually exclusive** events, $P(A \cap B) = 0$.

Making predictions using probability

If there are n trials of an experiment, and an event has probability p of occurring in each of the trials, then the number of times we *expect* the event to occur is np .

Independent events

Two events are **independent** if the occurrence of each of them does not affect the probability that the other occurs. An example of this is sampling **with replacement**.

For independent events A and B , $P(A \cap B) = P(A)P(B)$.

Dependent events

Two events are **dependent** if the occurrence of one of them *does* affect the probability that the other occurs. An example of this is sampling **without replacement**.

For dependent events A and B , $P(A \cap B) = P(A) \times P(B \text{ given that } A \text{ has occurred})$.

Conditional probability

For any two events A and B , “ $A \mid B$ ” represents the event “ A given that B has occurred”, and $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$.

DISCRETE RANDOM VARIABLES

A **random variable** represents the possible numerical outcomes of an experiment.

A **discrete random variable** can take any of a set of distinct values.

If X is a discrete random variable with possible values $\{x_1, x_2, \dots, x_n\}$ and corresponding probabilities $\{p_1, p_2, \dots, p_n\}$, then:

- $0 \leq p_i \leq 1$ for all $i = 1, \dots, n$
- $\sum_{i=1}^n p_i = p_1 + p_2 + \dots + p_n = 1$
- $\{p_1, p_2, \dots, p_n\}$ describes the **probability distribution** of X .

We can also describe the probability distribution of X using a **probability mass function** $P(x) = P(X = x)$.

The **expectation** of a discrete random variable X is $E(X) = \mu = \sum_{i=1}^n x_i p_i$.

A game where X is the gain to the player is said to be **fair** if $E(X) = 0$.

The **mode** is the data value x_i whose probability p_i is the highest.

THE BINOMIAL DISTRIBUTION

In a **binomial experiment** there are two possible results: success and failure.

Suppose there are n independent trials of the same experiment with the probability of success being a constant p for each trial. If X represents the number of successes in the n trials, then X has a **binomial distribution**, and we write $X \sim B(n, p)$.

The **binomial probability mass function** is $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$ where $x = 0, 1, 2, \dots, n$.

You should be able to use your calculator to find:

- $P(X = x)$ using the binomial probability distribution function
- $P(X \leq x)$ or $P(X \geq x)$ using the binomial cumulative distribution function.

If $X \sim B(n, p)$, then:

- $E(X) = \mu = np$
- $\text{Var}(X) = np(1 - p)$
- $\sigma = \sqrt{\text{Var}(X)} = \sqrt{np(1 - p)}$

THE NORMAL DISTRIBUTION

If the random variable X has a normal distribution with mean μ and variance σ^2 , we write $X \sim N(\mu, \sigma^2)$.

The probability density function is $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $x \in \mathbb{R}$.

$f(x)$ is a bell-shaped curve which is symmetric about $x = \mu$.

It has the property that:

- $\approx 68\%$ of all scores lie between $\mu - \sigma$ and $\mu + \sigma$
- $\approx 95\%$ of all scores lie between $\mu - 2\sigma$ and $\mu + 2\sigma$
- $\approx 99.7\%$ of all scores lie between $\mu - 3\sigma$ and $\mu + 3\sigma$.

You should be able to use your calculator to find normal probabilities for the situations:

- $P(X \leq a)$
- $P(X \geq a)$
- $P(a \leq X \leq b)$

You should also be able to use your calculator to find the scores corresponding to particular probabilities. These scores are known as **quantiles**.

HYPOTHESIS TESTING

Terminology

- A **statistical hypothesis** is a claim about a population parameter.
- The **null hypothesis** H_0 is a claim that the population parameter is *equal* to a particular value.
- The **alternative hypothesis** H_1 is a claim that the population parameter is *different* to the value specified by H_0 . For example, given the null hypothesis $H_0: \mu = \mu_0$, the alternative hypothesis could be:
 - ▶ $H_1: \mu > \mu_0$ (**one-tailed hypothesis**)
 - ▶ $H_1: \mu < \mu_0$ (**one-tailed hypothesis**)
 - ▶ $H_1: \mu \neq \mu_0$ (**two-tailed hypothesis**, as $\mu \neq \mu_0$ could mean $\mu > \mu_0$ or $\mu < \mu_0$).
- A **test statistic** is a random variable that summarises the information in a sample.
- The distribution of the test statistic under the assumptions of H_0 is called the **null distribution**.
- The **p-value** of a test statistic is the probability of a result that is as or more “extreme” being observed if H_0 is true.
- The **significance level** α of a statistical hypothesis test is the largest p-value that would result in rejecting H_0 . Any p-value less than or equal to α results in H_0 being rejected.

If a statistical hypothesis test has significance level α , the probability of a Type I error is α .

The significance level may be given as a decimal or a percentage.

General procedure

Step 1: Formulate **statistical hypotheses**.

Step 2: Choose a **significance level** for the test. This is a threshold for making a decision, like the confidence levels we saw previously.

Step 3: Use data from a sample to calculate a **test statistic**.

Step 4: Calculate a **p-value** for the test statistic. This is the probability of that test statistic occurring under the assumptions of one of the hypotheses.

Step 5: Make decisions about the hypotheses.

Step 6: Interpret the decision in the context of the problem.

The one-sample t -test

The **t -test** is used to test hypotheses about a population mean μ when:

- the population is normally distributed
- the population variance is **unknown**.

For a t -test of $H_0: \mu = \mu_0$ using a sample of size n with sample mean \bar{x} and sample standard deviation s :

- the **test statistic** is $t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$
- the **null distribution** is $T \sim t_{n-1}$
- the p -value calculation depends on H_1 :
 - ▶ If $H_1: \mu > \mu_0$, $p\text{-value} = P(T \geq t)$.
 - ▶ If $H_1: \mu < \mu_0$, $p\text{-value} = P(T \leq t)$.
 - ▶ If $H_1: \mu \neq \mu_0$, $p\text{-value} = 2 \times P(T \geq |t|)$.

The two-sample t -test

The **two-sample t -test** is used to compare the means of **two** samples from different populations.

If the populations have means μ_1 and μ_2 , the null hypothesis has the form:

$$\begin{aligned} H_0: \quad \mu_1 &= \mu_2 \quad \text{or equivalently} \\ H_0: \quad \mu_1 - \mu_2 &= 0 \end{aligned}$$

You should be able to use technology to calculate the test statistic and p -value.

In this course you are expected to assume **equal variances** and hence use the **pooled two-sample t -test** on your calculator.

The χ^2 goodness of fit test

The χ^2 goodness of fit test is used to determine whether a probability distribution fits a set of data.

Consider a scenario with k categories. Let p_i be the population proportion of individuals in category i , where $p_1 + p_2 + \dots + p_k = 1$.

The **hypotheses** in a χ^2 goodness of fit test have the form:

$$H_0: p_1 = p_{01}, p_2 = p_{02}, \dots, \text{ and } p_k = p_{0k}$$

$$H_1: \text{at least one of } p_i \neq p_{0i}$$

where p_{0i} is the population proportion of category i under the null hypothesis.

The **test statistic** for a χ^2 goodness of fit test is: $\chi^2_{\text{calc}} = \sum \frac{(f_{\text{obs}} - f_{\text{exp}})^2}{f_{\text{exp}}}$

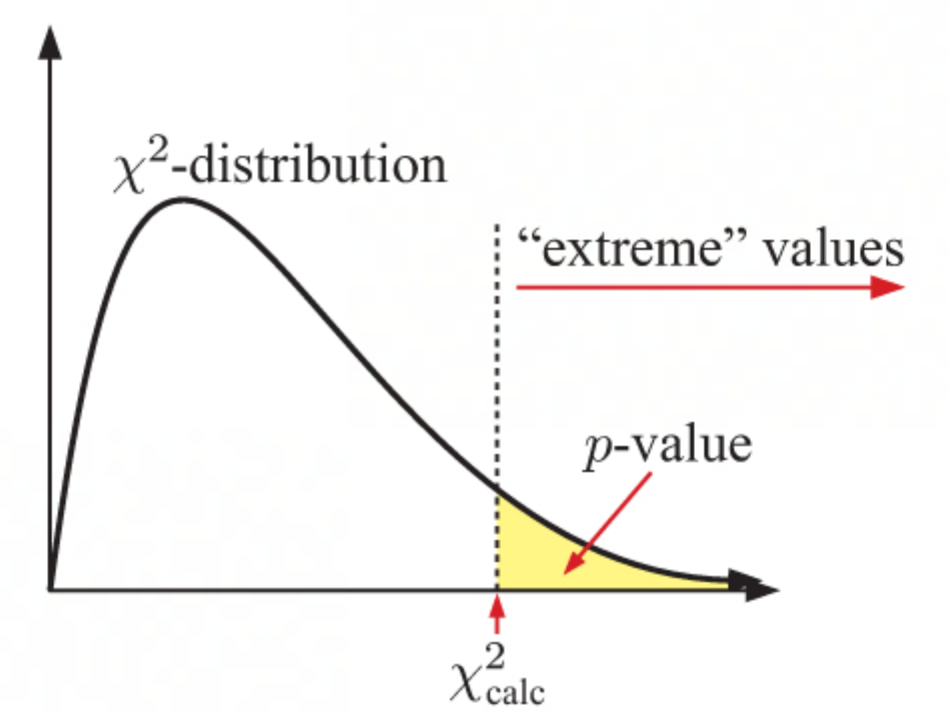
where f_{obs} is an **observed** frequency

f_{exp} is an **expected** frequency.

Degrees of freedom (df) refers to the number of values that are “free to vary”.

For a χ^2 goodness of fit test, **df = number of categories – 1**.

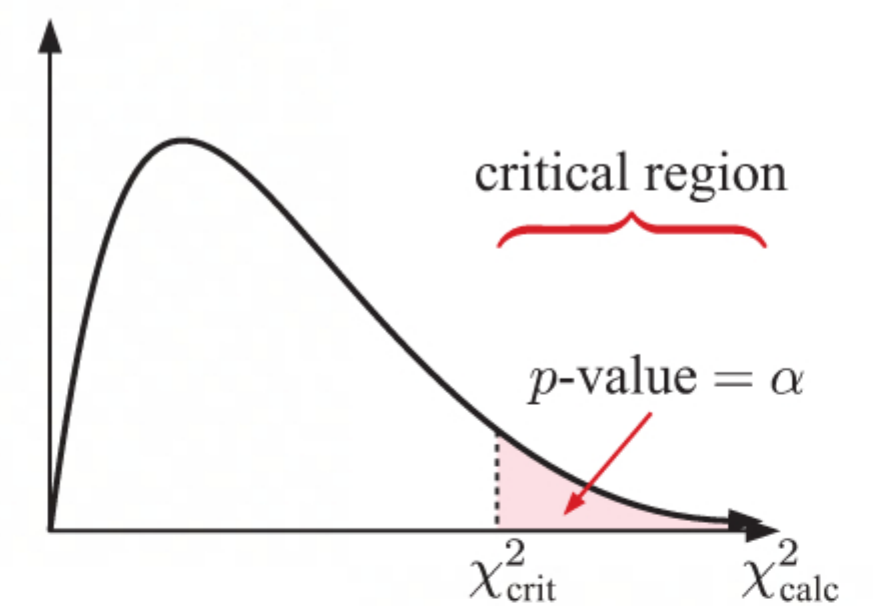
p-value = probability of observing a value greater than or equal to χ^2_{calc} .



For a χ^2 goodness of fit test, we denote the critical value as χ^2_{crit} .

Since we use the upper tail of the null distribution in calculating the p -value, the critical region is the set of values $\geq \chi^2_{\text{crit}}$.

The inequality $\chi^2_{\text{calc}} \geq \chi^2_{\text{crit}}$ is called the **rejection inequality**.



The χ^2 test for independence

The **χ^2 test for independence** is used to determine if two variables in a **contingency table** are independent or not. It is a special case of the χ^2 goodness of fit test.

The hypotheses for the χ^2 test for independence are H_0 : the variables are independent

H_1 : the variables are dependent

The test statistic for the χ^2 test for independence is calculated in a similar way to the χ^2 goodness of fit test. The expected frequency of each cell in the contingency table is given by $f_{\text{exp}} = \frac{\text{row sum} \times \text{column sum}}{\text{total}}$.

The p -value and critical value χ^2_{crit} for the χ^2 test for independence are calculated in the same way as the χ^2 goodness of fit test.

For a contingency table which has r rows and c columns, **df = $(r - 1)(c - 1)$** .

SKILL BUILDER QUESTIONS

- 1** Gerard wants to estimate the average height of the 500 students at his school. He randomly selects a sample of 10 students, and uses a tape measure to find the height of each student.

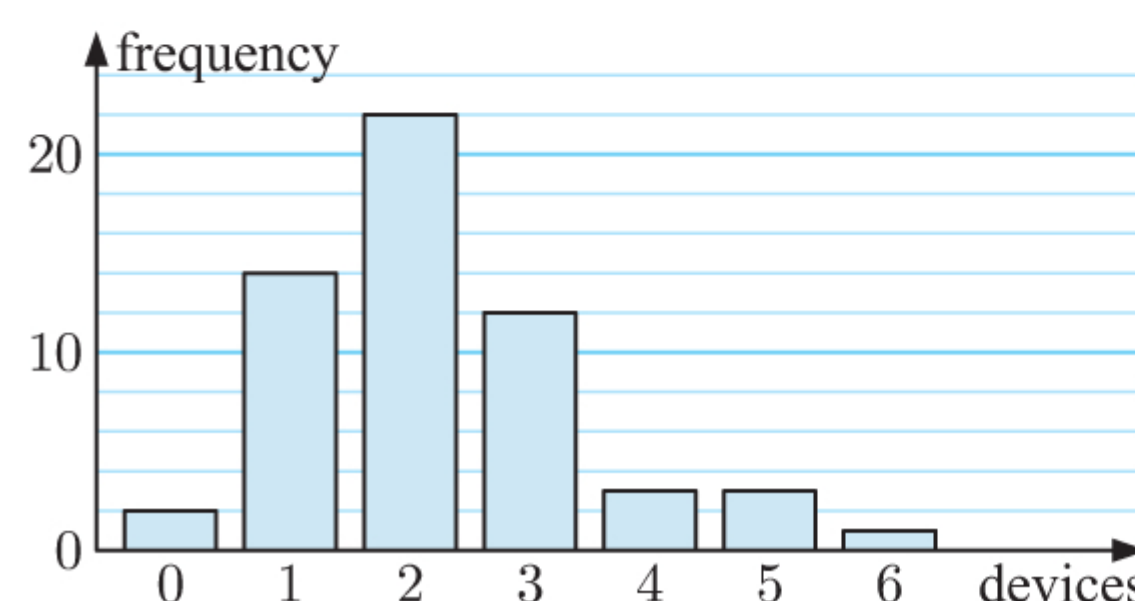
Explain why this approach may produce a: **a** coverage error **b** measurement error.

- 2** The students at Hoylebury Middle School are to be surveyed on their attitudes on wearing school uniform. The numbers of students in each year level are shown.

	Boys	Girls
Year 8	135	140
Year 9	130	145
Year 10	125	130

- a**
- i** What are the advantages of surveying 50 students?
 - ii** What are the disadvantages of surveying all students?
- b** A stratified sample system is used to select 50 students.
- i** How many Year 8 boys will be selected?
 - ii** How many girls will be selected in total?
- c** Explain why a stratified sample is better than a random sample in this case.
- 3** Marie is organising a staff lunch in a large office building. She asks the first 10 people to visit her office for their preferences, and then makes a decision.
- a** Explain why this is a convenience sample.
 - b** In what ways will Marie's sample be biased?
 - c** Suggest a more appropriate sampling method that Marie should use.
- 4** A ticket inspector checks the tickets of every 20th passenger leaving a train terminal, starting from the 8th passenger.
- a** Identify the sampling method used. Explain your answer.
 - b** List the next six passengers to be checked.
 - c** Given that 5000 passengers left the terminal that day, find the number of passengers checked.
- 5** Classify each variable as categorical, discrete, or continuous:
- a** The number of houses on a particular street.
 - b** The number of hours spent travelling on an airplane.
 - c** The brand of laptop someone uses.

- 6** A random sample of people were asked "How many devices have you used to browse the internet in the last month?". The results are displayed in the column graph.



- a** How many people were surveyed?
 - b** Find the mode of the data.
 - c** What percentage of people browsed the internet using 1 or 2 devices?
 - d** Describe the distribution of the data.
- 7** Each student in a class writes down the total number of children in their families:
- 1 2 4 3 2 1 1 2 1 5 4 2
2 1 3 2 3 4 1 2 1 2 2 1
- a** Explain why the data is discrete.
 - b** Construct a frequency table to organise the data.
 - c** Draw a column graph to display the data.
 - d** Describe the distribution of the data.
 - e** In what percentage of families are there 3 or more children?
- 8** The following marks out of 100 were obtained by students in a Chemistry examination:

79 29 40 33 75 74 64 40 53 95
60 86 58 51 66 53 87 43 91 58
57 68 32 65 77 44 72 24 55 77

- a** Construct a tally and frequency table for this data using class intervals 0 - 9, 10 - 19, ..., 90 - 99.
- b** Draw a column graph of the data.
- c** Write down the modal class.
- d** If the number of marks required to pass was 50, what percentage of students passed the examination?

- 9 The heights of a sample of emperor penguins were measured. The results are given in the table alongside.

Height (h cm)	Frequency
$105 \leq h < 110$	3
$110 \leq h < 115$	5
$115 \leq h < 120$	14
$120 \leq h < 125$	19
$125 \leq h < 130$	8
$130 \leq h < 135$	1

- Explain why *height* is a continuous variable.
- How many emperor penguins were measured?
- Construct a frequency histogram to display the data.
- Describe the distribution of the data.
- What is the modal class? Explain what this means.

- 10 The number of customers entering a convenience store each hour on a particular day were:

14, 23, 26, 34, 24, 18, 26, 16, 25

Without using technology, find the **a** mode **b** median **c** mean of the data.

- 11 An art gallery has added two new exhibits alongside their permanent collection. The number of tickets sold for each exhibit was counted every day for a month:

Exhibit A										Exhibit B									
42	49	55	48	62	81	91	50	60	59	59	51	60	44	57	90	98	50	62	55
47	73	84	89	55	59	35	42	51	83	44	62	75	99	57	57	49	53	71	70
75	28	30	19	39	45	69	65	27	32	68	32	33	24	47	43	61	42	52	46

- Find the:
 - mean number of visitors for each exhibit
 - median number of visitors for each exhibit.
 - Which exhibit was more popular? Explain your answer.
- 12 **a** The mean of 7 integers is 14. In ascending order, the integers are 9, 10, a , 13, b , 16, 21.
Find the values of a and b .
- b** In ascending order, a set of six numbers are: 1, 5, 9, 11, 16, p . The mean of the six numbers is the same as their median. Find p .
- 13 Miguel uses an application on his phone to find the amount of sleep he gets each night. The duration of his sleep, in hours, for the past 30 nights are:

7.5 6.8 7.8 6.3 8.6 9.1 7.1 5.8 7.7 7.3 7.7 7.4 11.5 7.1 7.4
8.0 7.6 7.1 9.1 8.0 7.5 7.4 7.5 8.1 8.6 8.7 6.8 7.4 7.7 8.5

- Calculate the mean and the median of the data.
 - Identify the outlier in this data set.
 - The outlier was the result of a recording error.
 - Calculate the mean and the median of the data with the outlier removed.
 - Which measure of centre is most affected if the outlier is removed?
- 14 Janice uses a pedometer to record the number of steps she took each day for 14 days.
- 10 613 5453 3526 6321 1468 3417 3504
4093 261 6560 5570 4253 2939 8888
- Calculate the mean and median steps taken.
 - Which measure is the most suitable to determine the typical number of steps taken by Janice each day? Justify your answer.

- 15** The frequency table alongside shows the number of touchdowns scored by teams in a professional gridiron league after one round.
- a** For this data set, find the:
- i** mean **ii** median **iii** mode.
- b** Construct a column graph for the data.
- c** Describe the distribution of the data.
- d** Which measure of centre is most appropriate for this data?

<i>Number of touchdowns</i>	<i>Frequency</i>
0	2
1	10
2	7
3	6
4	4
5	2
6	1

- 16** The frequency table alongside shows the number of cars owned by different families.
- a** Add a column to the table showing the *cumulative frequency* values.
- b** For this data set, calculate the:
- i** mean **ii** median **iii** mode.

<i>Number of cars</i>	<i>Frequency</i>
0	78
1	117
2	69
3	18
4	2
<i>Total</i>	284

- 17** After seven netball matches, Kai has averaged 11 goals per game.
- a** Find the value of a .
- b** How many goals will she need to score in the next game to improve her overall average to 12?

<i>Score</i>	7	9	a	13	16
<i>Frequency</i>	1	2	1	2	1

- 18** This table shows the weekly rent for a sample of studio apartments in Italy.
- a** Estimate the mean weekly rent.
- b** Find the probability that the weekly rent for a randomly chosen studio apartment will be €140 or greater.

<i>Weekly rent (€r)</i>	<i>Frequency</i>
$80 \leq r < 100$	3
$100 \leq r < 120$	15
$120 \leq r < 140$	26
$140 \leq r < 160$	30
$160 \leq r < 180$	14
$180 \leq r < 200$	1

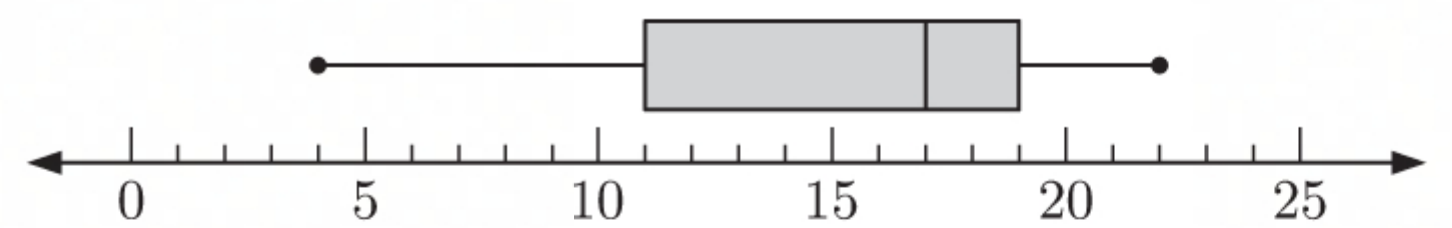
- 19** For each of the following data sets, find:
- i** the median **ii** Q_1 and Q_3 **iii** the range **iv** the interquartile range.
- a** 7, 8, 8, 10, 11, 13, 14, 14, 15, 18, 21 **b** 35, 32, 19, 26, 18, 22, 41, 43
- c** 38, 25, 14, 20, 39, 16, 52, 46, 37, 29, 18, 27

- 20** Cailan and Miles regularly play golf together, and have recorded their scores from their last 10 rounds:

Cailan: 84, 81, 86, 92, 85, 83, 80, 87, 90, 79
Miles: 87, 85, 83, 90, 88, 82, 84, 84, 91, 82

- a** Calculate the range and interquartile range for each data set.
- b** Which golfer had the lower:
- i** range **ii** interquartile range?
- c** Which measure of spread is more appropriate for determining who is generally the more consistent golfer? Explain your answer.
- 21** Consider this data set: 16, 20, 10, 16, 4, 12, 23, 18, 17, 9, 18, 16, 31, 26, 18, 14, 12, 14, 15
- a** Write the data set in order, and construct a five-number summary.
- b** Calculate the interquartile range.
- c** Calculate the upper and lower boundaries, and hence identify any outliers in the data set.
- d** Draw a box plot to represent the data.

- 22** A box plot has been drawn to show the heights of some petunia seedlings, in centimetres.



State the:

- a** minimum value **b** maximum value **c** median **d** upper quartile
e lower quartile **f** range **g** interquartile range.
- 23** After a 10 minute run, a class of 24 students measured their pulse rates. The results were:

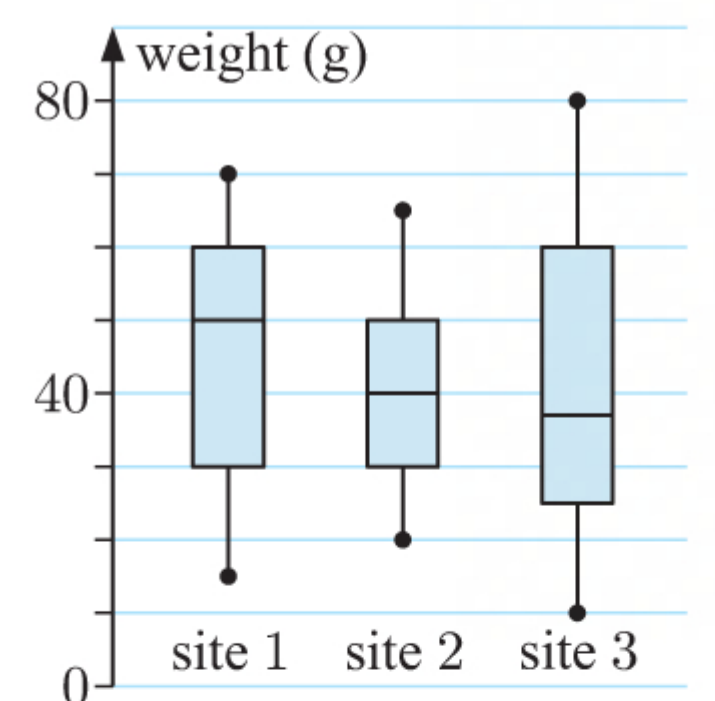
85 106 148 112 105 96 100 108 135 126 144 156
 98 108 112 128 148 140 120 123 133 144 118 125

- a** Construct a five-number summary for the data.
b Draw a box plot to represent the data.
c Find the range and interquartile range.
- 24** Employees in a law firm often work overtime. In a given week, the number of hours of overtime worked by the employees were:

0, 0, 1, 2, 4, 4, 4, 5, 5, 5, 5, 6, 6, 7, 9, 9, 11, 11, 12, 13, 15, 18, 22, 26, 34

- a** Calculate the value of:
i Q_1 **ii** the median **iii** Q_3
b Draw a box plot to illustrate the data.
c Based on your box plot, comment on whether the data appears symmetric or not. Explain your answer.
d For the given data, write down the:
i range **ii** interquartile range.
e Complete the following statements about the data:
i “The central 50% of employees work between and hours of overtime.”
ii “75% of employees work a maximum of approximately hours overtime.”

- 25** These parallel box plots show the weights of particular species of fungi collected from 3 different sites in a forest.



- a** Write down the five-number summary for site 1.
b Which site has the greatest range of weights?
c At which site do the weights of fungi have the least variation?
d Which site has the highest median weight of fungi?
e Which site has the highest proportion of weights above 40 grams?
- 26** A soft drink distributor is testing a new recipe for one of their best selling drinks. A randomly selected group of people are asked to taste the old and new recipes, and to give each a score out of 10. The results are given below:

Old recipe: 7 8 7.5 9 7 6 7 8 9 7 8 8
 New recipe: 6 8 7 9 7.5 4 6.5 7 8 5 7.5 8.5

- a** Find the five-number summary for each data set.
b Draw a parallel box plot for the data.
c Do you think the distributor should adopt this new recipe for their drink? Explain your answer.

- 27** The heights of a random sample of trees in an apple orchard are summarised in the table alongside.

Height (h m)	Frequency
$7 \leq h < 8$	8
$8 \leq h < 9$	59
$9 \leq h < 10$	74
$10 \leq h < 11$	22
$11 \leq h < 12$	1

- a** Construct a cumulative frequency graph for the data.
b Estimate the median height.
c Estimate the interquartile range.
d Estimate the 90th percentile. Interpret your answer.

28 The following data shows the time, in minutes, taken for 12 contestants to complete a task on a TV game show.

20, 32, 15, 40, 26, 25, 19, 28, 21, 25, 16, 22

- a Calculate the mean and standard deviation.
- b A contestant whose time is more than one standard deviation above the mean is immediately eliminated. List the times of the people who are immediately eliminated.
- c Recalculate the mean and standard deviation for the remaining times.
- d The remaining contestants whose times lie within one standard deviation of the new mean will participate in the next game. Find the number of contestants who will participate in the next game.

29 Anthony and Katherine are two musicians in an orchestra. They each recorded the number of hours they spent practising in the 10 days before a performance.

Anthony: $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 4, $4\frac{1}{2}$, 3, $3\frac{1}{2}$, 5, 6, 6

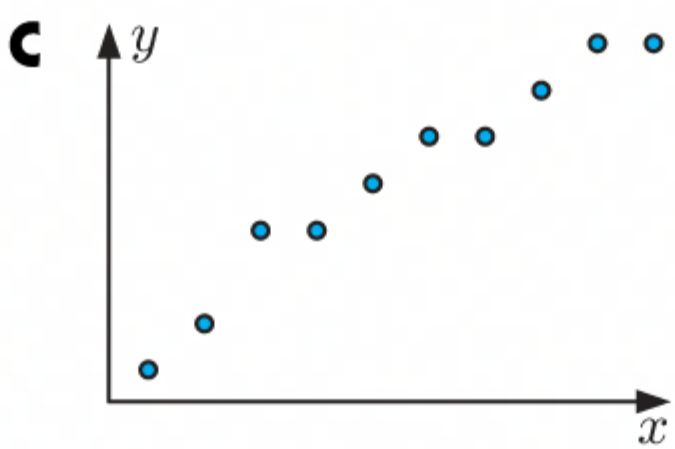
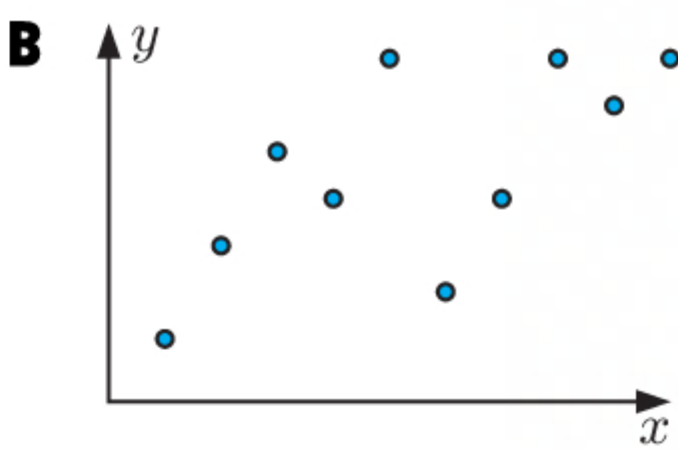
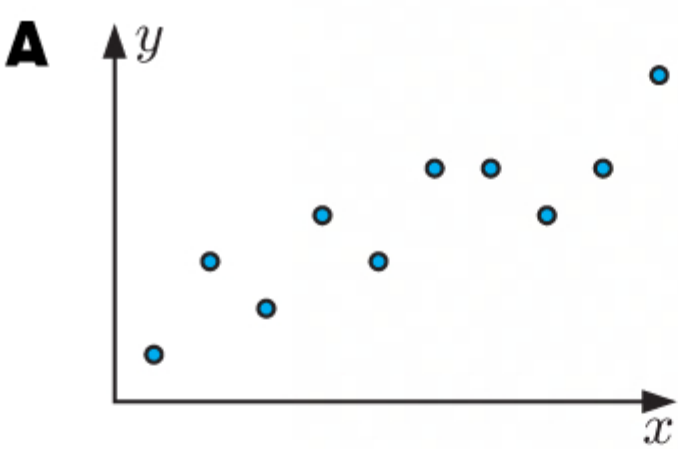
Katherine: 3, $3\frac{1}{2}$, 4, 3, 3, $3\frac{1}{2}$, 4, 4, $4\frac{1}{2}$, 4

- a Calculate the mean and standard deviation of each data set.
- b Which person generally practised for longer?
- c Which person practised more consistently?

30 This table shows the distribution of marks obtained on a logic test.
Use technology to find the mean and population standard deviation of the test scores.

Mark	3	4	5	6	7	8	9	10
Frequency	1	3	5	8	4	2	0	1

31 Consider the scatter diagrams below.



- a For each scatter diagram, determine whether the association between x and y is positive, negative, or zero.
- b Complete the table by matching each description with scatter diagram **A**, **B**, or **C**.

Strength of correlation	Scatter diagram
Weak	
Moderate	
Strong	

32 A journalist compares the scores given to two camera models by 6 online reviewers.

Camera A	8.5	8	9	7	8.5	7.5
Camera B	7	6	7.5	9	7.5	6

- a Draw a scatter diagram of the data.
- b Identify the outlier in the data.
- c It was found that the outlier was a recording error, and was removed.
 - i Describe the correlation between camera A's scores and camera B's scores.
 - ii Does an increase in camera A's scores cause an increase in camera B's scores? Explain your answer.

33 Ten students were given aptitude tests on language skills and mathematics. The table below shows the results:

Language (x)	12.5	15.0	10.5	12.0	9.5	10.5	15.5	10.0	14.0	12.0
Mathematics (y)	32	45	27	38	18	25	35	22	40	40

- a Plot the data on a scatter diagram.
- b Find the correlation coefficient r .
- c Use your results to comment on the statement: "Those who do well in languages also do well in mathematics."

34 Consider the following data on farm production.

Monthly rainfall (mm)	5	10	15	20	25	30
Crop yield (tonnes)	14	21	29	31	30	28

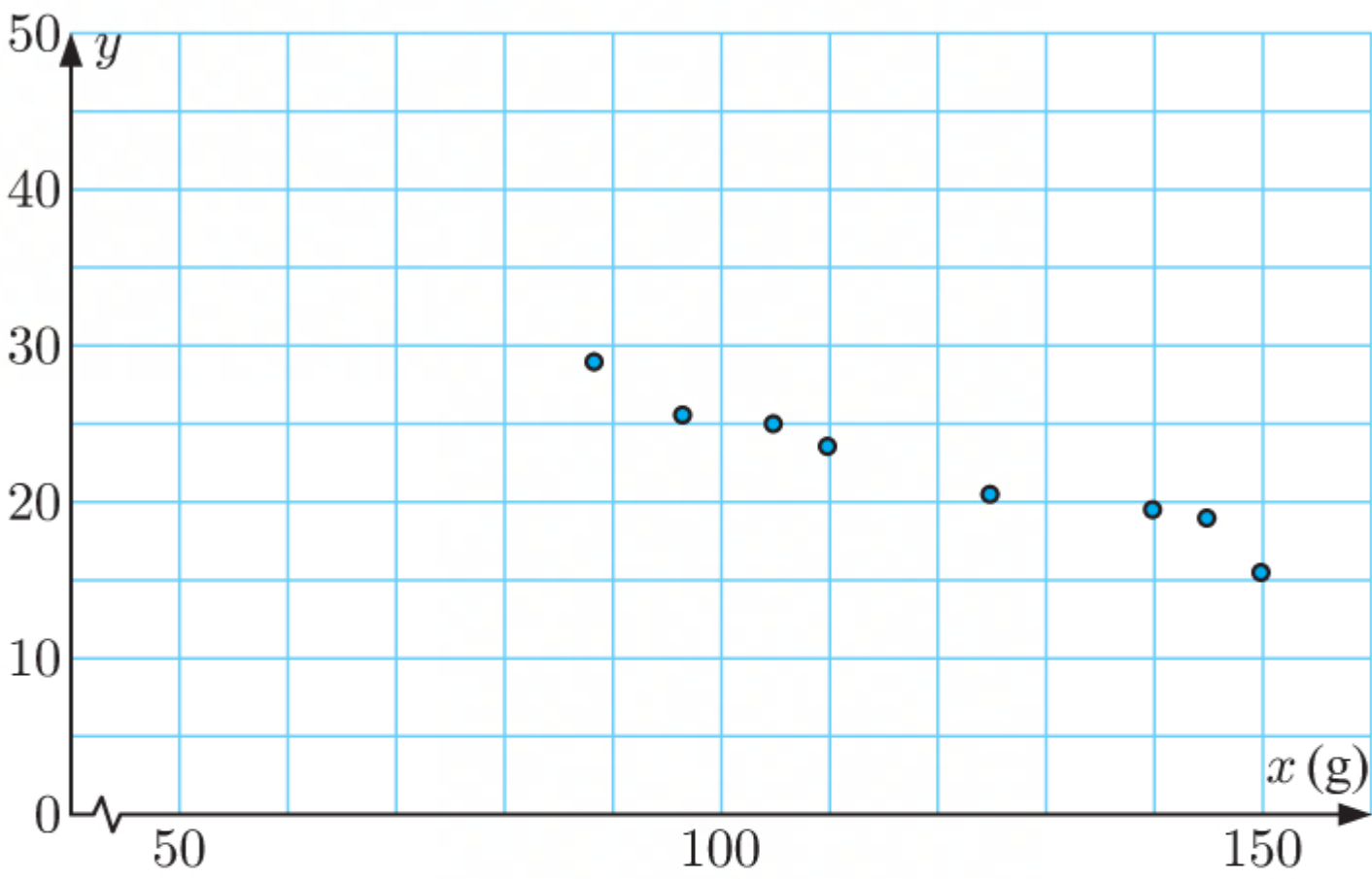
- a Calculate Pearson’s correlation coefficient r .
 - b What does the value of r suggest about the nature and strength of the relationship between monthly rainfall and crop yield?
 - c Draw the scatter diagram for the data.
 - d Explain why Pearson’s correlation coefficient may not be an appropriate measure for the data.
- 35 This table shows the exchange rate for Argentine pesos against the US dollar, and interest rates in Argentina at the corresponding times.

Exchange rate (pesos/USD)	2.85	2.95	2.90	2.75	2.65	2.80	3.05	2.98	2.95
Interest rate (%)	7.40	7.50	7.55	7.25	7.25	7.35	7.65	7.75	7.60

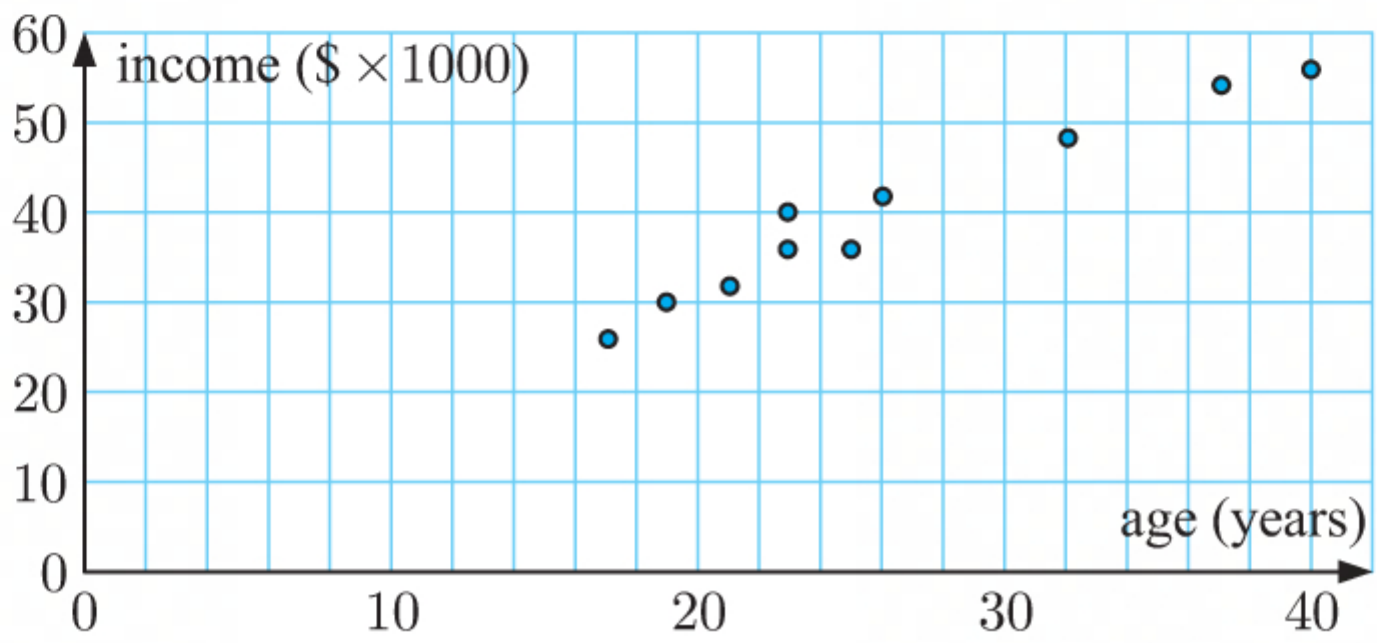
- a Draw a scatter diagram for the given data.
 - b Find the correlation coefficient between the two variables.
 - c Describe the nature and strength of the relationship between the variables.
- 36 In a sample of 2.5 kg bags of potatoes, the number of potatoes and their median weight is given below.

Median weight (x g)	88	97	105	110	125	140	145	150
Number in bag (y)	28	26	26	23	21	19	18	16

$\bar{x} = 120$ and $\bar{y} = 22.125$.



- a Copy the scatter diagram, and draw a line of best fit by eye.
 - b Hence estimate the number of potatoes in a bag if the median weight is:
 - i 100 grams
 - ii 70 grams.
 - c Which of the estimates in b is likely to be more reliable? Give a reason for your answer.
- 37 This scatter diagram shows the age and annual income of 10 randomly chosen individuals. The mean age is 27 and the mean income is \$40 000.
- a Describe the relationship between the age and annual income for these individuals.
 - b Do you think there is a causal relationship between the variables? Explain your answer.
 - c Draw a line of best fit by eye on the graph.
 - d Estimate the annual income for someone who is 30 years old. Comment on the reliability of your estimate.



- 38** A jeweller measured the volume and mass of some samples of silver. He suspects that one of the samples might be fake. The results are listed in the table.

Sample	A	B	C	D	E	F	G	H	I	J	K	L
Volume ($x \text{ cm}^3$)	3	6	4	7	16	8	5	12	9	6	10	11
Mass ($y \text{ g}$)	40	95	50	160	285	130	65	210	155	90	170	190

- Draw a scatter diagram for this data.
 - Calculate Pearson's product-moment correlation coefficient r .
 - Describe the relationship which appears to exist between the volume and mass of the samples of silver.
 - Do you agree with the jeweller that there is a fake sample?
 - Remove the suspect value from the data and find the equation of the regression line for the remaining data.
 - Use your equation to find the expected mass of the sample of silver with the same volume as the suspect sample.
- 39** 9 students sat a Mathematics examination. The number of hours that each of them studied and the results they obtained are shown in the table.

Study time ($x \text{ h}$)	7	6	3	16	15	11	18	32	20
Result ($y \%$)	56	42	25	80	65	60	85	96	90

- Write down the equation of the least squares regression line.
 - Describe the correlation between the variables.
 - Do you think there is a causal relationship between the variables? Explain your answer.
 - Tony's score in the examination was 70%. Use the line of best fit to estimate how long he studied for.
 - Interpret the y -intercept and the gradient of the equation of the line of best fit.
- 40** The average height h (in mm) of grass t days after being mowed, is shown in the table below.

Time ($t \text{ days}$)	0	1	2	3	4	5	6	7	8	9
Height ($h \text{ mm}$)	5	5.7	5.7	6.2	6.8	7.1	8	8.3	9	9.3

- Calculate Pearson's product-moment correlation coefficient r .
 - Explain the significance of the size and sign of r .
 - The regression line for h against t is $h \approx 0.4879t + 4.9145$. Use this equation to estimate the:
 - height of the grass after 14 days
 - time required for the grass height to reach 20 mm.
- 41** The following table lists the ages of contestants in a game, and the times they took to complete a task.

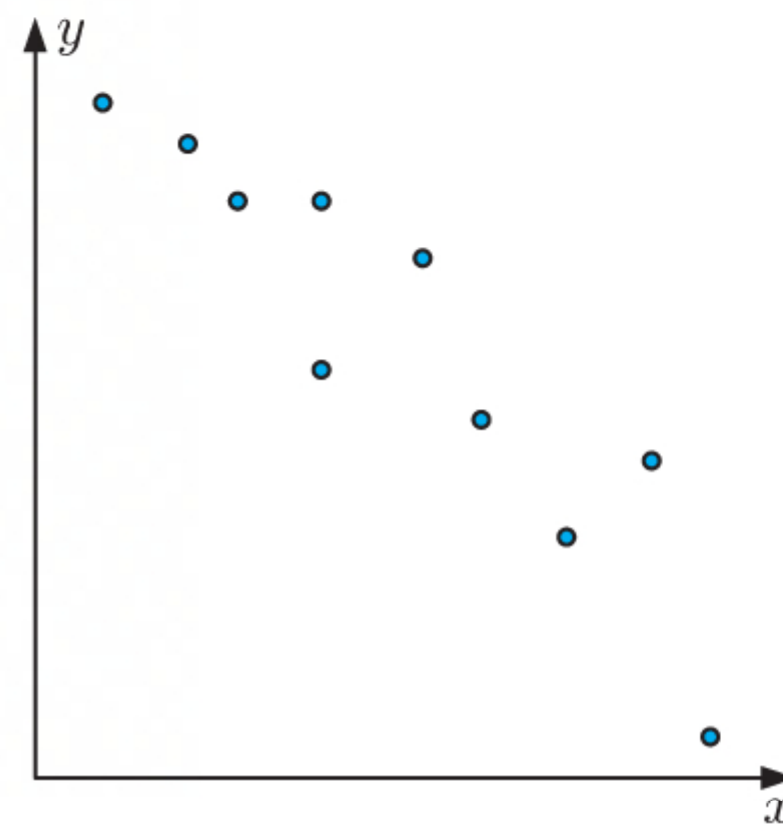
Age ($x \text{ years}$)	28	40	21	38	30	26	18	32	25	29	20	24
Time ($y \text{ min}$)	20	32	15	40	26	25	19	28	21	25	16	22

- Find the value of the correlation coefficient r , and explain what this value means.
 - Write down the equation of the linear regression line in the form $y = mx + c$.
 - Interpret the gradient of the regression line.
- 42** The lengths and weights of a zoo's pygmy shrews were recorded during the annual health check.

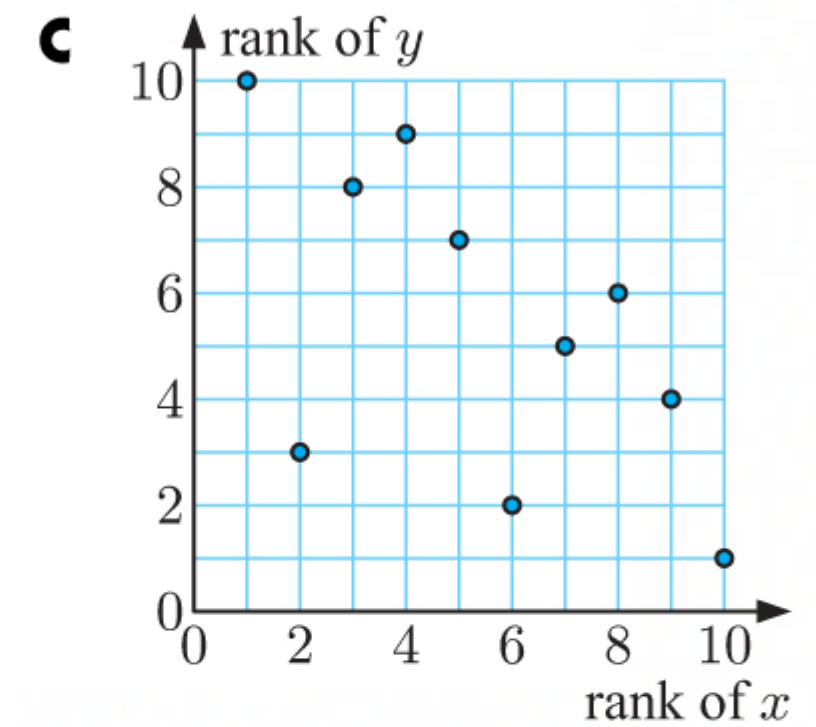
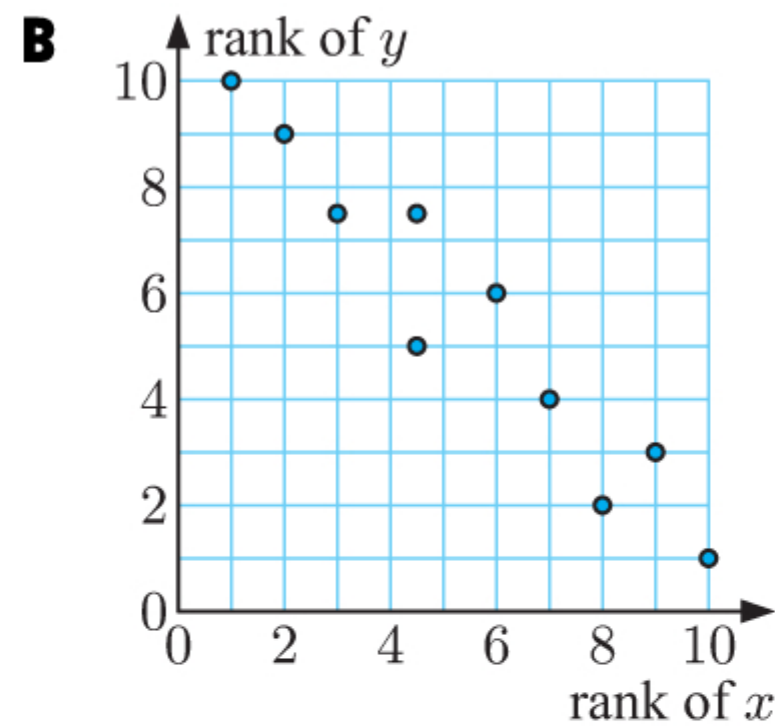
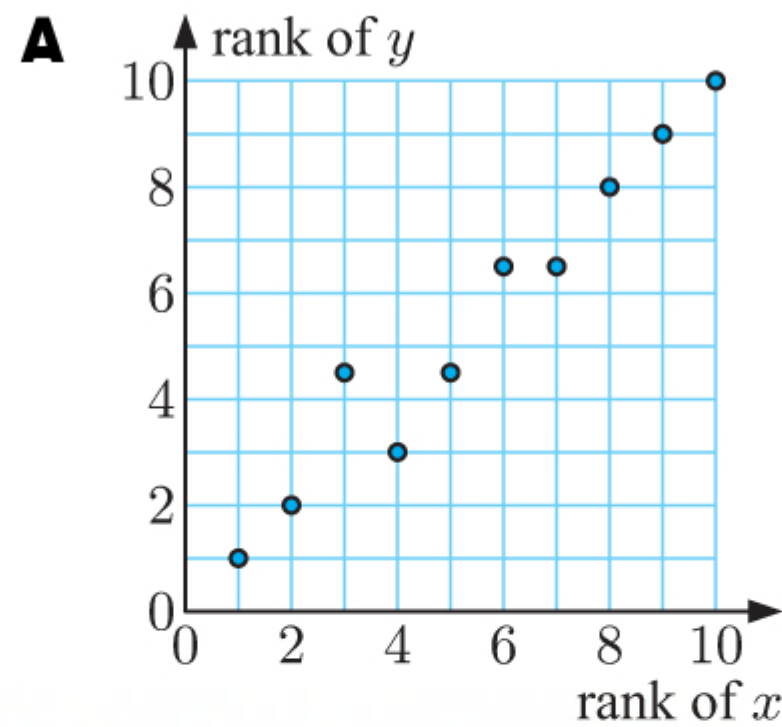
Length (mm)	95	83	91	82	75	62	79	63	81	69	94	88	72	77
Weight (g)	5.4	4.5	5.0	4.1	3.7	2.6	4.5	3.1	4.7	3.7	5.1	4.8	3.6	4.2

- Draw a scatter diagram for this data.
- Calculate Pearson's product-moment correlation coefficient r for the data.
- Hence describe the correlation between these two variables.
- Find the equation of the least squares regression line.
- Hence predict the weight of a pygmy shrew with length:
 - 110 mm
 - 70 mm
- Which of your predictions in **e** is more likely to be reliable? Explain your answer.

43 Consider the scatter diagram alongside.



a Which of the following scatter diagrams is the scatter diagram of the ranks?



b Hence identify the correct value of Spearman's rank correlation coefficient for this data:

A $r_s \approx -0.612$

B $r_s \approx 0.976$

C $r_s \approx -0.960$

44 This table shows the *number of matches played* and the *highest score* of a cricketer in her last 12 seasons.

<i>Number of matches played (x)</i>	11	5	10	16	2	1	8	20	15	2	4	4
<i>Highest score (y)</i>	92	65	71	82	21	7	55	85	79	18	60	51

a Draw a scatter diagram for the data.

b Calculate Pearson's coefficient r_p .

c Find the ranks for each of the variables.

d Calculate Spearman's rank correlation coefficient r_s .

e Describe the correlation between the variables.

45 Arthur records his time spent travelling to work over several months. Find, to 3 decimal places, the experimental probability that his next trip to work will last:

a 40 to 44 minutes

b at least 50 minutes

c between 35 and 49 minutes (inclusive).

<i>Time (min)</i>	<i>Frequency</i>
35 - 39	10
40 - 44	46
45 - 49	43
50+	15

46 An annual squash tournament groups players into 5 divisions according to their skill level.

The table shows the number of players at the tournament over 3 years.

Find the probability that a player:

a in the 2017 tournament played in division 1

b in any of the past tournaments played in division 3

c in the 2019 tournament did *not* play in division 2 or 4.

<i>Division</i>	2017	2018	2019
1	4	5	5
2	6	7	8
3	13	12	14
4	18	10	14
5	20	17	16
<i>Total</i>	61	51	57

47 A hospital recorded the age and gender of its 1020 melanoma patients over one year. The data is shown alongside.

a Complete the table.

b Find the probability that a randomly selected melanoma patient was:

i male

ii female and younger than 40

iii 60 or older, given they were female

iv male, given they were 40 or older.

	< 40	40 - 59	≥ 60	<i>Total</i>
<i>Male</i>	56	127		
<i>Female</i>	75	113	230	
<i>Total</i>				1020

- 48** A card is randomly selected from a deck of playing cards. Determine the probability that the card is:
- a** black **b** a club **c** a red number between 3 and 6 (inclusive)
- d** not a number **e** the ace of spades.

- 49** A die is rolled, and a square spinner with sectors 1, 2, 3, and 4 is spun.

- a** Draw a grid to illustrate the sample space of possible outcomes.
- b** Use your grid to find the probability of getting:
- i** two 1s **ii** two 5s **iii** a sum of 6
- iv** a 2 and a 3 **v** a 2 or a 3 (or both) **vi** exactly one 4.

- 50** Suppose $P(A) = 0.37$, $P(B) = 0.41$, and $P(A \cup B) = 0.78$.

- a** Find $P(A \cap B)$. **b** What can you say about A and B ?

- 51** A and B are mutually exclusive events. If $P(B) = 0.3$ and $P(A \cup B) = 0.55$, find $P(A)$.

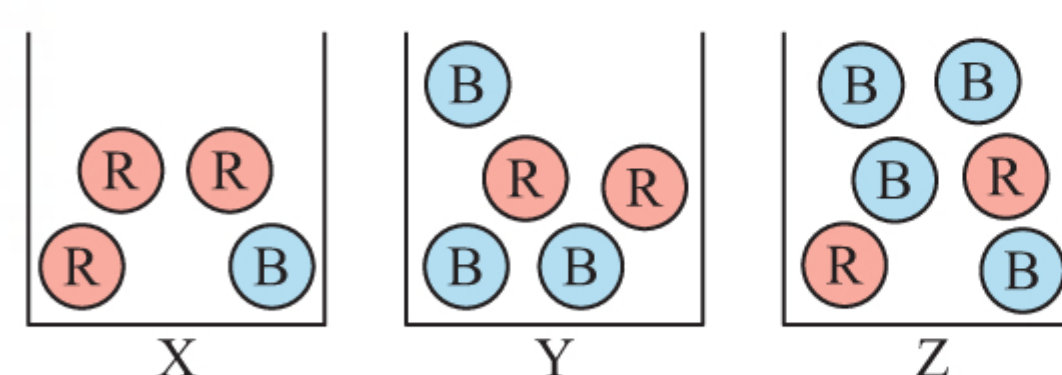
- 52** Given that $P(A) = \frac{23}{50}$, $P(B) = \frac{5}{7}$, and $P((A \cup B)') = \frac{1}{12}$, find $P(A \cap B)$.

- 53** One ball is drawn from each of the boxes shown.

- a** Draw a tree diagram to illustrate the situation.

- b** Find the probability that:

- i** exactly two red balls are drawn
- ii** blue balls are drawn from boxes X and Z
- iii** at most one blue ball is drawn.



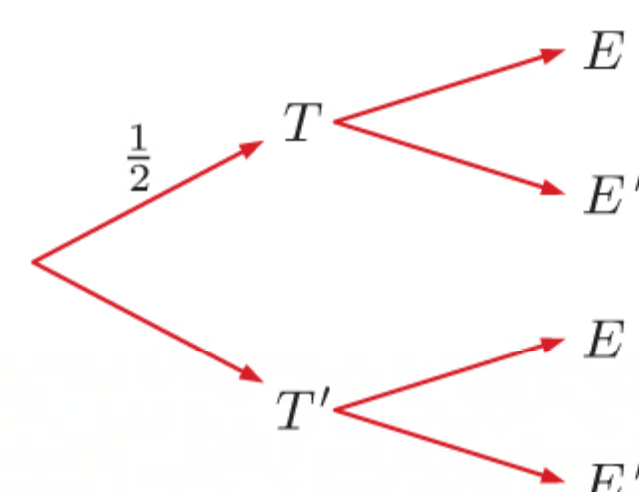
- c** Suppose an extra red ball is added to box Y . Which of the probabilities in **b** will be affected?

- 54** Suppose you toss a coin and roll a die simultaneously.

Let T represent a tail with the coin and E represent a 2 or a 5 with the die.

- a** Complete the tree diagram showing the probabilities of the different outcomes.

- b** Find: **i** $P(T \cap E')$ **ii** $P(T \cup E')$



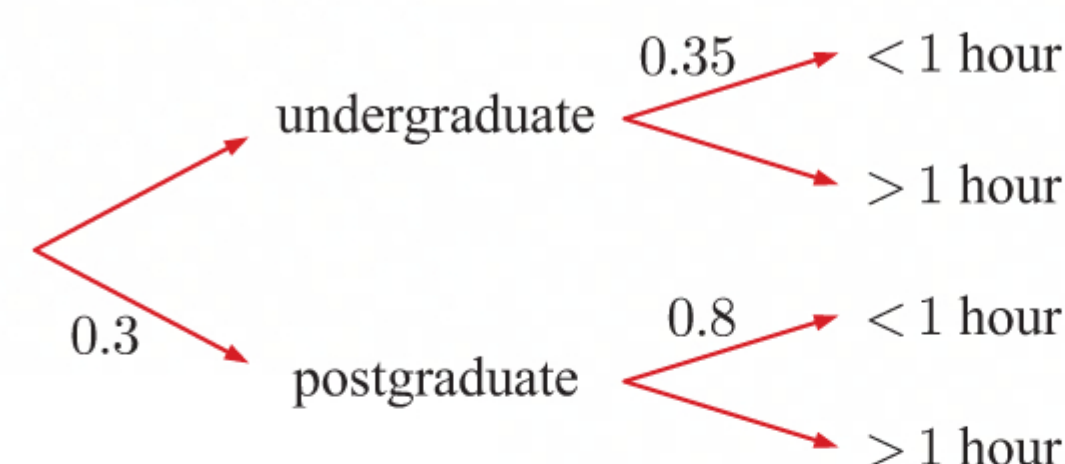
- 55** Twins Tom and Harry are keen archers. The probability that Tom successfully hits a target is 0.7. The probability that Harry successfully hits a target is 0.6. Suppose they both shoot at a target. Find the probability that:

- a** only one of them is successful **b** at least one of them is successful.

- 56** **a** Copy and complete this tree diagram about students at a university and how much time they spend at lunch.

- b** Find the probability that a student at the university:

- i** spends more than 1 hour at lunch
- ii** is not a postgraduate student who spends less than 1 hour at lunch.



- 57** A bag contains seven purple tickets and three red tickets. Michelle draws two tickets from the bag without replacement.

- a** Illustrate on a tree diagram the possible outcomes and the probabilities for each draw.

- b** Find the probability that Michelle will select:

- i** at least one red ticket **ii** one ticket of each colour **iii** a purple ticket second.

- 58** When Nick goes shopping, there is a 70% chance that Donna will join him. When Donna joins Nick, there is probability 0.3 that he purchases a packet of potato chips. When Nick shops alone, this probability rises to 95%.

Find the probability that:

- a** when Nick goes shopping, he purchases potato chips
- b** Donna joined Nick, given that Nick purchased potato chips.

- 59** Events A and B are independent. Given that $P(A \cup B) = 0.63$ and $P(B) = 0.36$, find $P(A)$.

- 60** A box of chocolates contains 6 dark brown, 4 light brown, and 2 white truffles. Two truffles are selected from the box without replacement.

Find the probability of selecting:

- a** 2 white truffles **b** different coloured truffles.

- 61** Find k in each of these probability distributions:

a

x	0	1	2	3
$P(X = x)$	k	0.2	0.5	0.1

b

x	2	4	6
$P(X = x)$	$2k$	0.1	$0.6 - k$

- 62** 40% of students in a class own an orange highlighter, 20% own a blue highlighter, and 50% do not own either coloured highlighter.

- a** Draw a Venn diagram to describe the situation.
b Find the probability that a randomly selected student:
i owns a blue highlighter, given they own an orange highlighter
ii owns an orange highlighter, given they do not own a blue highlighter.

- 63** In a class of 30 students, 17 have brown hair, 12 have blue eyes, and 4 have neither brown hair nor blue eyes.

- a** Display this information in a Venn diagram.
b A student is randomly selected. Find the probability that the student:
i has blue eyes but not brown hair **ii** has brown hair, given the student has blue eyes.

- 64** In tennis, the probability of Roger getting a “first serve” in is $\frac{7}{9}$. How many “first serves” would you expect Roger to get in out of 180 attempts?

- 65** **a** If 3 coins are tossed, find the probability that two fall heads and the other falls tails.
b Suppose 3 coins are tossed 400 times. On how many occasions would you expect to see exactly one tail?

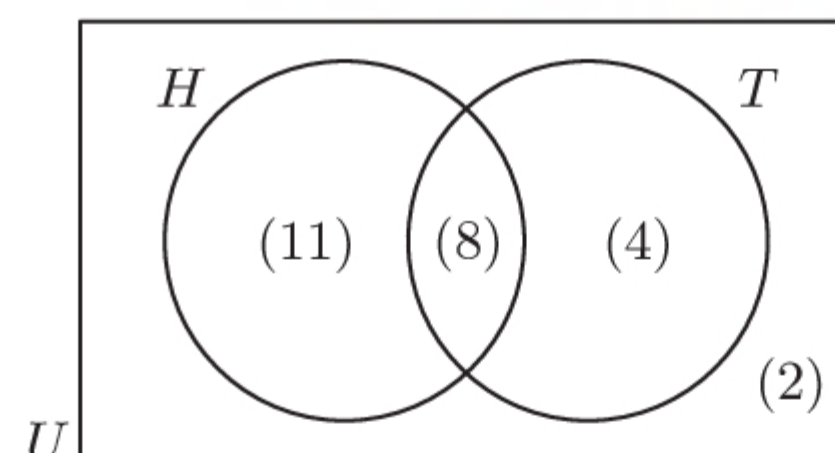
- 66** Two fair dice are rolled. Let X be the difference between the numbers rolled.

- a** Explain why X is a discrete random variable. **b** State the possible values of X .
c Find $P(X = 3)$.

- 67** This Venn diagram illustrates the number of students in a particular class who play hockey (H) and tennis (T).

A student from the class is picked at random. Find the probability that the student:

- a** plays hockey **b** does not play tennis
c plays at least one of the two sports
d plays tennis given that they play hockey.



- 68** A discrete random variable X has probability mass function $P(x) = \frac{a}{(x-3)^2}$ for $x = 0, 1, 2$.

- a** Find a . **b** Find $P(X = 2)$. **c** Find the mode and median of the distribution.

- 69** A bag contains 3 red tickets and 2 blue tickets. Tickets are selected from the bag, without replacement, until at least one ticket of each colour is selected. Let X be the total number of tickets selected.

- a** State the possible values of X . **b** Find the probability distribution of X .
c Find the mode of X . **d** Find the expected value of X .

- 70** Consider the probability mass function defined by $P(x) = P(X = x) = \frac{1}{24}(x+6)$ for $x = 1, 2, 3$.

- a** Find $P(x)$ for $x = 1, 2$, and 3. **b** Find the expected value of X .

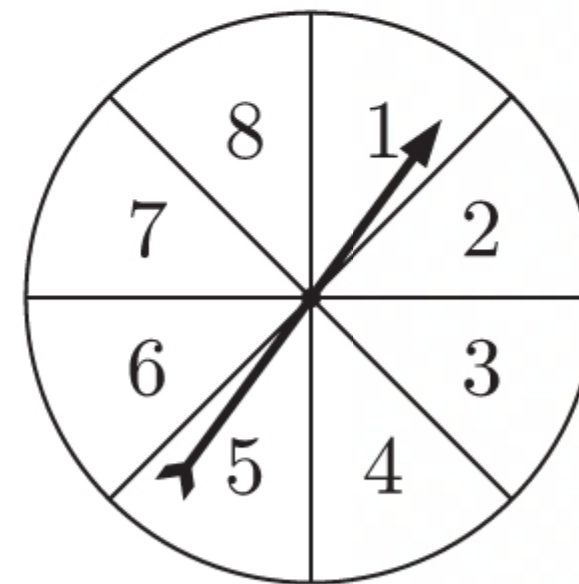
- 71** Each day, Russell drinks 0, 1, 2, 3, 4, or 5 cups of tea, with the probabilities shown.

Cups of tea	0	1	2	3	4	5
Probability	0.1	0.07	0.16	0.37	0.21	0.09

- a** Find the mode of the distribution.
b On average, how many cups of tea does Russell drink per day?

- 72** A bag contains 1 blue ticket, 3 red tickets, and 8 yellow tickets. A player randomly selects a ticket from the bag, and receives \$40 for a blue ticket, \$20 for a red ticket, and \$5 for a yellow ticket.
- Calculate the expected return for one trial of this game.
 - Given that the game costs \$15 to play, explain why it would not be advisable to play this game.
 - Find the number of extra red tickets that should be added to the bag to make the game fair.

- 73**
- Write down the first 4 rows of Pascal's triangle.
 - The spinner alongside is spun 4 times. Find the probability of getting:
 - exactly 3 numbers greater than 3
 - at least 2 numbers greater than 5
 - at most 1 number that is spelt with 3 letters.



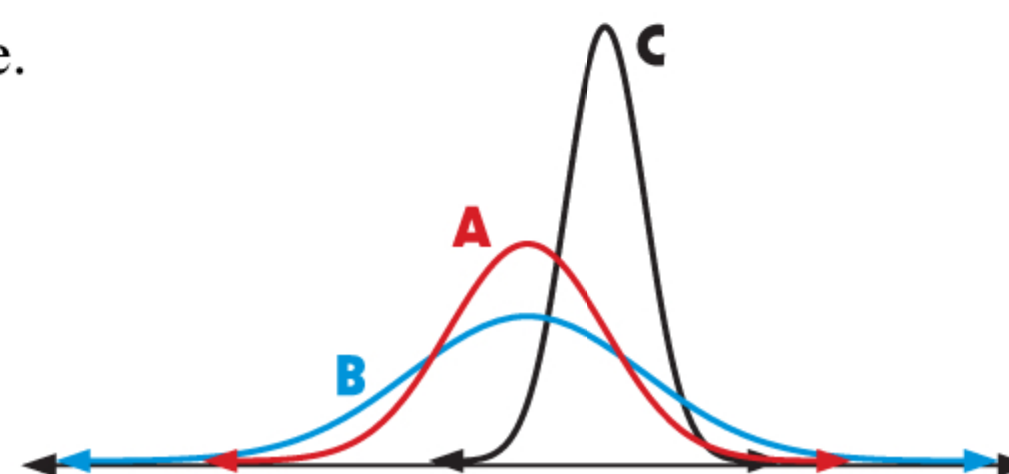
- 74** 80% of residents in a particular suburb oppose the construction of traffic lights at a particular intersection. A survey of 20 randomly selected residents is conducted.

Find the probability that:

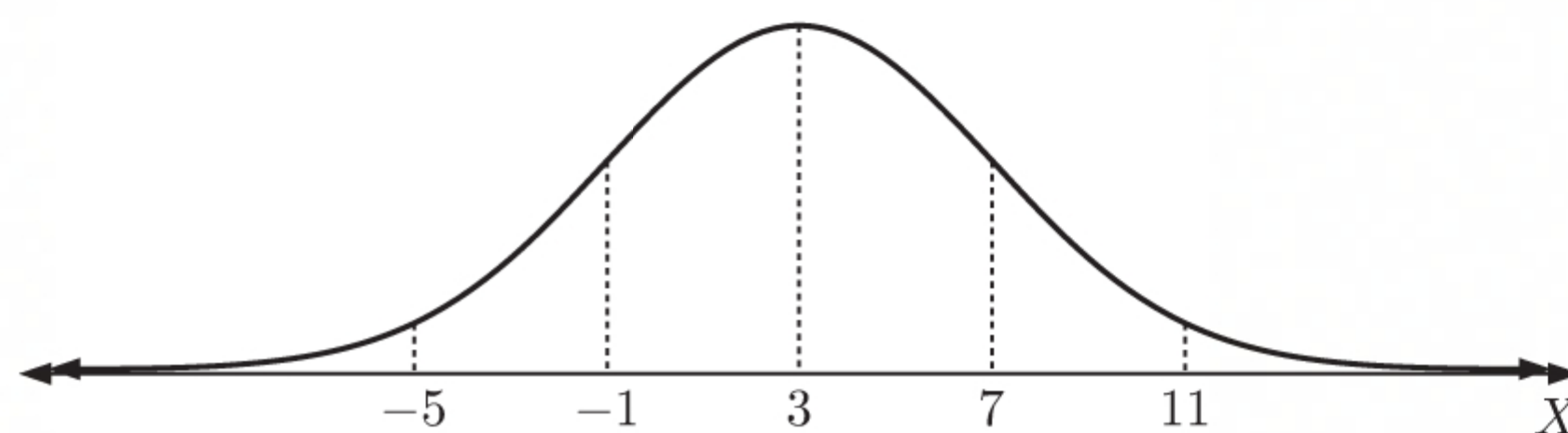
- exactly 16 residents oppose the construction
 - 16 or more residents oppose the construction
 - between 10 and 15 residents oppose the construction
 - more than 8 residents support the construction.
- 75** 5% of all items coming off a production line are defective. The manufacturer packages the items in boxes of six, and guarantees a refund if more than two items in a box are defective.
- On what percentage of boxes will the manufacturer have to pay a refund?
 - Patrick purchases 10 boxes. Find the probability that he will get a refund for exactly 1 box.
- 76** A company manufactures computer chips, and it is known that 3% of them are faulty. In a batch of 500 chips, find the probability that between 1 and 2 percent (inclusive) of them are faulty.
- 77** In a game, a player rolls a biased four-sided die. The probability of obtaining each possible score is shown in the table.
- | | | | | |
|-------------|----------------|-----|---------------|---------------|
| Score | 1 | 2 | 3 | 4 |
| Probability | $\frac{1}{12}$ | k | $\frac{1}{4}$ | $\frac{1}{3}$ |
- Find the value of k .
 - Let the random variable X denote the number of 2s that occur when the die is rolled 2400 times. Calculate the exact mean and standard deviation of X .
- 78** A multiple choice test consists of 30 questions with 5 answers to choose from. For each question, only one choice is correct. Let Y be the number of correct answers chosen if each answer is randomly guessed.
- Find the mean μ and standard deviation σ of Y .
 - Find $P(Y = 20)$.
 - Find $P(Y \geq \mu + 2\sigma)$.
- 79** Discuss whether the following variables are likely to be normally distributed. Sketch a graph to illustrate the possible distribution of each variable.
- the amount of sleep a person receives per night
 - the number of lollies in a sample of 500 g bags of lollies
 - the ages of people attending a high school musical.

- 80** Suppose $X \sim N(\mu, \sigma^2)$. Match each pair of parameters with the correct curve.

- $\mu = 4, \sigma = 1$
- $\mu = 2, \sigma = 2$
- $\mu = 2, \sigma = 3$



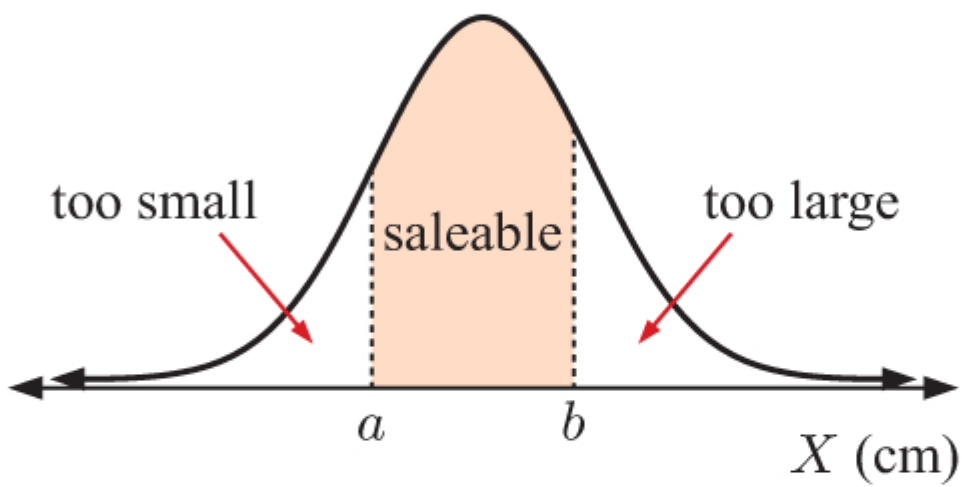
- 81** Consider the distribution curve of $X \sim N(3, 4^2)$ shown:



Copy the above graph, and on the same set of axes sketch the distribution curve for:

- a** $N(1, 4^2)$ **b** $N(3, 2^2)$ **c** $N(2, 64)$
- 82** Suppose a population is normally distributed with mean $\mu = 30$ and standard deviation $\sigma = 5$. Copy and complete:
- a** Approximately 68% of the population lies between and 35.
b Approximately 95% of the population lies between 20 and
c Approximately of the population lies between 15 and 45.
- 83** A normally distributed random variable X has a mean of 90. The probability $P(X < 85) \approx 0.1587$.
- a** Find $P(90 < X < 95)$. **b** Estimate the standard deviation of X .
- 84** Containers of a particular brand of ice cream have a capacity of 1050 mL. They are advertised as containing 1 litre of ice cream. The quantity of ice cream added to each container is normally distributed with mean 1020 mL and standard deviation 17 mL.
- a** Find the probability that the container has less than the advertised capacity.
b Find the percentage of containers that overflow.
c A sample of 75 containers are taken. Find the probability that at most three of the containers overflow.
- 85** The volume of drink dispensed by a coffee machine is normally distributed with mean 254 mL and standard deviation 2.3 mL.
- a** Find the probability that a randomly selected drink from the machine will have volume less than 254 mL.
b Find the percentage of drinks dispensed by the machine which have volume between 252 mL and 256 mL.
c A sample of 80 drinks is taken from the machine. Determine the number of drinks which will be expected to have volume at least two standard deviations above the mean.
d The machine operator guarantees that at least 95% of drinks will have volume at least 250 mL.
i Is the guarantee valid?
ii A technician adjusts the machine so the standard deviation is now 2.5 mL. What effect does this have on the operator's guarantee?
- 86** A machine fills bottles with tomato sauce. Each bottle is filled independently of all other bottles. The volume of sauce in each bottle is normally distributed with mean 500 mL and standard deviation 2.5 mL. Bottles are deemed to require extra sauce if the machine delivers less than 495 mL.
- a** Calculate the probability that a randomly selected bottle requires extra sauce.
b From a sample of 200 bottles, calculate the probability that at least 8 bottles require extra sauce.
- 87** The time taken for a skier to complete a particular downhill run is normally distributed with mean 45 seconds and standard deviation 4 seconds.
- a** Find the probability that the skier completes:
i one downhill run in under 40 seconds **ii** two consecutive downhill runs in under 40 seconds each.
b The skier completes a total of 60 independent runs. How many times would you expect the run to take between 44 seconds and 47 seconds?
- 88** The mean birth weight of babies in a population is normally distributed with mean 3.4 kg and standard deviation 300 grams.
- a** What proportion of babies in this population have birth weights:
i in excess of 4 kg **ii** between 3 kg and 4 kg?
b A *low birth weight* corresponds to any newborn weighing in the lowest 10% of birth weights. State the weight below which a baby is classified as having a *low birth weight*.

89 The length of a zucchini is normally distributed with mean 24.3 cm and standard deviation 6.83 cm. A supermarket buying zucchinis in bulk finds that 15% of them are too small and 20% of them are too large for sale. The remainder, with lengths between a cm and b cm, are able to be sold.



- a Find a and b .

b A zucchini is chosen at random. Find the probability that:

i it is of saleable length

ii its length lies between 20 cm and 26 cm

iii its length is less than 24.3 cm.

90 Consider the continuous random variable $X \sim N(44, 20)$. Suppose $P(X \leq m) = 0.65$ and $P(m \leq X \leq n) = 0.2$. Find $n - m$.

91 The times taken for the 200 runners in the school cross country event were normally distributed with mean 26 minutes and standard deviation 4 minutes.

- a Estimate the number of runners who completed the course in:

i less than 22 minutes

ii more than 27 minutes.

b The fastest 40% of runners finished quicker than what time?

92 Every day, Eugene walks a 5 km route and records his time. Last year, his average time was 40 minutes. Eugene wants to know if his time has improved this year. What set of hypotheses should be considered?

93 The minimum international standard diameter of a tennis ball is 6.541 cm.
A sample of 500 tennis balls manufactured by a company was taken, and the mean diameter of the sample was $\bar{x} = 6.548$ cm with sample standard deviation $s = 0.173$ cm.

It is known that the diameters of the tennis balls are normally distributed.
At the 5% level of significance, is there sufficient evidence to conclude that the tennis balls produced by this company do not meet the minimum international standard diameter?

94 Bao is a potato farmer. Between two harvests, he decides to change the fertiliser he uses. A sample of 100 potatoes is taken from each harvest, and the weights of each sample, in grams, are summarised below:

	Sample mean	Sample standard deviation
Before change	183	5.83
After change	184	2.35

Conduct a two-sample t -test to determine whether the change in fertiliser has increased the mean weight of potatoes. Use a 5% level of significance.

95 Brianna provides cookies for her colleagues to have at morning tea. Of the cookies she provides, 35% are choc-chip, 25% are oatmeal, 20% are shortbread, and the rest are butter.

The number of each type of cookie eaten over the past month are shown alongside.
Conduct an appropriate hypothesis test with a 1% level of significance to determine whether Brianna should change the proportion of cookies she provides.

Type of cookie	Number eaten
choc-chip	789
oatmeal	542
shortbread	423
butter	389
Total	2143

96 A survey of people found the following preferences for flavoured milk:

		Preferred milk flavour			
		Chocolate	Coffee	Strawberry	Caramel
Age	Adult	26	30	15	12
	Child	40	12	15	10

- It is claimed that the preferred flavour is independent of age.
- a Write suitable null and alternative hypotheses for a χ^2 test for independence.

b Find the value of the test statistic χ^2_{calc} .

c Given $\chi^2_{\text{crit}} = 7.81$, is there evidence at a 5% significance level to support the claim?

- 97** An experiment was conducted in an orchard to determine whether a fertiliser affected the yield of three varieties of oranges in the same way. The number of oranges per tree was calculated and the results were as follows:

	Variety A	Variety B	Variety C	Sum
Fertiliser	65	48	75	188
No fertiliser	40	54	58	152
Sum	105	102	133	340

The expected frequency table below shows some of the expected yields of each variety of orange, assuming the effect of the fertiliser is independent of the variety of orange.

	Variety A	Variety B	Variety C
Fertiliser			73.5
No fertiliser		45.6	59.5

- Write down a suitable null hypothesis H_0 and alternative hypothesis H_1 to test this independence.
- Copy and complete the expected frequency table.
- Write down the value of the test statistic χ^2_{calc} .
- Find the number of degrees of freedom.
- Find the p -value.
- At the 5% significance level, is the effect of the fertiliser independent of the variety of orange?

TOPIC 5: CALCULUS

RATES OF CHANGE

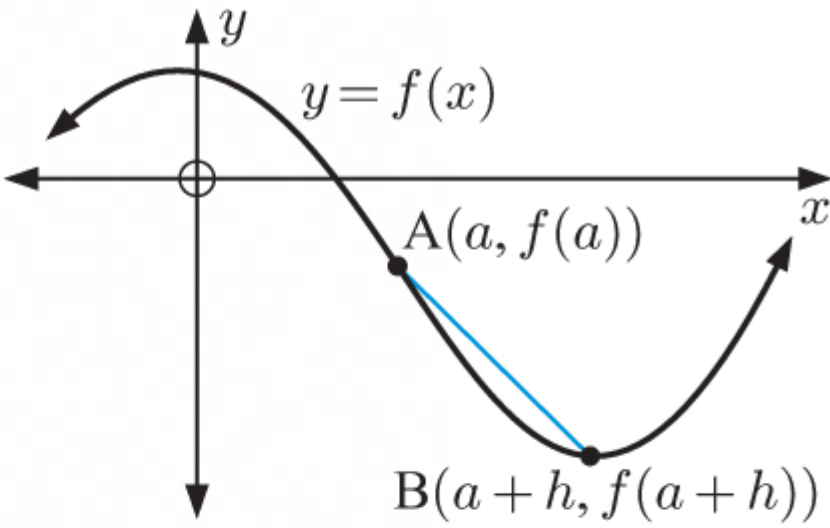
The **average rate of change** between two points on a graph is the **gradient of the chord** between them.

The **instantaneous rate of change** at a particular point is the **gradient of the tangent** to the graph at that point.

For the graph of a function $y = f(x)$, consider two points $A(a, f(a))$ and $B(a + h, f(a + h))$.

The gradient of the chord $[AB]$ is $\frac{f(a + h) - f(a)}{h}$.

As the point B is brought closer to the point A, the chord $[AB]$ more closely approximates the tangent at A. So, the gradient of the tangent at A is the **limit** of $\frac{f(a + h) - f(a)}{h}$ as h approaches 0, written $\lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}$.



THE DERIVATIVE FUNCTION

Given a function $f(x)$, the **derivative function** $f'(x)$ gives the gradient of the tangent to the curve for any value of x .

If we are given y in terms of x , the derivative function is $\frac{dy}{dx}$.

$\frac{dy}{dx}$ is the rate of change in y with respect to x .

If $\frac{dy}{dx}$ is positive, then as x increases, y also increases.

If $\frac{dy}{dx}$ is negative, then as x increases, y decreases.

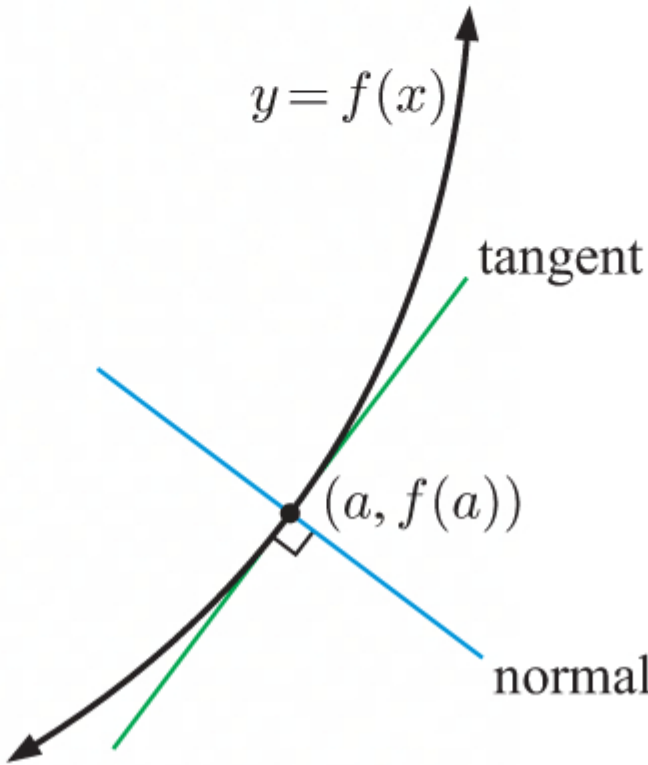
$f(x)$	$f'(x)$	Name of rule
c (a constant)	0	differentiating a constant
x^n	nx^{n-1}	differentiating x^n
$cu(x)$	$cu'(x)$	constant times a function
$u(x) + v(x)$	$u'(x) + v'(x)$	addition rule

PROPERTIES OF CURVES

Tangents and normals

For the curve $y = f(x)$:

- The gradient of the tangent at $x = a$ is $f'(a)$.
- The equation of the tangent at $x = a$ is $y = f'(a)(x - a) + f(a)$.
- The gradient of the normal at $x = a$ is $-\frac{1}{f'(a)}$.
- The equation of the normal at $x = a$ is $y = -\frac{1}{f'(a)}(x - a) + f(a)$.



Increasing and decreasing functions

$f(x)$ is **increasing** on an interval $S \Leftrightarrow f(a) \leq f(b)$ for all $a, b \in S$ such that $a < b$.

$f(x)$ is **decreasing** on $S \Leftrightarrow f(a) \geq f(b)$ for all $a, b \in S$ such that $a < b$.

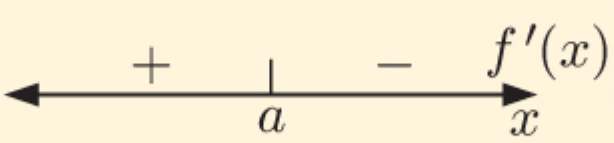
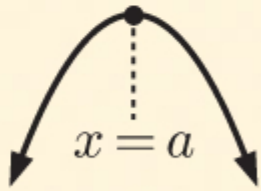
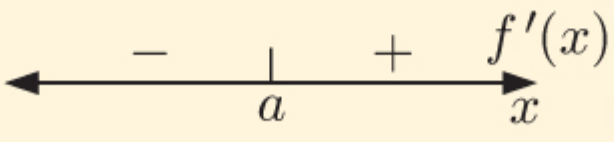
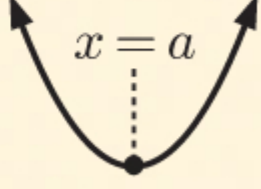
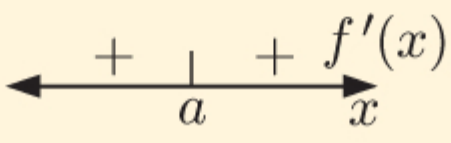
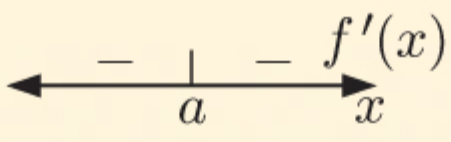
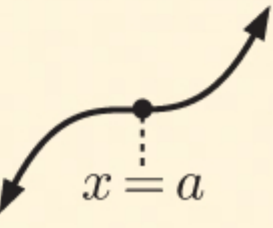
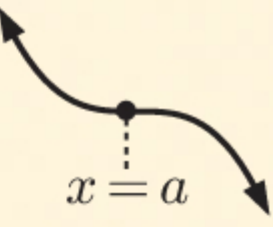
For most functions:

- $f(x)$ is increasing on $S \Leftrightarrow f'(x) \geq 0$ for all x in S .
- $f(x)$ is decreasing on $S \Leftrightarrow f'(x) \leq 0$ for all x in S .

Stationary points

A **stationary point** of a function is a point such that $f'(x) = 0$.

You should be able to identify and explain the significance of local and global maxima and minima, as well as stationary inflections.

Stationary point where $f'(a) = 0$	Sign diagram of $f'(x)$ near $x = a$	Shape of curve near $x = a$
local maximum		
local minimum		
stationary inflection	 or 	 or 

OPTIMISATION PROBLEMS

It is important to remember that a local minimum or maximum does not always give the minimum or maximum value of a function in a particular domain. You must check for other turning points in the domain, and the values of the function at the end points of the domain.

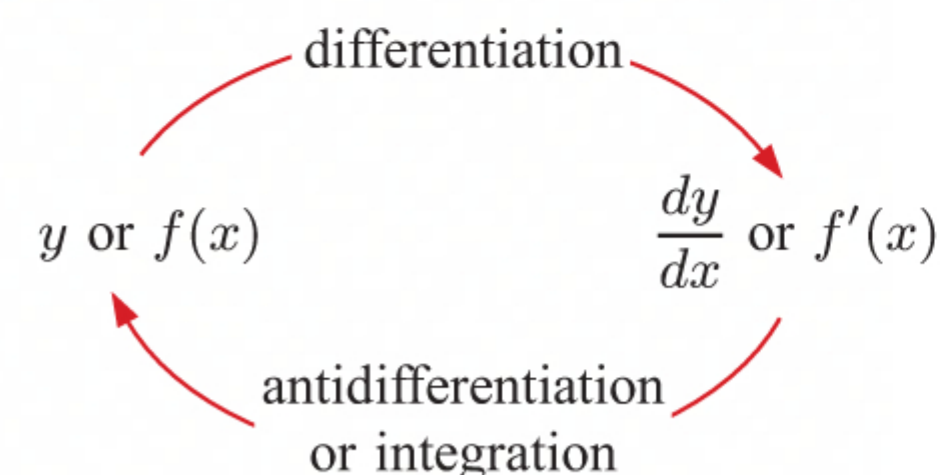
Optimisation problem solving method

- Step 1:* Draw a large, clear diagram of the situation.
- Step 2:* Construct a **formula** with the variable to be optimised as the subject. It should be written in terms of one convenient variable, for example x . You should write down what domain restrictions there are on x .
- Step 3:* Find the **first derivative** and find the value(s) of x which make the first derivative **zero**.
- Step 4:* For each stationary point, use the **sign diagram test** to determine whether you have a local maximum or local minimum.
- Step 5:* Identify the optimal solution, also considering end points where appropriate.
- Step 6:* Write your answer in a sentence, making sure you specifically answer the question.

INTEGRATION

Antidifferentiation or **integration** is the reverse process of differentiation.

The **antiderivative** or **integral** of $f(x)$ is the simplest function $F(x)$ such that $F'(x) = f(x)$.



Techniques for integration

When integrating, we use the rules for differentiation in reverse. Do not forget to include the **constant of integration**.

Function	Integral
k	$kx + c$
x^n	$\frac{x^{n+1}}{n+1} + c, \quad n \neq -1$

DEFINITE INTEGRALS

Fundamental Theorem of Calculus

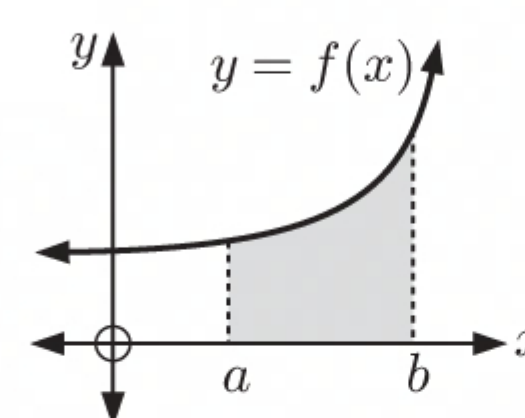
For a continuous function $f(x)$ with antiderivative $F(x)$, $\int_a^b f(x) dx = F(b) - F(a)$.

Properties of definite integrals

- $\int_a^a f(x) dx = 0$
- $\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$
- $\int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$
- $\int_b^a f(x) dx = -\int_a^b f(x) dx$
- $\int_a^b cf(x) dx = c \int_a^b f(x) dx$

Area under a curve

If $f(x)$ is a continuous *positive* function on the interval $a \leq x \leq b$, then $\int_a^b f(x) dx$ is the area under the curve between $x = a$ and $x = b$.

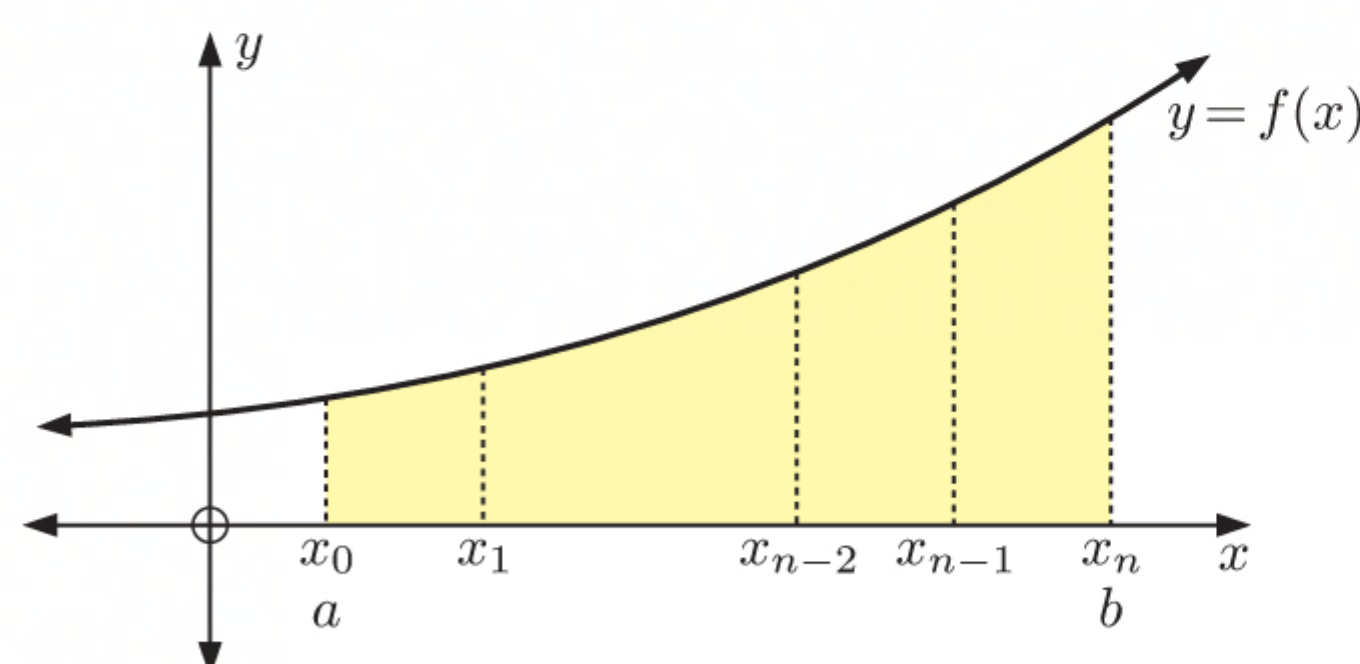


The trapezoidal rule

Suppose we divide the interval $a \leq x \leq b$ into n subintervals of equal width $h = \frac{b-a}{n}$.

The shaded area $\int_a^b f(x) dx$ is approximated by

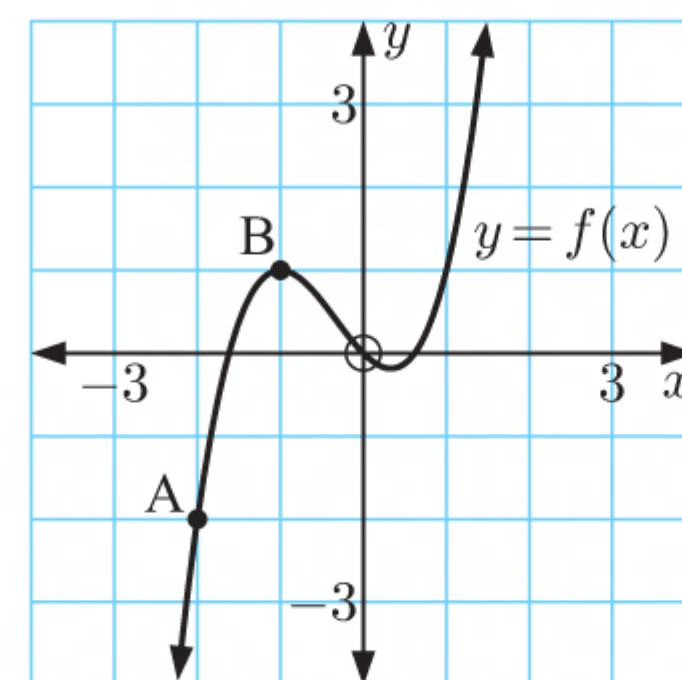
$$\int_a^b f(x) dx \approx \frac{h}{2} \left(f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right)$$



SKILL BUILDER QUESTIONS

- 1 Consider the graph of the function $y = f(x)$ alongside.

Find the average rate of change in $f(x)$ from A to B.

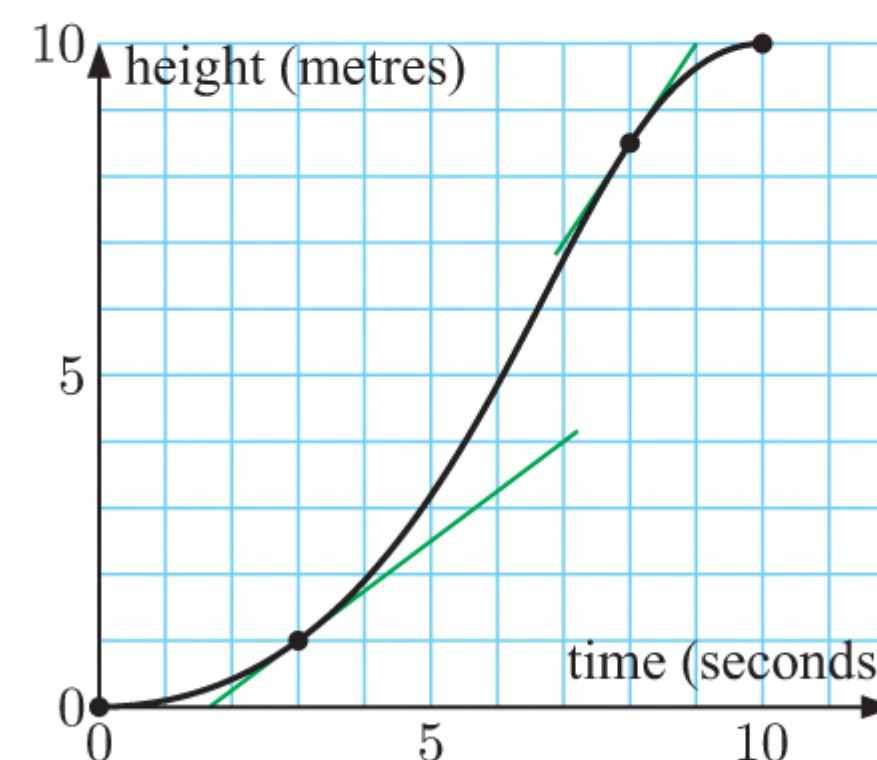


- 2 This graph shows the height of an elevator above ground level.

Use the tangents drawn to find the elevator's instantaneous speed after:

a 3 seconds

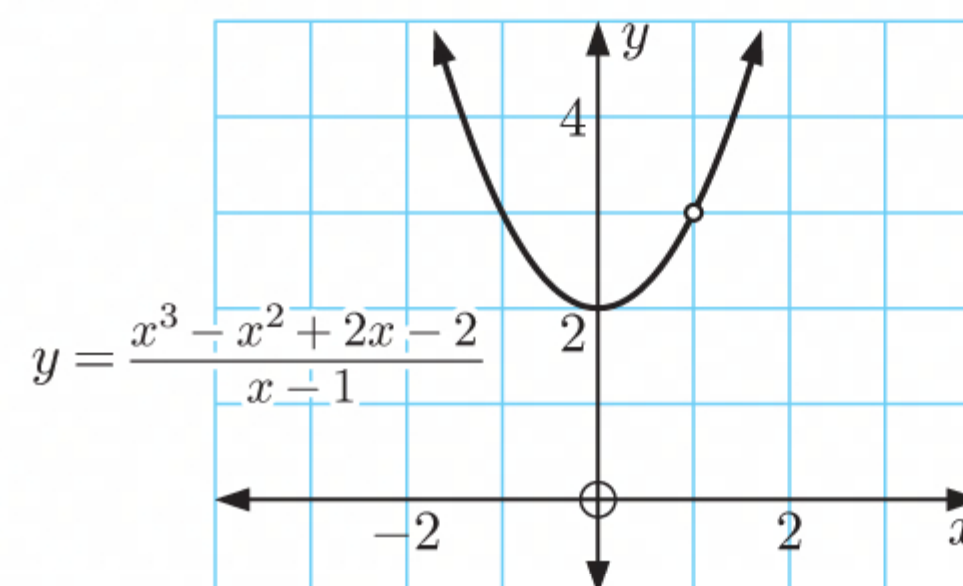
b 8 seconds.



- 3 The graph of $f(x) = \frac{x^3 - x^2 + 2x - 2}{x - 1}$ is shown alongside.

a Explain why the function is undefined at $x = 1$.

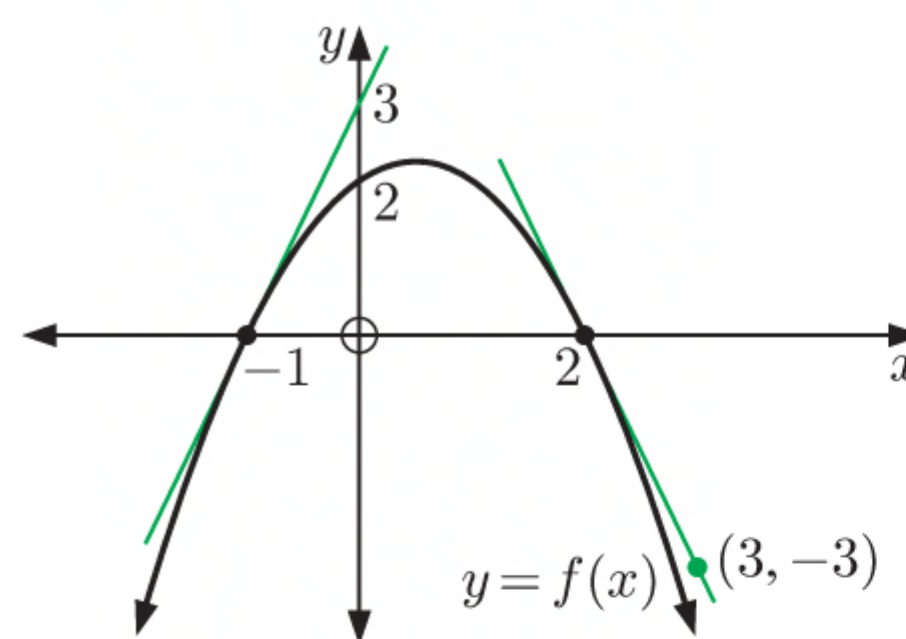
b Use the graph to find $\lim_{x \rightarrow 1} f(x)$.



- 4 For the given graph, find:

a $f'(-1)$

b $f'(2)$



- 5 Differentiate with respect to x :

a $6 - 3x + 2x^2$

b $\frac{1}{2}x^2 + 3x - 5$

c $\frac{1}{x} + \frac{1}{x^2}$

d $\frac{2x^2 + x + 1}{x}$

- 6 Find constants a and b such that:

a $f(x) = ax^2 + bx^3$, $f'(1) = -5$, and $f'(-1) = -13$

b $f(x) = ax + \frac{b}{x^2}$, $f(1) = 8$, and $f'(1) = -7$.

- 7 Find the gradient of the tangent to:

a $y = 3x - 2x^2$ at $x = 4$

b $y = \frac{x^2 + 4x - 1}{x^2}$ at $(1, 4)$.

- 8 The tangent to $f(x) = ax^2 + bx - 7$ at the point $(-1, -10)$ has gradient 1. Find a and b .

- 9 Find the coordinates of the point(s) on:

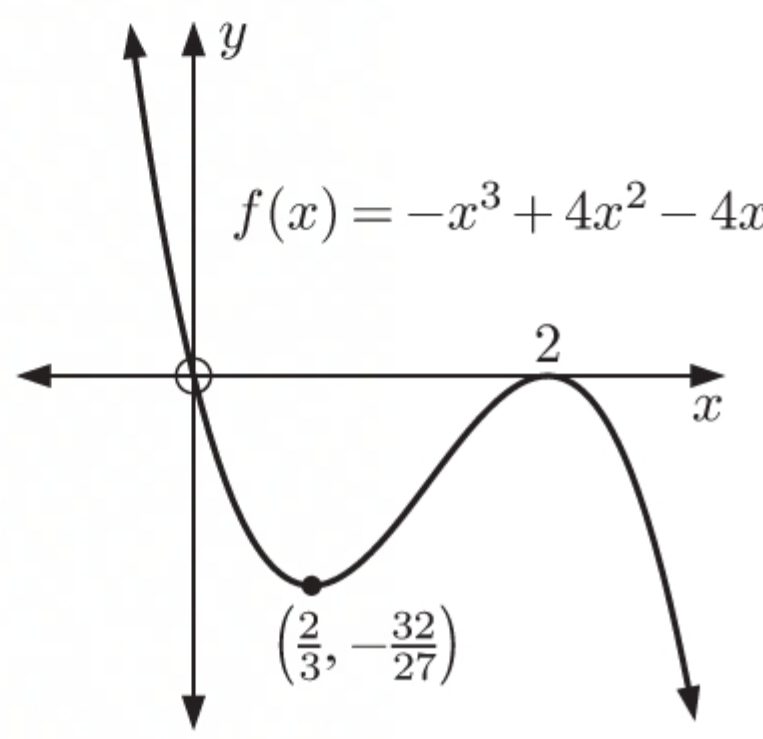
a $y = 3x^2 + 5x + 1$ where the tangent has gradient 11

b $f(x) = \frac{1}{2}x^3 - 4x - 2$ where the tangent has gradient $\frac{1}{2}$.

- 10 Find the equation of the tangent to:

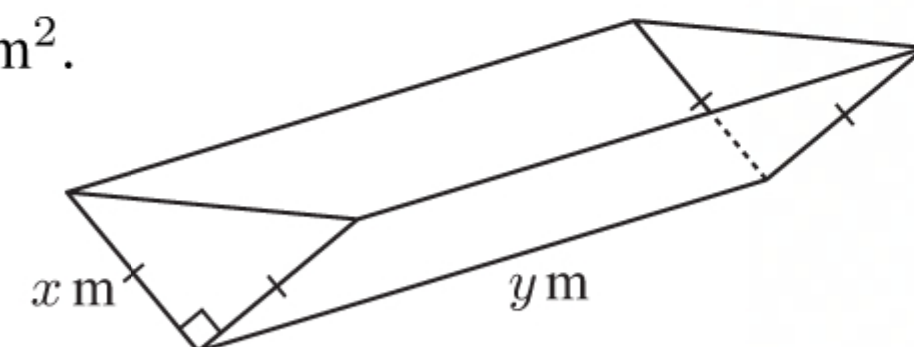
a $y = x^2 + 2x - 5$ at $x = 1$

b $y = 3 - \frac{2}{x}$ at $x = -2$.

- 11** Let $f(x) = -x^2 + 4x$.
- Find $f'(x)$.
 - Find the equation of the tangent to $y = f(x)$ at the point where $x = k$.
 - Suppose this tangent has positive gradient and passes through $(4, 9)$. Find the value of k .
- 12** Find where the tangent to $y = x^3 + 2x + 1$ at the point where $x = -1$, meets the curve again.
- 13** Consider the curve $y = \frac{a}{x} - x^2 + 1$ where $a \in \mathbb{R}$. The gradient of the tangent to the curve is -5 when $x = 2$.
- Find the value of a .
 - Determine the equation of the tangent to the curve at $x = 2$.
- 14** Find the equation of the normal to:
- $f(x) = x^3 - 2x$ at $x = 1$
 - $f(x) = \frac{3}{x} - \frac{6}{x^2}$ at $x = 2$.
- 15** The normal L_1 to the function $f(x) = ax^2 + bx$ at $x = 2$ has equation $x + 3y = -4$.
- Show that:
 - $4a + 2b = -2$
 - $4a + b = 3$
 - Hence find a and b .
 - The tangent L_2 to $y = f(x)$ at the point Q is parallel to L_1 . Find the coordinates of Q.
- 16** The graph of $f(x) = -x^3 + 4x^2 - 4x$ is shown alongside.
- Use the graph to write down the intervals where the function is:
 - increasing
 - decreasing.
 - Check your answer by finding $f'(x)$.
- 
- 17** Find the intervals where $f(x)$ is increasing or decreasing:
- $f(x) = 5 - 3x$
 - $f(x) = 2x^2 - 7x + 6$
 - $f(x) = -\frac{1}{x}$
 - $f(x) = 2x^3 - 9x^2 + 7x + 6$
- 18** For each of the following functions, find and classify all stationary points.
- $f(x) = x^3 - x^2$
 - $f(x) = x^4 - 2x^3 + 4x^2 - 8$
 - $f(x) = 2x + \frac{6}{x}$
- 19** $f(x) = 2x^3 + ax + b$ has a stationary point at $(1, 1)$.
- Find the values of a and b .
 - Find the position and nature of all stationary points.
- 20** Find the greatest and least values of:
- $f(x) = x^3 - 2x^2$ for $-1 \leq x \leq 1$
 - $f(x) = x^2 - \frac{27}{x}$ for $-6 \leq x \leq -1$
 - $f(x) = x^3 - 6x^2 + 12x - 10$ for $0 \leq x \leq 5$.
- 21** The cost of producing x bracelets is modelled by the function $C(x) = -0.2x^2 + 4x + 10$ dollars for $0 \leq x \leq 10$.
- Calculate $C(5)$ and explain what this represents.
 - Differentiate C with respect to x .
 - State the units that $C'(x)$ is measured in.
 - Find $C'(5)$, and interpret your answer.
 - Determine the maximum value of C on $0 \leq x \leq 10$.
- 22** Terry wants to fence off a rectangular garden plot of area 48 m^2 . Three sides will be fenced with strong wire mesh costing \$18 per metre, and the remaining side will be fenced with corrugated iron costing \$30 per metre.
- By letting x be the length in metres of the side fenced with corrugated iron, show that the cost of fencing is $C = 48\left(\frac{36}{x} + x\right)$ dollars.
 - Find the dimensions of the garden plot which will minimise the cost of fencing.
- 23** A man standing on a cliff above the ocean throws a ball high in the air. The height of the ball above the water t seconds after release is given by $h(t) = 100 + 32t - 4t^2$ m.
- Find $h'(t)$.
 - Find the maximum height above water reached by the ball.

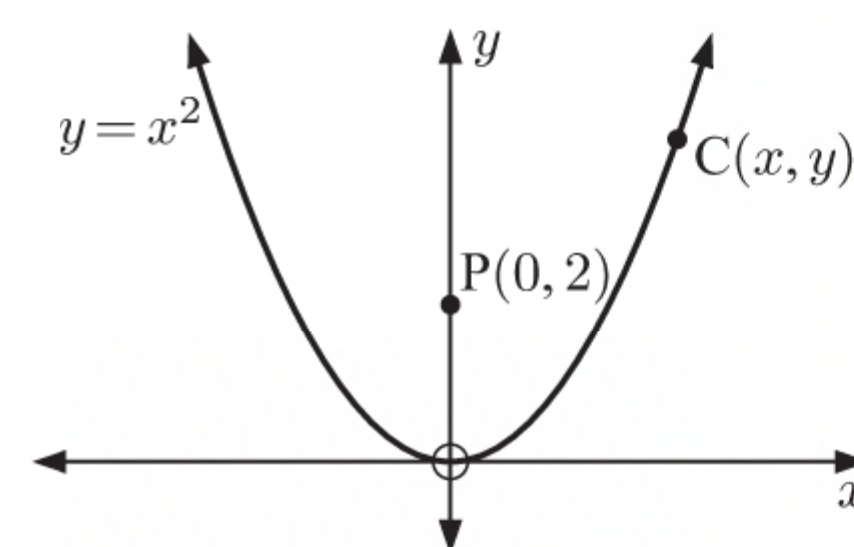
- 24** The diagram alongside shows an open trough. The total outside surface area is 27 m^2 .

- Show that $x^2 + 2xy = 27$.
- Find an expression for the volume V of the trough in terms of x only.
- Hence show that the volume of the trough is maximised if $x = y = 3$.



- 25** A comet travels in an orbit which can be described by the equation $y = x^2$ as shown in the diagram.

- Show that the distance of the comet at $C(x, y)$ from an observer at the point $P(0, 2)$ is given by $S(x) = \sqrt{x^4 - 3x^2 + 4}$.
- Find $\frac{d}{dx}(S^2)$, and hence find the values of x which minimise S^2 .
- Find the shortest and the greatest distance between the comet and the observer for $-2 \leq x \leq 2$.



- 26** Consider the function $f(x) = x^3 - 6x^2 + 11x + 7$.

A table of values for $f(x)$ is shown alongside.

- Using the trapezoidal rule with 5 subintervals, estimate the area between $y = f(x)$ and the x -axis for $1 \leq x \leq 2$.
- Find $\int_1^2 f(x) dx$ and interpret your answer.
- Hence find the percentage error in your estimate in **a**.

i	x_i	$f(x_i)$
0	1	13
1	1.2	13.288
2	1.4	13.384
3	1.6	13.336
4	1.8	13.192
5	2	13

- 27** Find the antiderivative of:

- $2x$
- $\frac{x^2}{3}$
- $\frac{3}{x^2}$

- 28** Differentiate $\frac{2}{x^2} - 3x$, and hence find $\int \left(\frac{8}{x^3} + 6 \right) dx$.

- 29** Find:

- $\int -3 dx$
- $\int \left(\frac{3}{x^2} + 2x^3 - 4 \right) dx$
- $\int \left(\frac{1}{x} + 2x \right)^2 dx$

- 30** Find y if:

- $\frac{dy}{dx} = 4x$
- $\frac{dy}{dx} = x^2 + \frac{1}{2}x + \frac{1}{3}$
- $\frac{dy}{dx} = \frac{3x^4 + 5}{x^3}$

- 31** Find $f(x)$ given that $f'(x) = 4x - 3x^2$ and $f(3) = -2$.

- 32** Find:

- $\int_0^1 (x^2 + x) dx$
- $\int_{-2}^1 (x^3 - 2x^2 - 4x + 9) dx$
- $\int_3^5 \left(\frac{8}{x^2} + 3x \right) dx$

- 33** Find:

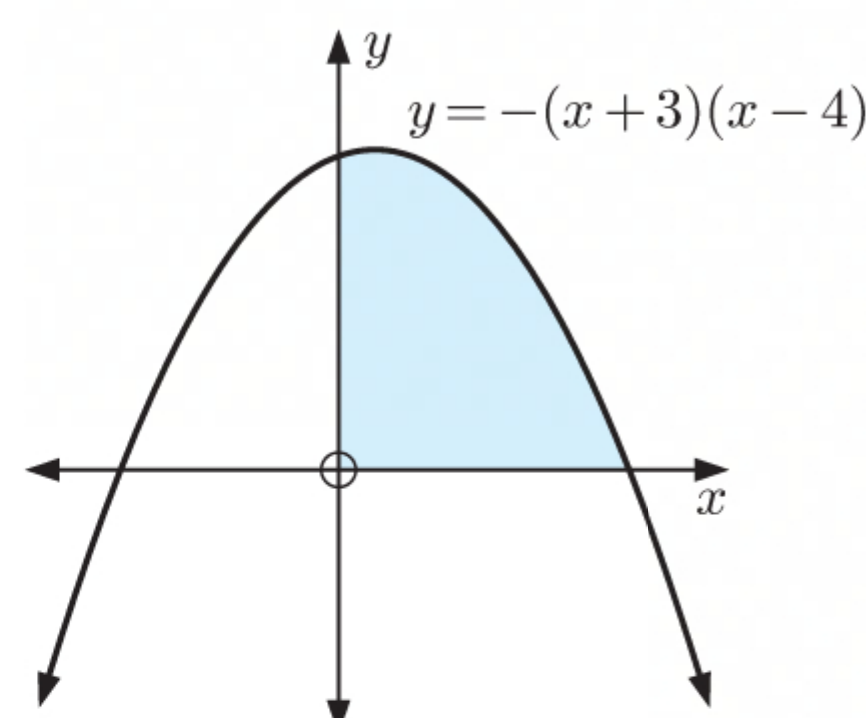
- $\int_{-1}^1 3x^2 dx$
- $\int_2^3 \frac{5x^3 - 2x}{x^5} dx$
- $\int_{-3}^0 (1 - 3x)^2 dx$

- 34** Use technology to evaluate, correct to 4 significant figures:

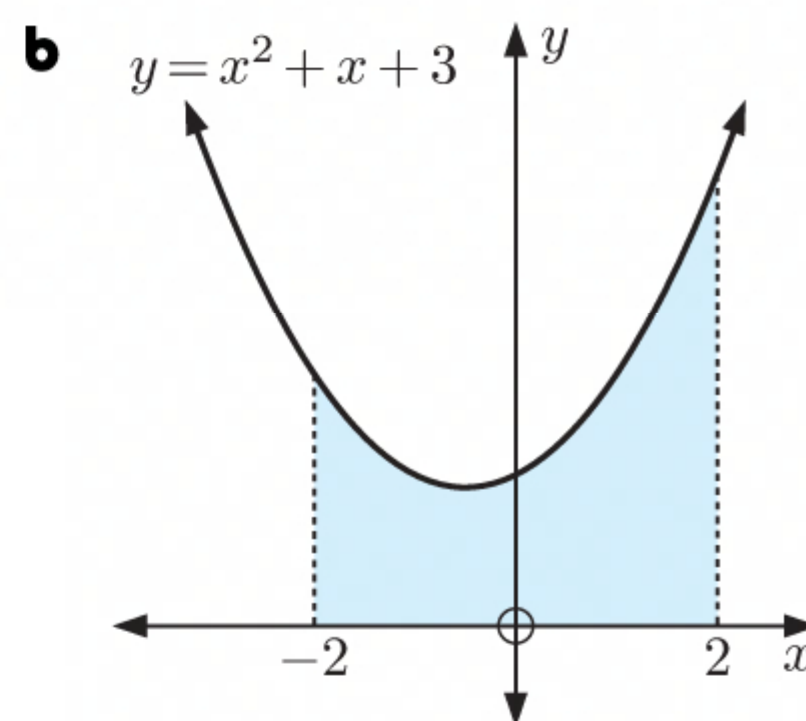
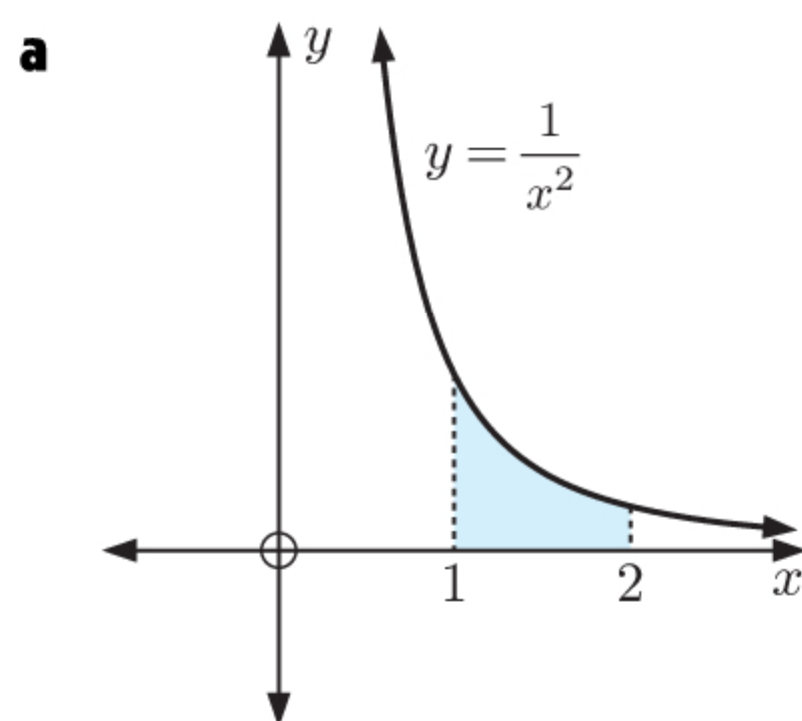
- $\int_1^2 (\sqrt{x} + x^2) dx$
- $\int_0^3 2^x dx$
- $\int_{-1.5}^{2.4} \frac{5x + 20}{x + 6} dx$

- 35** **a** Write an integral to represent the shaded area.

- b** Hence find the shaded area.

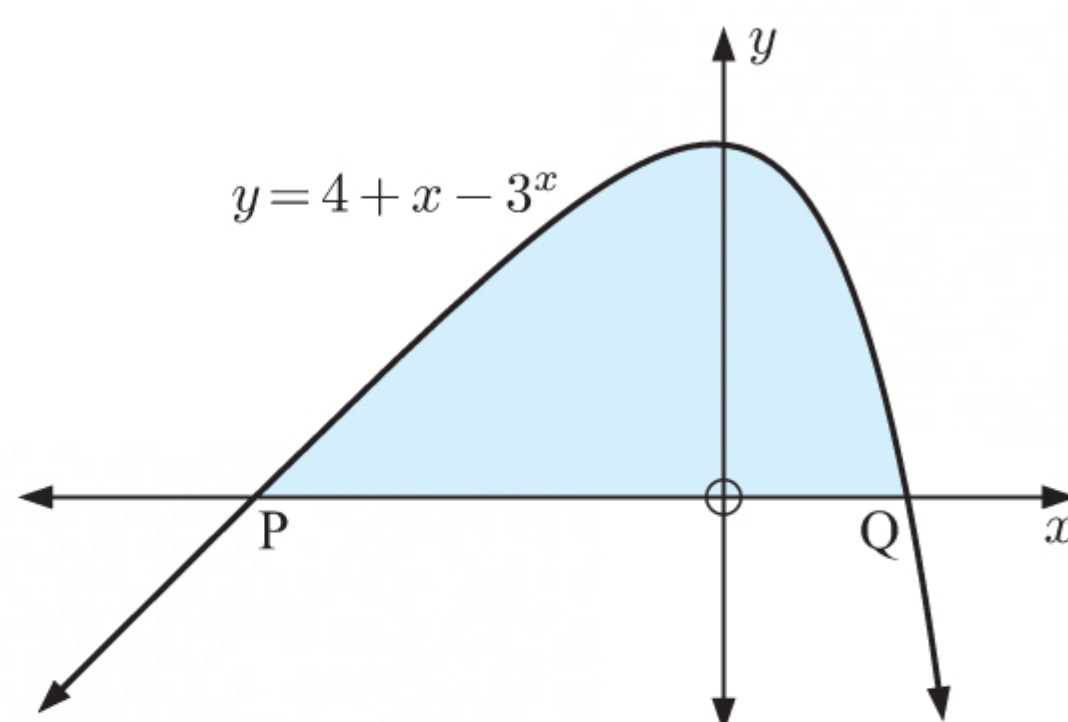


36 Find the shaded area:



37 Use technology to find:

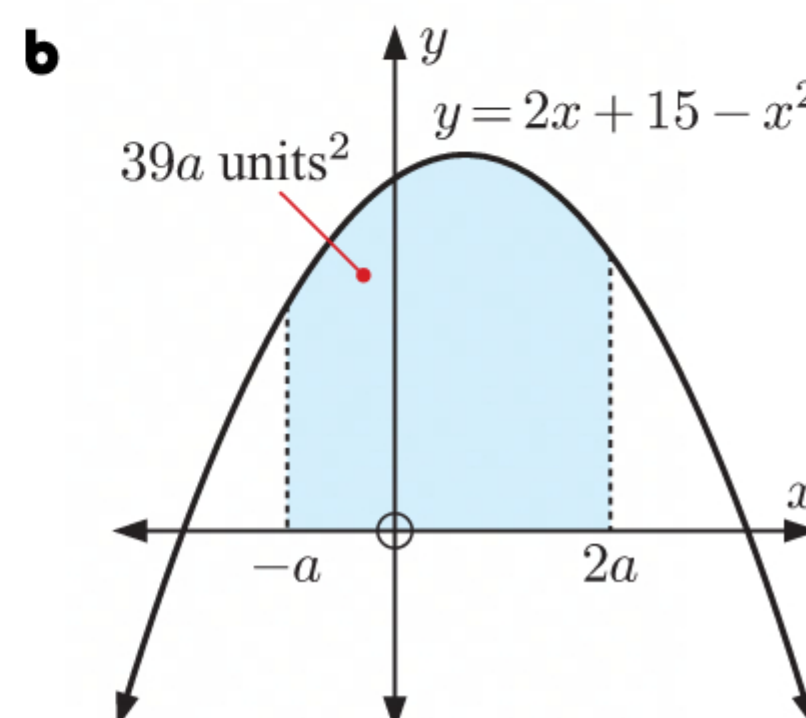
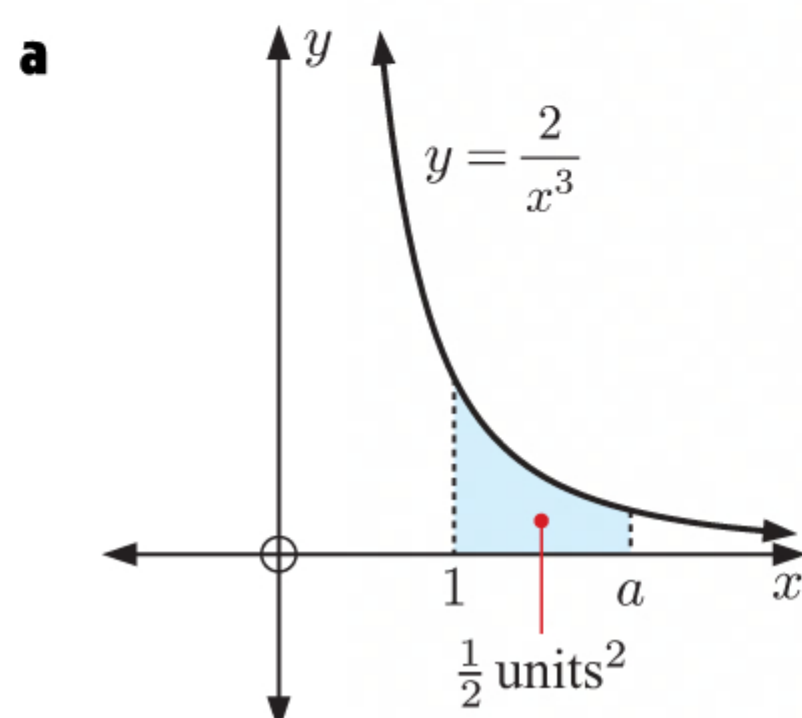
- a** the coordinates of P and Q
b the shaded area.



38 Find the area of the region bounded by:

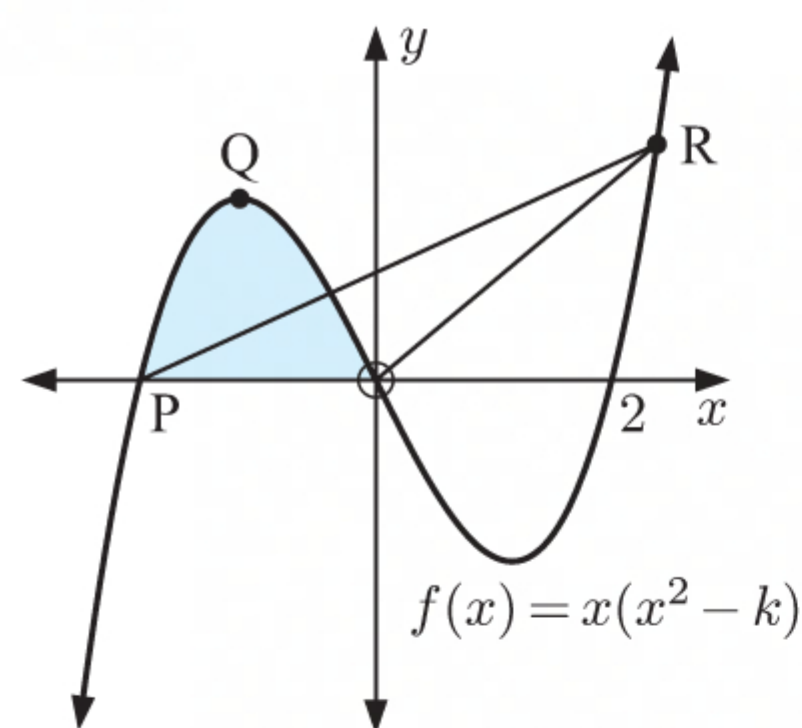
- a** $y = x^2 + 2x + 2$, the x -axis, $x = 0$, and $x = 3$
b $y = 4 - x^2$ and the x -axis
c $y = (2x + 5)^2$, the x -axis, and $x = -4$.

39 Find the exact value of a :



40 The function $f(x) = x(x^2 - k)$ is graphed alongside.

- a** Find the value of k .
b Find the coordinates of P.
c Find $f'(x)$.
d Hence find the exact x -coordinate of the local maximum Q.
e Find $\int f(x) dx$.
f Hence find the shaded area.
g The point R lies on the graph of $y = f(x)$ such that the area of triangle POR is equal to the shaded area. Find the coordinates of R.



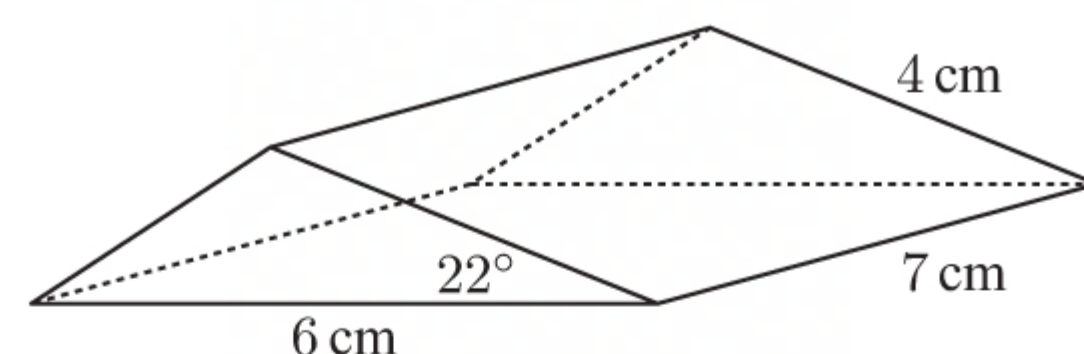
Mixed questions

MIXED QUESTIONS SET 1

- 1 Line L has equation $y = 3 - 2x$.
 - a If the point $P(3, k)$ lies on line L , determine the value of k .
 - b Write down the gradient of line L .
 - c Find the equation of the line perpendicular to L which passes through point P .

- 2 In this triangular prism, the side lengths are rounded to the nearest centimetre, and the angle is rounded to the nearest degree.

Find the boundary values for the prism's volume.



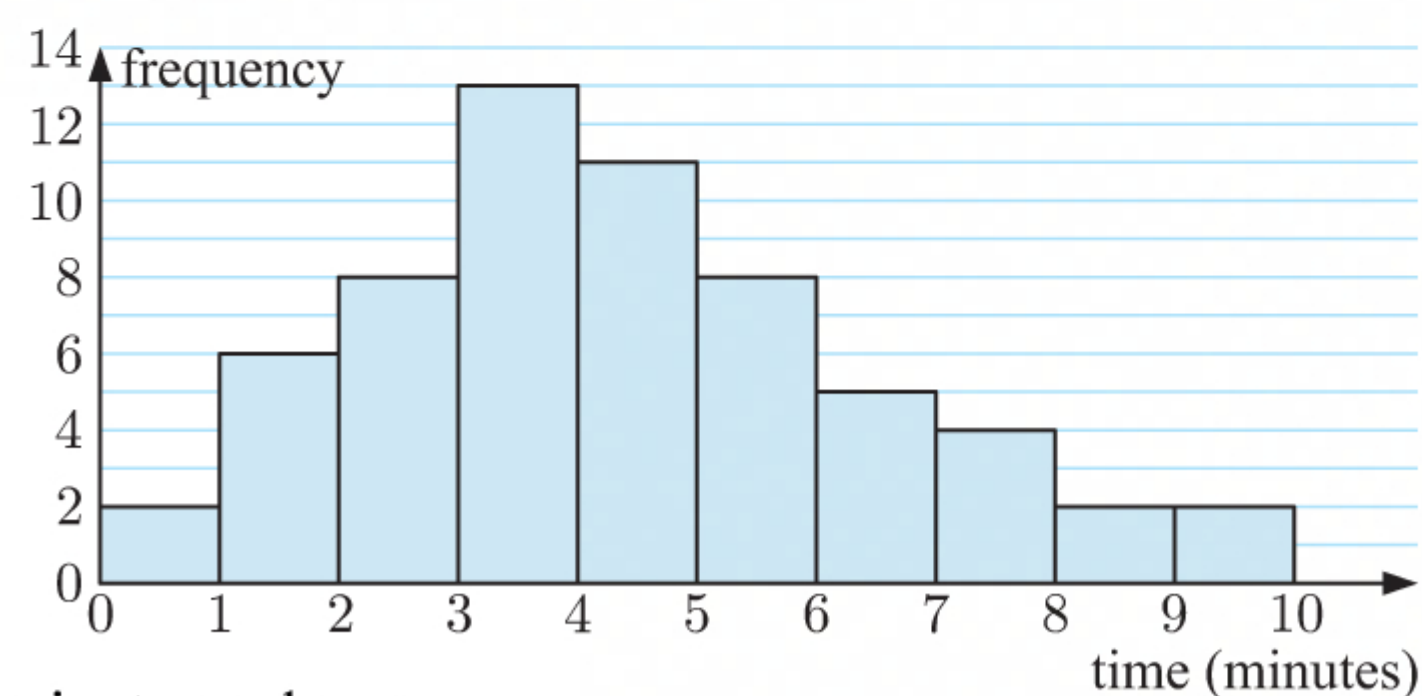
- 3 The height H of a small toy aeroplane, t seconds after it is thrown from the top of a building, is given by the function $H(t) = 80 - 5t^2$ metres, where $t \geq 0$.
 - a Find the initial height of the toy aeroplane.
 - b Determine the time it takes for the toy aeroplane to hit the ground.
 - c Find $H'(2)$, and explain what this value means.

- 4 The value of a car decreases by 10% each year. After 3 years its value is \$26 244.

- a Find the original value u_0 of the car.
- b Write a geometric sequence to describe the value of the car u_n after n years.
- c In what year will the value of the car fall below \$10 000?

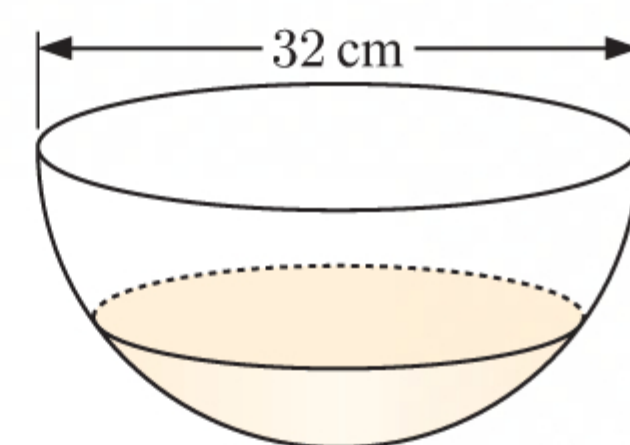
- 5 Before selecting a new mobile phone plan, George reviews the duration of calls he made over the last 3 months. George produced the histogram alongside to illustrate the data he collected.

- a Write down the modal class.
- b Organise the data into a frequency table.
- c Estimate the mean length of a phone call.
- d Estimate the probability that George's next call will last 6 minutes or longer.



- 6 A hemispherical mixing bowl has dimensions shown.

- a Find the capacity of the bowl.
- b Suppose the bowl is 20% full with cake batter.
 - i How many litres of cake batter does it contain?
 - ii The cake batter is poured into a cylindrical cake tin with diameter 25 cm. How high will it reach up the tin?



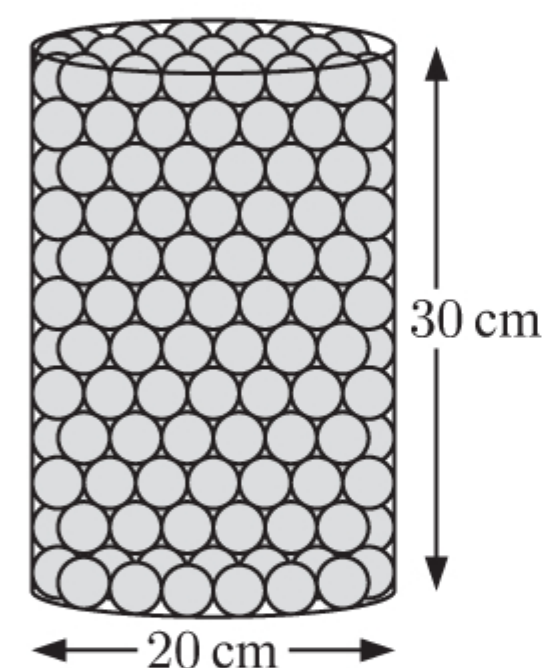
- 7 Charlie and Charlotte are on a road trip in Australia. They travel 36 km north-west from Wollongong to Picton, then 210 km south-west from Picton to Canberra.

- a How far is Canberra from Wollongong?
- b Find the bearing of Wollongong from Canberra.

- 8 A curve has gradient function $f'(x) = \frac{a}{x^2} + bx^2$ where a and b are constants. Find $f(x)$ given that $f(-1) = -7$, $f(1) = 7$, and $f(2) = 26$.

- 9 A jar 20 cm wide and 30 cm high is filled with marbles.

To estimate the number of marbles in the jar, Julie assumes that the jar is a perfect cylinder, and the marbles occupy 60% of the jar's volume.



- Use Julie's assumptions to construct a model for the number N of marbles with radius r cm in the jar.
 - Julie measures the radius of a marble in the jar to be 1.5 cm. Use Julie's model to estimate the number of marbles in the jar.
 - The marbles actually occupy about 64% of the jar's volume. Do you think the estimate in **b** is an overestimate or an underestimate? Explain your answer.
 - Given that there are actually 426 marbles in the jar, find the percentage error in the estimate in **b**.
- 10 The heights X of maize plants two months after planting are normally distributed with mean 40 cm and standard deviation 6.8 cm.
- Find:
 - $P(X < 25)$
 - a such that $P(X < 25) = P(X > a)$.
 - Six maize plants are randomly chosen. Find the probability that exactly four of them are more than 35 cm high.

- 11 A random variable X has the following distribution table:

x	-2	0	3	5
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{6}$	k	$\frac{1}{12}$

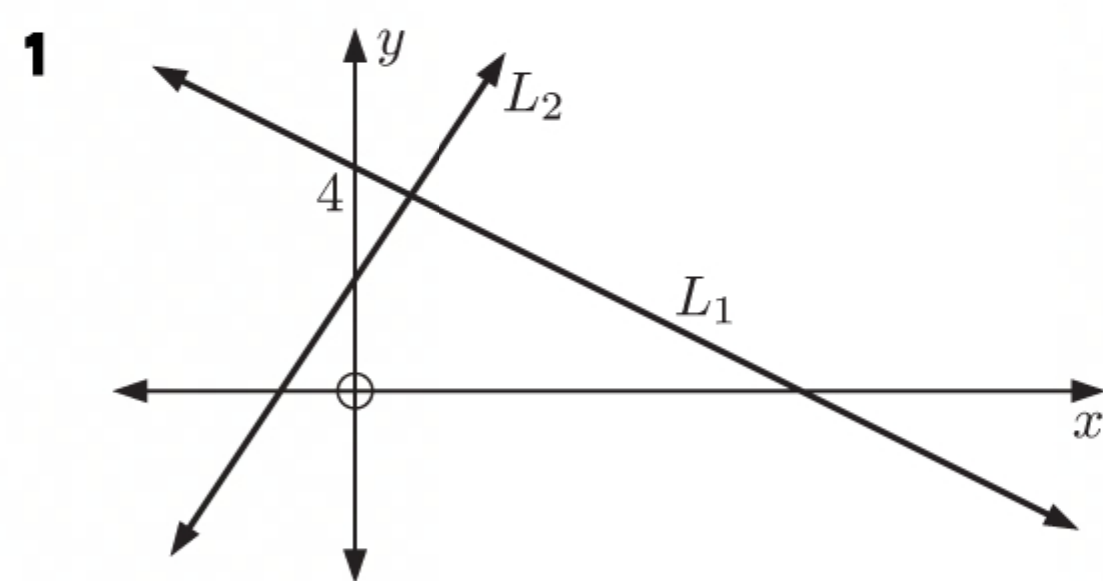
- Is the random variable X discrete or continuous?
 - Find k .
 - Find the mode and median of X .
 - Find $E(X)$.
- 12 This table shows the echo signal strength received from a radar detecting an object d km away.

Distance (d km)	5	10	20
Signal strength (S units)	40	2.5	0.156 25

The signal strength is inversely proportional to the fourth power of the distance.

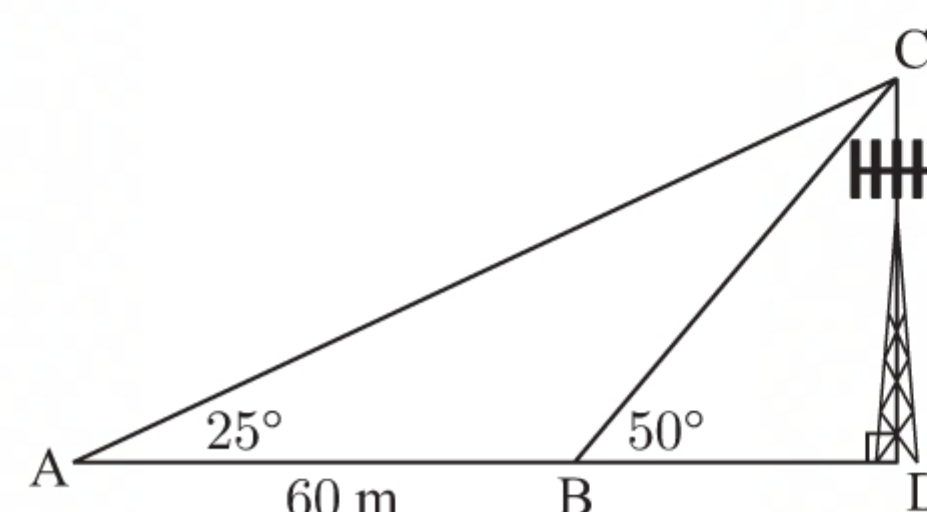
- Find the model connecting S and d .
- Find the signal strength for an object 18 km away.
- Find the percentage decrease in signal strength if the distance is tripled.

MIXED QUESTIONS SET 2



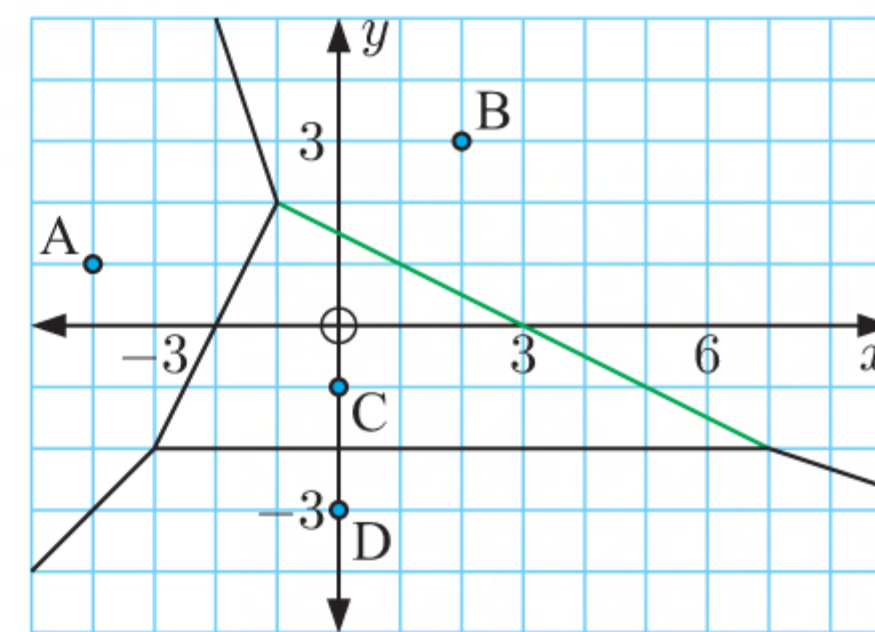
- L_1 has gradient $-\frac{1}{2}$ and passes through $(0, 4)$.
Find the equation of L_1 , giving your answer in the form $y = mx + c$.
- L_2 passes through $(-2, -1)$ and $(4, 8)$.
Find the point of intersection of L_1 and L_2 .

- 2 Lin wants to calculate the height of a mobile phone tower. He measures the angle of elevation from point A to the top of the tower C, then moves 60 m closer to point B and takes a second measurement. The information is given in the diagram alongside.

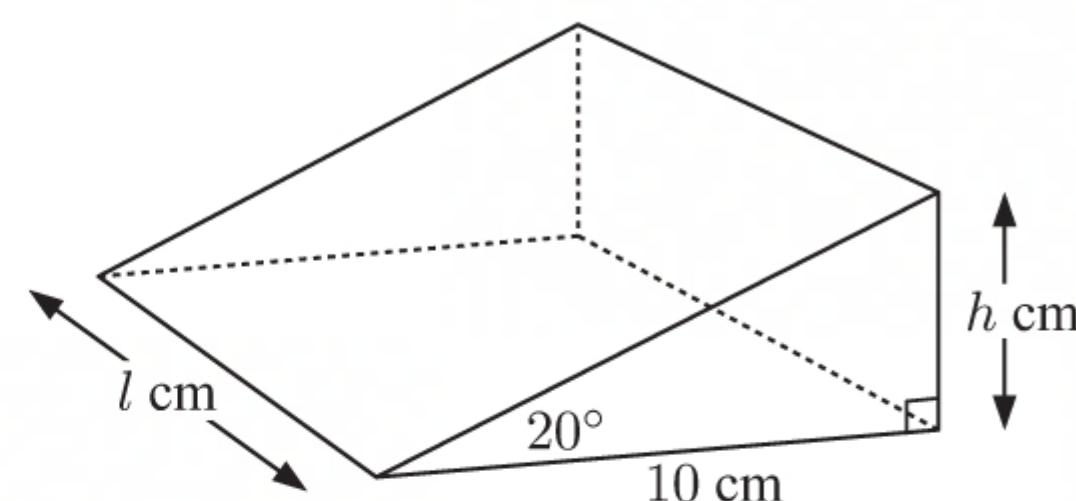


- Calculate the measure of \widehat{ACB} .
 - Determine the height of the tower.
- 3 Consider the arithmetic sequence with $u_5 = 18$ and $u_8 = 39$.
- Find the first term u_1 and common difference d .
 - Find the 12th term of the sequence.
 - Find the sum of the first 10 terms of the corresponding arithmetic series.
- 4 Consider the function $f(x) = ax^3 - bx^2$. The line $y = x - 6$ is a tangent to $y = f(x)$ at $x = 3$.
- Find the constants a and b .
 - Find the point where the tangent meets $y = f(x)$ again.
 - Graph $y = f(x)$ and $y = x - 6$ on the same set of axes.

- 5 Consider this Voronoi diagram for the sites $A(-4, 1)$, $B(2, 3)$, $C(0, -1)$ and $D(0, -3)$.



- Identify the site which is closest to:
 - $(2, 2)$
 - $(-5, -3)$
 - $(6, -4)$
 - Find the equation of the green edge. Write your answer in the form $ax + by + d = 0$.
 - Find the area of cell C.
- 6 A manufacturer produces wooden door-stops with the shape of the triangular prism shown.
- Calculate the height h correct to 4 significant figures.
 - Determine the area of the triangular end of the prism.
 - The volume of the door-stop is 60 cm^3 . Determine its length l .
 - Calculate the total surface area of each door-stop. Give your answer correct to 3 significant figures.
- 7 The management of a large shopping centre chain sent a survey team to one of its suburban shopping centres. Between 10 am and 3 pm on a very busy Thursday, 100 people in the main mall were asked the following multiple choice question:



“At which type of shopping centre do you prefer to shop?”

- A** suburban **B** central city **C** equally preferred **D** neither **E** no opinion

- Give *two* reasons why this survey is likely to contain a coverage error.
 - The results were: suburban 33%, central city 8%, equally preferred 51%, neither 4%, no opinion 4%.
Management concluded that “*more than four times as many people prefer suburban shopping to the central city*”. Explain why this conclusion is unreasonable.
- 8 The current in an electrical circuit t milliseconds after it is switched off is given by $I(t) = 40e^{-0.1t}$ amps.
- What current was flowing in the circuit initially?
 - What current was flowing in the circuit after 100 milliseconds?
 - Sketch $I(t)$ and $I = 1$ on the same set of axes.
 - How long did it take for the current to fall to 1 amp?
- 9 Soraya borrowed £7000 to go on a holiday. The bank charged an interest rate of $r\%$ p.a. compounded monthly. She will repay the loan with monthly repayments of £220 for 3 years.
- Find r .
 - Find the total interest charged on the loan.
 - Suppose Soraya chose to repay the loan over 4 years instead of 3 years.
 - Find Soraya’s monthly repayment.
 - How much *extra* interest will Soraya pay?

- 10 The number of people at a music festival t hours after 12 pm is modelled using the function $N(t) = at^3 + bt^2 + ct + d$. There were 2000 people at the festival at 12 pm. The number of people at the festival was increasing at 200 people per hour at 6 pm, and decreasing at 500 people per hour at 8 pm. The festival closed at 10 pm.
- State *four* conditions for $N(t)$ and $N'(t)$ you can deduce from the information given.
 - Hence find the function $N(t)$.
 - Predict the maximum number of people at the festival, and the time when this occurred.

- 11 A tinned food company examined a sample of its tins of corn and tins of pineapple for defects. The results are summarised in the table alongside.

	Defective	Not defective
Corn	37	581
Pineapple	24	617

- How many tins were included in the sample?
- Estimate the probability that the next randomly selected tin:
 - is not defective
 - is a defective tin of pineapple
 - is defective, given it is a tin of corn.

- 12** Monica is a police officer. She wants to investigate whether house break-ins in her city are equally likely to occur on each day of the week.

She compiles the following data for 140 randomly chosen house break-ins over the past year.

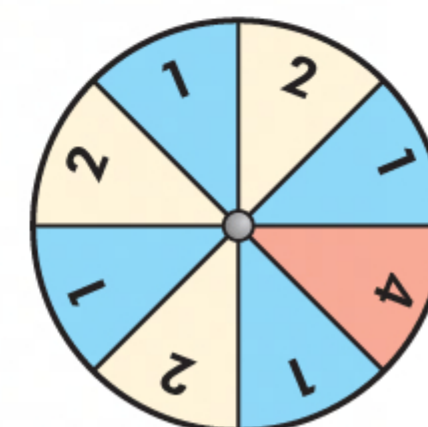
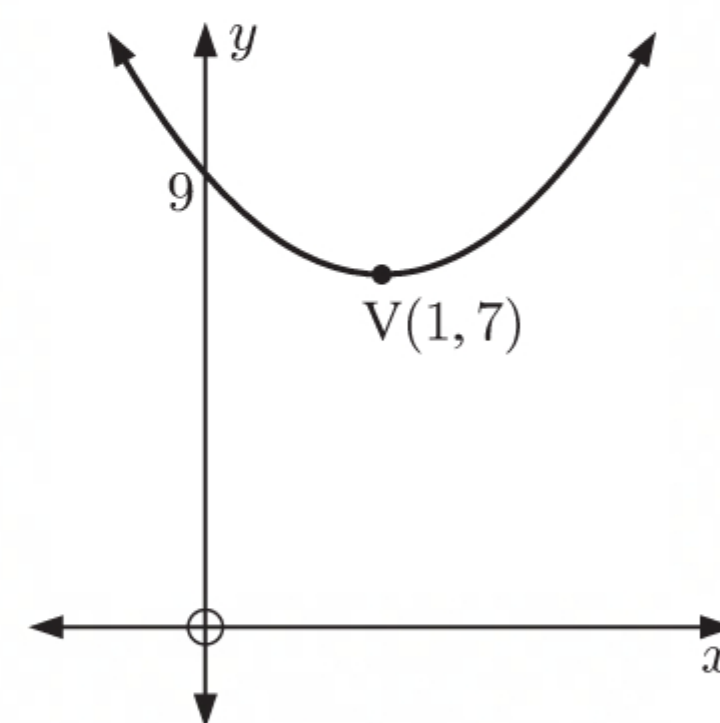
Day of the week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Number of break-ins	15	11	17	18	27	29	23

Monica will perform an appropriate χ^2 test at a 5% significance level. The critical value of χ^2 for this test is 12.59.

- Write down the hypotheses that Monica should test.
- Assuming the null hypothesis is true, how many break-ins from Monica's sample would be expected to occur on each day?
- Calculate the test statistic χ^2_{calc} .
- Hence determine whether the break-ins in this city are equally likely to occur on each day of the week.

MIXED QUESTIONS SET 3

- A 3 m long ladder is resting against the wall of a house. The ladder makes a 20° angle with the wall.
 - Calculate the distance from the foot of the ladder to the base of the wall.
 - The ladder suddenly slips so that its foot moves 1 m further away from the base of the wall. Calculate the angle that the ladder now makes with the wall.
- The fluoride concentration of lakes in a particular region was found to be 3×10^{-4} g per litre.
 - One lake has 5.6×10^8 litres of water. Find the amount of fluoride in the lake, giving your answer in scientific notation.
 - Another lake contains 4.13×10^7 g of fluoride. Find the volume of the lake.
- A graph of the quadratic $y = ax^2 + bx + c$ is shown alongside, including the vertex V and y-intercept.
 - Determine the value of c .
 - Use the axis of symmetry to write an equation involving a and b .
 - Use the point $(1, 7)$ to write another equation involving a and b .
 - Find a and b .
- A game is played in which the wheel shown is first spun by the player, and then by the game operator. The player wins \$ a if their spin is higher than the operator's. It costs \$ k to play the game. Find the relationship between a and k so that the game is fair.

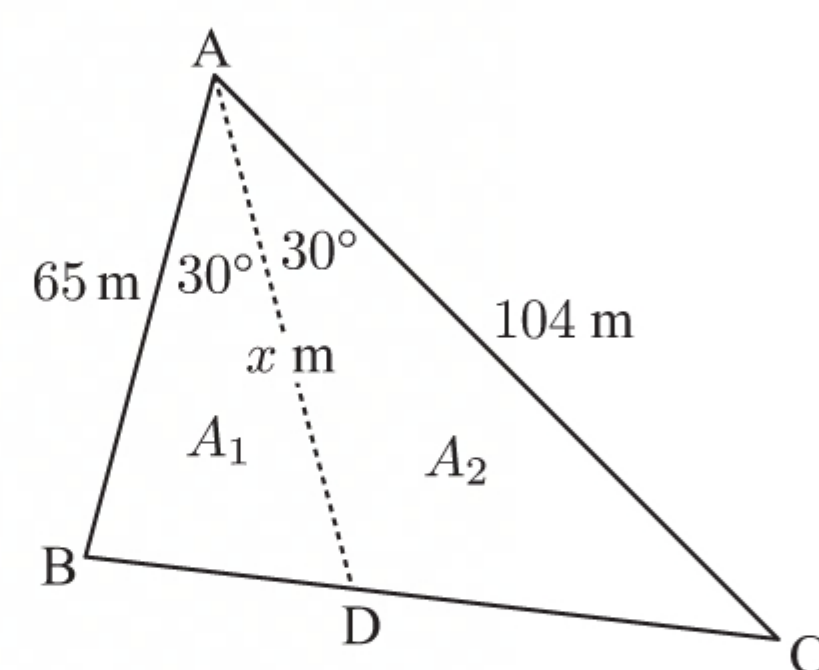


- Find the probability that a randomly chosen adult fish of this species is:
 - longer than 45 cm
 - between 35 cm and 50 cm long.
 - Determine the minimum length of the longest 10% of this species of fish.
 - A randomly selected fish is shorter than 48 cm. Find the probability that it is between 40 cm and 44 cm long.
- Suppose $f(x) = -x^2 + bx + 5$ for some constant $b \in \mathbb{R}$, and L is the tangent to $y = f(x)$ at $x = -1$.
 - Given that L has gradient 5, find the value of b .
 - The tangent to $y = f(x)$ at the point P is perpendicular to L . Find the coordinates of P.

- 7** The table shows the amount of petrol remaining in a motorbike's fuel tank and the number of kilometres travelled. The capacity of the tank is 10 litres.

Remaining fuel (x litres)	10	8	6	4	2	1
Distance (y km)	0	90	190	260	330	370

- a** Plot this data on a scatter diagram. **b** Find the equation of the regression line for y against x .
- c** Interpret the y -intercept of the regression line.
- d** The motorbike has travelled 220 km since its tank was refilled.
- i** Use your equation to estimate the amount of fuel left in the tank.
- ii** Find the average distance travelled per litre over the 220 km.
- 8** The probability of Mark waking up early is 0.8. If he wakes up early, he will pack lunch with probability 0.6. If he does not wake up early, he will pack lunch with probability 0.15.
- a** Display the sample space of possible outcomes on a tree diagram.
- b** Hence determine the probability that Mark will pack lunch today.
- 9** A farmer owns a triangular field ABC.
- D is the point on [BC] such that [AD] bisects \widehat{BAC} . The farmer divides the field into two parts A_1 and A_2 by constructing a straight fence [AD] of length x m.
- a** Use the cosine rule to calculate the length of [BC].
- b** Find the total area of the field.
- c** Find, in terms of x , the area of: **i** A_1 **ii** A_2 .
- d** Hence find x .
- 10** Let $D(t)$ m be the distance between two stunt motorcyclists t seconds after they start riding. It is known that the motorcyclists were initially 42 m apart, and that $D'(t) = 0.8t - 8$.
- a** Find $D(t)$.
- b** Find the distance between the motorcyclists after 2 seconds.
- c** Find the minimum distance between the motorcyclists, and the time at which it occurs.
- 11** Melinda bought a car valued for £55 000. She borrowed the money for the car over 7 years, with interest charged at 9.25% p.a. compounded monthly.
- a** Calculate her monthly repayments.
- b** Calculate the outstanding debt after two and a half years.
- c** The car depreciates at 15% p.a. After 7 years, Melinda sold the car at its depreciated value. Find the total cost of the car to Melinda, taking depreciation and interest into account.



- 12** A gardener has been asked to perform maintenance on Globe Park, a circular lawn with radius 30 m.

This involves:

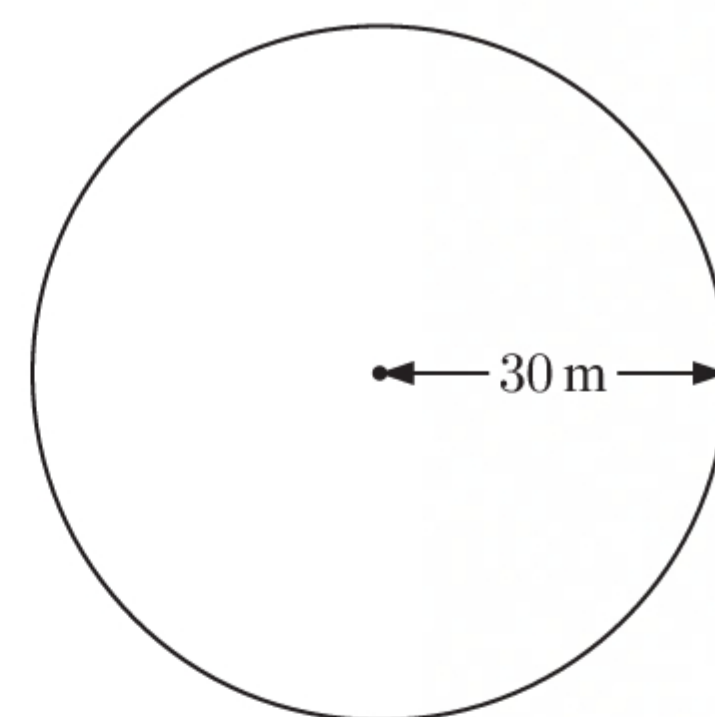
- mowing the interior of the lawn
- using a line trimmer to tidy the perimeter of the lawn.

From previous experience, the gardener knows that:

- a circular lawn with radius 10 m takes 30 minutes to maintain (1)
- a circular lawn with radius 20 m takes 110 minutes to maintain. (2)

The gardener believes the time to maintain a circular lawn with radius r m can be modelled by $T = ar + b$ minutes, where a and b are constants.

- a** Assuming the gardener's model is correct, write two equations connecting a and b .
- b** Hence find a and b .
- c** Explain why this model is not appropriate for small values of r .
- d** Use this model to predict how long it will take to maintain Globe Park.



- e** Given that it actually took 230 minutes to maintain Globe Park, find the percentage error in the prediction in **d**.
- f** The gardener's friend suggests a model of the form $T = pr^2 + qr$, where p and q are constants.
- Explain why a model of this form is reasonable.
 - Use the information from (1) and (2) to find p and q for this model.
 - Is this model better at predicting the time taken to maintain Globe Park? Explain your answer.

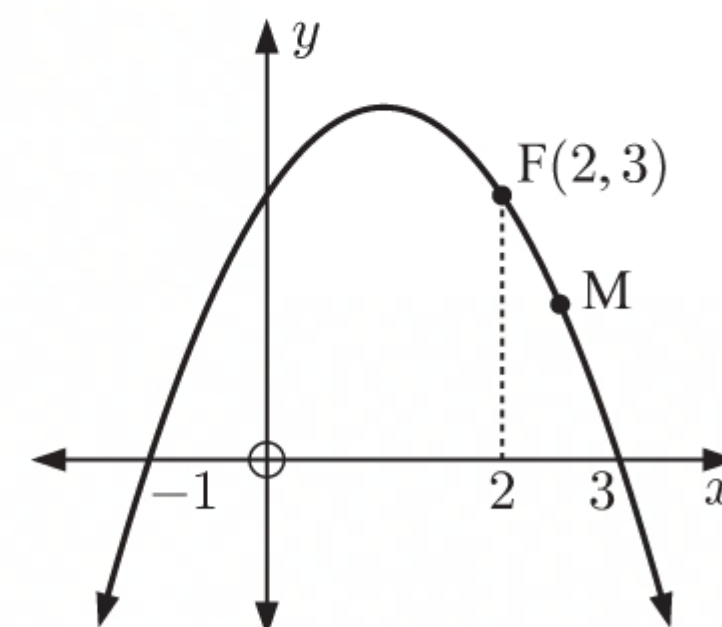
MIXED QUESTIONS SET 4

- 1** Eliza walked 6.1 km (rounded to 1 decimal place) in 82 minutes (rounded to the nearest minute). Find the *lower* bound for Eliza's average speed, in km h^{-1} .

- 2** Consider the graph of $y = 3 + 2x - x^2$.

The point F lies on the curve. Let M be a point close to F with x -coordinate $2 + h$.

- What is the y -coordinate of M ?
- Find the gradient of $[FM]$ in terms of h .
- Hence find the gradient of the tangent to the curve at F .



- 3** The data below are the recent sale prices, in thousands of dollars, of houses in two neighbourhoods.

<i>Neighbourhood A:</i>	275	281	320	265	305	258	310	430	285
	290	297	345	195	230	269	300	258	273
<i>Neighbourhood B:</i>	325	300	412	370	297	505	340	333	290
	428	305	520	360	410	275	320	431	410

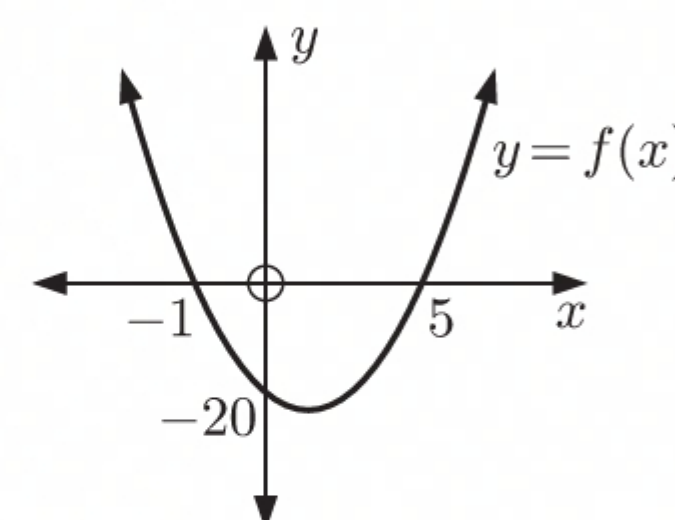
- Is the data discrete or continuous?
 - Use technology to find the five-number summary for each data set.
 - Display the data in a parallel box plot.
 - Compare and comment on the distributions of each data set.
- 4** Bags of rice are sold at a Jakartan wholesale market. The price per bag, P , if b bags are bought is shown below:

b (bags)	30	35	40	45	50
P (rupiah)	38 000	36 000	34 000	32 000	30 000

- Determine the function $P(b)$.
- Hence predict the total cost of purchasing 60 bags of rice.
- Do you think this model can be used to predict the cost of 150 bags of rice? Explain your answer.

- 5** The function f can be written in the form $f(x) = a(x - p)(x - q)$ where $p > q$.

- Write down the values of p and q .
- Find a .
- Write down the equation of the axis of symmetry.



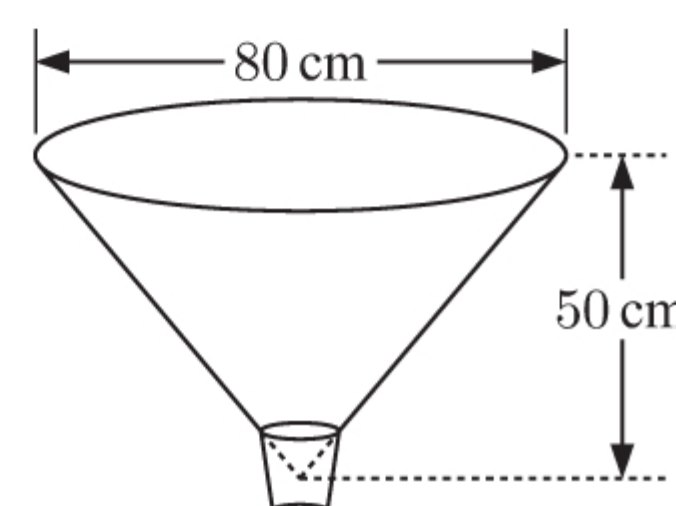
- 6** Trains A and B are 10 km apart, and are approaching the same train station.

Train A is 8 km from the train station on the bearing 071° . Train B is on the bearing 296° from the train station.

- Display this information on a diagram.
- Find the bearing of train B from train A.
- Train B is travelling at an average speed of 7 m s^{-1} . Find, to the nearest second, the time it will take for train B to reach the train station.

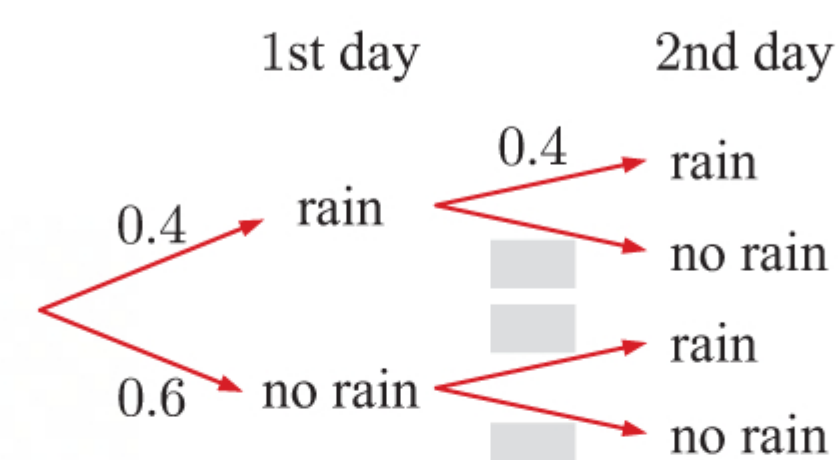
- 7** A conical funnel is 80 cm wide and 50 cm high.

- Estimate the capacity of the funnel in mL. Write your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.
- The funnel is half full with liquid, and its contents are poured into a cylindrical tube 20 cm wide. How high up the tube will the liquid reach?



- 8 A block of land is for sale for €225 000 with a 10% deposit required. Finance can be arranged over ten years at 5.99% p.a. interest compounded quarterly.
- Calculate the quarterly repayments necessary.
 - How much interest will be charged on the loan over the ten years?
 - Assuming inflation averages 3.5% per year, calculate the expected value of the block after ten years.

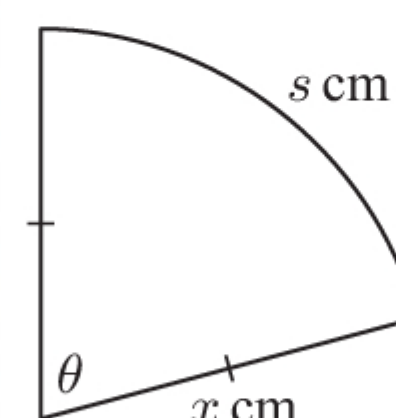
- 9 The probability of rain falling on any day in Dunedin is 0.4. The tree diagram shows the possible outcomes when two consecutive days are considered.



- Complete the tree diagram by filling in the missing probabilities.
- Hence determine the probability of:
 - rain on both days
 - no rain on exactly one day.
- Given that rain fell on at least one day, find the probability of rain on the second day.

- 10 A 40 cm piece of wire is bent to form a sector of a circle with radius x cm.

- Write θ in terms of x .
- Show that the area of the sector is given by $A = 20x - x^2$ cm².
- Find x and θ for which A is a maximum.



- 11 Before it is turned on, a refrigerator has an internal temperature of 27°C. Three hours later it has cooled to 6°C.

The internal temperature T (in °C) of the refrigerator t hours after being turned on is given by the function $T(t) = A \times B^{-t} + 3$, where A and B are constants.

- Determine the value of:
 - A
 - B .
- Find the internal temperature of the refrigerator 5 hours after being turned on.
- Write down the minimum temperature that the refrigerator could be expected to reach.

- 12 A company claims that their *Mega* speakers have a longer battery life than their rival's *Micro* speakers.

To test this claim, the battery lives of 12 *Mega* speakers and 10 *Micro* speakers were measured in hours.

Mega: 22.4, 23.5, 24.1, 22.3, 23.4, 22.9, 22.7, 21.4, 20.9, 22.1, 23.8, 22.9

Micro: 20.8, 21.2, 22.1, 20.7, 21.4, 22.2, 21.7, 23.5, 21.5, 22.5

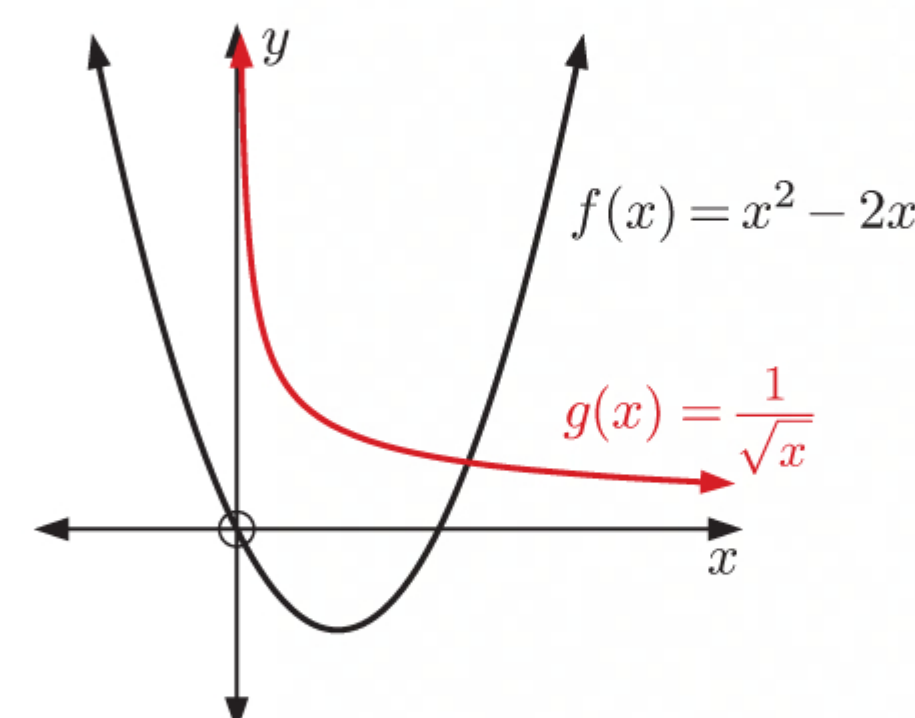
A two-sample t -test is performed at a 10% level of significance to determine whether the company's claim is valid.

- Write down the null and alternative hypotheses for this test.
- Calculate the test statistic and p -value.
- Determine whether the company's claim is valid.

MIXED QUESTIONS SET 5

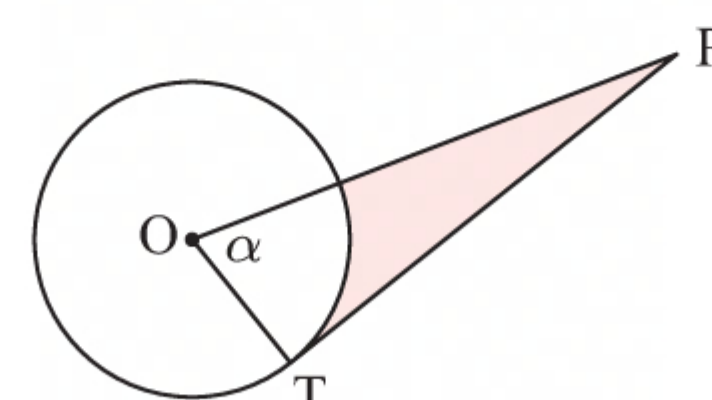
- 1 The graphs of $f(x) = x^2 - 2x$ and $g(x) = \frac{1}{\sqrt{x}}$ are shown alongside.

- Find $f(1)$ and $g(1)$.
- Explain why g is invertible but f is not.
- Find x such that $g^{-1}(x) = 4$.



- 2 [PT] is a tangent to the given circle. The circle has radius 9 cm and $OP = 30$ cm. Find:

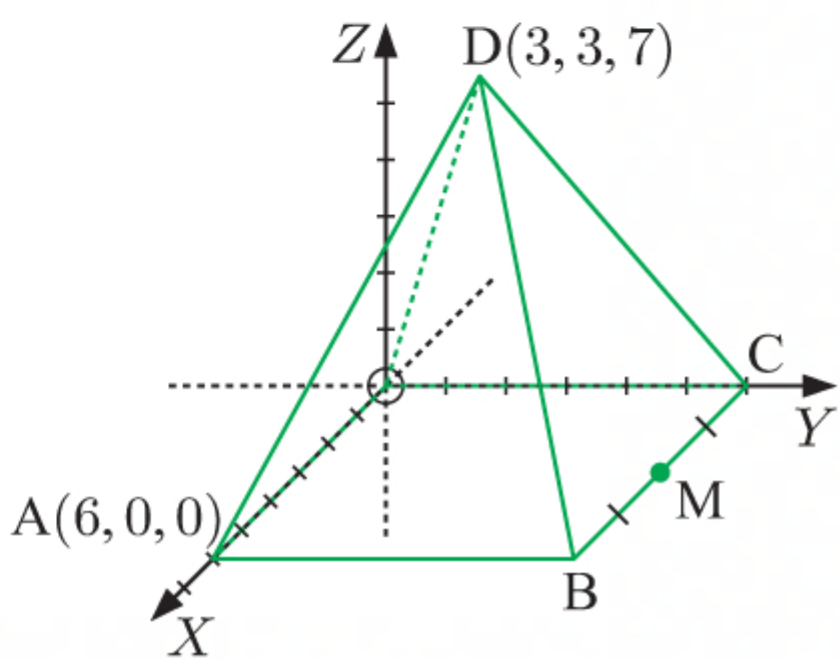
- α
- the area of the shaded region.



- 3 Twins Pierre and Francesca were each given \$100 on their 10th birthday. They immediately put their money into their individual money boxes. Each week throughout the next year they added a portion of their weekly pocket money. Pierre added \$10 each week. Francesca added 50 cents the first week, \$1 the next, \$1.50 the next, and so on, adding an extra 50 cents each subsequent week.
- a How much did Francesca add to her money box in the last week before her 11th birthday?
 - b Find the total amount that each child had added to his or her money box after 8 weeks.
 - c Who had more money in their money box after one year? Explain your answer.

4 Consider the square-based pyramid alongside. Find:

- a the coordinates of B and C
- b the volume of the pyramid
- c the coordinates of M
- d the surface area of the pyramid.



- 5 a Find the equation of the tangent to the curve $y = x^3 - 5$ at the point $(1, -4)$.
- b Where does this tangent cut the x -axis?
- 6 This table shows the surface area and fish population of eight lakes in a particular region.

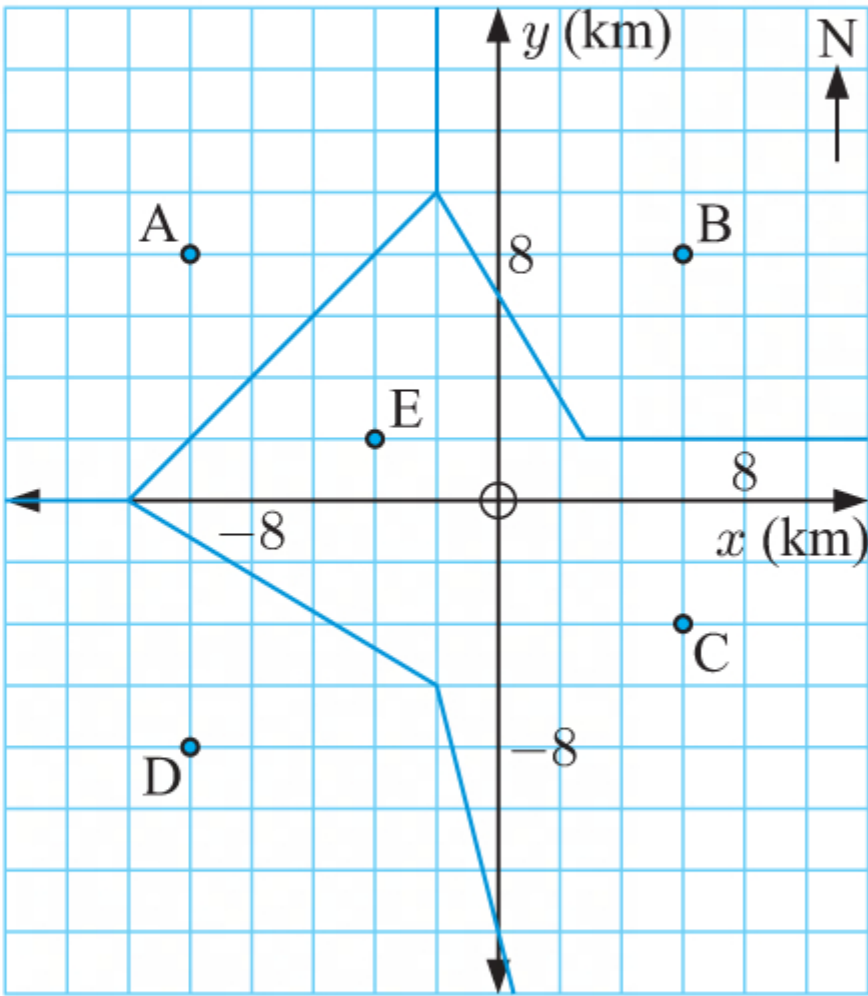
Lake	A	B	C	D	E	F	G	H
Surface area (x hectares)	25	10	35	16	19	27	14	16
Population (y)	5620	840	6125	1280	1805	3645	980	1110

- a Find Pearson’s product-moment correlation coefficient r_p .
 - b Copy and complete this table of ranks:
- | Lake | A | B | C | D | E | F | G | H |
|-------------|---|---|---|---|---|---|---|---|
| rank of x | | 1 | 8 | | | | | |
| rank of y | | 1 | | | | | 2 | |
- c Calculate Spearman’s rank correlation coefficient, r_s .
 - d Use r_p and r_s to describe the relationship between x and y .
 - e Suppose that, due to a recording error, the population of lake D was 1180 instead of 1280. Explain why this does not affect the value of r_s .

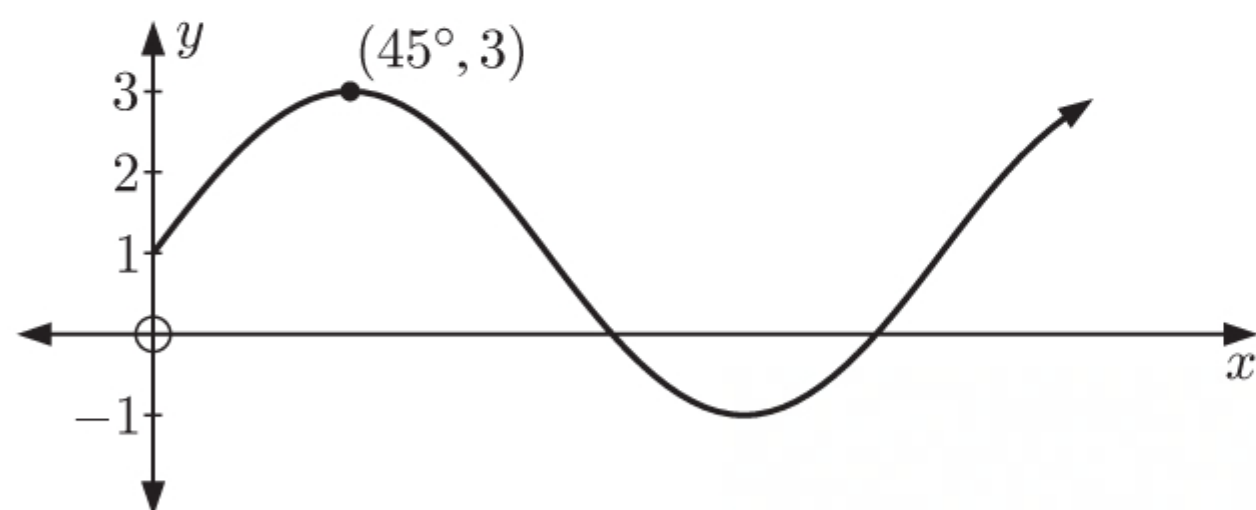
- 7 Suppose $f'(x) = (x^2 + 2)^2$ and that $f(1) = \frac{8}{15}$. Find $f(x)$.
- 8 A university club committee holds weekly meetings. Each committee member has a 70% chance of attending a given meeting. A meeting can only go ahead if at least 10 committee members are present.
- a If the club has 15 committee members, what percentage of meetings will go ahead?
 - b Find the smallest number of committee members required to ensure that at least 90% of the meetings will go ahead.

9 This incomplete Voronoi diagram shows petrol stations A, B, C, D, and E in a city.

- a Find the equation of the missing edge. Give your answer in the form $ax + by + d = 0$, $a, b, d \in \mathbb{Z}$.
- b In the context of the question, explain the significance of cell D.
- c Riley is currently equally closest to stations C and E, and is due south of O.
 - i Find Riley’s location.
 - ii Riley’s car has 6 km of petrol left. Will Riley be able to drive to a petrol station before his car runs out of petrol?



10

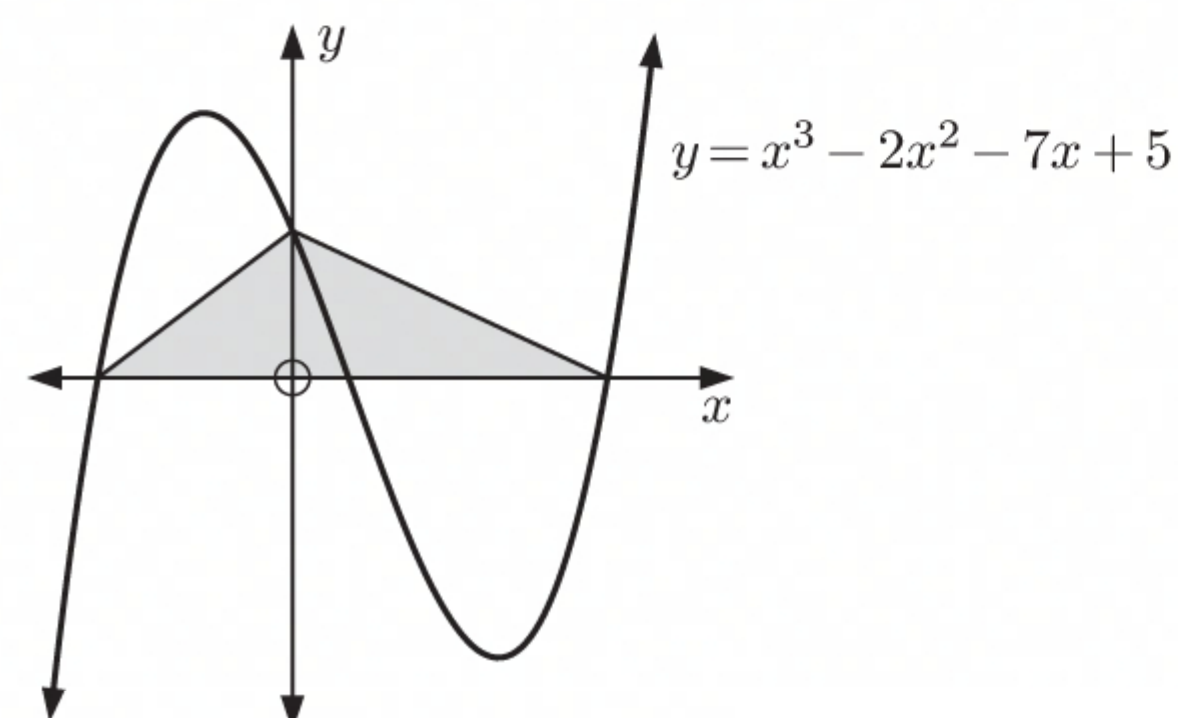


Find the equation of the sine function shown in the graph.

11

 a Use technology to find all solutions to $x^3 - 2x^2 - 7x + 5 = 0$.

b Hence find the area of the shaded triangle.



12

The distance travelled by two similar toy cars after rolling down a slope was measured 40 times each. The measurements were rounded to the nearest tenth of a metre.

Red car	3.6	4.6	5.6	6.4	4.2	5.3	6.1	4.5
	5.4	4.6	3.9	6.2	5.8	4.5	5.4	6.1
	4.5	5.6	5.7	4.8	3.9	5.6	6.1	5.9
	4.1	5.3	4.2	6.2	7.4	5.4	5.8	4.5
	3.9	5.4	5.7	4.8	5.4	5.7	6.1	6.4

Blue car	Number of rolls	40
	Median distance	4.8 m
	Shortest distance	3.2 m
	Longest distance	6.7 m
	Q_1 Lower quartile	4.1 m
	Q_3 Upper quartile	5.4 m

- Complete this table of cumulative frequencies for the red car data.
- Draw the cumulative frequency graph for the distance travelled by the red car.
- Use the graph to find the following statistics for the red car:
 - median distance
 - lower quartile
 - upper quartile
- Draw a parallel box and whisker diagram to display the data for both cars.
- Compare the statistics for distance travelled by the two toy cars. Is it reasonable to assume that the same machine manufactured these two toys? Explain your answer.

Distance (m)	Cumulative frequency
$3.5 \leq d < 4$	
$4 \leq d < 4.5$	
$4.5 \leq d < 5$	
$5 \leq d < 5.5$	
$5.5 \leq d < 6$	
$6 \leq d < 6.5$	
$6.5 \leq d < 7$	
$7 \leq d < 7.5$	

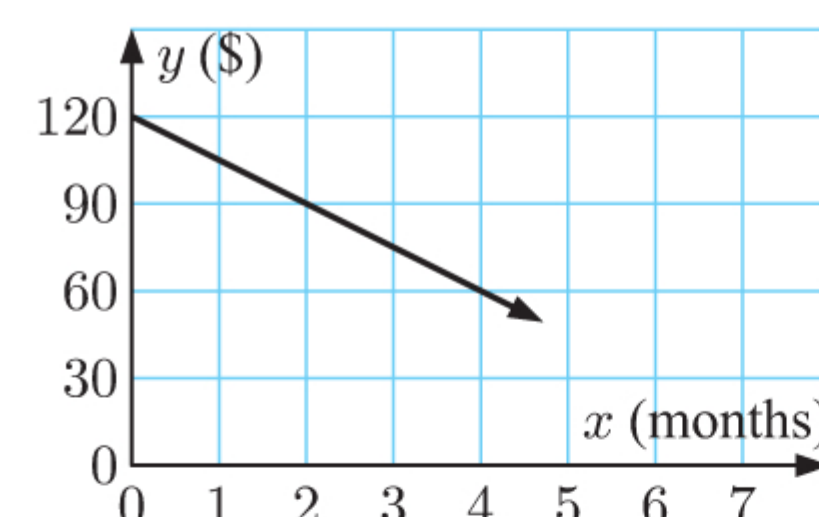
MIXED QUESTIONS SET 6

 1 Let $f(x) = 3 - 4^{-x}$.

- Points $A(2, p)$ and $B(-2, q)$ lie on $y = f(x)$. Determine p and q .
- For the graph of $y = f(x)$, determine the:
 - y -intercept
 - equation of the horizontal asymptote.
- Sketch the graph of $y = f(x)$, showing all details from above.
- Write down the range of $f(x)$.

 2 Michael purchases a music subscription at the start of the year. The graph shows the amount of money left in the subscription account after x months.

- Find the gradient and y -intercept of the line, and interpret your answers.
- Find the equation of the line.
- How long will it take for the account to run out of money?

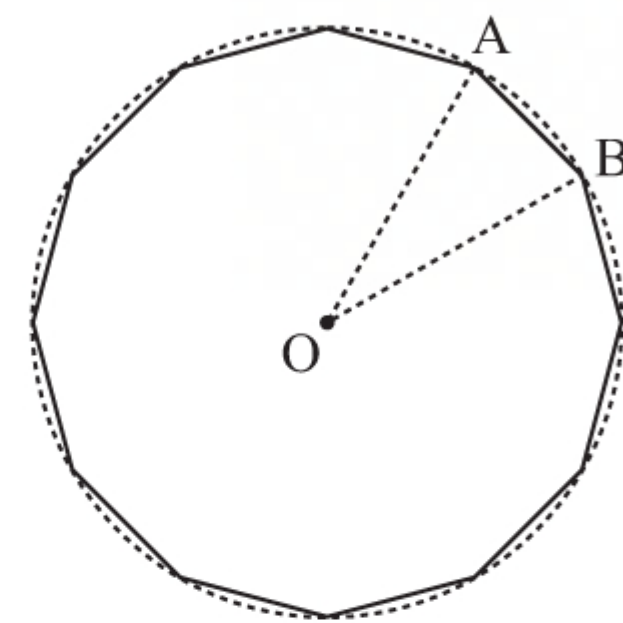


- 3 The normal to $y = \frac{a}{x} + 3$ at the point where $x = 2$ has gradient 2.

- Find a .
- Find the coordinates of the point where the normal at $x = 2$ meets the curve again.

- 4 A regular dodecagon (12-sided polygon) is inscribed in a circle of radius 6 cm. Points A and B are adjacent vertices of the dodecagon, and both lie on the circle.

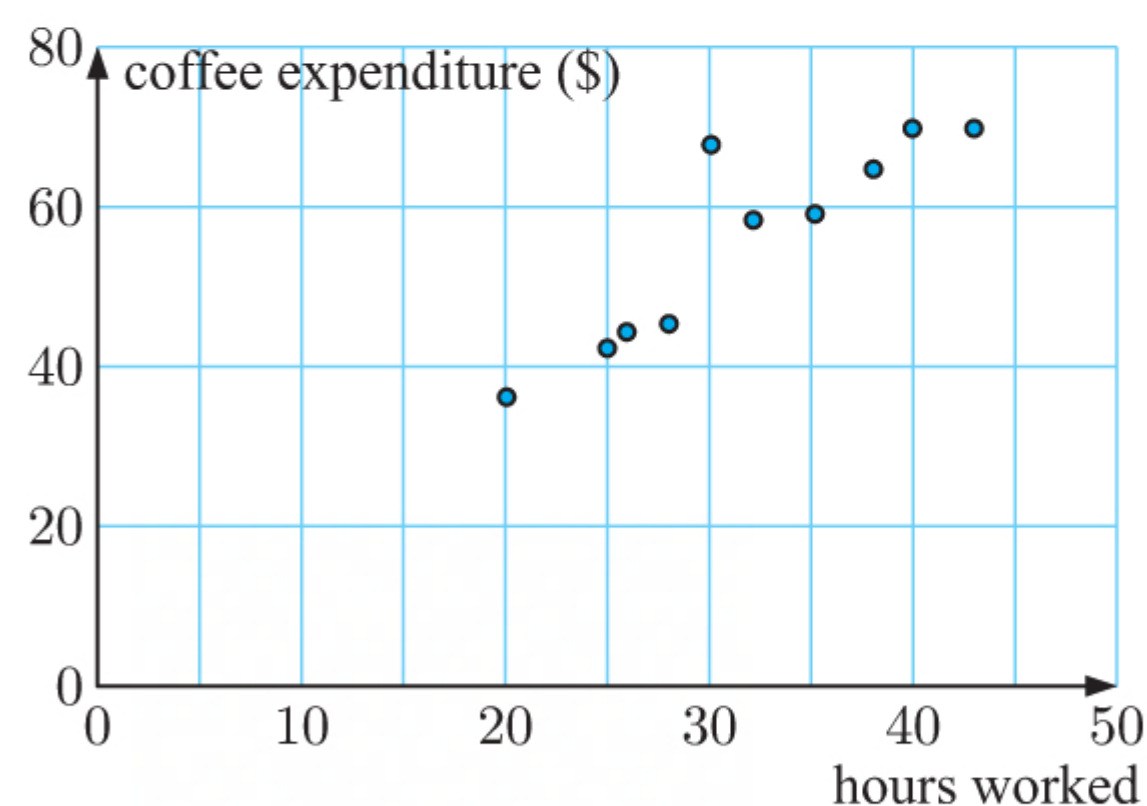
- Deduce that $\widehat{AOB} = 30^\circ$.
- Show that the area of triangle $AOB = 9 \text{ cm}^2$.
- Hence determine the area of the dodecagon.



- 5 This scatter diagram displays the amount James spends on coffee in the cafeteria against the number of hours he works in the week.

- James worked an average of 32 hours, and his average expenditure was \$56 per week. Plot the mean point $P(32, 56)$ on the graph.
- Draw a line of best fit by eye which passes through P.
- Use this line to predict the amount James will spend on coffee if he works a 35 hour week.
- Describe the nature and strength of the linear relationship between the variables. Comment on whether the prediction in c is reliable.

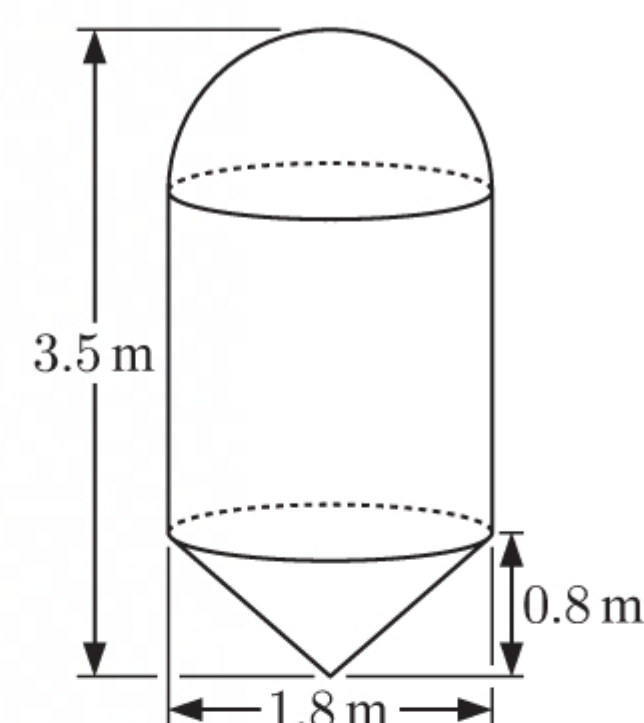
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GRAPH



- 6 A manufacturer states that the contents of its cereal boxes weigh an average of 320 g. A random sample of 24 boxes was weighed, with the following results recorded in grams:

312	320	326	330	326	322	326	330	331	315	323	316
315	325	311	320	308	325	320	332	316	309	314	324

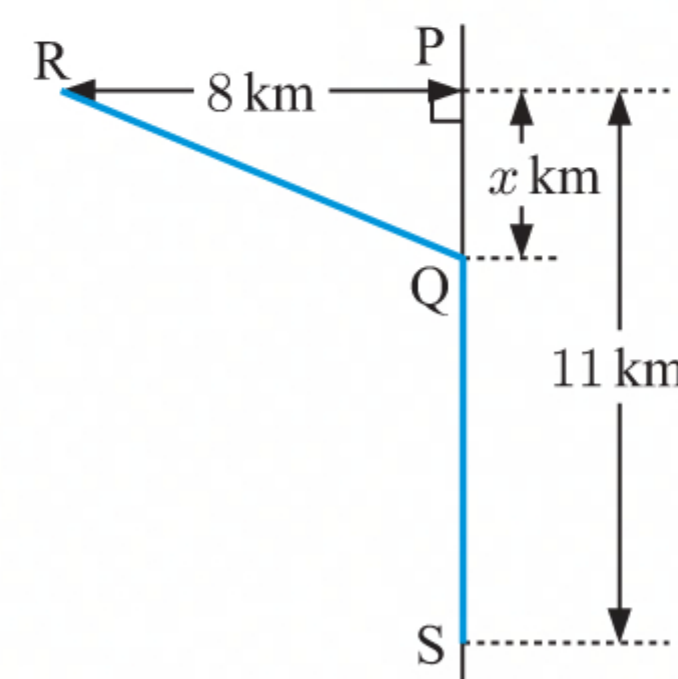
- Organise the data using a frequency table, with the class intervals $305 \leq w < 310$, $310 \leq w < 315$, and so on.
 - Draw a frequency histogram to display the data.
 - Describe the distribution of the data.
 - Find the modal class of this data.
 - Calculate the mean of the data. How does it compare to the manufacturer's claim?
- 7 A silo is made out of sheet metal using a hemisphere, a cylinder, and a cone.
- Find the height of the cylinder.
 - Find the total amount of sheet metal used.
 - Find the capacity of the silo in kL.



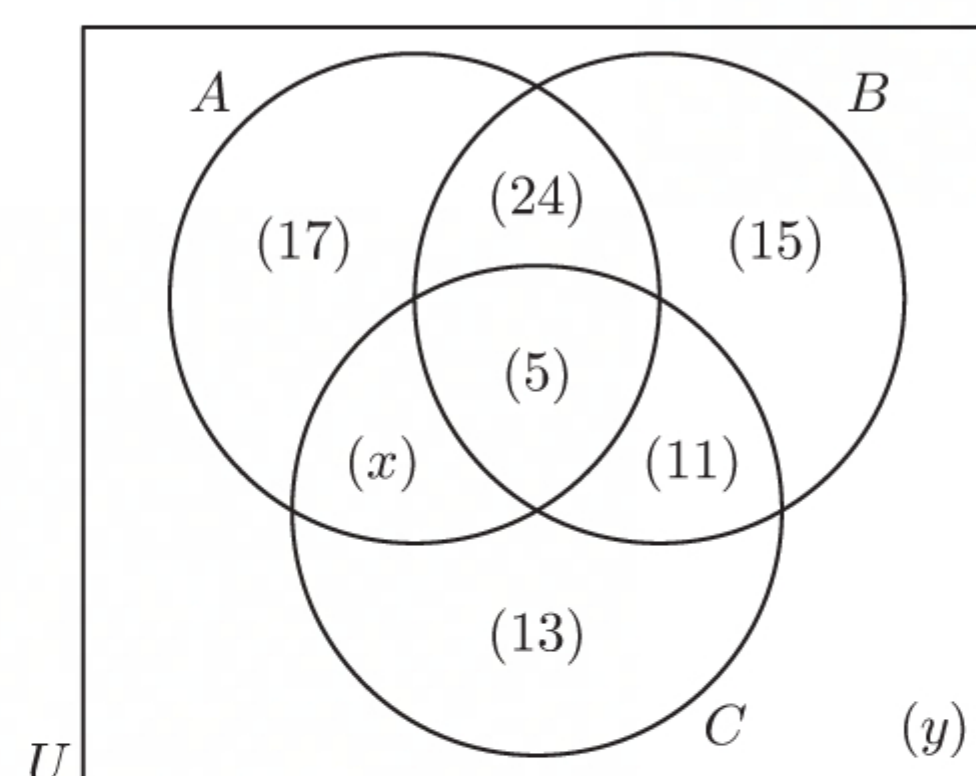
- 8 Let $P(x)$ be the number of prime numbers less than or equal to x . It is known that $P(x) \approx \frac{x}{\ln x}$ for large values of x .
- Estimate the number of prime numbers less than or equal to 1 000 000.
 - Given that there are actually 78 498 prime numbers less than or equal to 1 000 000, calculate the percentage error in your estimate in a to 1 decimal place.
 - Find x such that $P(x) \approx 2000$, and interpret your answer.
- 9 When an object falls from rest, the *distance* it has travelled is directly proportional to the square of the *time taken*.
An object dropped from rest travels 19.6 m in 2 seconds.
- How far will the object travel in 3 seconds?
 - How long will it take for the object to fall 100 m?

- 10** An offshore oil rig is at point R, 8 km from a straight shore. The point P is on the shore directly opposite the rig. A refinery is on the shore at S which is 11 km from P.

A pipeline is to be constructed under the sea from R to reach the shore at the point Q. From Q a pipeline is to be taken overland to S. The cost of the pipeline is \$5 million per km under the sea and \$3 million per km overland.

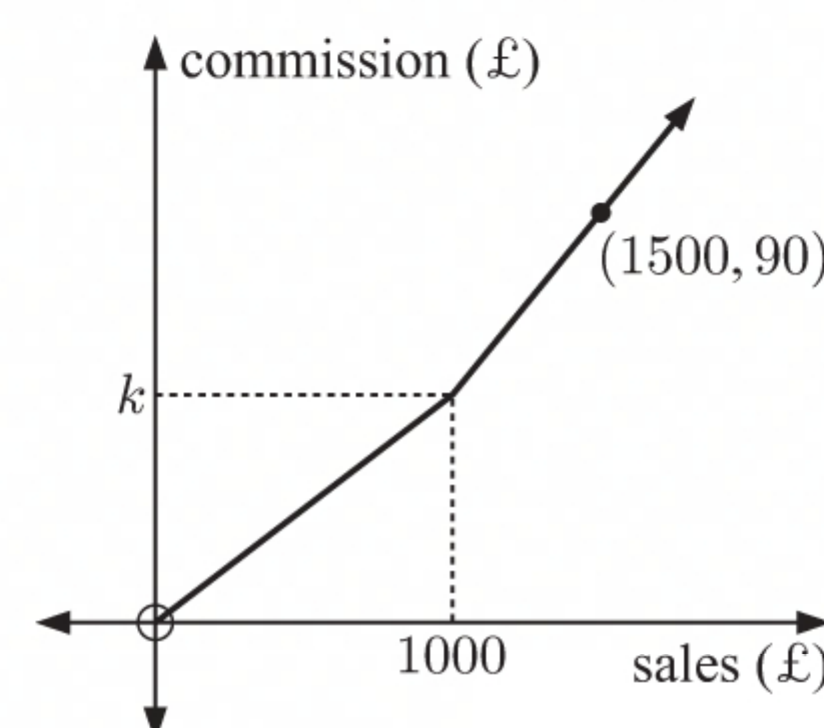


- a** If Q is x km from P, show that the cost to construct the pipeline from R to S is $C(x) = 5\sqrt{x^2 + 64} + 33 - 3x$ million dollars.
- b** Use technology to sketch the graph of $C(x)$ for $0 \leq x \leq 11$.
- c** Find the minimum cost of the pipeline.
- 11** Stig is 32 years old. He has \$97 000 in a savings fund that earns 4.9% p.a. interest compounding quarterly. He will make monthly contributions to the fund so that he can retire at age 55 years with \$1 000 000.
- a** How much should Stig contribute to his fund each quarter?
- b** How much interest is generated in the fund after age 32?
- c** After retiring at age 55 years, Stig rolls his \$1 000 000 in an annuity account earning 6.5% p.a. compounding monthly. How much can he withdraw per month if he wants his money to last for 30 years?
- d** Stig is used to living on \$2700 per month at age 32 and inflation has averaged 3.7% p.a. Will his standard of living be maintained at the time of his retirement? Explain your answer.
- 12** 100 diners at a restaurant were given a set three-course meal. After the meal, the diners were asked whether they liked each of the courses. The results are summarised alongside.
- a** Given that 48 people liked course A, find x and y .
- b** Which course was the most popular?
- c** Find the probability that a randomly selected diner liked:
- all of the courses
 - course B, but not course C
 - exactly two courses, given that the diner liked course C
 - none of the courses, given that the diner disliked course B.



MIXED QUESTIONS SET 7

- 1** Each day, a salesperson makes 5% commission on sales up to £1000, and a higher rate of commission on sales above £1000.
- a** Find the value of k .
- b** Find the higher rate of commission on sales above £1000.
- c** Find the commission earned when £1800 is made in sales.
- d** Find the value in sales needed to earn £150 commission.

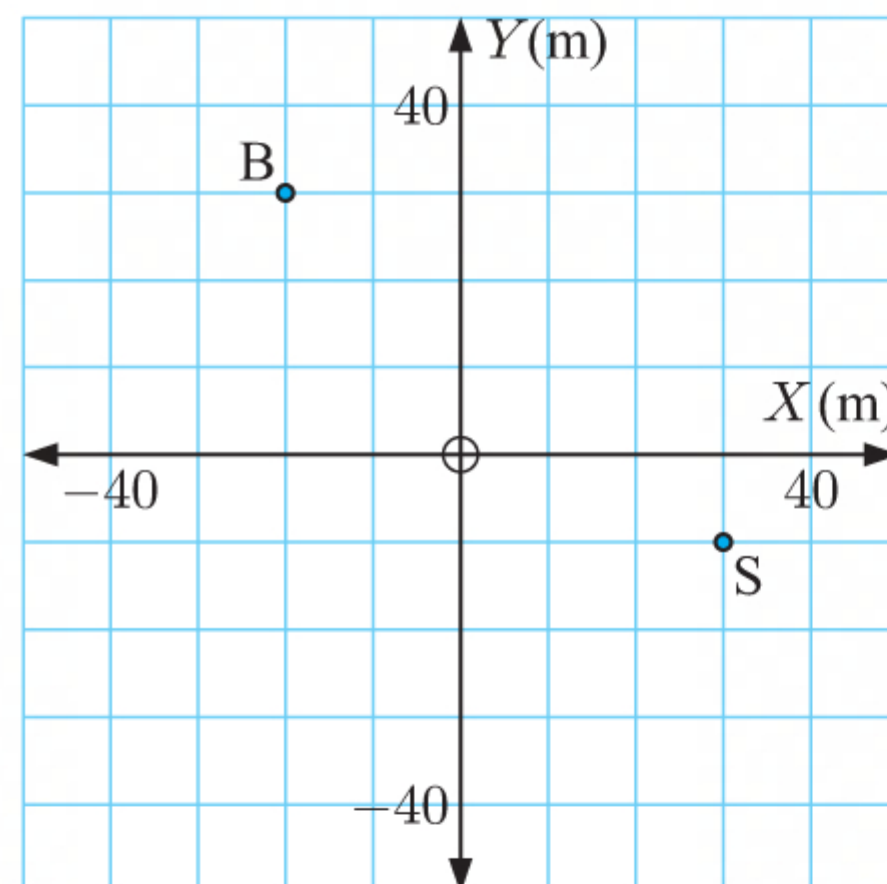


- 2** Cynthia invested \$2000 in an account that pays 4.4% p.a. interest compounded quarterly for 5 years.
- a** Find the final value of the investment.
- b** How much interest did Cynthia earn?
- c** Given that inflation averages 2.5% p.a. over the investment period, find the real value of the investment.

- 3** This grid shows the position of a boat B and a shipwreck S.

The boat's anchor is directly below the boat, 50 m below sea level. The shipwreck is 40 m below sea level. Suppose sea level has Z -coordinate 0.

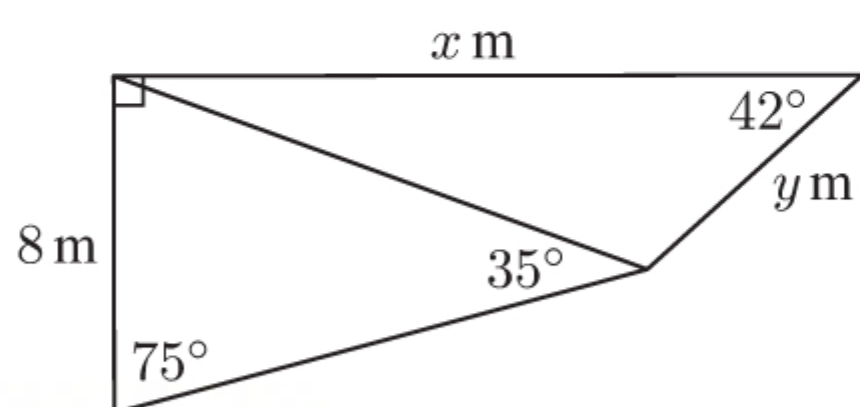
- a** Find the 3-dimensional coordinates of:
- the anchor
 - the shipwreck.
- b** A diver swims from the boat to the shipwreck. How far does the diver swim?
- c** Find:
- the angle of depression from the boat to the shipwreck
 - the angle of elevation from the anchor to the shipwreck.



- 4** 160 m of fence is used to enclose a rectangular field.

- a** Given that one side of the field has length x m, find the area of the field in terms of x .
- b** Find the dimensions of the field which would maximise the area.
- c** Suppose the actual area of the field is 1200 m^2 .
- Find the dimensions of the field.
 - The average production yield for this field is 6.5 kg m^{-2} . Determine the amount of production lost by not using the dimensions which maximise the area.

- 5** Find x and y in the given figure.



- 6** The heights of a sample of 80 children from a junior school were measured. The results are shown in the table alongside.

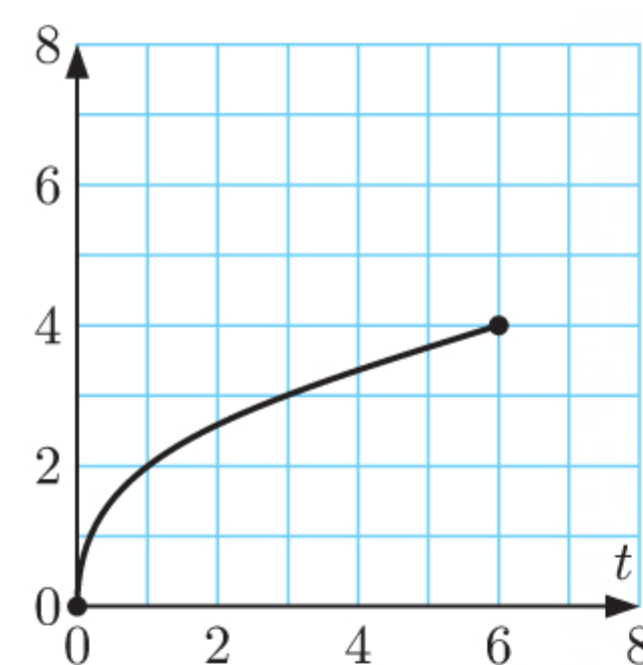
Estimate the: **a** mean **b** standard deviation.

Height (h cm)	Number of students
$80 \leq h < 90$	8
$90 \leq h < 100$	12
$100 \leq h < 110$	17
$110 \leq h < 120$	30
$120 \leq h < 130$	13

- 7** A and B are mutually exclusive events. If $P(B) = 0.3$ and $P(A \cup B) = 0.55$, find $P(A)$.

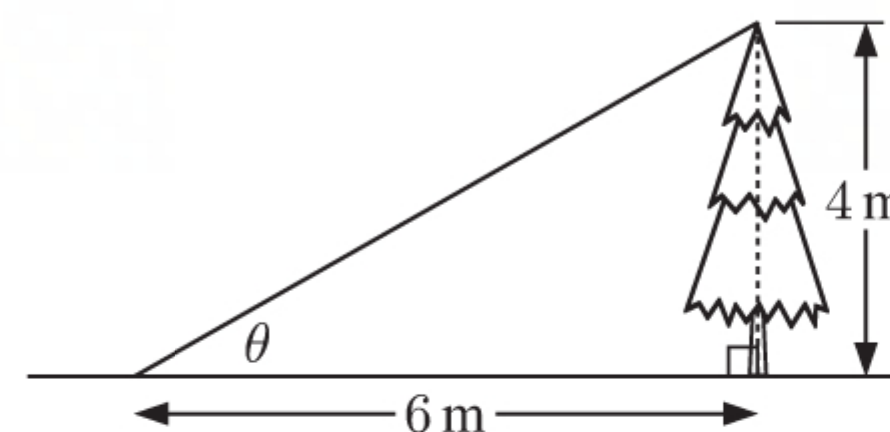
- 8** This graph shows the speed $S(t)$ of a cyclist during the first 6 seconds of motion.

- a** State the domain and range of $S(t)$.
- b** Copy the graph, and sketch the inverse function $S^{-1}(t)$.
- c** Find $S^{-1}(2)$, and explain its meaning.



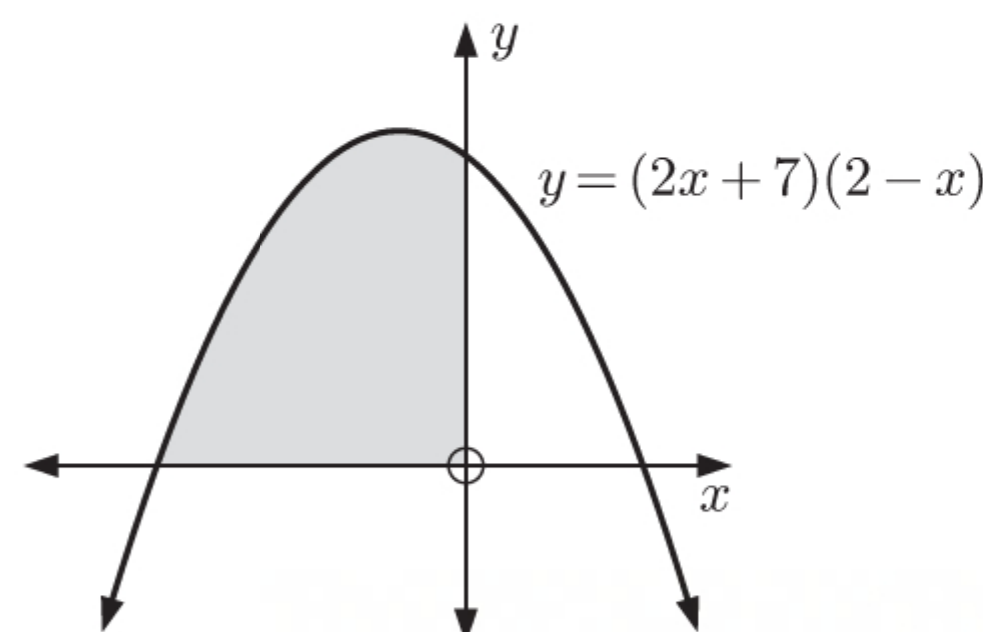
- 9** The measurements alongside have been rounded to the nearest metre.

- a** Use these measurements to estimate the angle of elevation θ .
- b** Find the boundary values of θ .
- c** Hence find the maximum percentage error in the estimate in **a**.



- 10** Consider the graph of $y = (2x + 7)(2 - x)$ shown alongside.

- a** Write down an integral for the area of the shaded region.
- b** Hence find the area of the shaded region.



11 This Voronoi diagram shows the locations of hospitals in a particular city.

a Copy the diagram and shade the cell with the missing hospital.

b Find the coordinates of the missing hospital X.

c Find the hospital closest to:

i $(-1, 2)$

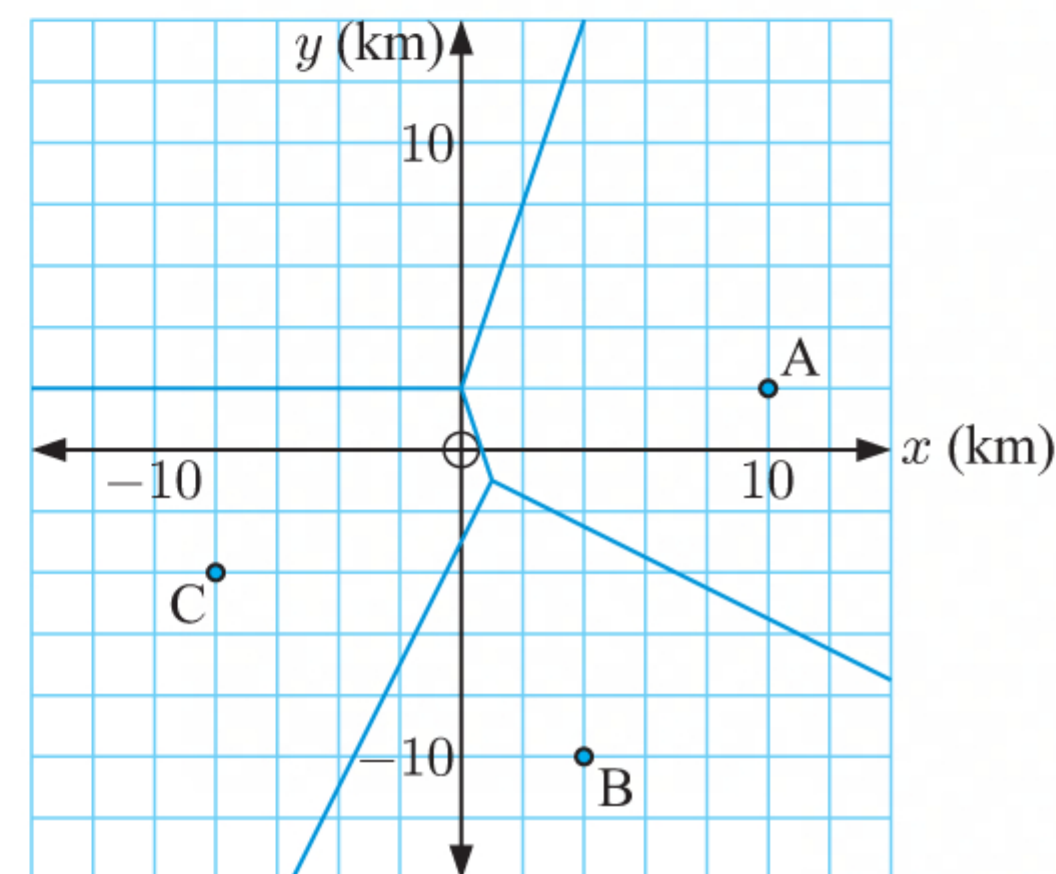
ii $(4, -1)$

d An ambulance is transferring a patient from hospital A to hospital X via the patient's house at $(2, 8)$. Assume the ambulance travels in a straight line for each leg of the journey.

i Find the distance from hospital A to the patient's house.

ii Explain why the distance in **i** is the same as the distance from the patient's house to hospital X.

iii The ambulance travels at an average speed of 40 km h^{-1} over the whole journey. Determine the total travel time to the nearest minute.



12 A bag contains 6 red balls and 4 white balls. A game is played in which the player draws 3 balls from the bag without replacement. The player wins if 3 red balls are drawn.

a Find the probability of the player winning a single game.

b Let X be the number of wins when the game is played 60 times.

i Find the mean μ and standard deviation σ of X .

ii Find $P(X = \mu)$.

iii Find $P(\mu - \sigma \leq X \leq \mu + \sigma)$.

MIXED QUESTIONS SET 8

1 The first four terms of a geometric sequence are 0.125, 0.25, 0.5, and 1.

a Write down the common ratio r .

b Find the 20th term u_{20} .

c Find the sum of the first 10 terms.

2 Two fair dice are rolled, and the difference between the scores is noted.

a Display the possible results on a 2-dimensional grid.

b Hence find the probability that the difference between the scores is 4.

3 Consider the function $f(x) = x^3 - 3x^2 - x + 3$, where f is defined on the domain $-2 \leq x \leq 3$, $x \in \mathbb{R}$.

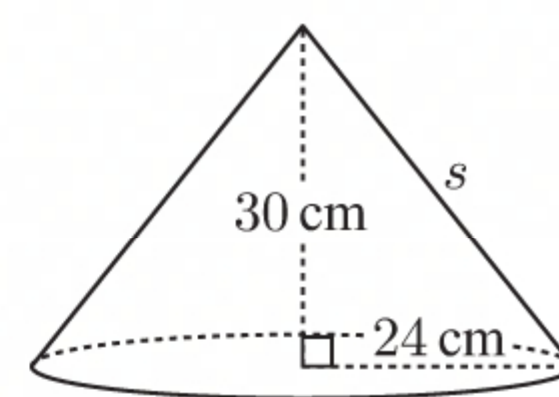
a Use technology to help sketch the graph of $y = f(x)$, showing any axes intercepts and turning points.

b Determine the range of f .

4 A solid right-circular cone has base radius 24 cm and vertical height 30 cm.

a Show that the slant height s is 38.4 cm, correct to 3 significant figures.

b Determine the total surface area of the cone. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.



5 A winemaker wants to examine the effect of weed spray in his vineyard. He randomly selects 50 sample spots, each of area 1 m^2 , and counts the number of weeds in each spot. The results are shown in the table alongside.

a Determine the value of p .

b Estimate the mean number of weeds per spot.

c What percentage of sample spots had fewer than 10 weeds?

Number of weeds	Frequency
0 - 4	9
5 - 9	15
10 - 14	10
15 - 19	p
20 - 24	5
25 - 29	2

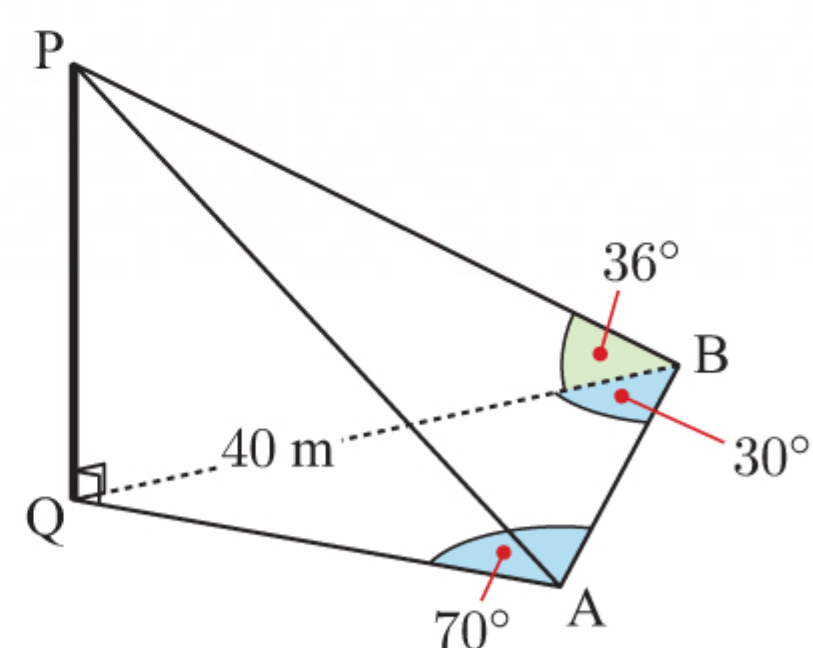
6 Consider the function $f(x) = \frac{6}{x^3} - \frac{2}{x}$.

a Show that $f'(x) = \frac{2(x+3)(x-3)}{x^4}$.

b Hence find intervals where $y = f(x)$ is increasing or decreasing.

- 8** The diagram shows a vertical pole [PQ], which is supported by two wires fixed to the horizontal ground at A and B. $\widehat{PBQ} = 36^\circ$, $\widehat{BAQ} = 70^\circ$, $\widehat{ABQ} = 30^\circ$, and the distance BQ is 40 m.

a the height of the pole
b the distance between A and B.



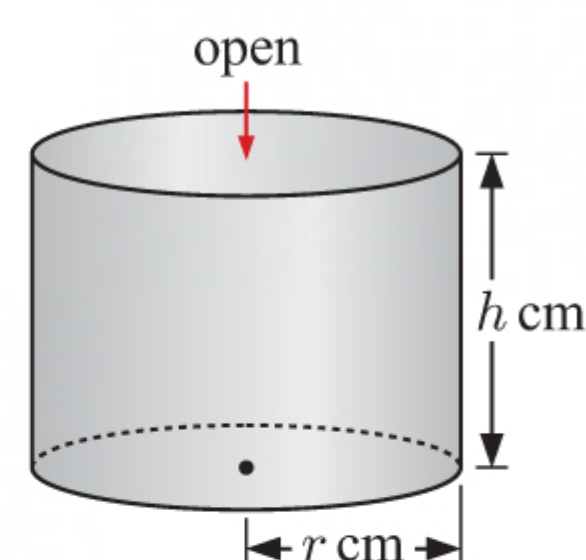
- She trials the longer bow with 10 shots, and the flight time in seconds for each arrow is recorded:

1.501	1.516	1.543	1.492	1.498
1.487	1.522	1.511	1.486	1.538

- a** State the null and alternative hypotheses.
- b** Calculate the:
i test statistic **ii** p -value.
- c** At the 1% significance level, determine whether the longer bow has decreased the flight time of Robin's arrows.

- a** Show that $h = \frac{500\,000}{\pi r^2}$.

- c** Use calculus to find the dimensions of the bin which minimises the amount of PVC plastic used.



- A bicycle wheel sits on the road so its valve is at the top. The tyre has inner radius 35 cm and outer radius 40 cm.

A diagram of a bicycle wheel with many spokes. A red circle is drawn around the hub. A red arrow points to a small valve on the rim. Another red arrow points to the bottom of the red circle. The wheel is shown on a grey ground surface.

- Find: **i** a **ii** d **iii** b

- | | | | | | |
|--------------------------|-------|-------|--------|--------|--------|
| θ | 3° | 22° | 37° | 52° | 74° |
| $A \text{ (m}^2\text{)}$ | 10.48 | 80.81 | 150.71 | 255.99 | 697.48 |

-

b David thinks that $A \propto \tan \theta$.

i Copy and complete the table.

θ	3°	22°	37°	52°	74°
$\tan \theta$					
$A \text{ (m}^2\text{)}$	10.48	80.81	150.71	255.99	697.48

ii Hence use technology to obtain a power model connecting A and $\tan \theta$.

iii Does the power model support David's claim?

iv Use the power model to estimate A when $\theta = 43^\circ$.

MIXED QUESTIONS SET 9

1 A quadratic function has the form $f(x) = ax^2 + bx + 7$. It is known that $f(2) = 7$ and $f(4) = 23$.

a Construct a set of simultaneous equations involving a and b .

b Find a and b .

c Hence calculate $f(-1)$.

2 Hayley and Patrick were training for a road cycling race. During the first week they both cycled 60 km. Hayley cycled an additional 20 km each subsequent week, whereas Patrick increased his distance by 20% each subsequent week.

a How far did each of them cycle in the 5th week of training?

b Who was the first to cycle 210 km in one week?

c Who cycled a greater total distance in the first 12 weeks? Explain your answer.

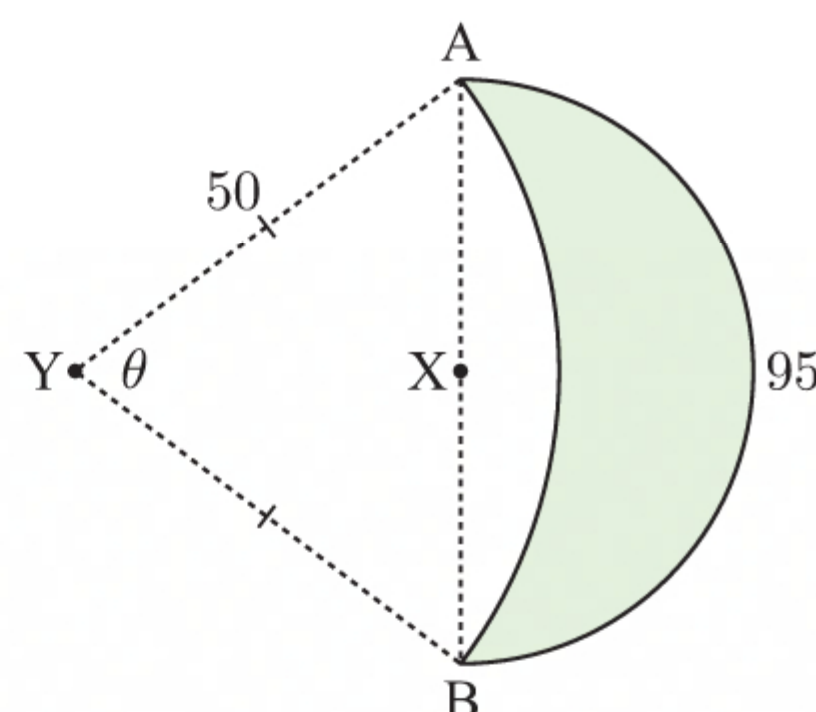
3 X and Y are the centres of the two arcs AB shown.

Find:

a the length AX

b the angle θ

c the shaded area.



4 a Use the trapezoidal rule to estimate the area between $f(x) = xe^x$ and the x -axis from $x = 0$ to $x = 1$ using:

i 2 subintervals

ii 6 subintervals.

b Given that the exact area is 1 unit², discuss the accuracy of your estimates in **a**.

5 The masses of sea lions on a particular island are normally distributed with mean 700 kg and standard deviation 80 kg.

a Given that 65% of the sea lions weigh less than a kg, find a .

b Find the probability that a randomly selected sea lion weighs more than 600 kg.

c Let Y be the number of sea lions in a group of 20 who weigh less than 600 kg.

i Find the mean and standard deviation of Y .

ii Find $P(Y > 3)$.

6 The following data shows Craig's weekly grocery bills, in dollars, for the last 5 months.

181, 155, 163, 200, 149, 185, 160, 159, 164, 171,
173, 212, 303, 191, 169, 161, 207, 140, 132, 165

a Find the median, lower quartile, and upper quartile of the data set.

b Find the interquartile range of the data set.

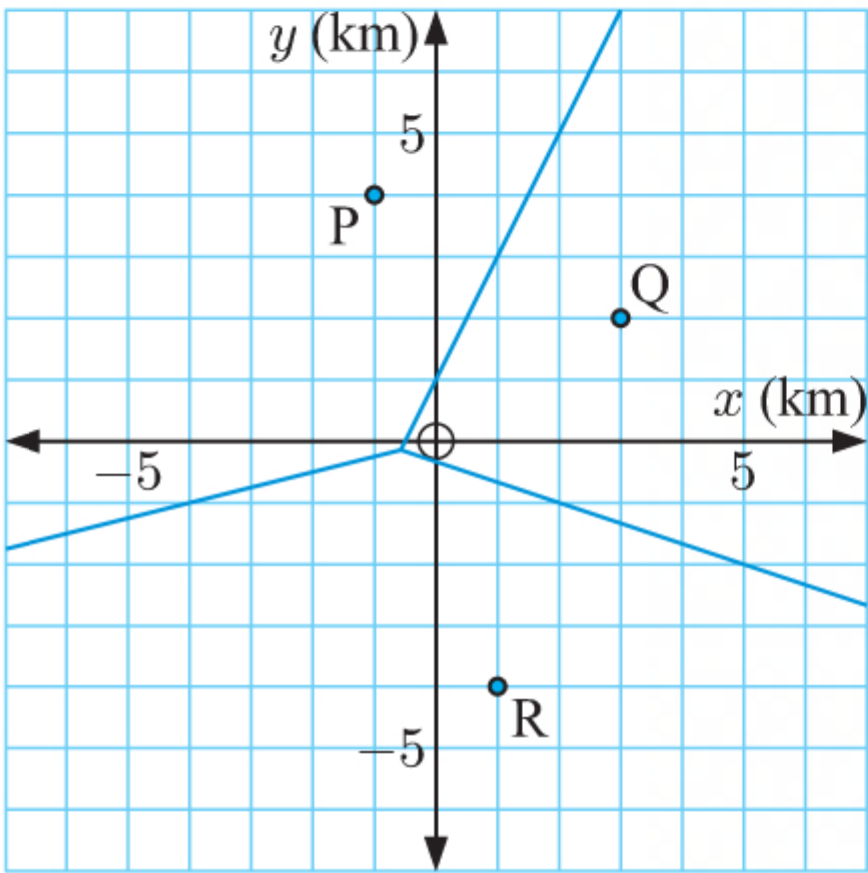
c The bill of \$303 occurred when Craig bought groceries for a large Christmas lunch. Show that this value is an outlier.

d Draw a box plot of the data set.

- 7 The weight of a radioactive substance after t years is given by $W(t) = 5 \times (0.965)^t$ grams, $t \geq 0$.
- a Find the percentage decrease in weight of the substance each year.
 - b Find the weight of the substance after 300 years. Write your answer in the form $a \times 10^k$ where $1 \leq a < 10$, $k \in \mathbb{Z}$.
 - c How long will it take for the weight to fall below 1 g?

- 8 The lead concentration of soil was measured at sites P, Q, and R on a farm. The results are shown below, in parts per million.

Site	Lead concentration (ppm)
P	28
Q	47
R	62



- a Use nearest neighbour interpolation to estimate the lead concentration at:
 - i $(2, 4)$
 - ii $(-2, -3)$
 - iii $(-4, -1)$
 - b An additional measurement revealed a lead concentration of 55 ppm at $S(-1, -6)$.
 - i Redraw the Voronoi diagrams with site S added.
 - ii Where necessary, update your estimates in a.
- 9 Consider the cubic function $y = x^3 + ax^2 + bx + 3$.
- a The tangent to the function at $(1, 8)$ has equation $y = 2x + 6$. Determine the values of a and b .
 - b Find the equation of the normal to the function at $x = -1$.
- 10 Raman transfers €500 000 of savings into an annuity fund which earns 4.6% p.a. interest compounded monthly. She wants to withdraw €3000 per month.
- a How long will her money last?
 - b Find the balance of the fund after 4 years.
 - c After 4 years, Raman decides that she only needs her money to last a further 15 years. How much can she withdraw each month for the remaining 15 years?
- 11 ABC is an equilateral triangle with sides 10 cm long. P is a point within the triangle. P is 7 cm from A and 4 cm from B.
- a Draw a diagram clearly showing all of the information provided.
 - b Calculate:
 - i \widehat{BAP}
 - ii \widehat{CAP}
 - c Hence find the length of [CP].

- 12 Students in an art course were asked to select one specialisation: painting, sketching, or sculpting. The number of students who chose each activity is shown alongside.

A χ^2 test for independence is to be used to determine whether gender is independent of the specialisation chosen. The test is performed at the 1% significance level, with corresponding critical χ^2 value 9.210.

	Painting	Sketching	Sculpting
Male	30	35	15
Female	20	15	25

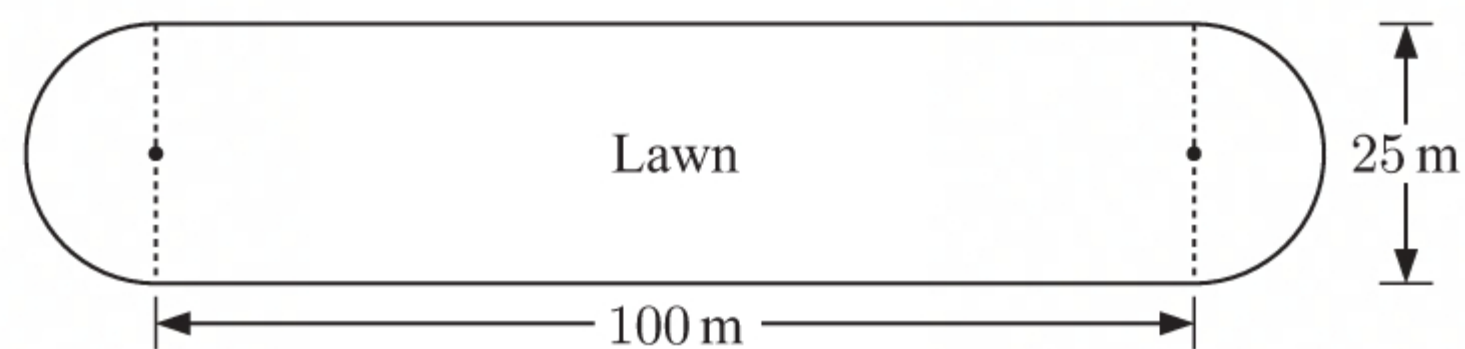
- a Assuming gender is independent of the specialisation chosen, find the expected number of male sculptors.
- b Calculate the value of the χ^2 test statistic.
- c Hence determine whether gender is independent of the specialisation chosen. Explain your reasoning.

MIXED QUESTIONS SET 10

- 1 Gordon needs to fertilise the large lawn shown in the diagram.

The ends of this lawn are semi-circular, and the middle is rectangular.

Gordon calculates the area of the lawn using correct formulae, but approximates the value of π to one significant figure, so $\pi \approx 3$.



- Show that the area of the lawn is $(156.25\pi + 2500) \text{ m}^2$.
 - Write down Gordon's approximation for the area of the lawn. Give your answer to the nearest whole number.
 - Calculate the percentage error in Gordon's approximation. Round your answer to 2 significant figures.
- 2 Two year 7 students are selected each week to hoist the flag before the start of class. Year 7 has been divided into 2 classes: class A has 30 students, and class B has 27 students.

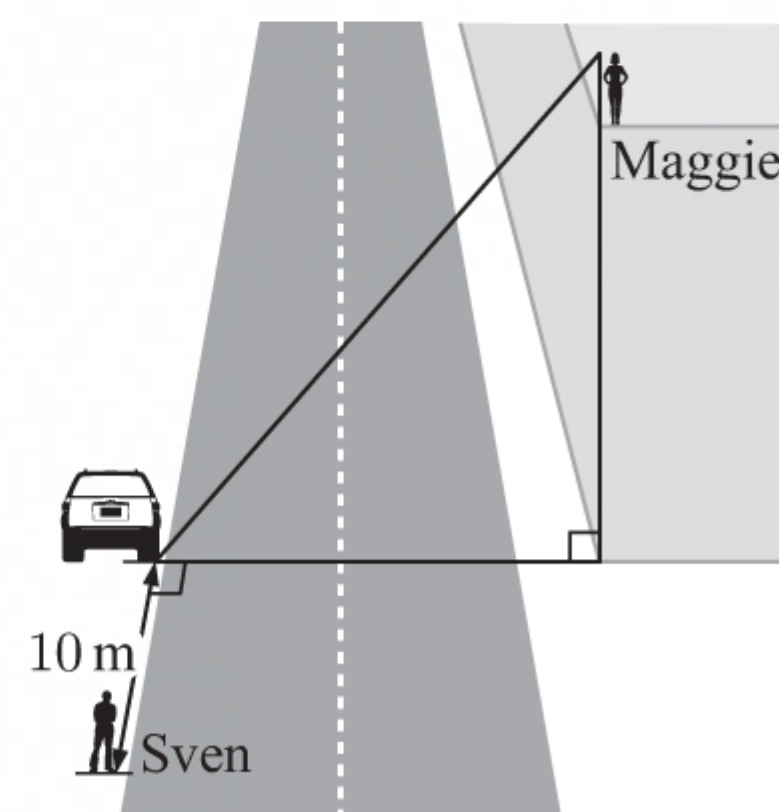
- Find the probability that, in any given week, the two selected students selected are in the same class.
- Over the course of 20 weeks, how many times do you expect that the two selected students are in the same class?

- 3 The volume of water in a tank is given by $V(t) = 10t^2 - \frac{1}{3}t^3$ litres, where t is the time in minutes and $0 \leq t \leq 30$.

- Find $V(5)$ and explain what this means.
- Find $V'(t)$. Do not forget to include units.
- Find t when $V'(t) = 0$.
- Find $V'(5)$ and $V'(25)$.
- Determine the time(s) at which the volume is increasing by 75 litres per minute.

- 4 Maggie is 155 cm tall and is standing on top of a building 50 m tall. A car is parked on the far kerb of the road, directly opposite Maggie. To see the car, Maggie looks down at an angle 67° below horizontal.

- How far is the car from the base of the building?
- Maggie's friend Sven is walking on the same side of the road that the car is parked. He is currently 10 m from the car.
 - Find the distance between Maggie and Sven.
 - At what angle must Sven look up to see Maggie?



- 5 A scientist measures the pressure of a fixed mass of gas, as the volume it occupies is increased.

Volume ($V \text{ m}^3$)	0.5	0.8	1.2	1.5	2.4
Pressure ($P \text{ kPa}$)	7.2	4.5	3	2.4	1.5

- Draw a scatter diagram of the data. Explain why an inverse variation model appears to be appropriate.
 - Given V and P are inversely proportional, determine the model connecting V and P .
 - Find the pressure of the gas when it occupies 3 m^3 .
- 6 Kapil invested 2000 rupees in a bank account on January 1st 2012. The account pays 8.25% per annum compounded annually.
- Find the total value of Kapil's investment on January 1st 2019.
 - Would it have been a better option for Kapil to invest his money in an account paying 8% per annum interest compounded monthly? Justify your answer.

- 7 The population P of a species after n months follows the rule $P = 1000 + ae^{kn}$.

The initial population was 2000, and after 1 year the population was 4000.

- Find:
 - a
 - k .
- How long will it take for the population to reach 15 000?

- 8** Two four-sided dice are rolled simultaneously. The faces of each die are labelled 1, 2, 3, and 4.

Let S represent the sum of the results from the dice.

- a** The grid alongside shows the values of S for each possible set of results. Copy the grid and fill in the missing values.

- b** Find: **i** $P(S = 5)$ **ii** $P(S > 5)$ **iii** $P(S = 5 \mid S > 3)$

- c** Suppose points are awarded according to the value of S .

Value of S	$S = 2$	$3 \leq S \leq 5$	$S > 5$
Number of points won or lost	32	16	-8

- i** Determine the expected number of points for each roll of the dice.

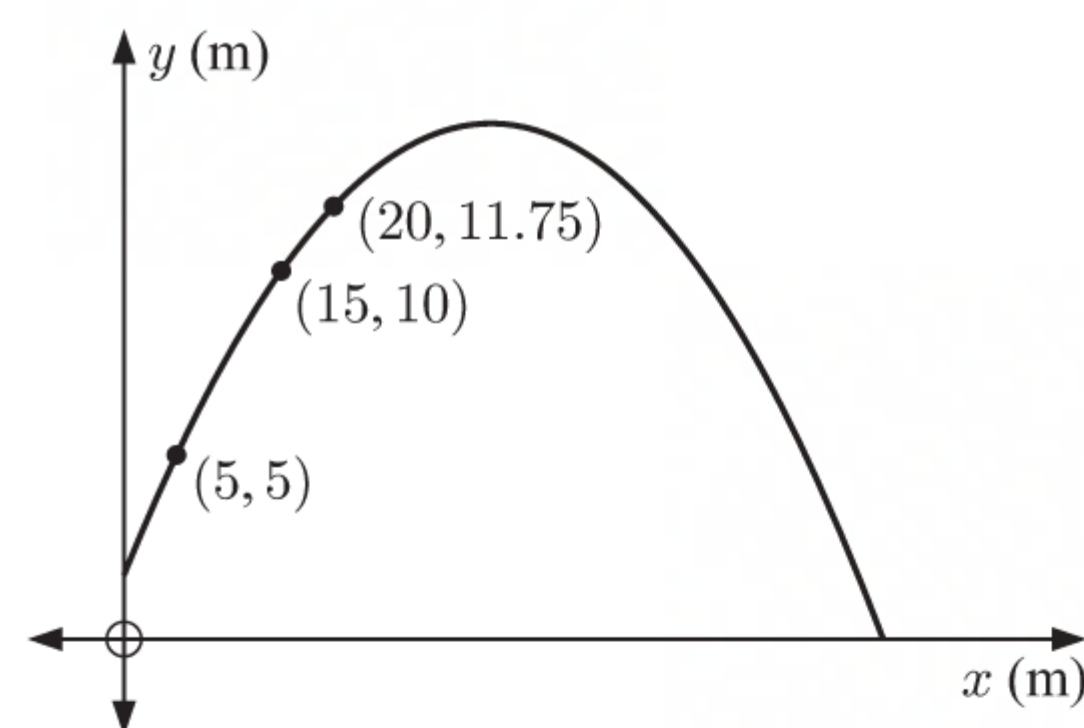
- ii** What should the number of points for $S = 2$ be changed to so that the expectation for the game is zero?

	Die 2			
	1	2	3	4
Die 1	1	2	3	4
	2	3	4	5
	3	4	5	
	4	5		

- 9** This graph shows the path travelled by a discus thrown during an athletics event.

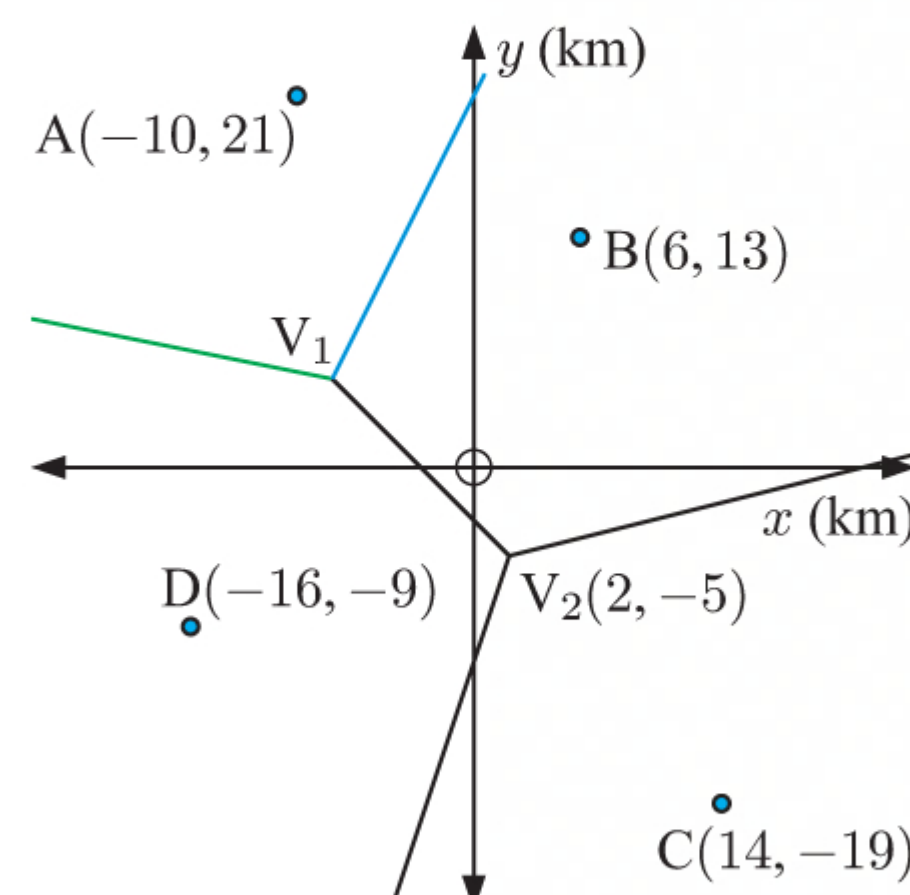
x and y are connected by the quadratic model $y = ax^2 + bx + c$.

- a** Write down three equations involving a , b , and c .
b Use technology to find a , b , and c .
c Find the maximum height reached by the discus.
d How far did the discus travel horizontally before it hit the ground?



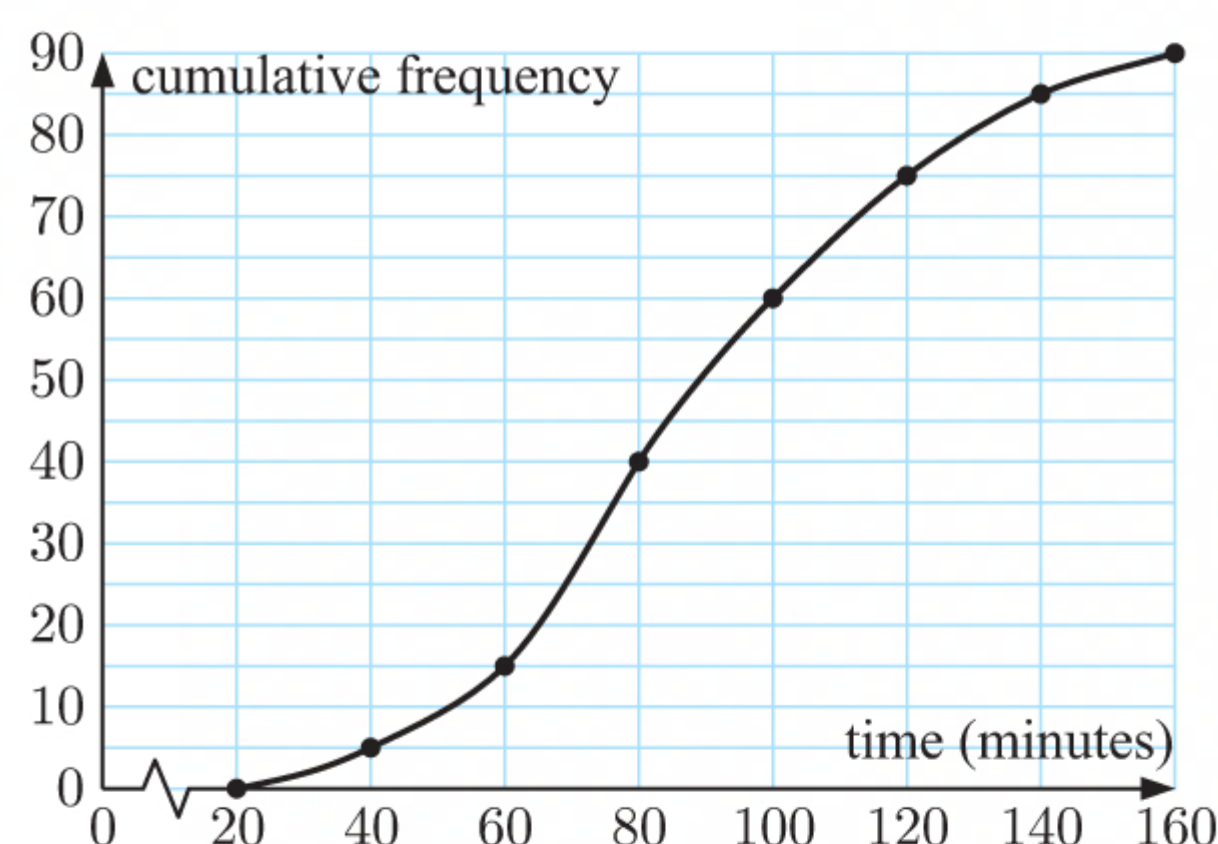
- 10** Consider this Voronoi diagram for the towns A, B, C, and D.

- a** Find the equation of:
i the blue edge **ii** the green edge.
b Hence find the coordinates of V_1 .
c Ivan wants to open a weekend retreat, located so that it is as far as possible from the nearest town.
i Find the optimal position for the retreat.
ii Find the towns which are closest to the retreat in this case.



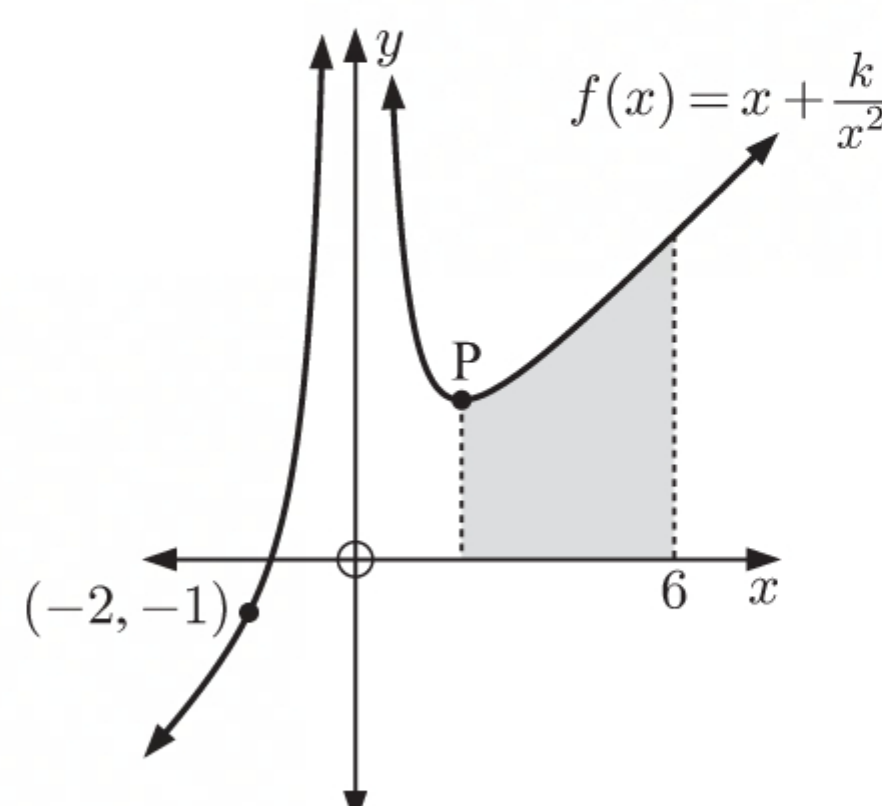
- 11** The lengths, in minutes, of games in a chess tournament are displayed in this cumulative frequency graph.

- a** How many games were played during the tournament?
b Find the median game length.
c Estimate the interquartile range for the data.
d 10% of the games took less than k minutes. Estimate the value of k .
e Draw a frequency histogram to represent the data.



- 12** The function $f(x) = x + \frac{k}{x^2}$ is graphed alongside.

- a** Find k .
b Find $f'(x)$.
c Hence find the coordinates of P.
d Find $\int f(x) dx$.
e Hence find the shaded area.



Trial examination 1

PAPER 1

CALCULATOR, 90 MINUTES

1 [Maximum mark: 5]

A sunflower has a height of 90 cm when Elodie plants it in her garden. The sunflower's height increases at a constant rate of 6 cm per week.

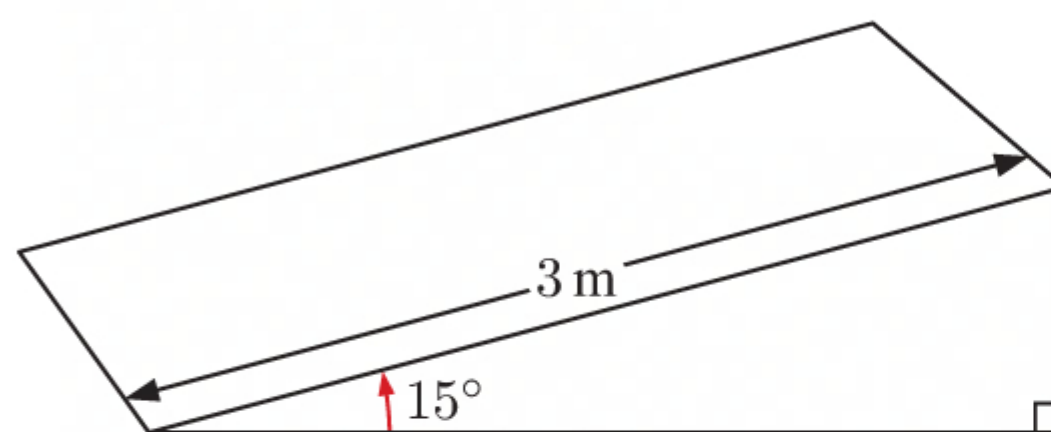
- a** Find the height of the sunflower 15 weeks after Elodie planted it. [1]

Let $H(t)$ be the height of the sunflower, in cms, t weeks after it is planted.

- b** Write down an expression for the height in the form $H(t) = at + b$. [1]
c Find an expression for t in terms of H . [1]
d Given that the maximum height the sunflower grows to is 3 metres write down the range of t . [2]

2 [Maximum mark: 4]

A ramp, in the shape of a triangular prism, is to be constructed to provide easier access to a building. The ramp has a length of 3 metres and is at an angle of 15° to the horizontal as shown in the diagram below.



Find the area of the triangular side of the ramp in m^2 .

3 [Maximum mark: 4]

A survey by the Activities Department at a high school found that 75% of students had signed up for Sports while 50% of students had signed up for Music. Only 10% of students did not sign up for either of these activities.

- a** Find the probability that a student has signed up for both Sports and Music. [2]
b Given that a student did not sign up for Music find the probability that they did sign up for Sports. [2]

4 [Maximum mark: 5]

Saul is shooting baskets from the free throw line on the basketball court. From previous experience he estimates the probability that he scores a basket with a single shot is 60%. He lines up 7 basketballs and shoots each one from the free throw line. The random variable X is the number of baskets he scores from his 7 shots.

- a** Describe the distribution of the random variable that you would use to model this situation together with any assumptions you are making in using it. [2]
b Find the probability that Saul scores less than 3 baskets. [1]
c Find $E(X)$ and $\text{Var}(X)$. [2]

5 [Maximum mark: 6]

Anya wants to take out a loan of \$5000 to buy a car. A finance company offers the following repayment scheme. The loan will be repaid in monthly payments, over 3 years with an interest rate of 4.1% p.a., compounded monthly.

- a** Find the amount Anya must pay each month. [2]
b Calculate the amount still remaining to be repaid after Anya has been making payments for one year. [2]

After one year the finance company changes its interest rate on the loan to 5.3% p.a., compounded monthly.

- c** Find Anya's new monthly payment for the remaining two years of the loan. [2]

6 [Maximum mark: 6]

Sam owns a small boat and so wishes to investigate the depth of water in his local harbour. On the 1st June, during low tide at 09:18 the depth was 2.5 m and during the subsequent high tide at 15:30 the depth was 4.9 m. Letting $w(t)$ be the depth of water (metres) at time t (hours after midnight), Sam finds a model for the depth of water on 1st June using a function of the form $w(t) = a \sin(bt) + d, 0 \leq t \leq 24$.

- a Show that $d = 3.7$. [1]
- b Find the value of a . [1]
- c State the period of the function and hence state the value of b correct to 2 decimal places. [2]

To take his boat out Sam needs the depth in the harbour to be above 3.3 metres.

- d Find the latest time on 1st June that Sam can return to the harbour. [2]

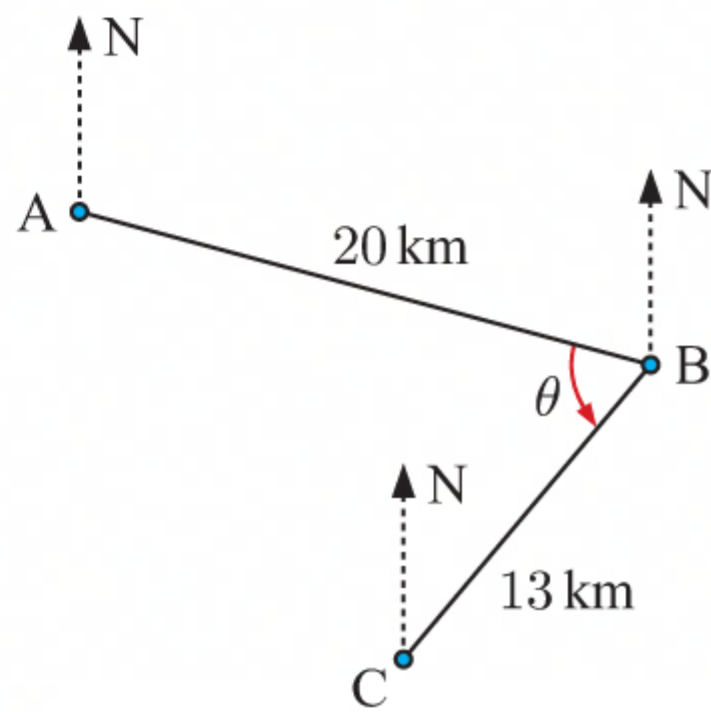
7 [Maximum mark: 6]

The curve with equation $y = x^2 + \frac{4}{x}, x \neq 0$ passes through the point A with coordinates $(2, k)$.

- a Find the value of k . [1]
- b Find an expression for $\frac{dy}{dx}$. [2]
- c Show that the equation of the tangent at the point A passes through the origin. [3]

8 [Maximum mark: 7]

A ship sails from a port A for 20 km on a bearing of 105° until it arrives at port B. It then sails on a bearing of 220° for 13 km to reach the port C.



- a Show that the angle on the diagram, $\theta = 65^\circ$. [1]
- b Find the distance the ship must sail from port C to return to port A. [3]
- c Find the bearing the ship must sail on from port C to return to port A. [3]

9 [Maximum mark: 6]

A hotel manager collects data on the age of their guests and the feedback they provide on their experience of staying at the hotel. The results are shown in the table below.

		Feedback		
		Negative	Neutral	Positive
Age	Under 40	19	23	52
	40 - 60	9	18	64
	Over 60	12	25	76

Test at the 10% level whether the age of the guests and the feedback they provide on their stay are independent of each other.

State clearly the null and alternative hypotheses, the degrees of freedom, the χ^2 -value, the p -value and, with reason, the conclusion of the test.

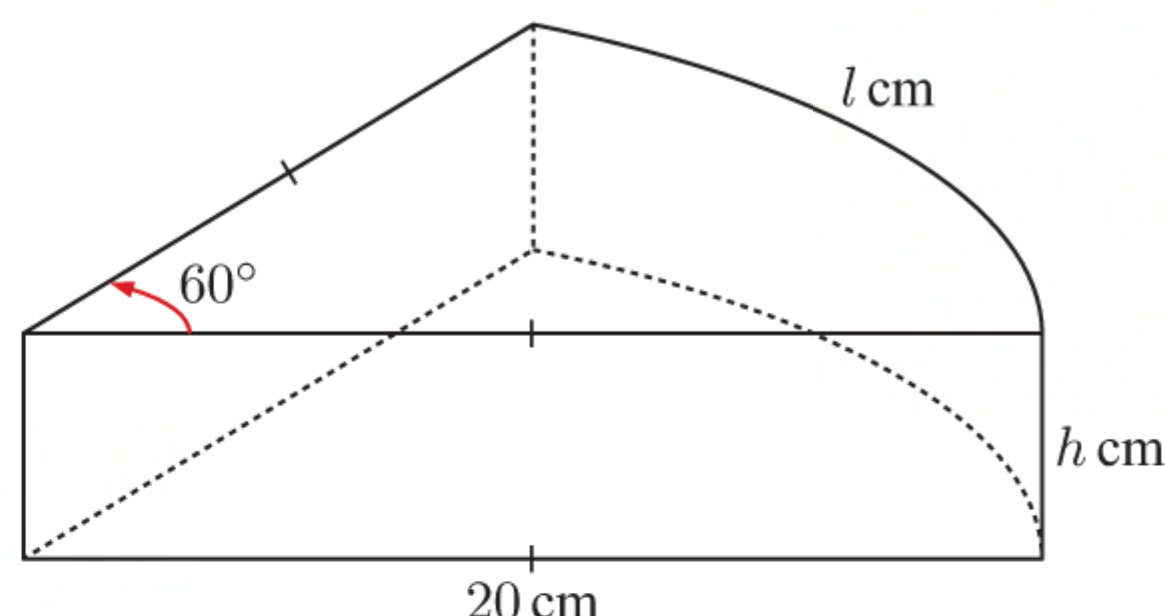
10 [Maximum mark: 5]

A pot of soup is boiled and left to cool down to room temperature. The temperature T in $^\circ\text{C}$ can be modelled by $T = 22 + 78(0.85)^t, t > 0$ at time t minutes after boiling.

- a State the temperature of the room suggested by this model. [1]
- b Find the temperature of the soup after 5 minutes. [2]
- c Find the time taken for the temperature of the soup to drop below 30°C . [2]

11 [Maximum mark: 7]

A cheese company is creating a box for a special edition piece of cheese. The box is a prism with a cross-section in the shape of a sector. The radii of the sector are 20 cm, separated by an angle of 60 degrees. The volume of the box is 1250 cm^3 with a height of h cm as shown below.



- Find the area of the sector which forms the top of the box, correct to 2 decimal places. [2]
- Find the value of h , correct to 2 decimal places. [1]
- Find l , the arc length of the top of the box, correct to 2 decimal places. [1]
- Hence, find the total surface area of the box to the nearest cm^2 . [3]

12 [Maximum mark: 5]

A manufacturing company suspects that one of their machines is working inefficiently and they are therefore considering replacing it with a newer model. They obtain a new machine on a one week loan and take a sample of production times (in mins) from both the old and new machines.

Old machine	137	143	148	139	143	146	140	142		
New machine	135	141	137	144	134	140	142	139	138	142

The manufacturing company wishes to carry out a t -test to determine whether the new machine will give a lower production time than the old machine. The significance level of the test is 5%.

- State the null and alternative hypotheses for the test. [1]
- Find the t -value and p -value of the test. [2]
- State, with reason, the conclusion of the test. [2]

13 [Maximum mark: 6]

Pumpkins produced by a farm have masses which can be modelled by a normal distribution with mean of 4.7 kg and a standard deviation of 0.9 kg.

The farmer does not sell any pumpkins which have a mass of less than 3 kg.

The heaviest 10% of pumpkins are sold as 'XL Pumpkins' by the farmer.

- Find the probability that a randomly chosen pumpkin from the farm is below 3 kg. [2]
- Find the lowest mass for a pumpkin to be sold as an 'XL Pumpkin'. [2]
- Two pumpkins are selected at random. Find the probability that one is too light to sell and the other is an 'XL Pumpkin'. [2]

14 [Maximum mark: 8]

Ann and Greg work for the same company which pays them at the end of each month. They sign the following contracts for 24 months, starting in January 2020 and finishing in December 2021.

Ann: Paid \$1000 in January 2020, increasing by \$50 in each subsequent month.

Greg: Paid \$600 in January 2020, increasing by 10% in each subsequent month.

- Find Greg's monthly salary for February 2021. [2]
- Find the total amount received by Ann in the first year of her contract. [2]
- Find the month and year in which the total amount received from Greg's contract first exceeds the total amount received from Ann's contract. [4]

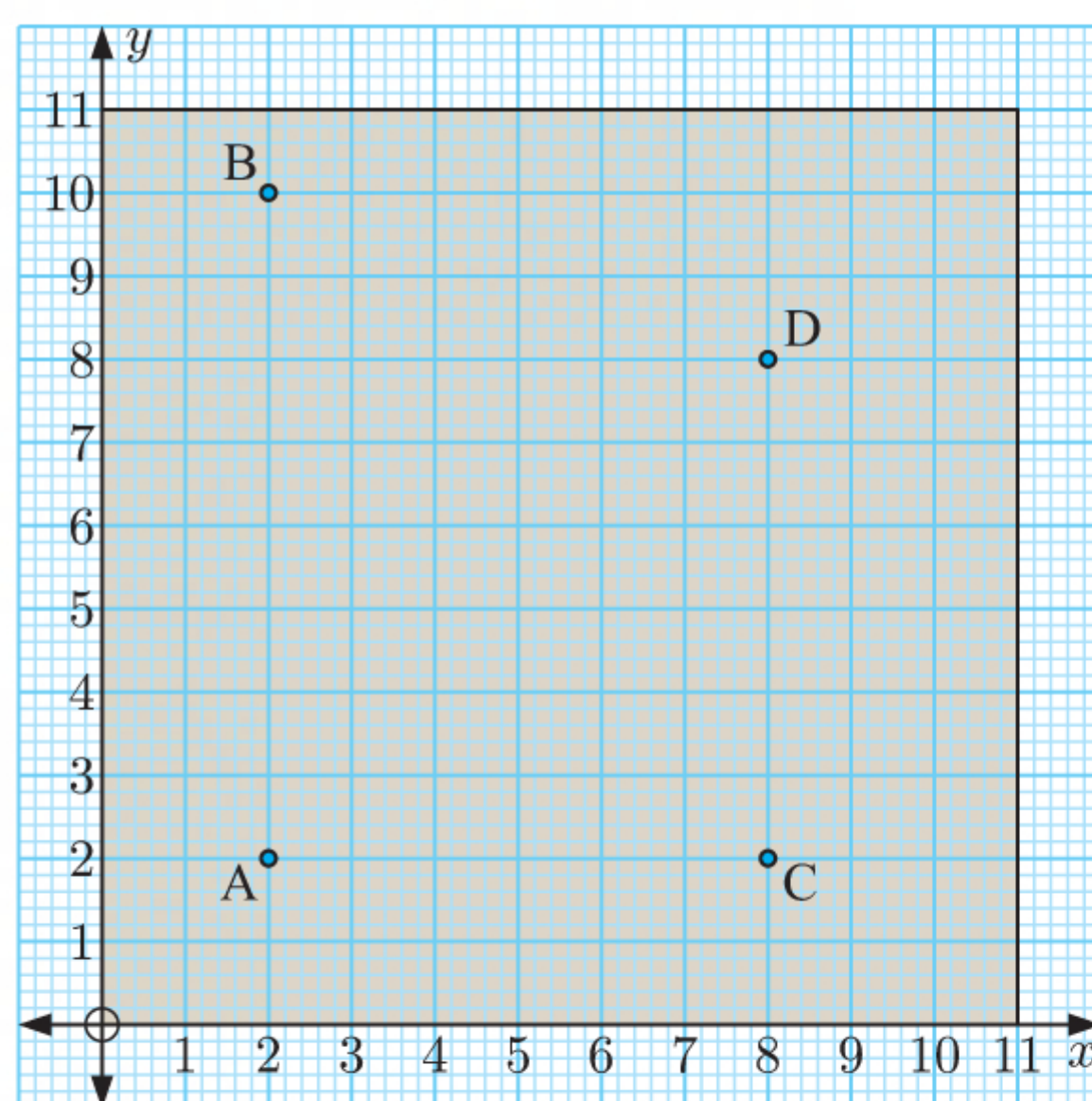
PAPER 2**CALCULATOR, 90 MINUTES****1 [Maximum mark: 14]**

A farmer's field is modelled by the square shown below with each unit representing 10 metres. Feeding stations for the cattle are positioned at points A(2, 2), B(2, 10), C(8, 2), and D(8, 8) within the field.

- a** Find the midpoint of the line segment [BD]. [1]
- b** Calculate the gradient of the line segment [BD]. [2]
- c** Show that the equation of the perpendicular bisector of the line segment [BD] is $y - 3x + 6 = 0$. [3]

It is assumed that cattle will feed at the nearest station so the farmer wishes to construct a Voronoi diagram.

- d** Given that the perpendicular bisector of the line segment [AD] is $y = 10 - x$ construct the Voronoi diagram for the sites A, B, C, and D, showing all edges clearly on a diagram like the one below. [5]



The farmer now wishes to place a fifth feeding station inside the boundaries of the field. The new feeding station should be at least 10 m away from the boundary wall and placed so it is as far away as possible from any of the present feeding stations A, B, C, or D.

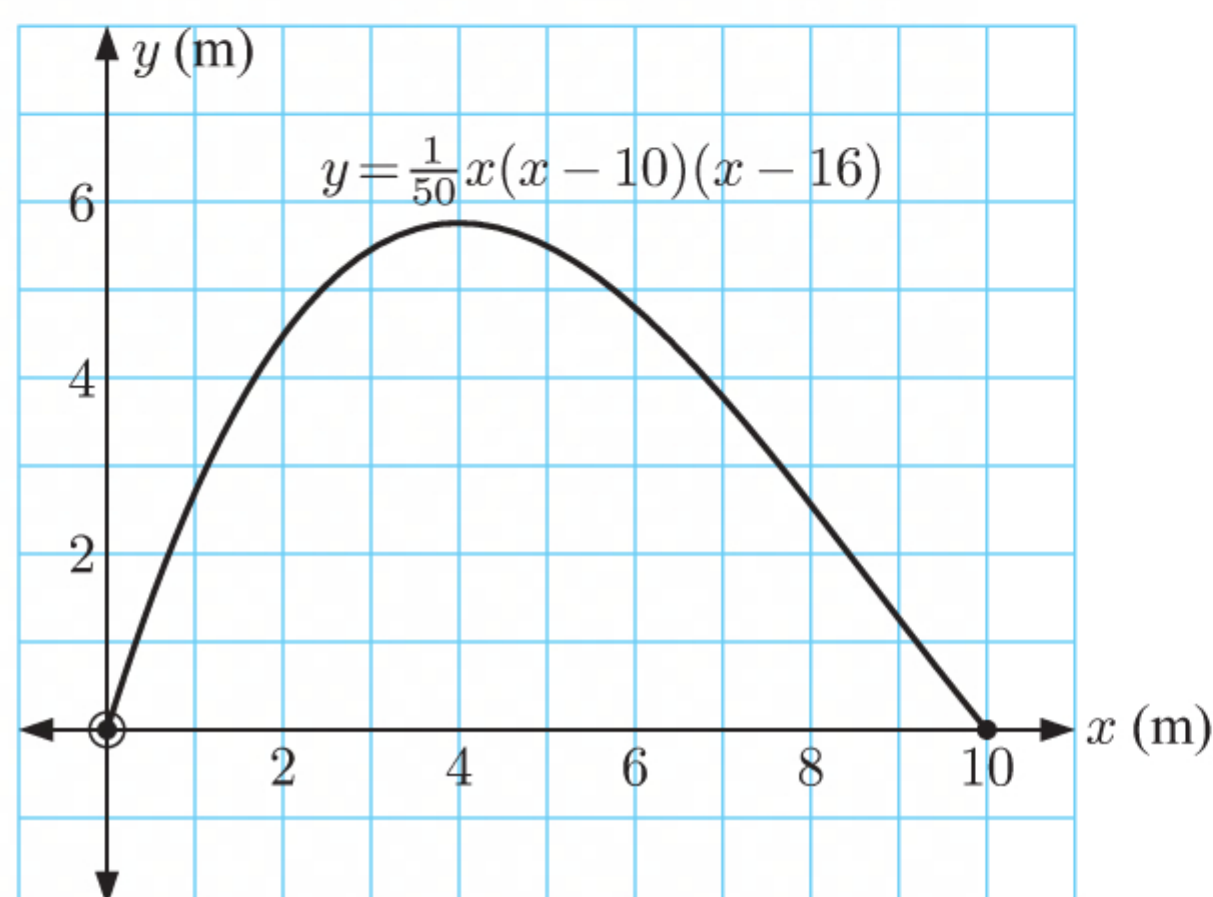
- e** Determine the optimum location of the new feeding station. [3]

2 [Maximum mark: 16]

A single lane road tunnel is 100 m long. Its constant cross-section is shown on the graph below and is modelled by the function

$$y = \frac{1}{50}x(x - 10)(x - 16), \quad 0 \leq x \leq 10$$

where x and y are horizontal and vertical distances measured in metres.



- a** Find an expression for $\frac{dy}{dx}$. [3]
- b** Hence, or otherwise, find the greatest height of the tunnel to the nearest cm. [2]
- c** An extra large truck has a rectangular cross-section with height 5.3 m and width 2.8 m and wishes to pass through the tunnel. Is this possible? Show the reasoning for your answer. [3]

Using the model, Engineer A finds the volume of the tunnel exactly. Engineer B uses the Trapezoidal rule to estimate the volume of the tunnel.

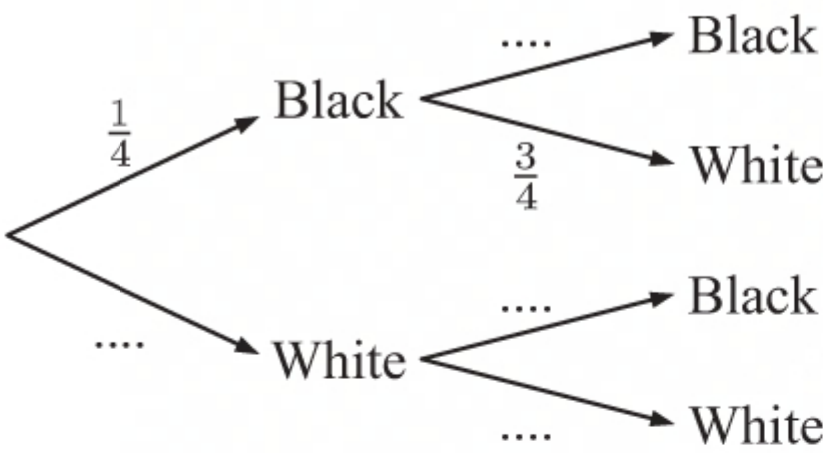
- d** Find the volume of the tunnel in m^3 , obtained by Engineer A. Give your answer correct to 4 significant figures. [3]
- e** Find the volume of the tunnel in m^3 , obtained by Engineer B using the Trapezoidal rule with 5 strips. [3]
- f** Find the percentage error in Engineer B’s answer in comparison to the answer obtained by Engineer A. [2]

3 [Maximum mark: 17]

The spinner shown opposite has four equally likely sections, three of which are white and one black.



- a** The spinner is spun twice. Assuming the spins of the spinner are independent, copy and complete the tree diagram below. [2]



The random variable X is the number of blacks obtained from two spins of the spinner.

- b** Copy and complete the table alongside: [2]

x	0	1	2
$P(X = x)$			$\frac{1}{16}$

- c** Find $E(X)$. [2]

At a fairground game it costs \$3 to spin the spinner twice. The player then receives the following amount back:

0 blacks \Rightarrow \$0
1 black \Rightarrow \$5
2 blacks \Rightarrow \$10

- d** Ben plays the fairground game 10 times. Find the expected loss he will make. [3]

A fairground inspector suspects that the spinner may be biased. To test this he plays the game 100 times and records the following results.

0 blacks	1 black	2 blacks
51	46	3

- e** Using your answer to **b**, complete the table for the expected number of blacks from 100 games. [2]

0 blacks	1 black	2 blacks

The inspector conducts a χ^2 goodness of fit test. The test is carried out at a 5% significance level.

- f** Write down the number of degrees of freedom. [1]
- g** Find the χ^2 -value and p -value for the test. [3]
- h** State the conclusion of the test. Give a reason for your answer. [2]

4 [Maximum mark: 17]

Sami wishes to investigate the performance of the 120 students in her grade on their recent Mathematics and Physics tests. She takes a sample of 12 students from the grade whose Physics test scores, to the nearest percent, are shown in the table below.

<i>Student</i>	A	B	C	D	E	F	G	H	I	J	K	L
<i>Physics (p%)</i>	9	33	56	58	61	72	69	72	79	72	80	85

- a** Describe how the 12 students could have been selected if Sami used convenience sampling. [1]
- b** Describe how the 12 students could have been selected if Sami used systematic sampling. [1]
- c** Calculate the median and interquartile range for the Physics test scores in the sample. [3]

An outlier is defined as a data item which is more than $1.5 \times$ interquartile range (IQR) from the nearest quartile.

- d** Draw a box and whisker plot of the Physics test scores in the sample indicating clearly any data items which are outliers. [3]

The table below shows the corresponding Mathematics test scores, also to the nearest percent, for the 12 students in the sample.

<i>Student</i>	A	B	C	D	E	F	G	H	I	J	K	L
<i>Physics (p%)</i>	9	33	56	58	61	72	69	72	79	72	80	85
<i>Mathematics (m%)</i>	20	29	30	41	53	53	58	64	71	80	85	96

- e** Calculate Pearson’s product-moment correlation coefficient for the data. [2]
- f** Sami’s friend scored 68% for the Mathematics test, but was absent for Physics. By calculating the regression line of p against m , make a prediction for their score on the Physics test. [3]
- g** Calculate Spearman’s rank correlation coefficient for the data. [3]
- h** Give a possible reason why your answer for **g** is greater than your answer for **e**. [1]

5 [Maximum mark: 16]

In 1980 ornithologists introduced a new species of bird into a large area of forest. 99 birds were initially introduced, and there were 184 birds in 1985.

The size of the population of the species, t years after 1980, can be described using a model of the form

$$N(t) = a \times b^t, \quad t \geq 0.$$

- a** Give the values of the parameters a and b . [2]
- b** Hence, state the annual growth rate of the population as a percentage. [1]
- c** Use your model to predict the size of the population of the species in the year 2000. [2]

In 2000 a logging company began to operate in the forest causing significant deforestation and damaging the habitat of the species of bird. Ornithologists adjusted their model to the following function to model the size of the population of the species, **where t is again the number of years after 1980.**

$$N(t) = 1180 \times (0.83)^{(t-20)}, \quad t \geq 20$$

- d** Use this new model to find the size of the population in 2010. [2]
- e** In which year does the model suggest that the size of the population of the species will first drop below 30 birds? [3]
- f** In the period from 1980 to 2020 for how many years was the size of the population of the species greater than 500? [3]

In 2020 conservationists succeed in stopping the logging in the forest. Ornithologists then introduce another 100 birds of the species into the forest to add to the remaining population. They now expect the habitat of the species of bird to recover and the total population to increase at a rate of 8% per year.

- g** Estimate the size of the population in 2030. [3]

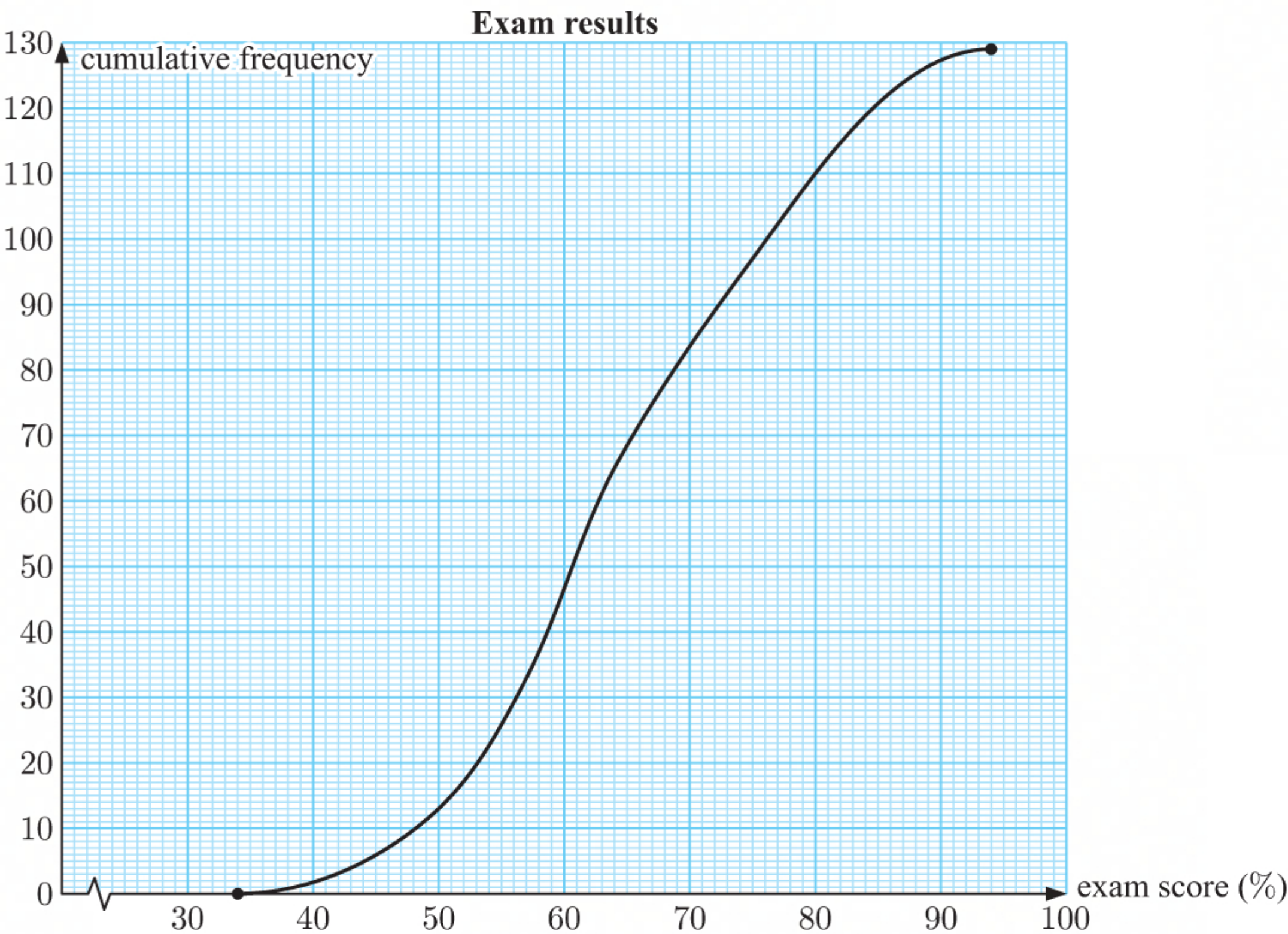
Trial examination 2

PAPER 1

CALCULATOR, 90 MINUTES

1 [Maximum mark: 6]

The cumulative frequency graph below displays the performances of 129 students in an examination.



- a Find the percentage of students who obtained at least 50% for their examination.

[1]
- b Construct a box and whisker diagram to summarise the results.

[3]
- c Find the interquartile range for the results and explain what it means.

[2]

2 [Maximum mark: 6]

The rent Georgio pays for his apartment increases by 2.5% each year. In 2020, Georgio pays a total of €12 000 in rent.

- a Explain why Georgio’s rent follows a geometric sequence, and find its common ratio.

[2]
- b How much rent will Georgio pay for his apartment in 2025?

[2]
- c Find the total amount Georgio will pay in rent from 2020 to 2025 inclusive.

[2]

3 [Maximum mark: 6]

The manager of a shopping mall wants to survey customers to ask how many times per week, on average, they visit the mall. She knows from a previous survey that the age of her customers follows the distribution alongside.

Age	Percentage
under 30	19.7%
31 - 50	30.4%
51 - 70	28.3%
over 70	21.6%

The manager uses a stratified sample of 250 customers to obtain these results:

Visits per week	1	2	3	4	5	6	7
Customers	55	x	56	42	y	17	6

The mean number of visits per week for these customers was found to be 3.08 .

- a Find the number of customers sampled from the age range 51 - 70.

[2]
- b Write a pair of equations involving x and y .

[2]
- c Find x and y by solving your pair of equations simultaneously.

[2]

4 [Maximum mark: 3]

The volume of a sphere with radius r is given by $V(r) = \frac{4}{3}\pi r^3$, $r > 0$.

- a** If the radius of a sphere is doubled, what happens to its volume? [1]
- b** Rearrange the formula to write r as a function of V . Explain what this function means. [2]

5 [Maximum mark: 5]

A village in Peru breeds guinea pigs. If the population has sufficient resources, it will double in size every 2 months. Initially, there are 24 guinea pigs.

Suppose the population of guinea pigs is modelled by

$P = A \times 2^{kt}$ where P is the population
 t is the time in months
 A, k are constants.

- a** Find the value of A . [1]
- b** Find the value of k . [2]
- c** Estimate the number of guinea pigs in the population after 11 months. [2]

6 [Maximum mark: 5]

A survey was conducted amongst the teachers at a large school to determine their preferred pet. The results are sorted according to the classes they teach.

		Pet preference			
		Cat	Dog	Bird	Other
Teacher type	Arts & Humanities	12	17	16	11
	Sciences	18	9	5	8
	Sports	5	8	3	2

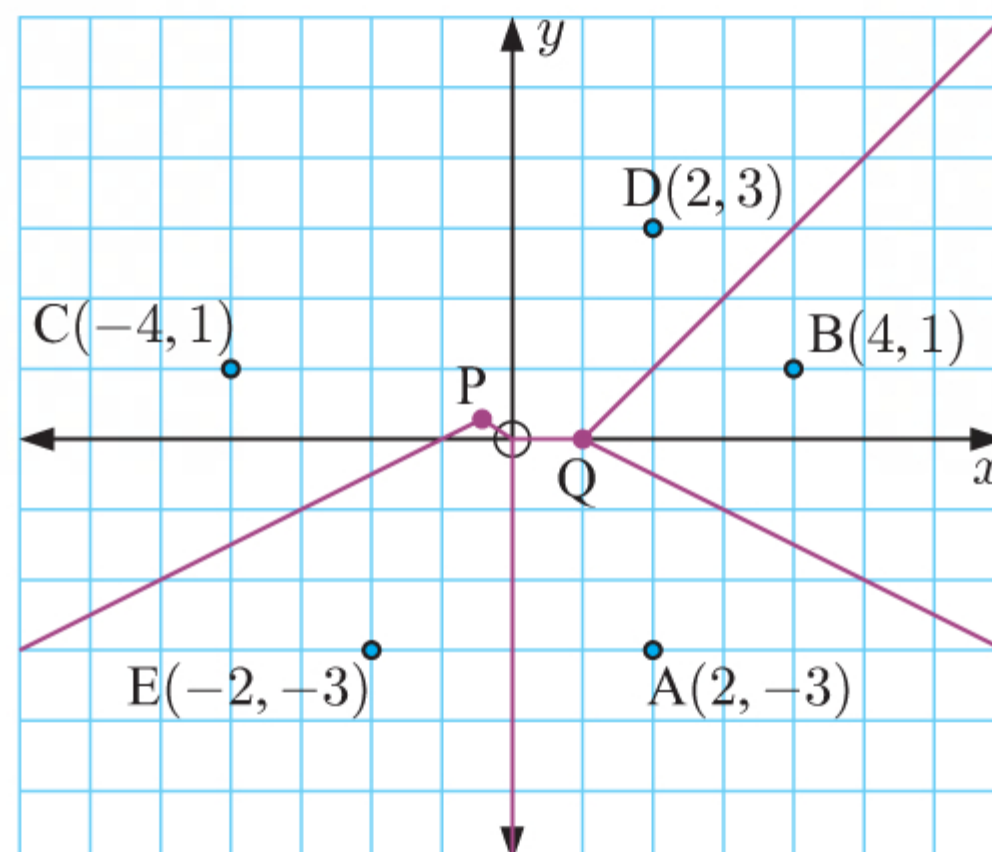
A χ^2 test for independence is to be conducted at a 5% level of significance to determine whether teacher type and pet preference are independent.

- a** State the null hypothesis. [1]
- b** Calculate χ^2_{calc} . [2]
- c** The critical value for this test is 12.59. Determine whether the null hypothesis should be accepted. [2]

7 [Maximum mark: 8]

Hamsika wants to open a new Indian restaurant not more than 4 km from the centre of her city. It should be strategically placed to avoid competition from existing Indian restaurants.

The map below shows the centre of the city (the origin) and the existing Indian restaurants A to E. The grid units are in km.



- a** All but one edge of a Voronoi diagram has been drawn on the grid. [3]
Find the equation of the missing edge, giving your answer in the form $ax + by + d = 0$, $a, b, d \in \mathbb{Z}$.
- b** The equation of OP is $y = -\frac{2}{3}x$. [2]
Find exactly the coordinates of P.
- c** Suggest the most appropriate location for Hamsika's restaurant. Justify your answer. [3]

8 [Maximum mark: 6]

The famous bell Big Ben is housed in Elizabeth Tower, London. Each face of the tower features an analogue clock whose centre is 53 m above the ground. The minute hand of each clock is 4.3 m long.

The height of the tip of the minute hand above the ground t minutes after midnight is given by

$$h(t) = a \cos(bt)^\circ + d \text{ metres}$$

where a , b , d are constants.

a Find, with reasons, the value of:

i a [1]

ii d [1]

iii b [2]

b Find the height of the tip of the minute hand 23 minutes past the hour. [2]

9 [Maximum mark: 6]

The weights of a sample of newborn calves are recorded in the table below:

Male weight (kg)	38.2	41.7	31.9	46.8	37.5	43.0	
Female weight (kg)	36.8	37.2	42.6	39.1	45.7	42.2	34.3

A hypothesis test at a 5% level of significance is conducted to see whether the mean weight μ_m of male calves, is the same as the mean weight μ_f of female calves.

a State the null and alternative hypotheses. [2]

b Assuming the weights of both male and female calves are normally distributed with the same standard deviation, calculate the p -value for this test. [2]

c State, with a reason, whether the null hypothesis should be accepted. [2]

10 [Maximum mark: 6]

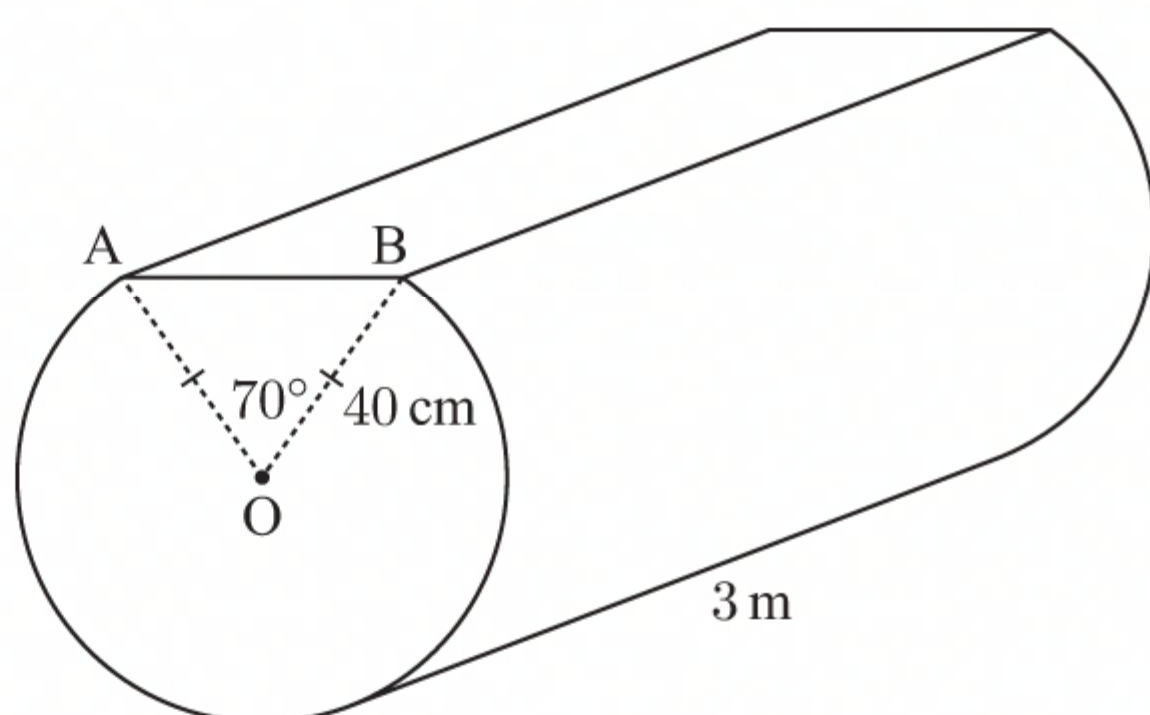
Consider the quadratic function $f(x) = -(x + 2)(x - 1)$.

a Find the axes intercepts of $y = f(x)$. [2]

b Sketch the graph of $y = f(x)$. [2]

c Find the area enclosed by $y = f(x)$ and the x -axis. [2]

11 [Maximum mark: 5]



A cylindrical log is cut as shown to form a park bench.

Find:

a the area of triangle OAB in m^2 [2]

b the volume of the log in m^3 . [3]

12 [Maximum mark: 5]

When people go shopping in the fish market, the number of types of fish each person buys has the following distribution:

Types of fish	0	1	2	3	4	5
Probability	k	0.46	0.15	0.06	0.03	0.01

a Find the proportion of people who leave without buying any fish. [1]

b Calculate the expected number of fish types a person will buy. [2]

c Three people leaving the fish market are selected at random. Find the probability that at least one has bought more than one type of fish. [2]

13 [Maximum mark: 9]

Orienteers Morris and Eleanor leave point P at the same time. Morris runs 3.1 km on the bearing 148° , then a further 2.2 km on the bearing 214° , to point F. Eleanor runs directly to point F.

- a** Find the distance Eleanor runs. [3]
- b** Find the bearing on which Eleanor runs. [3]
- c** Morris averages 8.4 km h^{-1} whereas Eleanor averages 6.9 km h^{-1} . Who arrives first and by how much? [3]

14 [Maximum mark: 4]

The donations received by an animal welfare foundation each week are normally distributed with mean €13 600 and standard deviation €3100.

- a** Find the probability that in a randomly selected week, the foundation receives more than €10 000. [2]
- b** In how many weeks of a 52 week year would you expect the foundation to receive more than €10 000? [2]

PAPER 2**CALCULATOR, 90 MINUTES****1 [Maximum mark: 14]**

At age 30, Corina borrows some money to purchase a new boat. She is offered a 5 year loan at an interest rate of 10.1% p.a., compounded quarterly. Her monthly loan repayments are \$297.57.

- a** How much money did Corina borrow? [3]
- b** Find the interest paid on the loan. [2]

At age 60, Corina retires with \$865 400 in a savings fund. She finds an annuity fund for her money which returns 5.2% p.a. compounded monthly.

- c** Corina wants the money to last until she is 90 years old. How much can she afford to withdraw each month? [3]
- d** At age 66, Corina realises that with rising medical and utility costs, she will need to withdraw more money each month. She decides to withdraw \$5400.
 - i** How much money is in the account at the time Corina makes this decision? [3]
 - ii** What age will Corina be when her annuity runs out? [3]

2 [Maximum mark: 20]

A chain of department stores surveys all its staff to better understand their external care responsibilities. They found that 46.8% of their staff were responsible for children under the age of 18, and 11.5% were carers for their parents or other adults. 43.4% of their staff had no external care responsibilities.

- a** Construct a Venn diagram to display this information, letting C be the event that a staff member is responsible for a child under the age of 18, and P be the event that a staff member is a carer for a parent or other adult. [4]
- b** Find the probability that a randomly selected staff member is responsible for a child under the age of 18 *and* is a carer for a parent or other adult. [1]
- c** A randomly chosen staff member is not responsible for a child under the age of 18. Find the probability that they do not have a parent or other adult in their care. [3]
- d** The chain of stores has a total of 58 463 staff. Estimate the number of staff who are responsible for a child under the age of 18 *but are not* carers for parents or other adults. [2]

- e A sample of 500 staff is randomly selected to answer a more extensive questionnaire. Before they begin, the Human Resources manager wants to be confident that the sample will be a good representation of the population by conducting a χ^2 goodness of fit test with a 5% level of significance.

The table alongside summarises the people selected in the sample:

Group	Frequency
$C \cap P$	9
$C \cap P'$	236
$C' \cap P$	
$C' \cap P'$	205
Total	500

- i How many people in the sample are not responsible for a child under the age of 18 but *are* carers for a parent or other adult? [1]
- ii Write down the hypotheses for the χ^2 goodness of fit test. [2]
- iii State the degrees of freedom for the test. [1]
- iv Construct a table of expected frequencies for each group using the proportions in the whole population. [2]
- v Calculate the test statistic χ^2_{calc} for the test. [2]
- vi Given that the critical value for this test is $\chi^2_{\text{crit}} \approx 7.81$, write down the conclusion of the test. [2]

3 [Maximum mark: 17]

The Eiffel Tower in Paris is 324 m high, and has base $125 \text{ m} \times 125 \text{ m}$.



- a A tourist shop sells scale models of the tower which have base $8 \text{ cm} \times 8 \text{ cm}$, and which are encased in a glass pyramid with dimensions 2% bigger than the model.
- i Find the height of the model, to the nearest mm. [2]
- ii Find the volume of the glass pyramid which encases the model. [4]
- b The shop owner is keen to maximise the profit he makes from selling the models.
- He knows that the more he orders, the cheaper the models will be for him to buy. However, he will eventually buy more than he can sell, at which point he would make less profit.

His orders from the previous years have generated the profits shown in the table:

Quantity	Profit (€)
4000	32 000
9000	85 500
12 000	115 200

The shop owner decides to fit a cubic model of the form

$$P(x) = ax^3 + bx^2 + cx + d$$

for the profit P when he buys x thousand models.

- i Explain why $d = 0$. [1]
- ii Construct three linear equations for a , b , and c . Hence find $P(x)$. [4]
- iii Find $P'(x)$. [2]
- iv Solve $P'(x) = 0$. [2]
- v Hence find, to the nearest hundred, the optimum number of models the owner should buy, and the profit he will make in this case. [2]

4 [Maximum mark: 12]

In a bone mineral density (BMD) test, a person’s BMD is a t -score which is the number of standard deviations the person is above or below the young healthy adult mean.

The mass and BMD of a sample of university age students were measured, and are summarised in the table:

<i>Student</i>	A	B	C	D	E	F	G	H	I	J
<i>Mass (x kg)</i>	83.2	64.1	75.4	80.9	91.2	57.5	104.6	71.0	66.7	88.4
<i>BMD (y)</i>	0.0	0.2	−0.3	−0.5	1.4	−1.2	0.4	−0.6	0.4	0.8

- a

Calculate Pearson’s product-moment correlation coefficient r_p for the variables.
Hence interpret the relationship between the variables.

[2]
- b

Find the least-squares regression line connecting the variables.

[2]
- c

Use your least-squares regression line to estimate the BMD of a student with mass 92 kg. Discuss the reliability of your estimate.

[3]
- d

Copy and complete this table showing the *ranks* for x and y .

[2]

<i>Student</i>	A	B	C	D	E	F	G	H	I	J
<i>rank of mass x</i>		2				1			3	
<i>rank of BMD y</i>				3		1		2		

- e

Calculate Spearman’s rank correlation coefficient r_s for the data, and describe what this means.

[3]

5 [Maximum mark: 17]

When a skydiver jumps out of a plane, their speed after t seconds is given by the function $v(t)$.

After 1 second, the skydiver is falling at 9.8 m s^{-1} .

- a

The distance the skydiver has fallen after τ seconds is the area between $v(t)$ and the t -axis on the interval $0 \leq t \leq \tau$.
Write this distance as an integral.

[1]
- b

If we ignore air resistance, v increases in proportion to t .

i

Find the proportionality constant, and hence write v as a function of t .

[3]

ii

Find, by integration, the distance the skydiver falls in the first 2 seconds.

[3]
- c

As the speed of the skydiver increases, air resistance quickly becomes important, and we need to include it in our model. A more accurate model for higher speeds has the form $V(t) = 53(1 - e^{-kt})$ where $k \in \mathbb{R}$.

i

Find $V(0)$ and explain why this is consistent with the previous model.

[2]

ii

Find the value of k , to 3 decimal places.

[2]

iii

Sketch the graph of $V(t)$. Discuss what happens to the speed of the skydiver over time.

[3]

iv

Calculate the distance the skydiver falls in the first 5 seconds, giving your answer to the nearest metre.

[3]

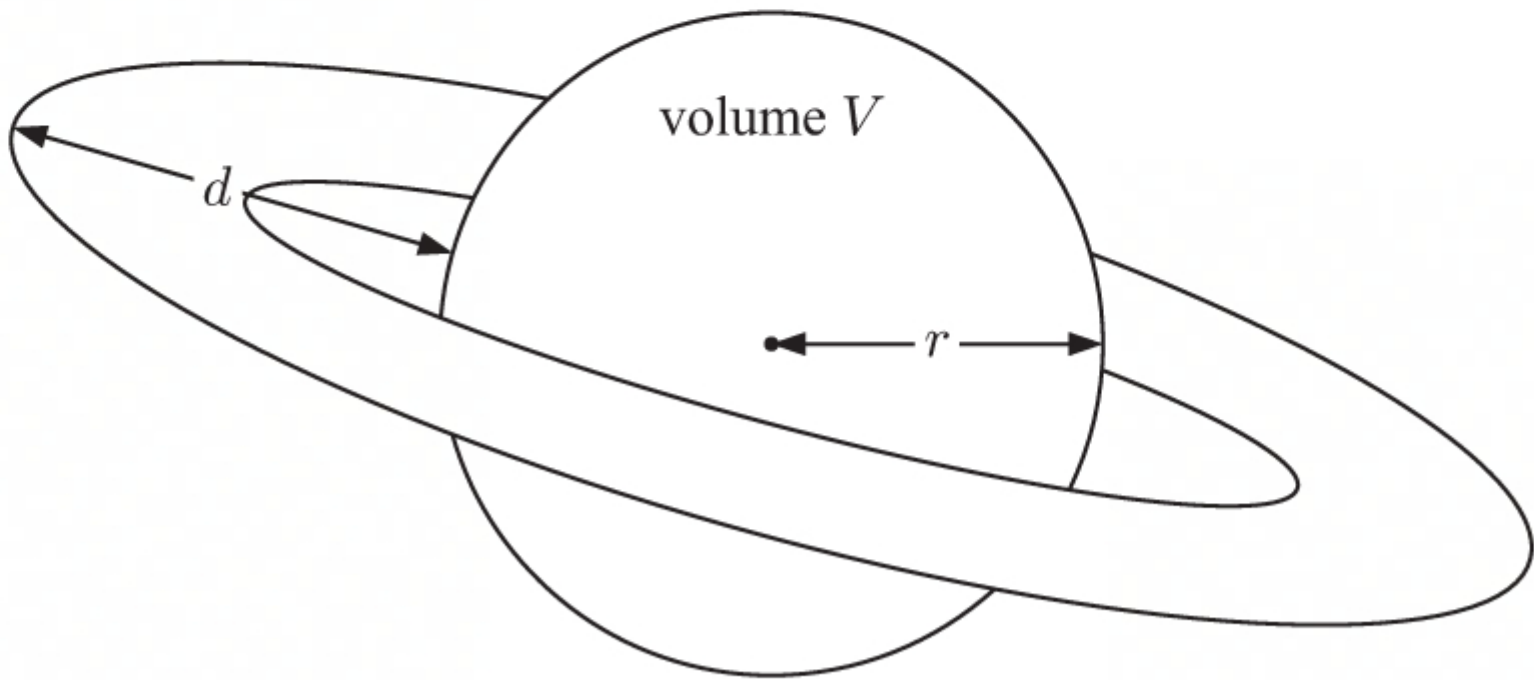
Trial examination 3

PAPER 1

CALCULATOR, 90 MINUTES

1 [Maximum mark: 5]

The data below is given by NASA for the planet Saturn.



Equatorial radius	$r \approx 5.8232 \times 10^4 \text{ km}$
Volume	$V \approx 8.2713 \times 10^{14} \text{ km}^3$
Ring system	up to $d \approx 2.82 \times 10^5 \text{ km}$ from planet

- a Use the equatorial radius given to estimate the volume of Saturn. [2]
- b Estimate the circumference of the orbit of an ice-covered rock at the far edge of Saturn’s rings. [3]

2 [Maximum mark: 5]

In a board game, a player is required to roll a pair of ordinary dice.

- a A pair of dice is rolled. Find the probability that the sum of the dice is 5. [2]
- b A pair of dice is rolled 10 times. Find the probability that their sum will be 5 at least twice. [3]

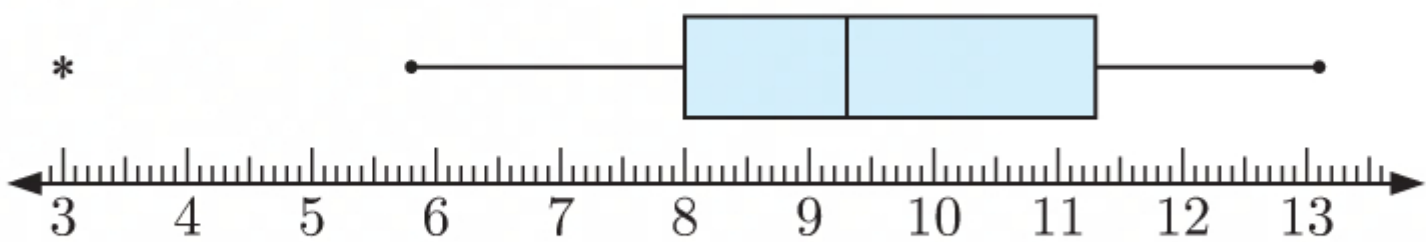
3 [Maximum mark: 11]

A nursery wants to compare the growth of seedlings under different lighting conditions. Two samples A and B are compared after 30 days.

- a The heights (in cm) of the sample A seedlings are:
6.8 7.8 8.2 8.5 8.7 9.6 9.9 10.0 10.1 10.3 11.3 11.5 12.1 13.2 14.2

i Find the five-number summary for this data. [3]

ii Assuming there are no outliers construct a box plot to display the data. [3]
- b The box plot for the sample B seedlings is:



- i Find the range and interquartile range for this sample. [2]
- ii Compare the two samples to decide which growing conditions are more favourable. [3]

4 [Maximum mark: 5]

Jody asks 8 office workers in a big city about the distance they travel to work each day, and the time it takes them to get there. The workers record their journeys to work by GPS.

Distance travelled to work (x km)	2.2	6.8	15.4	3.1	5.6	9.0	17.2	1.4	4.1
Travel time (y minutes)	16	27	43	14	26	32	61	12	19

- a Calculate Pearson’s correlation coefficient r for the data. [1]
- b Calculate the y against x regression line for the given data. [2]
- c Hence estimate the distance a worker travels to the office, if it takes them 50 minutes to get to work. [2]

5 [Maximum mark: 6]

A geologist is studying a radioactive sample he has recently collected. The initial weight of the sample is 18.61 g. When each atom emits its α -particle, the mass of the atom is reduced by 2.2%.

The mass of the sample is given by $M(t) = A + B(\frac{1}{2})^{\frac{t}{400}}$, $t \geq 0$ days.

- a Explain why A is the final mass of the sample after every atom in the sample has emitted its α -particle, and find its value. [2]
- b Find the value of B . [2]
- c Estimate the mass of the sample after 2 years. [2]

6 [Maximum mark: 5]

Sam and Markus are on holiday exploring the coast at Wollongong. Sam has climbed the 25 m high lighthouse, which stands at the top of a cliff. Markus is in a boat some distance offshore. He measures the angle of elevation to the base of the lighthouse is 6.4° , and the angle of elevation to Sam is 10.3° .

- a Find the distance from Markus to the base of the cliff below the lighthouse. [3]
- b Find the height of the cliff. [2]

7 [Maximum mark: 4]

The size or magnitude of an earthquake is described using a logarithmic scale called the Richter scale.

If an earthquake releases E joules of energy, then the magnitude M of the earthquake is given by

$$M = \frac{2}{3} \log_{10} E - 3.6.$$

- a Find the magnitude of an earthquake which releases 7.8×10^{13} J of energy. [2]
- b How much energy is released by an earthquake of magnitude 2.6? [2]

8 [Maximum mark: 8]

Boxes of chocolate frogs are each sold with a collectible card featuring a famous professor. It is claimed that the cards are distributed as follows:

Professor	D	M	T	F	S
Percentage	60%	20%	10%	7%	3%

Eager student H buys a crate containing 150 boxes. The following cards are found:

Professor	D	M	T	F	S
Observed frequency	85	37	12	12	4

To investigate whether his sample is consistent with the claim, student H conducts a χ^2 goodness of fit test at a 5% significance level.

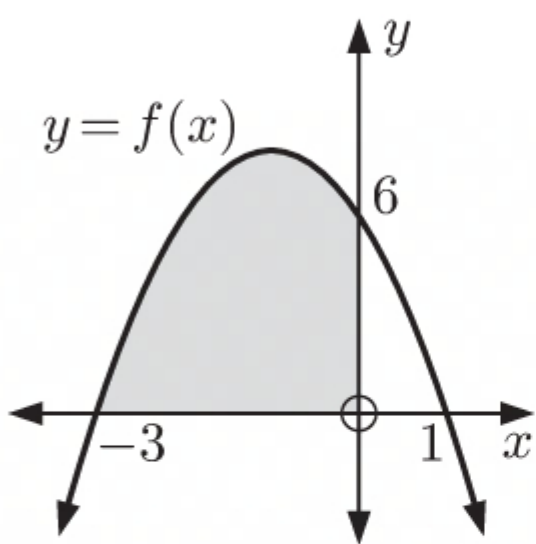
- a Write down the null hypothesis. [1]
- b Copy and complete the following table of expected frequencies: [2]

Professor	D	M	T	F	S
Expected frequency					

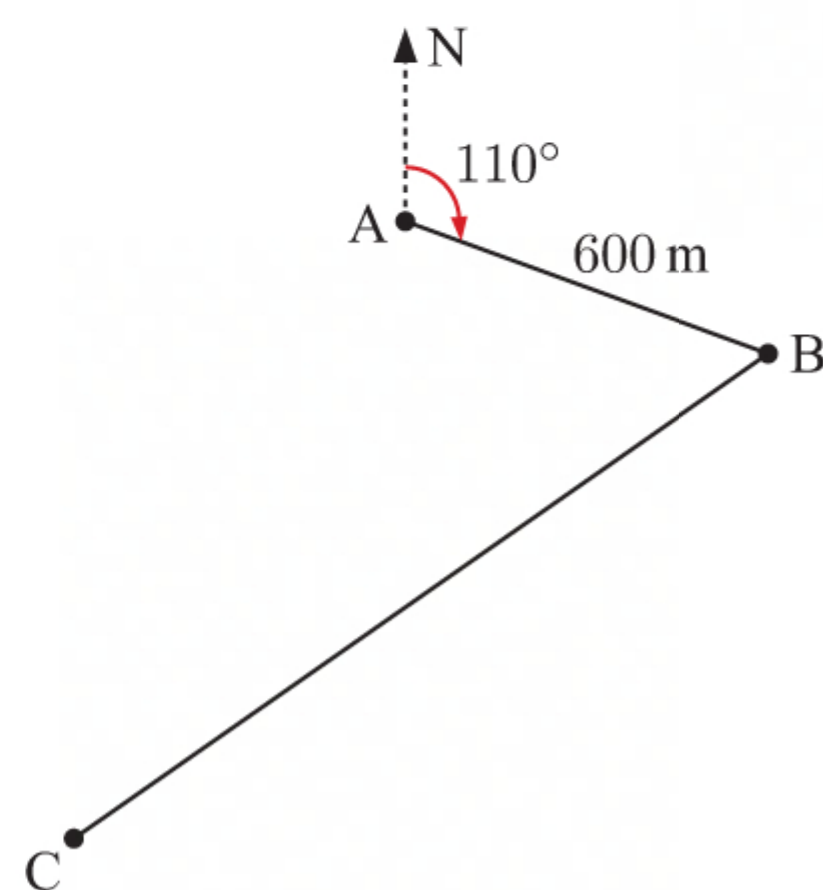
- c Write down the number of degrees of freedom. [1]
- d Find the p -value for the test. [2]
- e State the conclusion of the test, giving a reason for your answer. [2]

9 [Maximum mark: 5]

The quadratic function $f(x) = k(x - a)(x + 3)$ has the graph shown.



- a State the values of a and k . [2]
- b Find the shaded area by integration. [3]

10 [Maximum mark: 6]

Alan runs at 3 m s^{-1} from A to B. At exactly the same time, Belinda starts cycling at 8 m s^{-1} on the bearing 230° from B to C.

- a** Find \widehat{ABC} . [2]
- b** Find the distance between Alan and Belinda after 2 minutes. [4]

11 [Maximum mark: 6]

A warehouse buys a new forklift for \$36 995. After 3 years its value has depreciated to \$21 600.

- a** Find the annual rate of depreciation. [3]
- b** Assuming this rate of depreciation continues, what do you expect the forklift to be worth 10 years after its purchase? [3]

12 [Maximum mark: 5]

A group of adults were surveyed regarding their opinions of their government's handling of the COVID-19 pandemic. The results are shown in the table:

Age group	Opinion		
	poor	fair	excellent
18 - 25	14	12	6
26 - 39	19	20	11
40 - 59	8	15	13
60+	12	17	9

Genevieve conducts a χ^2 test for independence at a 10% level of significance.

- a** State the null hypothesis. [1]
- b** Calculate the p -value for this test. [2]
- c** State, giving a reason, whether the null hypothesis should be accepted. [2]

13 [Maximum mark: 6]

In Florence, Italy, a street vendor sells replicas of the statue of David by Michelangelo. The replicas for sale have the following dimensions:

Size	extra small	small	medium	large	extra large
Height (cm)	8.0	10.0	12.5	16.0	25.0
Volume (cm ³)	8.2	16.0	31.3	65.5	250

- a** Find the variation model which best fits the data, including the value of r . [2]
- b** Do you think the replicas are mathematically *similar* to one another? Explain your answer. [2]
- c** The real statue of David is approximately 5.17 m high, with volume 2.098 m^3 . [2]
- Are the replicas to scale? Explain your answer.

14 [Maximum mark: 3]

When Rachel types 100 words, she makes typographical errors with the probabilities given in the table:

<i>Number of errors</i>	0	1	2	3	4	5
<i>Probability</i>	0.21	0.36	k	0.16	0.04	0.01

- a Find the value of k . [1]
- b On average, how many typographical errors does Rachel make per 100 words? [2]

PAPER 2

CALCULATOR, 90 MINUTES

1 [Maximum mark: 16]

The mass of cherries in a harvest is normally distributed with mean 5.38 g and standard deviation 0.62 g.
Let X be the mass of a randomly selected cherry.

- a Find the value of k such that $P(X < k) = 0.4$. Explain what this value means. [3]
- b Christina takes a sample of 50 cherries.

i Find the probability that a randomly selected cherry has mass greater than 6 g. [1]

ii How many cherries would Christina expect to have mass greater than 6 g? [2]

iii Find the probability that at least 4 cherries will have mass greater than 6 g. [2]
- c Pedro takes a sample of 10 cherries and analyses their sugar content. He records the following results:

<i>Mass (g)</i>	5.17	5.84	6.01	5.74	4.88	5.41	5.62	4.78	4.89	5.20
<i>Sugar content (g)</i>	0.66	0.83	0.92	0.75	0.59	0.71	0.75	0.60	0.58	0.65

- i Sketch the scatter diagram for the data. [2]
- ii Explain why Spearman's rank correlation coefficient might be appropriate for this data. [1]
- iii Find the ranks for each variable. [2]
- iv Calculate Spearman's rank correlation coefficient r_s . [2]
- v Hence describe the correlation between the variables. [1]

2 [Maximum mark: 15]

Herlina lives in a fishing village in Indonesia. She wants to model the tides in the harbour so she can help fishermen avoid the coral reefs. She sits by the pier one day and records the water depth on the jetty post at various intervals between high tides. Her results are given in the table below:

<i>Time after first high tide (t hours)</i>	0	0.8	1.8	2.6	4.2	5.4	6.2	7.4	8.6	9.6	10.2	11.0	11.6	12.4
<i>Water depth (h m)</i>	3.3	3.2	2.8	2.4	1.5	1.0	0.9	1.1	1.7	2.3	2.6	3.0	3.2	3.3

From a scatterplot of the data, Herlina decides that a trigonometric model is appropriate.

- a To model the water depth h at time t hours, Herlina could choose a cosine model of the form $h(t) = a \cos(bt)^\circ + d$ metres, or a sine model of the form $h(t) = a \sin(bt)^\circ + d$ metres. [2]
Which model is more appropriate? Explain your answer.
- b Use the high and low tide measurements to estimate the amplitude of the model, and hence state the value of the appropriate constant. [2]
- c Use the data to estimate the mean water depth, and hence state the value of the appropriate constant. [2]
- d Use the data to estimate the period between high tides, and hence state the value of the appropriate constant. [2]
- e Use your model to estimate the depth of the water 14 hours after high tide. [2]
- f The fishermen have told Herlina that their boats can only pass over the reef when the water depth at the pier is at least 1.4 m. Estimate the average number of hours per day, for which the fishing boats can pass over the reef. [4]

- g** Herlina realises that not all of the fishing boats are the same, and so it would be most helpful to the fishermen if she gave them a trigonometric function for the depth of water $r(t)$ above the reef. Write a suitable function for Herlina, given that the top of the reef is 55 cm higher than the sea floor beside the pier. [1]

3 [Maximum mark: 15]

Portia has €10 000 she wants to invest. She is given two options:

A: 5% per annum simple interest paid quarterly.

B: 4.4% per annum interest compounded quarterly.

- a** Identify which investment would result in an arithmetic sequence and which would result in a geometric sequence. [2]
- b** Write a formula for the value of the simple interest investment after n quarters. [2]
- c** **i** Write a formula for the value of the compound interest investment after n quarters. [2]
ii Hence or otherwise, find the value of this investment after 7 quarters. [1]
- d** Find how long Portia would need to invest her money, for the compound interest investment to be the better option. [3]
- e** Portia chooses the compound interest investment, and invests her money for 15 years. At this time, she withdraws the total and places it in an annuity fund which returns 2.8% p.a. compounded monthly.
- i** Find the starting balance for the annuity fund. [2]
- ii** How much can Portia withdraw each month, for the fund to last 5 years? [3]

4 [Maximum mark: 17]

James buys bales of wool from different producers, then sells the wool to clothing manufacturers.

James needs to be confident that the bales he sells have a mean pressed weight of more than 160 kg. He therefore tests the weight of every 20th bale that is sent to him.

- a** State the type of sampling James is using. [1]
- b** During one month, the weights in kg of the bales James tests are:
- | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|
| 166.5 | 158.2 | 170.7 | 153.4 | 165.2 | 168.6 | 161.3 |
| 155.9 | 162.3 | 164.7 | 159.6 | 167.1 | 158.3 | 162.7 |
- i** Find the sample mean and sample standard deviation. [2]
- ii** What type of hypothesis test should be used to determine whether James can confidently sell the bales he is receiving? Explain your answer. [2]
- iii** State the null and alternative hypotheses for this test. [2]
- iv** Conduct the appropriate hypothesis test at a 5% level of significance to determine whether James can confidently sell the bales he is receiving. [3]
- c** In the same month, James' business partner Susan samples 12 bales she has received. She finds their mean pressed weight is 164.5 kg. James wants to test whether the pressed weights of the bales he is receiving are different from those Susan is receiving.
- i** What type of hypothesis test should James perform? [1]
- ii** State one assumption James will need to make in his test. [1]
- iii** State the null and alternative hypotheses for this test. [2]
- iv** Conduct the appropriate hypothesis test at a 5% level of significance to determine whether the mean pressed weights of the bales are different. [3]

5 [Maximum mark: 17]

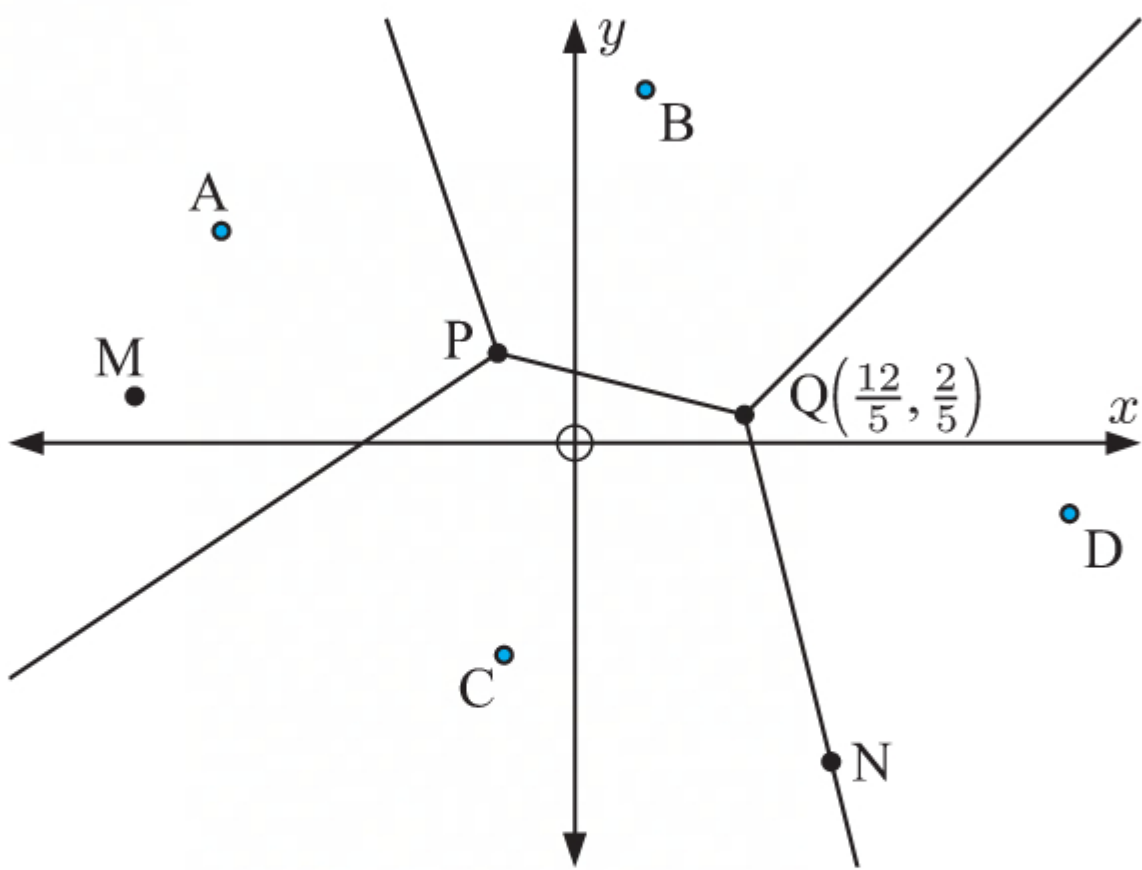
An alpine ski region has four weather stations whose coordinates and altitudes are given in the table below. The grid units are kilometres.

Weather station	Coordinates	Altitude (m)
A	(−5, 3)	2635
B	(1, 5)	z
C	(−1, k)	2307
D	(7, −1)	2683

The Voronoi diagram alongside has been constructed to help people understand which weather station will be closest to them.

The points P and Q are the intersections of the Voronoi edges.

The points M and N are accommodation for skiers.



- a Find the equation of the perpendicular bisector of line segment AB. [3]
- b The equation of the perpendicular bisector of line segment AC is $y = \frac{2}{3}x + 2$.
 - i Find the value of k . [2]
 - ii Find the coordinates of point P. [3]
- c Use nearest neighbour interpolation to estimate the altitude of point:
 - i M [1]
 - ii N [1]
- d Using nearest neighbour interpolation, the altitude of Q is estimated to be 2602. Find the value of z . [2]
- e An extra weather station is to be added in between the existing ones.
 - i Where should it be placed, so it is as far as possible from each of the existing weather stations? [3]
 - ii Sketch a new Voronoi diagram for the weather stations. You do *not* need to include the coordinates of each point. [2]

Trial examination 4

PAPER 1

CALCULATOR, 90 MINUTES

1 [Maximum mark: 6]

A minor sector has radius 5 cm and the angle subtended at the centre is 128° .

- a Calculate the perimeter of the minor sector. [3]
- b Calculate the area of the major sector. [3]

2 [Maximum mark: 6]

Students in a small class took a multiple choice test for Mathematics out of 20, and a multiple choice test for Physics out of 10. The results are shown below:

Mathematics (x)	1	3	4	6	8	9	11	14
Physics (y)	1	2	4	4	5	7	8	9

- a Calculate the Pearson's product-moment correlation coefficient, r . [2]

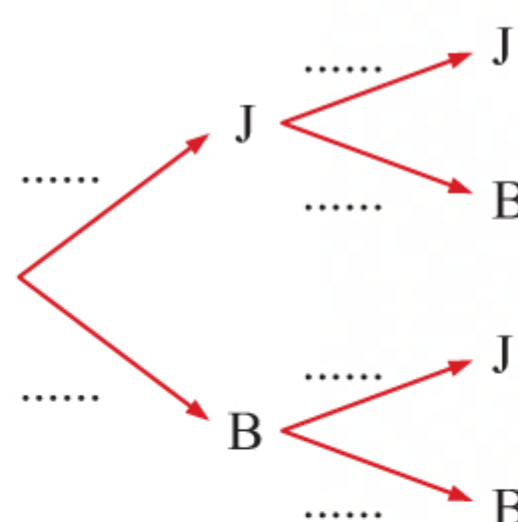
The relationship between x and y can be modelled by the equation $y = ax + b$.

- b Find the values of a and b . [2]
- c One student was sick for the Physics test but scored 10 in the Mathematics test. Use your regression line to predict what they would have scored in the Physics test to the nearest integer. [2]

3 [Maximum mark: 6]

Julie and Bill are playing tennis. The probability that Julie wins the first game is $\frac{1}{3}$. If Bill wins a game the probability that he wins the next game is 0.8. If Julie wins a game the probability that she wins the next game is 0.5. Let J be the event that Julie wins a game, and B be the event that Bill wins a game.

- a Copy and complete the tree diagram below. [2]



- b Find the probability that Julie wins the second game. [2]
- c Given that Julie wins the second game, find the probability that Julie won the first game. [2]

4 [Maximum mark: 5]

Steve took a Chemistry multiple choice test. It consisted of 30 questions with 5 different answers for each question. Each question had exactly one correct answer. Steve guessed the answers to all of the questions.

- a Find the probability that he answered exactly 12 of the questions correctly. [3]

If Steve answers less than 10 questions correctly, then he needs to re-sit the test.

- b What is the probability he must re-sit the test? [2]

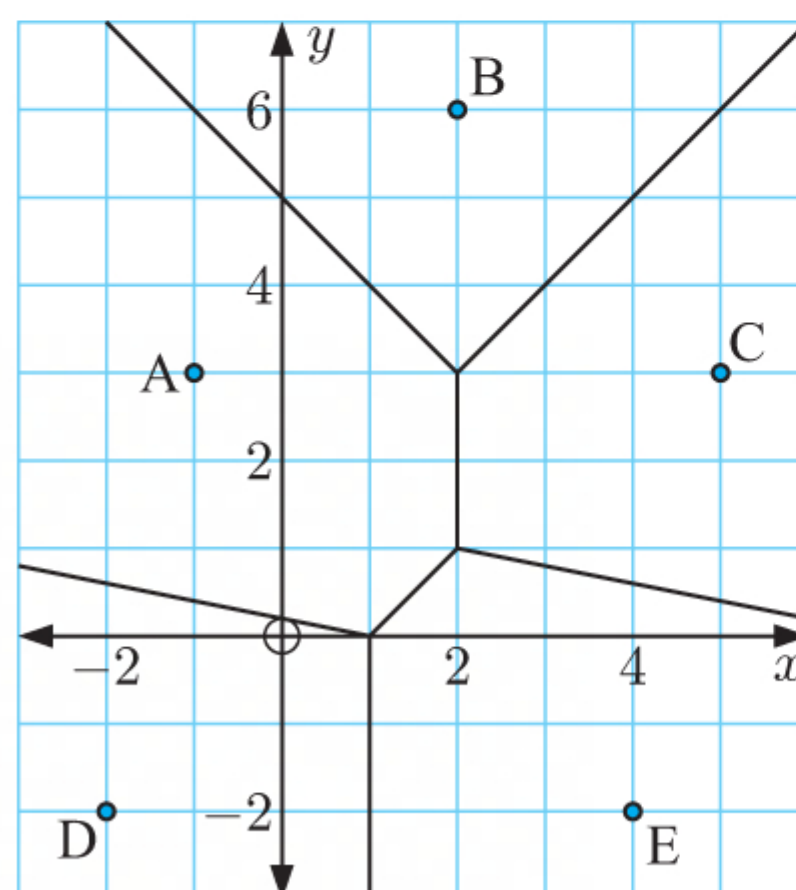
5 [Maximum mark: 5]

The function $f(x)$ is defined by $f(x) = x^2 - 6x + 20$.

- a Find $f(5)$. [1]
- b Find the range of the function. [2]
- c Find the smallest value of a such that the function $g(x) = x^2 - 6x + 20$, $x \geq a$, is one-to-one. [2]

6 [Maximum mark: 4]

The Voronoi diagram with points $A(-1, 3)$, $B(2, 6)$, $C(5, 3)$, $D(-2, -2)$, and $E(4, -2)$ below represents the locations of recycling centres in a city.



- a** Charlotte lives at $(4, 1)$. Which recycling centre should she visit? [1]
- b** A new recycling centre is to be built as far away as possible from the existing recycling centres. Assuming it must be located inside the region defined by the existing recycling centres, where should the new recycling centre be placed? [3]

7 [Maximum mark: 6]

A savings scheme is offering an interest rate of 2.5% per annum, compounded half-yearly. Astria wants to save £10 000. She works out that she can save £500 a year, which she will deposit on the 1st of January each year.

- a** How much will she have saved after 5 years? [3]
- b** How many years will it take for Astria to save the full amount? Give your answer as an integer. [3]

8 [Maximum mark: 4]

The life expectancy of a person in the United States of America, t years after 1900, is given by

$$L = 10.5 + 13.9 \ln(t + 10) \text{ years, } t \geq 0.$$

- a** Estimate the life expectancy in the United States of America in 1975. [2]
- b** In what year was the life expectancy 60 years? [2]

9 [Maximum mark: 5]

The volume, $V \text{ cm}^3$, of a tin of radius $r \text{ cm}$ and with fixed surface area, is given by the formula

$$V = 250r - \pi r^3.$$

- a** Find $\frac{dV}{dr}$. [1]
- b** Find the positive value of r when $\frac{dV}{dr} = 0$. [2]
- c** Find the value of V which corresponds to this value of r . [2]

10 [Maximum mark: 7]

The number of words in the first 20 sentences of Chapter 1 of *The Hunger Games* by Suzanne Collins are as follows:

12, 17, 12, 4, 7, 7, 9, 18, 13, 18, 7, 5, 11, 14, 14, 3, 5, 25, 9, 10

- a** Calculate the mean for the data. [2]
- b** Calculate the IQR for the data. [2]
- c** What is the largest data value that is *not* an outlier? [3]

11 [Maximum mark: 7]

Sarah will cycle 2000 miles over a number of days for charity. She cycles 12 miles on day 1, and increases this distance by 10% each day.

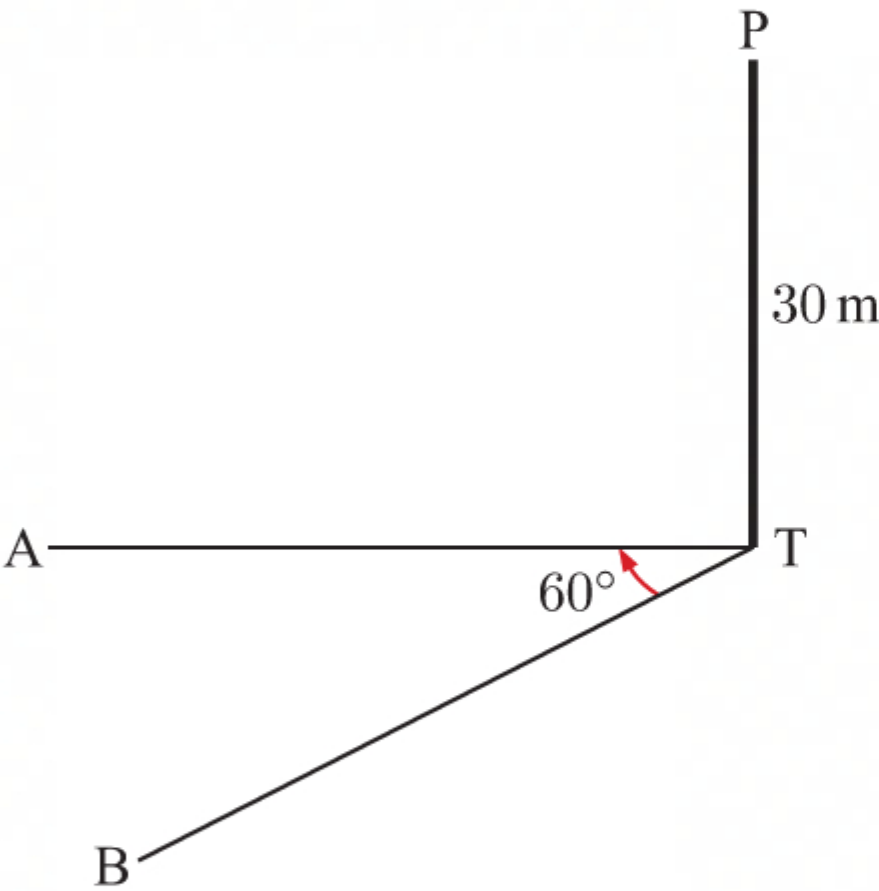
- a** How many days will it take for her to complete this challenge? [4]
- b** What is the greatest number of miles that she will complete in a single day? [3]

12 [Maximum mark: 7]

- A theory claims that, when sweet peas with red flowers and sweet peas with white flowers are crossed, the next generation of sweet peas have red, white, and pink flowers in the proportions $\frac{1}{6}$, $\frac{1}{6}$, and $\frac{2}{3}$ respectively.
- The outcomes in an actual experiment are as follows: 24 with red flowers, 34 with white flowers, and 62 with pink flowers.
- A χ^2 goodness of fit test at a 1% significance level was conducted on the data.
- a State the null and alternative hypotheses. [2]
 - b State the number of degrees of freedom. [1]
 - c Find the p -value. [2]
 - d What is the conclusion of the test? Give reasons for your answer. [2]

13 [Maximum mark: 8]

The diagram below shows a tower, PT, with height 30 m and points A and B which lie on the same horizontal plane as the base of the tower. The angle of elevation to the top of the tower is 15° from A, and 13° from B. The angle \widehat{BTA} is 60° as shown.



Calculate the distance AB.

14 [Maximum mark: 4]

A renowned butcher is blindfolded and asked to taste and arrange eight cuts of meat in order of price. The correct order is A, B, C, D, E, F, G, H while the order chosen by the butcher was A, (B, D), C, G, (E, F, H). The brackets indicate cuts of meat which the butcher assigned the same price.

- a Copy and complete this table of rankings: [2]

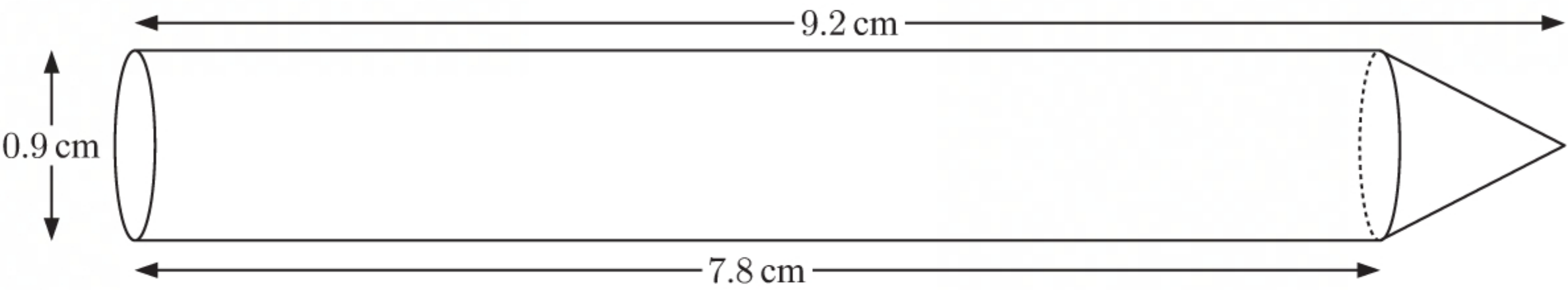
	A	B	C	D	E	F	G	H
True rank	1	2						
Butcher's rank	1							

- b Determine the value of r_s as a measure of the correlation between the butcher's opinion and the correct order. [2]

PAPER 2 CALCULATOR, 90 MINUTES

1 [Maximum mark: 17]

Lucy was assigned a project on 3D shapes and decided to model her crayon as a cylinder and a cone. She drew the diagram below and added measurements to 1 decimal place.

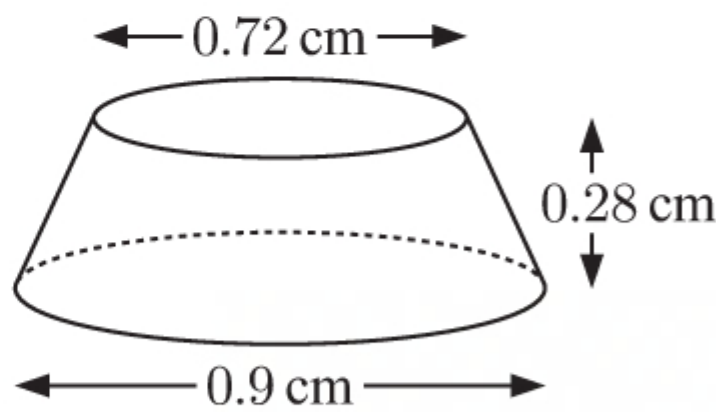


- a Calculate the height of the cone. [1]
- b Find the total volume of the crayon. [4]
- c Calculate the slant height of the cone. [2]

A label is put around the crayon so that only the cone and base of the crayon is visible.

- d What percentage of the crayon’s surface area is being covered by the label? [6]

After a few weeks of use the top of the crayon looks like this:



- e Calculate the remaining volume of the crayon. [4]

2 [Maximum mark: 14]

A survey was done in a school in which every 5th student was asked how much screen time they had had the previous evening. The results are shown below:

	Year 9	Year 10	Year 11	Year 12
Less than 1 hour	4	6	8	11
Between 1 and 3 hours	6	8	6	14
More than 3 hours	9	7	7	9

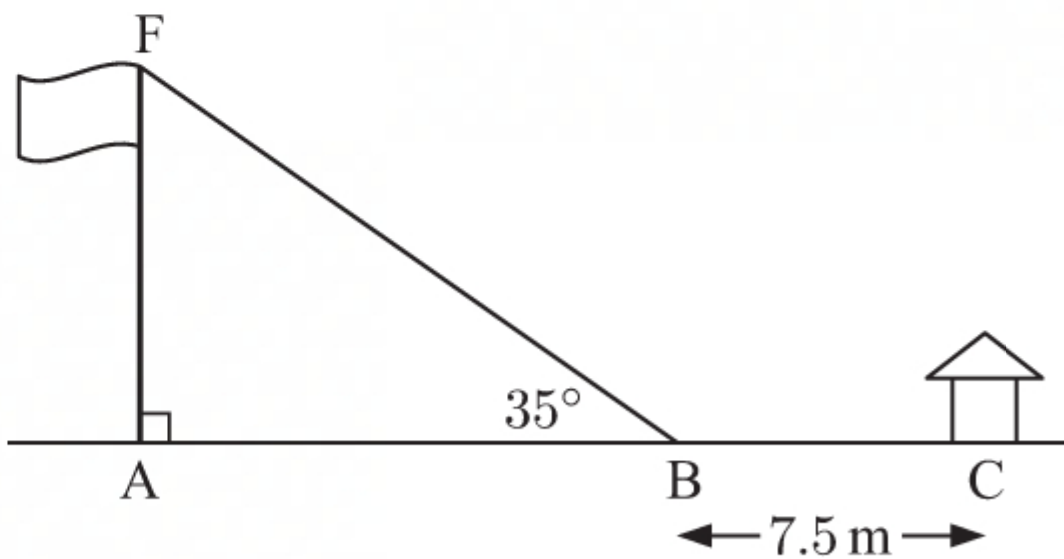
- a Name the sampling method used in this survey. [1]
- b Find the probability that a student chosen at random had more than 3 hours of screen time. [2]
- c Find the probability that a student chosen at random was in Year 11 and had less than one hour of screen time. [2]
- d Given that a student chosen at random is in Year 12, find the probability that they had more than 3 hours of screen time. [3]

One of the teachers in the school wanted to test whether the amount of screen time was dependent on the Year group a student was in. He performs a χ^2 test at a 10% significance level.

- e Write down the null hypothesis for this test. [1]
- f Write down the number of degrees of freedom. [1]
- g Find the p -value for this test. [2]
- h State the conclusion of the test, giving a reason for your answer. [2]

3 [Maximum mark: 13]

A vertical flagpole is positioned at A and is supported by a wire which runs from the top of the flagpole to the ground at B. The wire makes an angle of 35° with the ground as shown in the diagram below:



- a Find the angle of depression from the top of the flagpole to B. [1]

There is a storm forecast for the upcoming week. The site manager wants to add another wire for extra support from the top of the flagpole to his cabin at C. The angle that the wire will make with the ground is 29° .

- b Calculate the length of the wire that is required to go from the top of the flagpole to C. [5]
- c Calculate the height of the flagpole. [3]

Every morning the site manager must walk from his cabin to the base of the flagpole to raise the flag, then return to his cabin. Once the working day is finished, he must walk there again to bring the flag down before returning to his cabin to lock up.

- d Calculate the **total distance** the site manager walks each day to raise and bring down the flag. [4]

4 [Maximum mark: 15]

Clare has forgotten her graphics calculator and instead just has a normal calculator in her bag. She therefore has to estimate the area under the curve $y = \sqrt{5x - 1}$ between $x = 2$ and $x = 6$ using the trapezoidal rule.

- a** Copy and complete the table below giving your answers to 4 decimal places. [2]

x	2	2.5	3	3.5	4	4.5	5	5.5	6
$y = \sqrt{5x - 1}$	3		3.7417		4.3589		4.8990		5.3852

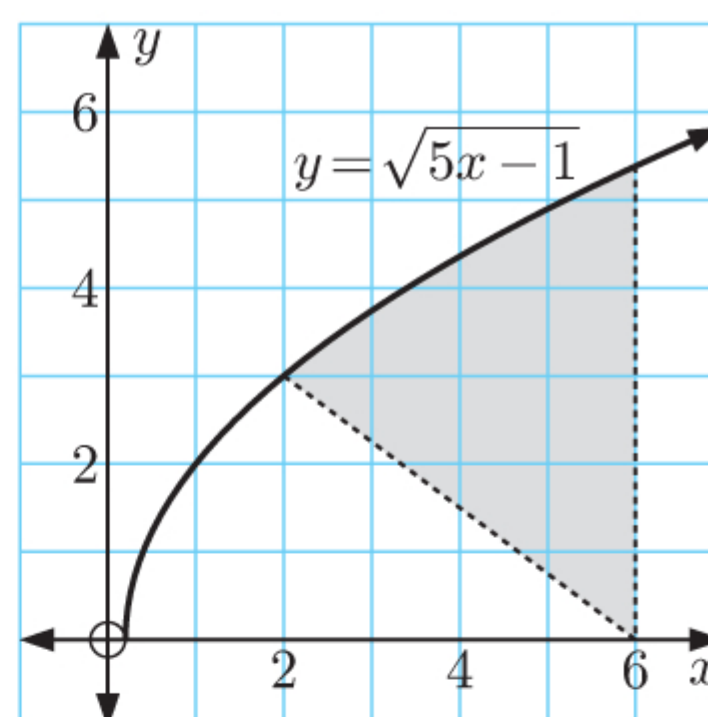
- b** Using the table in part **a** and the trapezoidal rule, estimate $\int_2^6 \sqrt{5x - 1} dx$ to 3 decimal places. [4]

Clare's friend Roy lets Clare borrow his graphics calculator when he was not using it.

- c** Calculate the value of $\int_2^6 \sqrt{5x - 1} dx$, giving your answer to 3 decimal places. [2]

- d** Calculate the percentage error in Clare's estimate using the value in part **c**. [3]

e [4]



Clare has designed a pool in the shape of the shaded region above, where the units are metres.

Use your answer to part **c** to find the area of the pool, to 3 decimal places.

5 [Maximum mark: 21]

A farmer supplies eggs to his local shop. Last year the weights of eggs, X , were normally distributed with a mean of 51.6 g and a standard deviation of 7.5 g.

- a** Sketch this information on a diagram showing ± 1 standard deviation. [3]

- b** Find the percentage of eggs that weighed less than 49 g. [2]

Large eggs are classified as having a weight greater than k grams.

- c** Given that large eggs made up 27.87% of eggs, find the value of k . [2]

40 eggs were selected at random.

- d** Find the expected number of large eggs in the selection. [3]

- e** Find the probability that exactly $\frac{2}{5}$ of the eggs were large. [3]

The farmer claims that the eggs he produced this year are heavier than last year. Below is a random sample of the weights of 30 eggs:

33 45 54 70 36 37 55 50 81 32 40 37 58 57 53
52 51 40 65 75 78 68 63 53 54 52 55 56 53 40

- f** Calculate the mean and standard deviation for this sample. [3]

The farmer performs a t -test at a 10% significance level to test his claim.

- g i** State the null and alternative hypotheses for this test. [1]

- ii** Find the p -value and t -statistic. [2]

- h** Conclude, giving reasons for your answer, whether the farmer was correct in his claim. [2]

Worked solutions

TOPIC 1 SKILL BUILDER QUESTIONS

1 diameter = 6.24 cm

$$\therefore \text{radius} = \frac{6.24}{2} = 3.12 \text{ cm}$$

$$\therefore \text{area} = \pi \times (3.12)^2 \\ \approx 30.581\,519\,5 \dots$$

a area $\approx 30.6 \text{ cm}^2$ {3 significant figures}

b area $\approx 30.5815 \text{ cm}^2$ {4 decimal places}

2 a 382×21
 $\approx 400 \times 20$
 ≈ 8000

b 6.91×0.875
 $\approx 7 \times 0.9$
 ≈ 6.3

c $38\,107 \div 213$
 $\approx 40\,000 \div 200$
 ≈ 200

3 The measuring device is accurate to $\pm \frac{0.1}{2} \text{ km h}^{-1} = \pm 0.05 \text{ km h}^{-1}$.

$$\therefore \text{the range of values is } 141.6 \pm 0.05 \text{ km h}^{-1}.$$

The cricket player's actual bowling speed lies between 141.55 km h^{-1} and 141.65 km h^{-1} .

$$\therefore 141.55 \text{ km h}^{-1} < s < 141.65 \text{ km h}^{-1}$$

4 The width of the block could be from $16\frac{1}{2} \text{ m}$ to $17\frac{1}{2} \text{ m}$.

The length of the block could be from $21\frac{1}{2} \text{ m}$ to $22\frac{1}{2} \text{ m}$.

$$\therefore \text{the lower boundary of the area is } 16\frac{1}{2} \times 21\frac{1}{2} = 354.75 \text{ m}^2$$

$$\text{and the upper boundary of the area is } 17\frac{1}{2} \times 22\frac{1}{2} = 393.75 \text{ m}^2.$$

$$\therefore 354.75 \text{ m}^2 < A < 393.75 \text{ m}^2$$

5 a Volume $\approx 15 \text{ cm} \times 12 \text{ cm} \times 8 \text{ cm}$
 $\approx 1440 \text{ cm}^3$

b i Actual volume = $15.3 \text{ cm} \times 11.8 \text{ cm} \times 8.4 \text{ cm}$
 $= 1516.536 \text{ cm}^3$

ii Absolute error = $|V_A - V_E|$
 $= |1440 - 1516.536| \text{ cm}^3$
 $= |-76.536| \text{ cm}^3$
 $= 76.536 \text{ cm}^3$

$$\begin{aligned} \text{Percentage error} &= \frac{|V_A - V_E|}{V_E} \times 100\% \\ &= \frac{76.536}{1516.536} \times 100\% \\ &\approx 5.05\% \end{aligned}$$

6 a Area $\approx \pi \times 7^2$
 $\approx 49\pi$
 $\approx 154 \text{ cm}^2$

b The radius length could be from $6\frac{1}{2} \text{ cm}$ to $7\frac{1}{2} \text{ cm}$.

$$\therefore \text{the lower boundary of the area is } \pi \times (6\frac{1}{2})^2 = 42.25\pi \text{ cm}^2 \approx 133 \text{ cm}^2$$

$$\text{and the upper boundary of the area is } \pi \times (7\frac{1}{2})^2 = 56.25\pi \text{ cm}^2 \approx 177 \text{ cm}^2.$$

c If the exact area V_E was $42.25\pi \text{ cm}^2$, the percentage error = $\frac{|V_A - V_E|}{V_E} \times 100\%$
 $= \frac{|49\pi - 42.25\pi|}{42.25\pi} \times 100\%$
 $\approx 16.0\%$

If the exact area V_E was $56.25\pi \text{ cm}^2$, the percentage error = $\frac{|V_A - V_E|}{V_E} \times 100\%$
 $= \frac{|49\pi - 56.25\pi|}{56.25\pi} \times 100\%$
 $\approx 12.9\%$

$$\therefore \text{the maximum percentage error in the estimate} \approx 16.0\%.$$

$$\begin{aligned} 7 \quad a \quad 64 &= 8^2 \\ &= (2^3)^2 \\ &= 2^6 \end{aligned}$$

$$\begin{aligned} b \quad 125 \times 5^k &= 5^3 \times 5^k \\ &= 5^{3+k} \end{aligned}$$

$$\begin{aligned} c \quad \frac{9^m}{81^n} &= \frac{9^m}{(9^2)^n} \\ &= \frac{(3^2)^m}{(3^2)^{2n}} \\ &= \frac{3^{2m}}{3^{4n}} \\ &= 3^{2m-4n} \end{aligned}$$

$$\begin{aligned} 8 \quad a \quad (-3m^3)^4 &= (-3)^4 \times (m^3)^4 \\ &= 81m^{12} \end{aligned}$$

$$\begin{aligned} b \quad \left(\frac{xy^2}{2}\right)^5 &= \frac{(xy^2)^5}{2^5} \\ &= \frac{x^5y^{10}}{32} \end{aligned}$$

$$\begin{aligned} c \quad 7s^2t \times (4st^3)^3 &= 7s^2t \times 4^3s^3t^9 \\ &= 7 \times 64s^5t^{10} \\ &= 448s^5t^{10} \end{aligned}$$

$$\begin{aligned} 9 \quad a \quad 4^0 + 4^{-1} &= 1 + \frac{1}{4} \\ &= \frac{5}{4} \end{aligned}$$

$$\begin{aligned} b \quad \left(2\frac{3}{4}\right)^{-2} &= \left(\frac{11}{4}\right)^{-2} \\ &= \left(\frac{4}{11}\right)^2 \\ &= \frac{4^2}{11^2} \\ &= \frac{16}{121} \end{aligned}$$

$$\begin{aligned} c \quad 2^2 + 2^1 + 2^{-1} &= 4 + 2 + \frac{1}{2} \\ &= 6 + \frac{1}{2} \\ &= \frac{13}{2} \end{aligned}$$

$$\begin{aligned} 10 \quad a \quad (x^2 + x^{-2})^2 &= (x^2)^2 + 2x^2 \times x^{-2} + (x^{-2})^2 \\ &= x^4 + 2 + x^{-4} \end{aligned}$$

$$\begin{aligned} b \quad (x^4 - x^2)(x^3 + 3) &= x^4 \times x^3 + 3x^4 + (-x^2) \times x^3 + (-x^2) \times 3 \\ &= x^7 + 3x^4 - x^5 - 3x^2 \end{aligned}$$

$$\begin{aligned} 11 \quad a \quad a^2b^{-3} &= a^2 \times \frac{1}{b^3} \\ &= \frac{a^2}{b^3} \end{aligned}$$

$$\begin{aligned} b \quad \frac{2m^{-2}n^3}{m^5n^{-5}} &= 2 \times m^{-2-5} \times n^{3-(-5)} \\ &= 2 \times m^{-7} \times n^8 \\ &= 2 \times \frac{1}{m^7} \times n^8 \\ &= \frac{2n^8}{m^7} \end{aligned}$$

$$\begin{aligned} c \quad \frac{12a^{-3}}{b^{-5}} &= 12 \times a^{-3} \times \frac{1}{b^{-5}} \\ &= 12 \times \frac{1}{a^3} \times b^5 \\ &= \frac{12b^5}{a^3} \end{aligned}$$

$$\begin{aligned} 12 \quad a \quad 42000 &= 4.2 \times 10\,000 \\ &= 4.2 \times 10^4 \end{aligned}$$

$$\begin{aligned} b \quad 0.0000678 &= 6.78 \times 0.000\,001 \\ &= 6.78 \times 10^{-5} \end{aligned}$$

$$\begin{aligned} c \quad 526000000 &= 5.26 \times 100\,000\,000 \\ &= 5.26 \times 10^8 \end{aligned}$$

13 Using technology:

$$a \quad (3.57 \times 10^6) \times (2.38 \times 10^3) = 8.4966 \times 10^9$$

$$c \quad (0.000\,08)^4 = 4.096 \times 10^{-17}$$

$$b \quad \frac{4.61 \times 10^{-7}}{3.45 \times 10^8} \approx 1.34 \times 10^{-15}$$

$$\begin{aligned} 14 \quad a \quad 8x - 2 &= 3x^2 \\ \therefore -3x^2 + 8x - 2 &= 0 \end{aligned}$$

Using technology, $x \approx 0.279$ or 2.39

$$\begin{aligned} b \quad 3x^3 + 7x^2 - 3x &= 2 \\ \therefore 3x^3 + 7x^2 - 3x - 2 &= 0 \end{aligned}$$

Using technology, $x \approx 0.667$, -2.62 , or -0.382

$$15 \quad a \quad \begin{cases} 2x - 3y = 2 \\ 5x + 3y = 5 \end{cases}$$

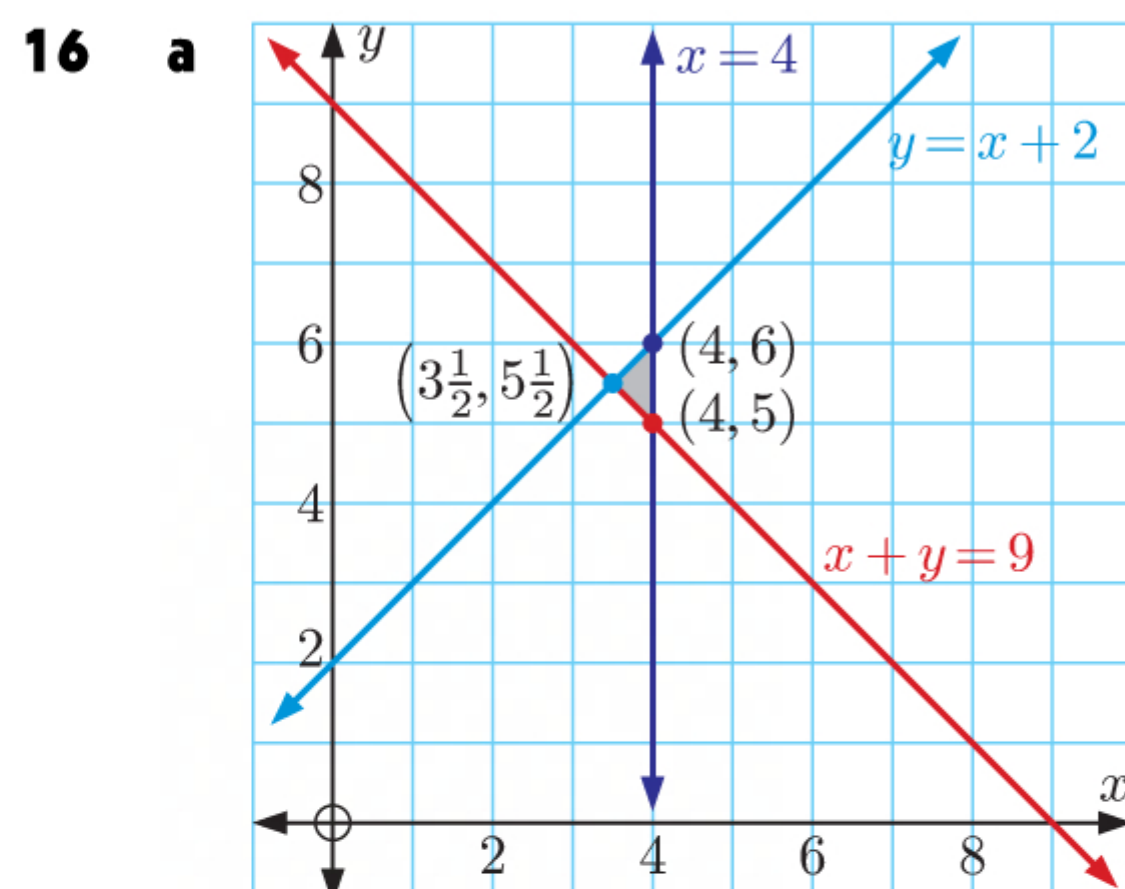
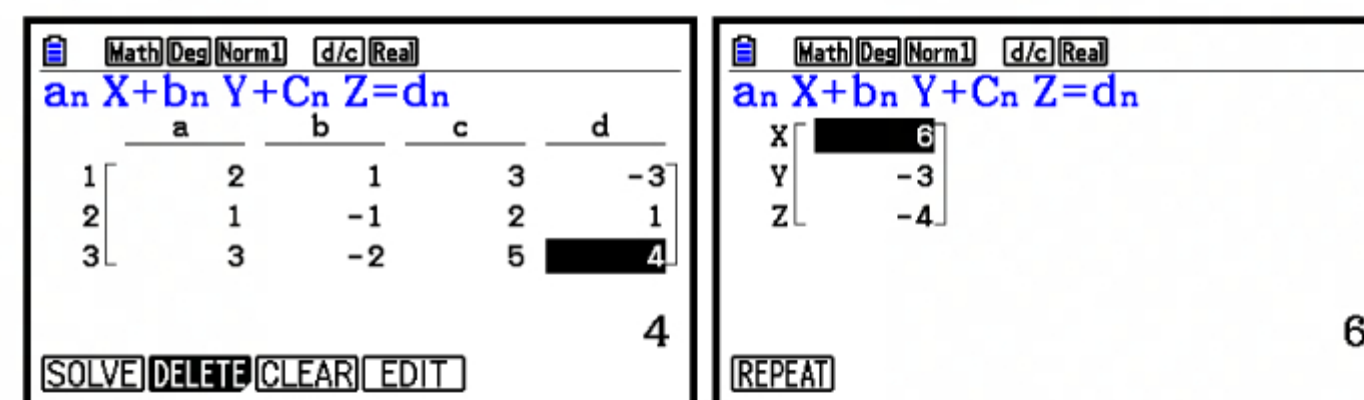
So, the solution is $x = 1$, $y = 0$.

$$b \quad \begin{cases} 3x - 7y = -8 \\ 6x + 11y = 12 \end{cases}$$

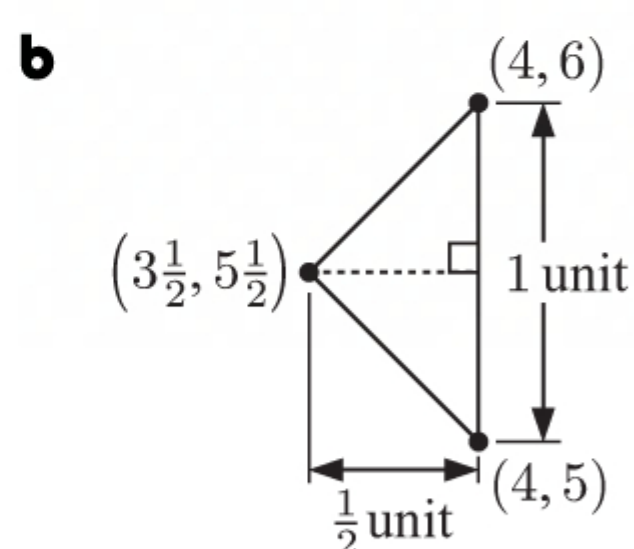
So, the solution is $x \approx -0.0533$, $y = 1.12$.

$$\mathbf{c} \quad \begin{cases} 2x + y + 3z = -3 \\ x - y + 2z = 1 \\ 3x - 2y + 5z = 4 \end{cases}$$

So, the solution is $x = 6$, $y = -3$, $z = -4$.



The vertices of the triangle are $(4, 6)$, $(4, 5)$, and $(3\frac{1}{2}, 5\frac{1}{2})$.



$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 1 \times \frac{1}{2} \\ &= \frac{1}{4} \text{ units}^2 \end{aligned}$$

17 8, 13, 18, 23, 28,

a $13 - 8 = 5$

$18 - 13 = 5$

$23 - 18 = 5$

$28 - 23 = 5$

The difference between successive terms is constant.

\therefore the sequence is arithmetic with $u_1 = 8$ and $d = 5$.

b $u_n = u_1 + (n - 1)d$

$\therefore u_n = 8 + 5(n - 1)$

$\therefore u_n = 3 + 5n$

d i Let $u_n = 153$

$\therefore 3 + 5n = 153$

$\therefore 5n = 150$

$\therefore n = 30$

$\therefore 153$ is a member of the sequence, and in fact is the 30th term.

c $u_{42} = 3 + 5(42)$
 $= 213$

ii Let $u_n = 4067$

$\therefore 3 + 5n = 4067$

$\therefore 5n = 4064$

$\therefore n = 812\frac{4}{5}$

But n must be an integer, so 4067 is not a member of the sequence.

18 a If 3, k , 11 are consecutive terms of an arithmetic sequence, then

$k - 3 = 11 - k$ {equating differences}

$\therefore 2k = 14$

$\therefore k = 7$

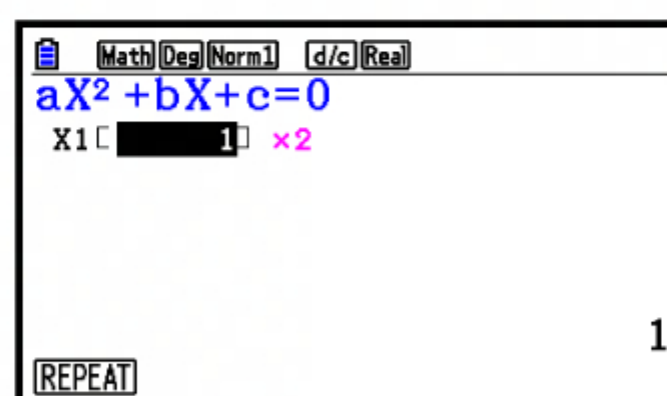
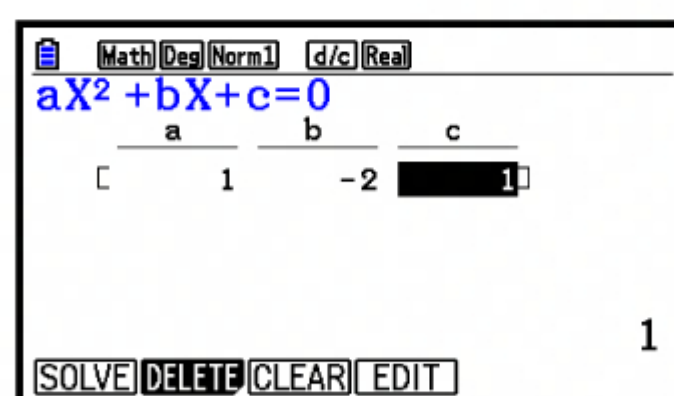
b If -2 , $k + 4$, $k^2 + 11$ are consecutive terms of an arithmetic sequence, then

$k + 4 - (-2) = k^2 + 11 - (k + 4)$ {equating differences}

$\therefore k + 6 = k^2 + 11 - k - 4$

$\therefore k^2 - 2k + 1 = 0$

$\therefore k = 1$ {using technology}



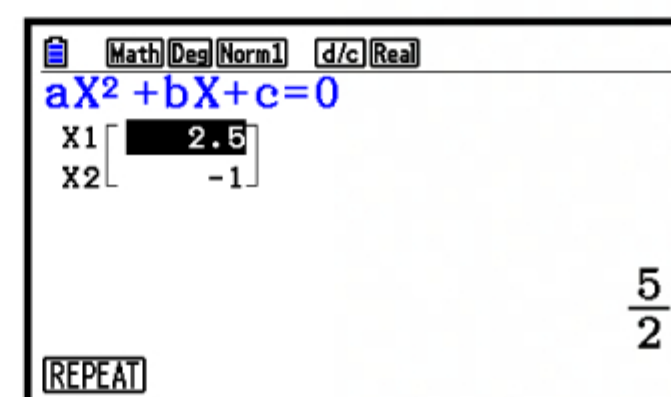
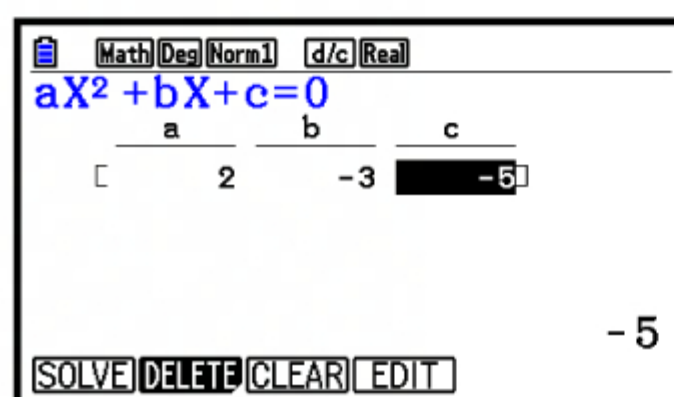
- c** If $k - 5$, $2k$, $2k^2$ are consecutive terms of an arithmetic sequence, then

$$2k - (k - 5) = 2k^2 - 2k \quad \{\text{equating differences}\}$$

$$\therefore k + 5 = 2k^2 - 2k$$

$$\therefore 2k^2 - 3k - 5 = 0$$

$$\therefore k = -1 \text{ or } \frac{5}{2} \quad \{\text{using technology}\}$$



19 a Average mass = $\frac{\text{total mass} - \text{mass of cage}}{\text{number of hamsters}}$

$$= \frac{1400 - 800}{5}$$

$$= \frac{600}{5}$$

$$= 120 \text{ g}$$

b $u_n = 800 + 120n$

20 a $u_5 = u_1 r^4 = 324 \quad \dots (1)$

and $u_{10} = u_1 r^9 = 78\,732 \quad \dots (2)$

Now $\frac{u_1 r^9}{u_1 r^4} = \frac{78\,732}{324} \quad \{(2) \div (1)\}$

$$\therefore r^5 = 243$$

$$\therefore r = \sqrt[5]{243}$$

$$\therefore r = 3$$

Using (1), $u_1(3)^4 = 324$

$$\therefore 81u_1 = 324$$

$$\therefore u_1 = 4$$

Thus $u_n = 4 \times 3^{n-1}$

b $u_8 = u_1 r^7 = -10 \quad \dots (1)$

and $u_{12} = u_1 r^{11} = -160 \quad \dots (2)$

Now $\frac{u_1 r^{11}}{u_1 r^7} = \frac{-160}{-10} \quad \{(2) \div (1)\}$

$$\therefore r^4 = 16$$

$$\therefore r = \pm \sqrt[4]{16}$$

$$\therefore r = \pm 2$$

If $r = 2$, then using (1), $u_1(2)^7 = -10$

$$\therefore 128u_1 = -10$$

$$\therefore u_1 = \frac{-10}{128} = -\frac{5}{64}$$

Thus $u_n = -\frac{5}{64} \times 2^{n-1}$

If $r = -2$, then using (1), $u_1(-2)^7 = -10$

$$\therefore -128u_1 = -10$$

$$\therefore u_1 = \frac{10}{128} = \frac{5}{64}$$

Thus $u_n = \frac{5}{64} \times (-2)^{n-1}$

21 $2, 2\sqrt{3}, 6, 6\sqrt{3}$

a $\frac{2\sqrt{3}}{2} = \sqrt{3}, \quad \frac{6}{2\sqrt{3}} = \frac{\cancel{2} \times 3}{\cancel{2}\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}, \quad \frac{6\sqrt{3}}{6} = \sqrt{3}$

Consecutive terms have a common ratio of $\sqrt{3}$.

\therefore the sequence is geometric with $u_1 = 2$ and $r = \sqrt{3}$.

b $u_n = u_1 r^{n-1}$

$$= 2(\sqrt{3})^{n-1}$$

c $u_{10} = 2(\sqrt{3})^{10-1}$

$$= 2(\sqrt{3})^9$$

$$= 2\sqrt{3} \times (\sqrt{3})^8$$

$$= 2\sqrt{3} \times ((\sqrt{3})^2)^4$$

$$= 2\sqrt{3} \times 3^4$$

$$= 162\sqrt{3}$$

d We need to find n such that $u_n = 2(\sqrt{3})^{n-1} > 1000$.

Using a graphics calculator with $Y_1 = 2 \times \sqrt{3} \wedge (X - 1)$, we view a table of values:

X	Y1
11	486
12	841.77
13	1458
14	2525.3

The first term to exceed 1000 is $u_{13} = 1458$.

22 There is a fixed percentage increase each year, so the population forms a geometric sequence with $u_0 = 217$ and $r = 1.42$.

\therefore the population after n years is $u_n = 217 \times (1.42)^n$.

a i $u_5 = 217 \times (1.42)^5$
 ≈ 1252.86

The expected population size after 5 years is approximately 1250 birds.

ii $u_{10} = 217 \times (1.42)^{10}$
 ≈ 7233.41

The expected population size after 10 years is approximately 7230 birds.

b We need to find when $217 \times (1.42)^n = 30\,000$.

Using technology, $n \approx 14.1$, so it will take approximately 14.1 years for the population to reach 30 000.

Eq: $217 \times 1.42^x = 30000$
$x = 14.05663401$
Lft = 30000
Rgt = 30000
REPEAT

23 a If the interest rate per annum is 7.2%, then the interest rate per month $i = \frac{7.2\%}{12} = 0.6\% = 0.006$.

$$\begin{aligned} r &= 1 + i \\ &= 1 + 0.006 \\ &= 1.006 \end{aligned}$$

b The interest is calculated monthly, so $n = 3 \times 12 = 36$ time periods.

$$\begin{aligned} u_{36} &= u_0 \times r^{36} \\ &= 500 \times (1.006)^{36} \\ &\approx 620.15 \end{aligned}$$

The value of the account after 3 years is €620.15.

c $\text{real value} \times (1.02)^3 = €620.15$
 $\therefore \text{real value} = \frac{€620.15}{(1.02)^3}$
 $= €584.38$

24 There is 1 time period every 3 months, so $n = \frac{33}{3} = 11$ time periods.

Each time period the investment increases by $i = \frac{8\%}{4} = 2\%$.

$$\begin{aligned} \therefore \text{the amount after 33 months is } u_{11} &= u_0 \times (1 + i)^{11} \\ &= 3500 \times (1.02)^{11} \quad \{2\% = 0.02\} \\ &\approx 4351.81 \end{aligned}$$

The maturing value of the account is £4351.81.

25 The initial investment u_0 is unknown.

There are $r = 5 \times 12 = 60$ time periods.

Each time period the investment increases by $i = \frac{4.8\%}{12} = 0.4\%$.

$$\begin{aligned} \text{Now } u_{60} &= u_0 \times (1 + i)^{60} \\ \therefore 30\,000 &= u_0 \times (1.004)^{60} \quad \{0.4\% = 0.004\} \\ \therefore u_0 &= \frac{30\,000}{(1.004)^{60}} \approx 23\,610.14 \end{aligned}$$

\therefore I need to invest \$23 610.14 now.

$$\begin{aligned} 26 \quad a \quad u_3 &= u_0 \times (1 - d)^3 \\ &= 2000 \times (0.7)^3 \quad \{30\% = 0.3\} \\ &= 686 \end{aligned}$$

So, after 3 years the value of the television is £686.

$$27 \quad a \quad N = 4.5 \times 12 = 54, \quad PV = -10\,000, \quad PMT = 0, \quad FV = 12\,000, \quad P/Y = 12, \quad C/Y = 12$$

$$\therefore I\% \approx 4.06$$

The account paid about 4.06% interest per annum.

$$b \quad I\% = 4.06, \quad PV = -10\,000, \quad PMT = 0, \quad FV = 20\,000, \quad P/Y = 12, \quad C/Y = 12$$

$$\therefore N \approx 205.2$$

It will take 206 months or 17 years and 2 months for Lauren to double her deposit.

[Norm] [End]	
Compound Interest	
n	=54
I%	=4.058437598
PV	=-10000
PMT	=0
FV	=12000
P/Y	=12
n	I% PV PMT FV AMORTIZ

[Norm] [End]	
Compound Interest	
n	=205.2174663
I%	=4.06
PV	=-10000
PMT	=0
FV	=20000
P/Y	=12
n	I% PV PMT FV AMORTIZ

$$28 \quad a \quad \text{The common difference} \\ d = -6.$$

$$\begin{aligned} b \quad u_n &= u_1 + (n - 1)d \\ \therefore u_{20} &= u_1 + 19d \\ &= 51 + 19 \times -6 \\ &= 51 - 114 \\ &= -63 \end{aligned}$$

$$\begin{aligned} c \quad S_n &= \frac{n}{2}(u_1 + u_n) \\ \therefore S_{20} &= \frac{20}{2}(51 - 63) \\ &= 10 \times -12 \\ &= -120 \end{aligned}$$

$$29 \quad a \quad \text{The series is arithmetic with } u_1 = 11, \quad d = 4, \quad \text{and } n = 20.$$

$$\begin{aligned} \text{Now } S_n &= \frac{n}{2}(2u_1 + (n - 1)d) \\ \therefore S_{20} &= \frac{20}{2}(2 \times 11 + 19 \times 4) \\ &= 10(22 + 76) \\ &= 980 \end{aligned}$$

$$b \quad 7 + 12.5 + 18 + 23.5 + \dots + 106$$

The series is arithmetic with $u_1 = 7$, $d = 5.5$, and $u_n = 106$.

First we need to find n .

$$\begin{aligned} \text{Now } u_n &= 106 \\ \therefore u_1 + (n - 1)d &= 106 \\ \therefore 7 + 5.5(n - 1) &= 106 \\ \therefore 5.5(n - 1) &= 99 \\ \therefore n - 1 &= 18 \\ \therefore n &= 19 \end{aligned}$$

$$\begin{aligned} \text{Using } S_n &= \frac{n}{2}(u_1 + u_n) \\ \therefore S_{19} &= \frac{19}{2}(7 + 106) \\ &= \frac{19}{2} \times 113 \\ &= 1073.5 \end{aligned}$$

$$c \quad 1 - 2 + 3 - 4 + 5 - 6 + 7 - \dots \text{ to 100 terms can be expressed as two separate arithmetic series:}$$

$$\begin{aligned} &1 + 3 + 5 + 7 + \dots \text{ where } u_1 = 1, \quad d = 2, \quad n = 50 \\ \text{and } &-2 - 4 - 6 - 8 - \dots \text{ where } u_1 = -2, \quad d = -2, \quad n = 50 \end{aligned}$$

$$\begin{aligned} \text{Using } S_n &= \frac{n}{2}(2u_1 + (n - 1)d), \quad \text{the sum of the first series} = \frac{50}{2}(2(1) + 49(2)) \\ &= 25(2 + 98) \\ &= 2500 \end{aligned}$$

$$\begin{aligned} \text{and the sum of the second series} &= \frac{50}{2}(2(-2) + 49(-2)) \\ &= 25(-4 - 98) \\ &= -2550 \end{aligned}$$

$$\begin{aligned} \therefore \text{the sum of both series} &= 2500 + (-2550) \\ &= -50 \end{aligned}$$

So, $1 - 2 + 3 - 4 + 5 - 6 + 7 - \dots$ to 100 terms is -50 .

- d** The integers from 1 to 200 which are not divisible by 3 are 1, 2, 4, 5, 7, 8, ..., 200.

The sum of these integers can be expressed as two separate arithmetic series A and B :

$$S_A = 1 + 4 + 7 + \dots + 196 + 199 \quad \text{where } u_1 = 1, d = 3, u_n = 199$$

$$\text{and } S_B = 2 + 5 + 8 + \dots + 197 + 200 \quad \text{where } u_1 = 2, d = 3, u_n = 200$$

$$\text{Now for } S_A, u_n = u_1 + (n-1)d \quad \text{and for } S_B, u_n = u_1 + (n-1)d$$

$$\therefore 199 = 1 + 3(n-1)$$

$$\therefore 200 = 2 + 3(n-1)$$

$$\therefore 198 = 3(n-1)$$

$$\therefore 198 = 3(n-1)$$

$$\therefore 66 = n-1$$

$$\therefore 66 = n-1$$

$$\therefore n = 67$$

$$\therefore n = 67$$

$$\text{Using } S_n = \frac{n}{2}(u_1 + u_n), \quad S_A = \frac{67}{2}(1 + 199) = 6700 \quad \text{and} \quad S_B = \frac{67}{2}(2 + 200) = 6767$$

$$\text{The total sum} = S_A + S_B$$

$$= 6700 + 6767$$

$$= 13\,467$$

30 a $u_7 = 1 \quad \therefore u_1 + 6d = 1 \quad \{\text{using } u_n = u_1 + (n-1)d\}$

$$u_{15} = -23 \quad \therefore u_1 + 14d = -23$$

Using technology to solve these equations simultaneously, we find that $u_1 = 19$ and $d = -3$.

b $u_n = u_1 + (n-1)d$
 $\therefore u_{27} = 19 + 26(-3) \quad \{\text{using a}\}$
 $= -59$

c $S_n = \frac{n}{2}(u_1 + u_n)$
 $\therefore S_{27} = \frac{27}{2}(19 + (-59)) \quad \{\text{from b}\}$
 $= \frac{27}{2} \times (-40)$
 $= -540$

31 a $S_n = \frac{n}{2}(2u_1 + (n-1)d)$
 $\therefore -210 = \frac{n}{2}(2 \times 18 - 3(n-1))$
 $\therefore \frac{n}{2}(36 - 3n + 3) = -210$
 $\therefore \frac{n}{2}(39 - 3n) = -210$

b From **a**, $\frac{n}{2}(39 - 3n) = -210$
 $\therefore n(39 - 3n) = -420$
 $\therefore 39n - 3n^2 = -420$
 $\therefore 3n^2 - 39n - 420 = 0$

Using technology, $n = 20 \quad \{n > 0\}$

- 32 a** The sequence is arithmetic with $u_1 = 7$ and $d = 3$.

$$\text{Now } S_n = \frac{n}{2}(2u_1 + (n-1)d)$$

$$\therefore S_n = \frac{n}{2}(2(7) + 3(n-1))$$

$$\therefore S_n = \frac{n}{2}(14 + 3n - 3)$$

$$\therefore S_n = \frac{n}{2}(11 + 3n)$$

b $S_n = 140$
 $\therefore \frac{n}{2}(11 + 3n) = 140 \quad \{\text{using a}\}$
 $\therefore n(11 + 3n) = 280$
 $\therefore 11n + 3n^2 = 280$
 $\therefore 3n^2 + 11n - 280 = 0$

Using technology, $n = 8 \quad \{n > 0\}$

- 33 a** The series is geometric with $u_1 = 10$, $r = \frac{1}{2}$, and $n = 8$.

$$\begin{aligned}\text{Now } S_n &= \frac{u_1(r^n - 1)}{r - 1} \\ \therefore S_8 &= \frac{10((\frac{1}{2})^8 - 1)}{\frac{1}{2} - 1} \\ &= 19.921\,875\end{aligned}$$

- b** The series is geometric with $u_1 = 2$, $r = 5$, and $n = 10$.

$$\begin{aligned}\text{Now } S_n &= \frac{u_1(r^n - 1)}{r - 1} \\ \therefore S_{10} &= \frac{2(5^{10} - 1)}{5 - 1} \\ &= 4\,882\,812\end{aligned}$$

- c** $\sum_{k=1}^{20} 3 \times (-2)^{k+2} = 3 \times (-2)^3 + 3 \times (-2)^4 + \dots + 3 \times (-2)^{22}$

The series is geometric with $u_1 = 3 \times (-2)^3 = -24$, $r = -2$, and $n = 20$.

$$\begin{aligned}\text{Now } S_n &= \frac{u_1(r^n - 1)}{r - 1} \\ \therefore S_{20} &= \frac{-24((-2)^{20} - 1)}{-2 - 1} \\ &= 8\,388\,600\end{aligned}$$

- 34 a** $10 + 14 + 18 + 22 + \dots + 138$ is arithmetic with $u_1 = 10$, $d = 4$

$$\begin{aligned}\text{Now } u_1 + (n - 1)d &= 138 \\ \therefore 10 + 4(n - 1) &= 138 \\ \therefore 4(n - 1) &= 128 \\ \therefore n - 1 &= 32 \\ \therefore n &= 33\end{aligned}$$

$$\begin{aligned}\text{So, the sum is } \frac{n}{2}(u_1 + u_{33}) &= \frac{33}{2}(10 + 138) \\ &= \frac{33}{2}(148) \\ &= 2442\end{aligned}$$

- b** $6 - 12 + 24 - 48 + 96 - \dots + 1536$ is geometric with $u_1 = 6$, $r = -2$

$$\begin{aligned}\text{Now } u_1 r^{n-1} &= 1536 \\ \therefore 6 \times (-2)^{n-1} &= 1536 \\ \therefore n &= 9 \quad \{\text{using technology}\}\end{aligned}$$

$$\begin{aligned}\text{So, the sum is } \frac{u_1(1 - r^n)}{1 - r} &= \frac{6(1 - (-2)^9)}{1 - (-2)} \\ &= \frac{6}{3}(1 - (-2)^9) \\ &= 2 \times 513 \\ &= 1026\end{aligned}$$

- 35 a** $N = 20 \times 12 = 240$, $I\% = 7.2$, $PV = 120\,000$, $FV = 0$, $P/Y = 12$, $C/Y = 12$

$$\therefore PMT \approx -944.82$$

The monthly repayment is \$944.82.

Norm1	End
Compound Interest	
n = 240	
I% = 7.2	
PV = 120000	
PMT = -944.8191588	
FV = 0	
P/Y = 12	
n I% PV PMT FV AMORTZ	

- b** $N = 12$, $I\% = 7.2$, $PV = 120\,000$, $PMT = -944.82$, $P/Y = 12$, $C/Y = 12$

$$\therefore FV \approx 117\,211.33$$

After 1 year, \$117 211.33 is still owing on the loan.

Norm1	End
Compound Interest	
n = 12	
I% = 7.2	
PV = 120000	
PMT = -944.82	
FV = -117211.3264	
P/Y = 12	
n I% PV PMT FV AMORTZ	

- c i** Amount paid = $\$944.82 \times 12$
= \$11 337.84

$$\text{ii } \$120\,000 - \$117\,211.33 = \$2788.67$$

- iii** The loan does not decrease by the full amount of the monthly repayment as the payment is used to pay interest as well as to reduce the principal.

- d i** $N = 19 \times 12 = 228$, $I\% = 6.95$, $PV = 117\,211.33$, $FV = 0$, $P/Y = 12$, $C/Y = 12$

$$\therefore PMT \approx -927.42$$

The new monthly repayment is \$927.42.

Norm1	End
Compound Interest	
n = 228	
I% = 6.95	
PV = 117211.33	
PMT = -927.4156112	
FV = 0	
P/Y = 12	
n I% PV PMT FV AMORTZ	

- ii** $I\% = 6.95$, $PV = 117\,211.33$, $PMT = -944.82$, $FV = 0$, $P/Y = 12$, $C/Y = 12$

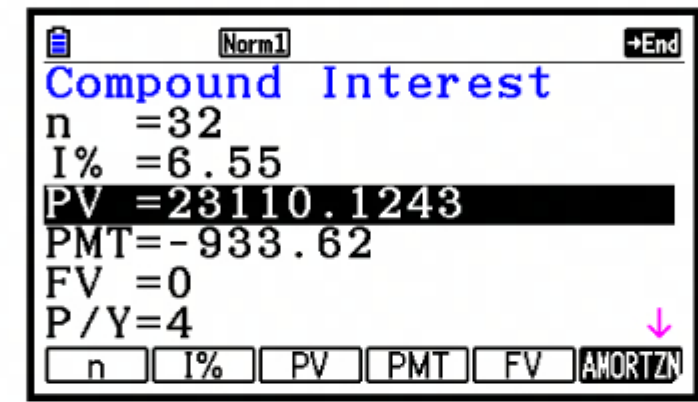
$$\therefore N \approx 219.5$$

It will take 220 months to pay off the rest of the loan with the original repayments.

\therefore the loan will be paid off 8 months earlier.

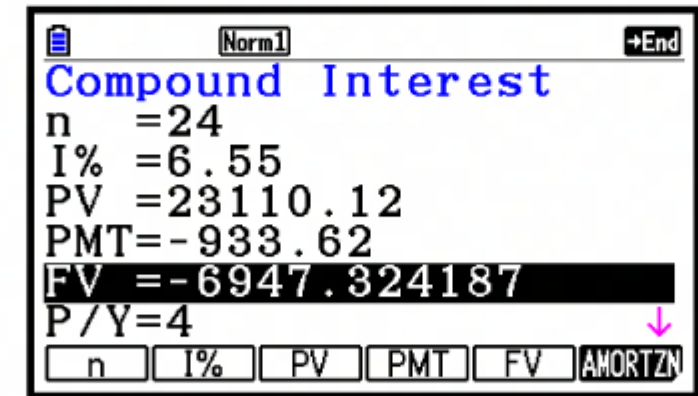
Norm1	End
Compound Interest	
n = 219.500585	
I% = 6.95	
PV = 117211.33	
PMT = -944.82	
FV = 0	
P/Y = 12	
n I% PV PMT FV AMORTZ	

- 36 a** $N = 8 \times 4 = 32$, $I\% = 6.55$, $PMT = -933.62$, $FV = 0$, $P/Y = 4$, $C/Y = 4$
 $\therefore PV \approx 23\,110.12$
 Oscar borrowed \$23 110.12.

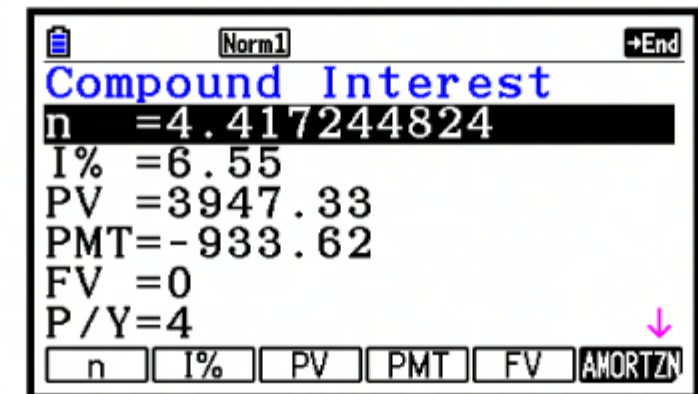


b $\text{Interest} = \$933.62 \times 8 \times 4 - \$23\,110.12$
 $= \$6765.72$

- c i** $N = 6 \times 4 = 24$, $I\% = 6.55$, $PV = 23\,110.12$, $PMT = -933.62$, $P/Y = 4$, $C/Y = 4$
 $\therefore FV \approx -6947.33$
 So, the outstanding balance is \$6947.33.



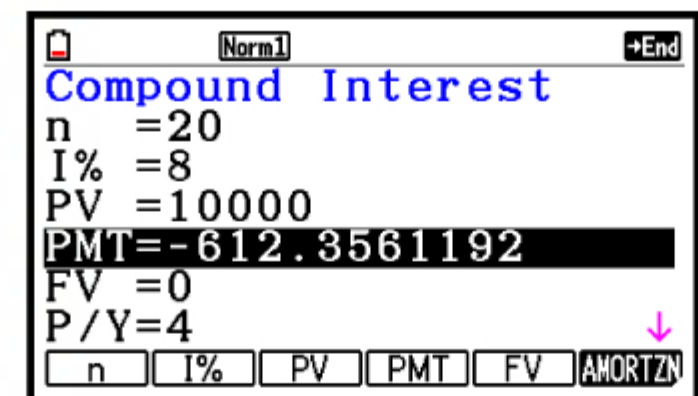
- ii** $I\% = 6.55$, $PV = 6947.33 - 3000$, $PMT = -933.62$, $FV = 0$, $P/Y = 4$, $C/Y = 4$
 $\therefore N \approx 4.417$



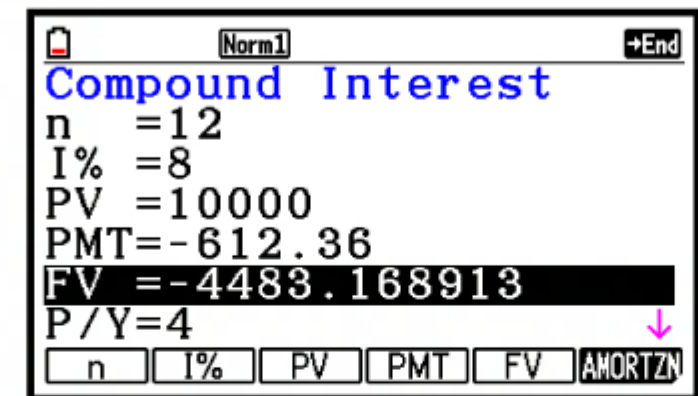
It will take another 5 quarters to pay off the rest of the loan.

$$\begin{aligned}\therefore \text{time saved} &= 8 \times 4 - (6 \times 4 + 5) \\ &= 32 - 29 \\ &= 3 \text{ quarters}\end{aligned}$$

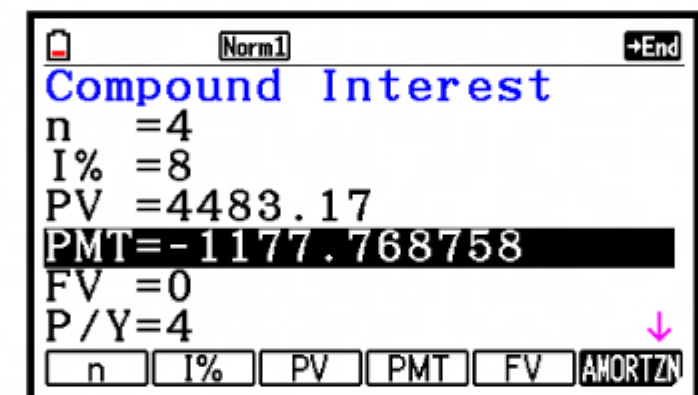
- 37 a** $N = 5 \times 4 = 20$, $I\% = 8$, $PV = 10\,000$, $FV = 0$, $P/Y = 4$, $C/Y = 12$
 $\therefore PMT \approx -612.36$
 The quarterly repayment is \$612.36.



- b** $N = 3 \times 4 = 12$, $I\% = 8$, $PV = 10\,000$, $PMT = -612.36$, $P/Y = 4$, $C/Y = 12$
 $\therefore FV \approx -4483.17$
 The balance of the loan is \$4483.17.



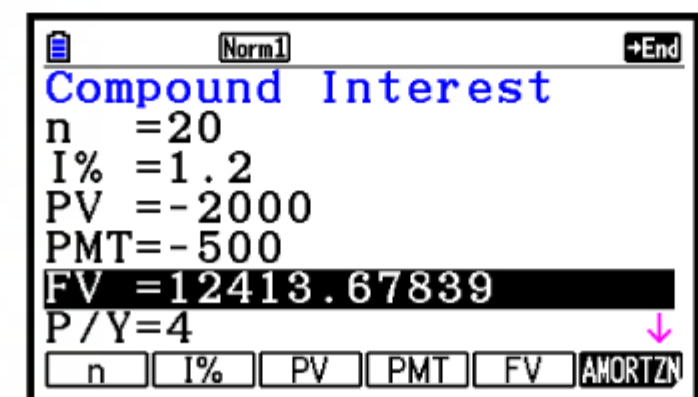
- c** $N = 1 \times 4 = 4$, $I\% = 8$, $PV = 4483.17$, $FV = 0$, $P/Y = 4$, $C/Y = 12$
 $\therefore PMT \approx -1177.77$
 The quarterly repayment must increase to \$1177.77.



- 38 a** $\text{real interest rate multiplier} \times 1.003 = 1.012$
 $\therefore \text{real interest rate multiplier} = \frac{1.012}{1.003}$
 $\approx 1.008\,97$

$$\therefore \text{real interest rate} \approx 0.897\% \approx 0.9\%$$

- b** $N = 5 \times 4 = 20$, $I\% = 1.2$, $PV = -2000$, $PMT = -500$, $P/Y = 4$, $C/Y = 4$
 $\therefore FV \approx 12\,413.68$



After 5 years, Cassie will have €12 413.68 in her savings account.

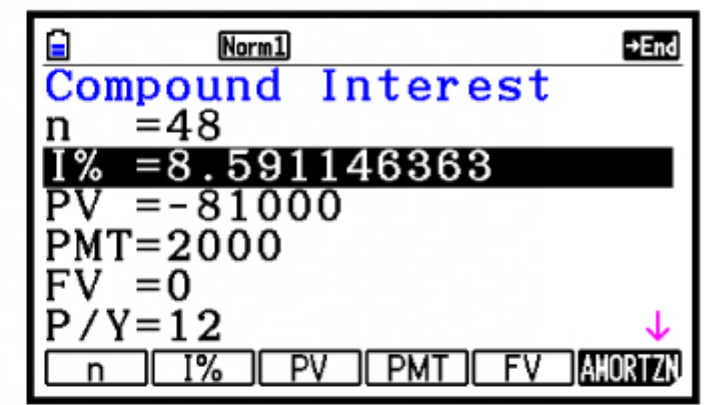
$$\text{Now real value} \times (1.003)^{5 \times 4} = €12\,413.68$$

$$\begin{aligned}\therefore \text{real value} &= \frac{€12\,413.68}{(1.003)^{20}} \\ &\approx €11\,691.81\end{aligned}$$

- 39 a** $N = 4 \times 12 = 48$, $PV = -81\,000$, $PMT = 2000$, $FV = 0$, $P/Y = 12$, $C/Y = 12$

$$\therefore I\% \approx 8.60$$

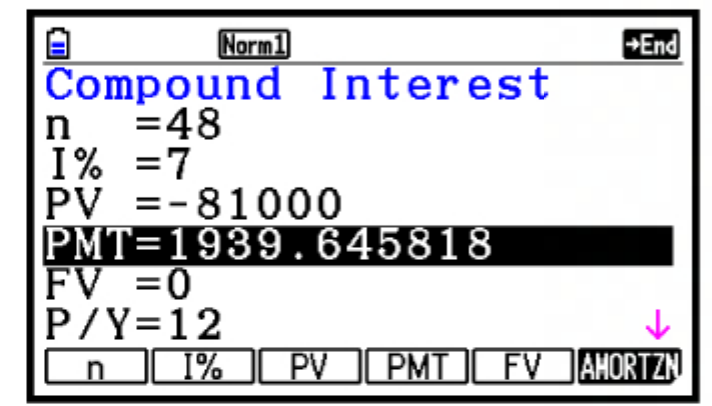
Bill needs to receive 8.60% p.a. compounded monthly.



- b** $N = 48$, $I\% = 7$, $PV = -81\,000$, $FV = 0$, $P/Y = 12$, $C/Y = 12$

$$\therefore PMT \approx 1939.64$$

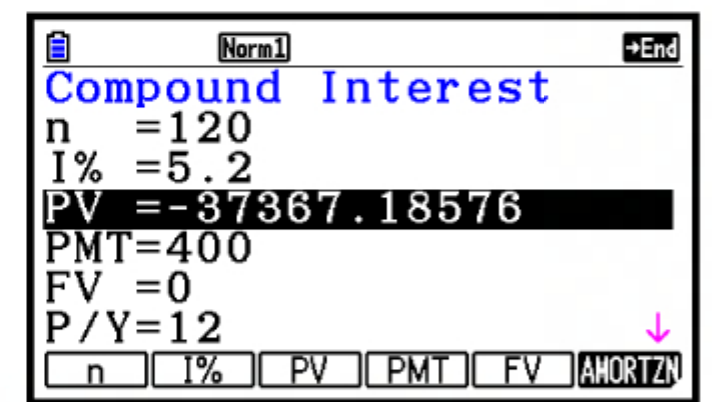
Bill would receive \$1939.64 per month.



- 40 a** $N = 10 \times 12 = 120$, $I\% = 5.2$, $PMT = 200 \times 2 = 400$, $FV = 0$, $P/Y = 12$, $C/Y = 12$

$$\therefore PV \approx 37\,367.19$$

So \$37 367.19 should be invested now to provide such an annuity.

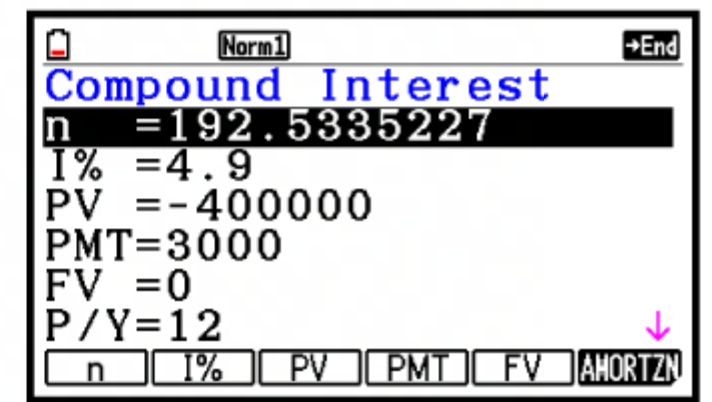


- b** Total interest = $\$200 \times 2 \times 12 \times 10 - \$37\,367.19$
 $= \$48\,000 - \$37\,367.19$
 $= \$10\,632.81$

- 41 a** $I\% = 4.9$, $PV = -400\,000$, $PMT = 3000$, $FV = 0$, $P/Y = 12$, $C/Y = 4$

$$\therefore N \approx 192.5$$

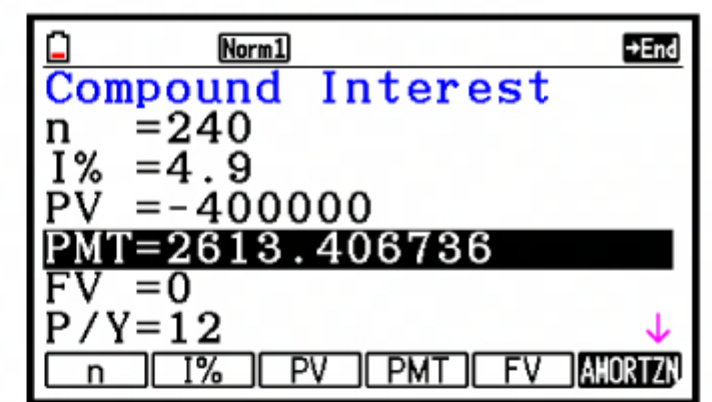
Anne's money will last for 193 months, or 16 years and 1 month.



- b** $N = 20 \times 12 = 240$, $I\% = 4.9$, $PV = -400\,000$, $FV = 0$, $P/Y = 12$, $C/Y = 4$

$$\therefore PMT \approx 2613.40$$

Anne should withdraw \$2613.40 each month.



42 a $\log(10^9 \times 1000^b)$
 $= \log(10^9 \times (10^3)^b)$
 $= \log(10^9 \times 10^{3b})$
 $= \log(10^{9+3b})$
 $= 9 + 3b$

b $\log\left(\frac{10^n}{100}\right)$
 $= \log\left(\frac{10^n}{10^2}\right)$
 $= \log(10^{n-2})$
 $= n - 2$

c $\log(2^t \times 5^t)$
 $= \log((2 \times 5)^t)$
 $= \log(10^t)$
 $= t$

43 a $2 = 10^{\log 2}$
 $\approx 10^{0.3010}$

b $200 = 10^{\log 200}$
 $\approx 10^{2.3010}$

c $0.02 = 10^{\log 0.02}$
 $\approx 10^{-1.6990}$

44 a $\ln(e^k \times e^4)$
 $= \ln(e^{k+4})$
 $= k + 4$

b $\ln\left(\frac{e}{e^m}\right)$
 $= \ln(e^{1-m})$
 $= 1 - m$

c $e^{2 \ln 6}$
 $= (e^{\ln 6})^2$
 $= 6^2$
 $= 36$

d $e^{-\ln 3}$
 $= (e^{\ln 3})^{-1}$
 $= 3^{-1}$
 $= \frac{1}{3}$

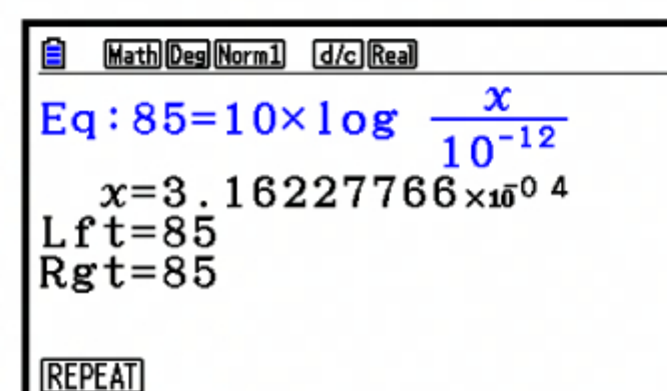
45 a $47 = e^{\ln 47}$
 $\approx e^{3.8501}$

b $500 = e^{\ln 500}$
 $\approx e^{6.2146}$

c $0.023 = e^{\ln 0.023}$
 $\approx e^{-3.7723}$

46 a When $I = 3 \times 10^{-2}$, $L = 10 \log\left(\frac{3 \times 10^{-2}}{10^{-12}}\right)$
 $\approx 105 \text{ dB}$

b When $L = 85$, $85 = 10 \log\left(\frac{I}{10^{-12}}\right)$
 \therefore using technology, $I \approx 3.16 \times 10^{-4}$



TOPIC 2 SKILL BUILDER QUESTIONS

1 a The equation of the line is $y - 6 = -2(x - (-5))$
 $\therefore y - 6 = -2(x + 5)$
 $\therefore y - 6 = -2x - 10$
 $\therefore y = -2x - 4$

b The equation of the line is $y = \frac{5}{8}x + 5$.

2 a The line is parallel to $2x - y = -3$ or $y = 2x + 3$ which has gradient 2.

\therefore the line has gradient 2 and passes through $(5, 3)$.

\therefore the equation of the line is $y - 3 = 2(x - 5)$
 $\therefore y - 3 = 2x - 10$
 $\therefore y = 2x - 7$

b The line is perpendicular to $y = -4x + 3$, which has gradient -4 .

\therefore the line has gradient $\frac{1}{4}$ and passes through $(-1, 5)$.

\therefore the equation of the line is $y - 5 = \frac{1}{4}(x - (-1))$
 $\therefore y - 5 = \frac{1}{4}(x + 1)$
 $\therefore y - 5 = \frac{1}{4}x + \frac{1}{4}$
 $\therefore y = \frac{1}{4}x + \frac{21}{4}$

3 Substituting $x = -1$ and $y = -6$ into the equation gives $7(-1) - (-6) = k$
 $\therefore k = -7 + 6 = -1$

4 a The gradient is $\frac{y_1 - y_2}{x_1 - x_2} = \frac{10 - 4}{-1 - (-3)} = \frac{6}{2} = 3$

b The equation of the line is $y = 3x + c$.

The line passes through $(-3, 4)$, so $4 = 3(-3) + c$
 $\therefore c = 13$

\therefore the equation is $y = 3x + 13$.

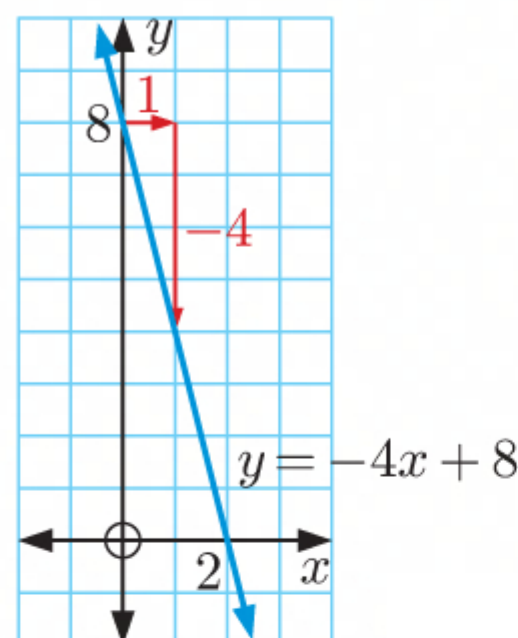
c When $x = 0$, $y = 13$ \therefore the y -intercept is 13.

When $y = 0$, $3x + 13 = 0$
 $\therefore x = -\frac{13}{3}$

\therefore the x -intercept is $-\frac{13}{3}$.

5 a For $y = -4x + 8$:

- the y -intercept is 8
- the gradient is -4 .



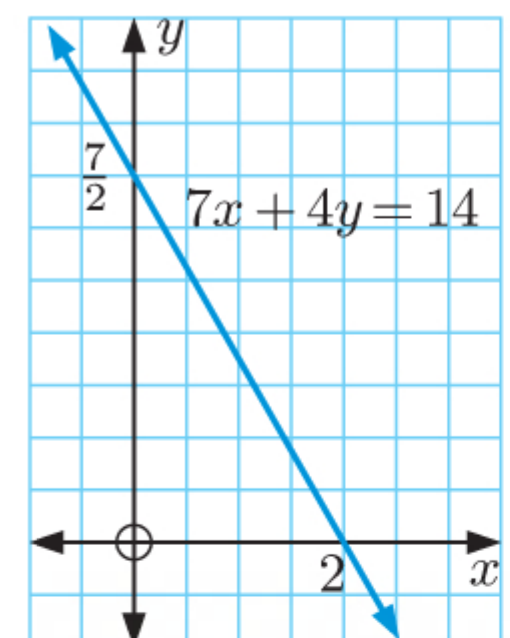
b For $7x + 4y = 14$:

When $x = 0$, $4y = 14$
 $\therefore y = \frac{7}{2}$

So, the y -intercept is $\frac{7}{2}$.

When $y = 0$, $7x = 14$
 $\therefore x = 2$

So, the x -intercept is 2.



6 $y = 3x + 4$

a The gradient is 3.

The y -intercept is 4.

When $y = 0$, $3x + 4 = 0$
 $\therefore x = -\frac{4}{3}$

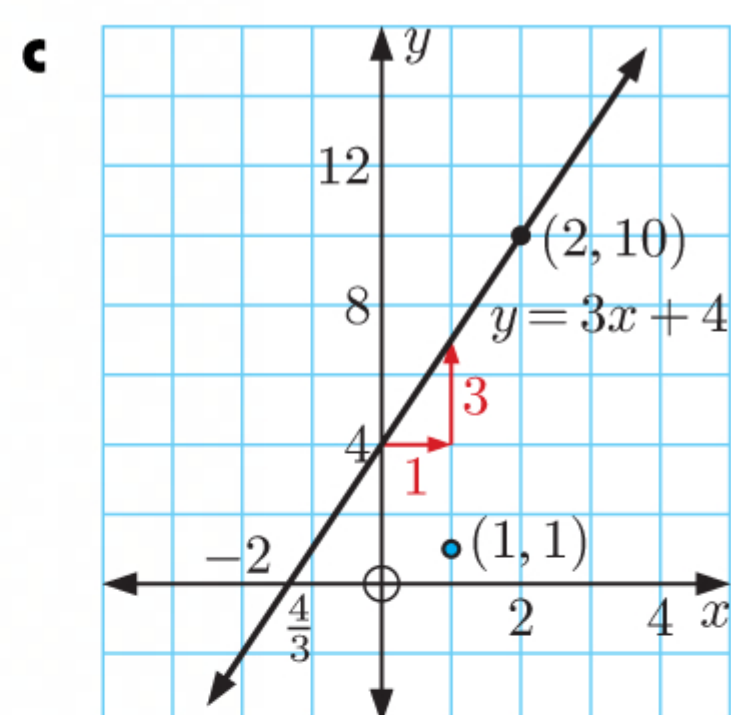
\therefore the x -intercept is $-\frac{4}{3}$.

b i When $x = 1$, we have $y = 3(1) + 4$
 $= 3 + 4$
 $= 7$ ✗

So, $(1, 1)$ does *not* lie on the line.

ii When $x = 2$, we have $y = 3(2) + 4$
 $= 6 + 4$
 $= 10$ ✓

So, $(2, 10)$ does lie on the line.



- 7 a** x adult tickets at \$30 each and y child tickets at \$15 costs \$120 in total.

$$\therefore 30x + 15y = 120$$

- b** When $y = 4$, $30x + 15(4) = 120$

$$\therefore 30x + 60 = 120$$

$$\therefore 30x = 60$$

$$\therefore x = 2$$

\therefore Tammy bought 2 adult tickets.

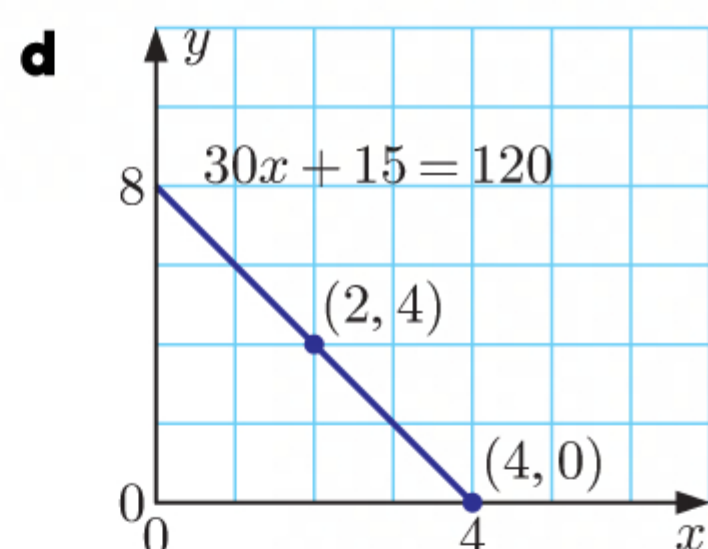
- c** When $y = 0$, $30x + 15(0) = 120$

$$\therefore 30x = 120$$

$$\therefore x = 4$$

\therefore the x -intercept of the line $30x + 15y = 120$ is 4.

If Tammy did not buy any child tickets, then she bought 4 adult tickets.



- 8** $y = \frac{3}{2}x + 1$ has gradient $\frac{3}{2}$

$\therefore ax + by = 20$ has gradient $-\frac{2}{3}$.

$$\therefore -\frac{a}{b} = -\frac{2}{3}$$

$$\therefore 3a = 2b$$

$$\therefore a = \frac{2}{3}b \quad \dots (*)$$

Now $ax + by = 20$ passes through $(2, 2)$

$$\therefore a(2) + b(2) = 20$$

$$\therefore 2a + 2b = 20$$

$$\therefore a + b = 10$$

$$\therefore \frac{2}{3}b + b = 10 \quad \{\text{using } (*)\}$$

$$\therefore \frac{5}{3}b = 10$$

$$\therefore b = 6$$

Substituting $b = 6$ into $(*)$ gives $a = \frac{2}{3}(6) = 4$.

So, $a = 4$ and $b = 6$.

- 9 a** $4x(x + 7) = 0$

$$\therefore x(x + 7) = 0$$

$$\therefore x = 0 \quad \text{or} \quad x + 7 = 0 \quad \{\text{null factor law}\}$$

$$\therefore x = 0 \quad \text{or} \quad x = -7$$

- b** $5(x + 6)(3x - 5) = 0$

$$\therefore (x + 6)(3x - 5) = 0$$

$$\therefore x + 6 = 0 \quad \text{or} \quad 3x - 5 = 0 \quad \{\text{null factor law}\}$$

$$\therefore x = -6 \quad \text{or} \quad x = \frac{5}{3}$$

- c** $-2(x - 3)(4x + 3)^2 = 0$

$$\therefore (x - 3)(4x + 3)^2 = 0$$

$$\therefore x - 3 = 0 \quad \text{or} \quad 4x + 3 = 0 \quad \{\text{null factor law}\}$$

$$\therefore x = 3 \quad \text{or} \quad x = -\frac{3}{4}$$

10 a $\frac{2}{x} = 5x - 3$
 $\therefore 2 = 5x^2 - 3x$

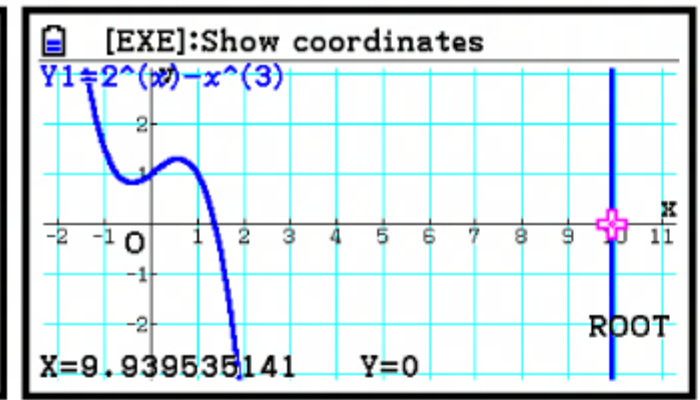
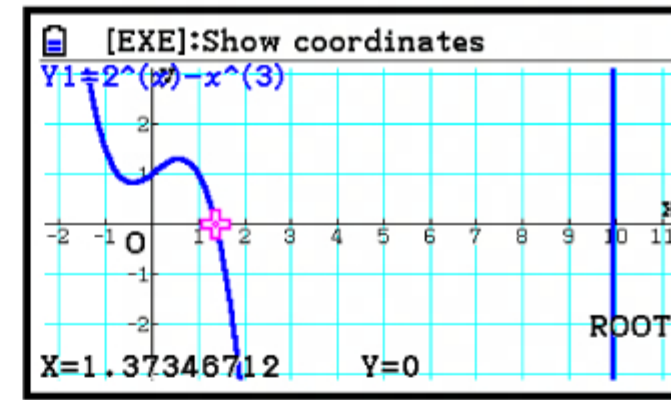
$\therefore 5x^2 - 3x - 2 = 0$

Using technology, $x = -0.4$ or 1

b We graph $y = 2^x - x^3$

The x -intercepts are ≈ 1.37 and 9.94 .

\therefore the solutions are $x \approx 1.37$ or 9.94 .



11 a When $x = -1$, $y = 2(-1)^2 + 5(-1) - 1$
 $= 2 - 5 - 1$
 $= -4$
 $\therefore (-1, -4)$ satisfies the function $y = 2x^2 + 5x - 1$.

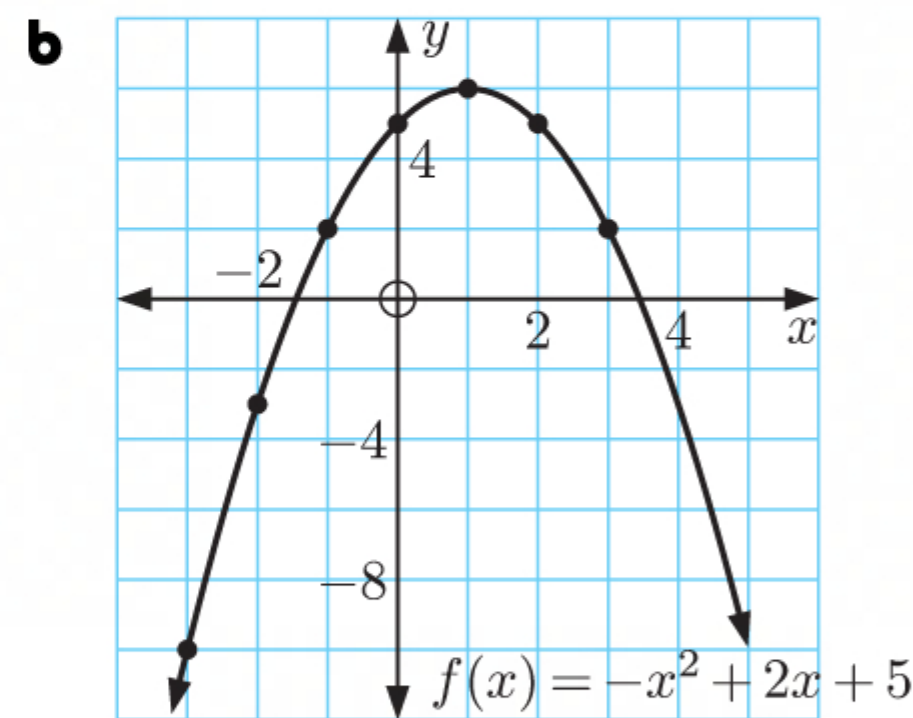
b When $x = 10$, $y = \frac{1}{2}(10)^2 - 6(10) - 3$
 $= 50 - 60 - 3$
 $= -13$

$\therefore (10, -19)$ does not satisfy the function $y = \frac{1}{2}x^2 - 6x - 3$.

12 $f(x) = -x^2 + 2x + 5$

a

x	-3	-2	-1	0	1	2	3
$f(x)$	-10	-3	2	5	6	5	2



13 a $y = x^2 - x - 12$
 When $y = 0$, $x^2 - x - 12 = 0$
 $\therefore x = 4$ or -3 {technology}
 \therefore the zeros are 4 and -3 .

b $f(x) = 5x - x^2$
 When $f(x) = 0$, $5x - x^2 = 0$
 $\therefore x = 5$ or 0 {technology}
 \therefore the zeros are 5 and 0 .

c $y = 8x^2 - 2x - 3$
 When $y = 0$, $8x^2 - 2x - 3 = 0$
 $\therefore x = \frac{3}{4}$ or $-\frac{1}{2}$ {technology}
 \therefore the zeros are $\frac{3}{4}$ and $-\frac{1}{2}$.

14 a $y = (2x - 1)(x + 3)$
 When $x = 0$, $y = (-1)(3) = -3$
 \therefore the y -intercept is -3 .
 When $y = 0$, $(2x - 1)(x + 3) = 0$
 $\therefore 2x - 1 = 0$ or $x + 3 = 0$ {null factor law}
 $\therefore x = \frac{1}{2}$ or -3
 \therefore the x -intercepts are $\frac{1}{2}$ and -3 .

b $f(x) = (x + 1)^2$
 $\therefore f(0) = (1)^2 = 1$
 \therefore the y -intercept is 1 .
 When $f(x) = 0$, $(x + 1)^2 = 0$
 $\therefore x + 1 = 0$
 $\therefore x = -1$
 \therefore the x -intercept is -1 .

c $y = 3x^2 + 4x - 4$

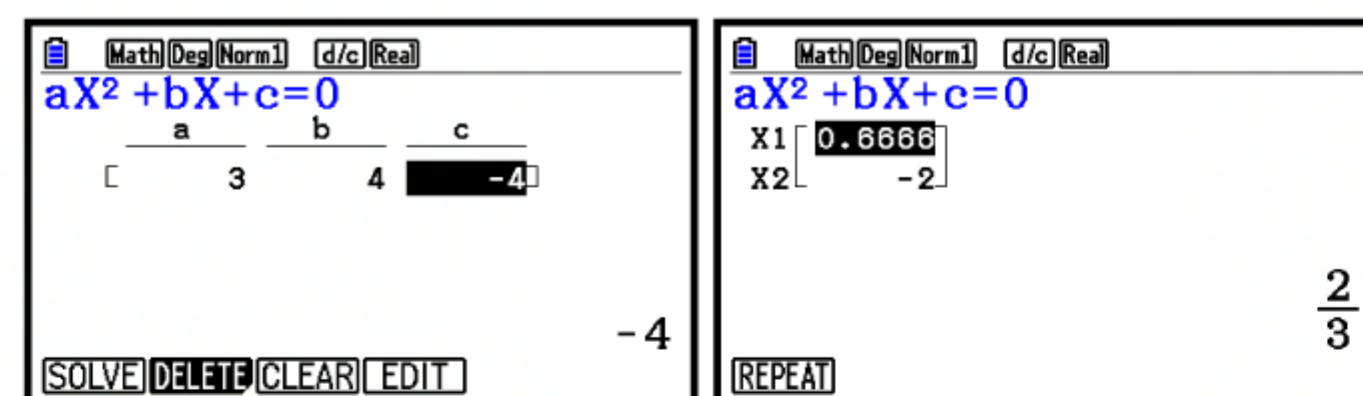
When $x = 0$, $y = -4$

\therefore the y -intercept is -4 .


When $y = 0$, $3x^2 + 4x - 4 = 0$

$\therefore x = \frac{2}{3}$ or -2 {technology}

\therefore the x -intercepts are $\frac{2}{3}$ and -2 .



15 a $y = x^2 - 2x - 8$ has $a = 1$.

Since $a > 0$, the parabola has shape .

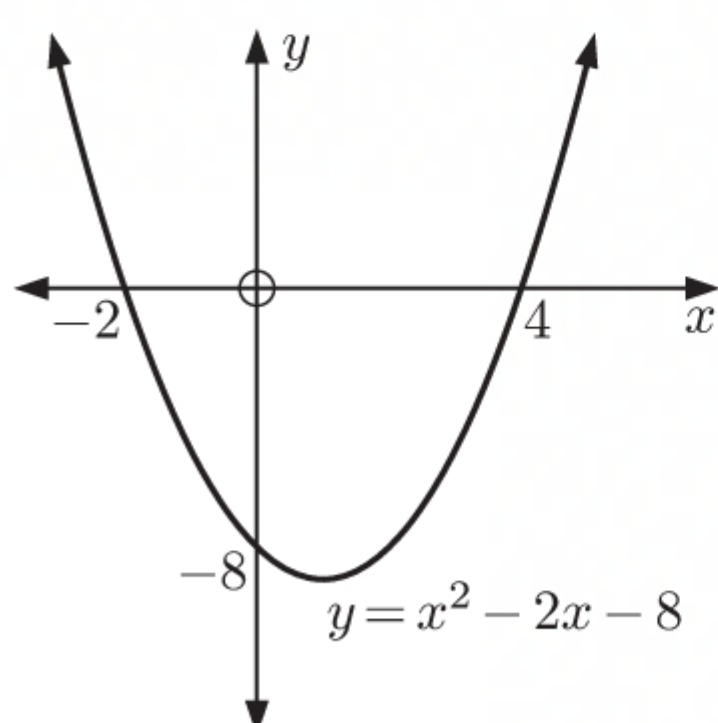
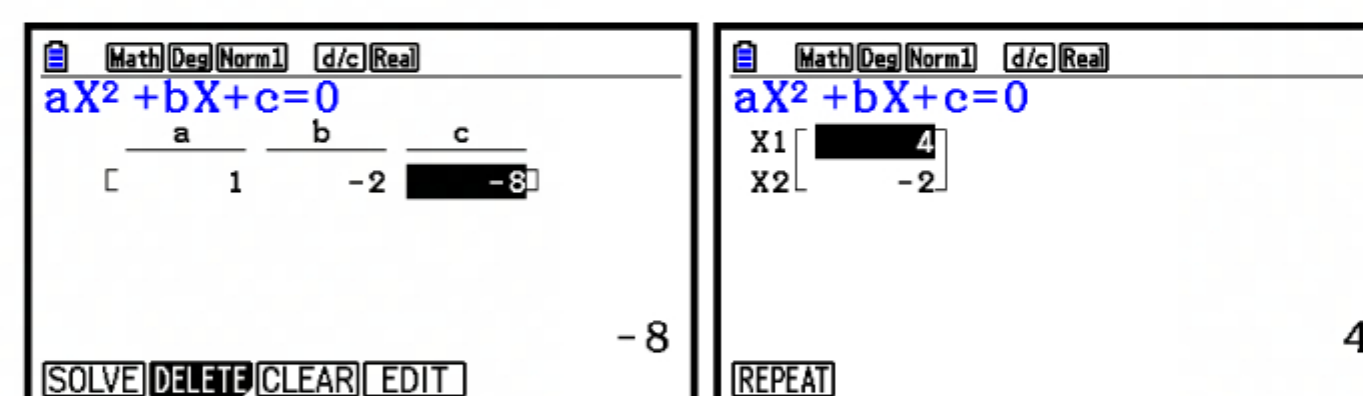
When $x = 0$, $y = -8$

\therefore the y -intercept is -8 .

When $y = 0$, $x^2 - 2x - 8 = 0$

$\therefore x = 4$ or -2 {technology}


\therefore the x -intercepts are 4 and -2 .



b $f(x) = -(2x + 1)(x - 3)$

$= -(2x^2 - 5x - 3)$

$= -2x^2 + 5x + 3$ has $a = -2$.

Since $a < 0$, the parabola has shape .

$f(0) = 3$

\therefore the y -intercept is 3 .

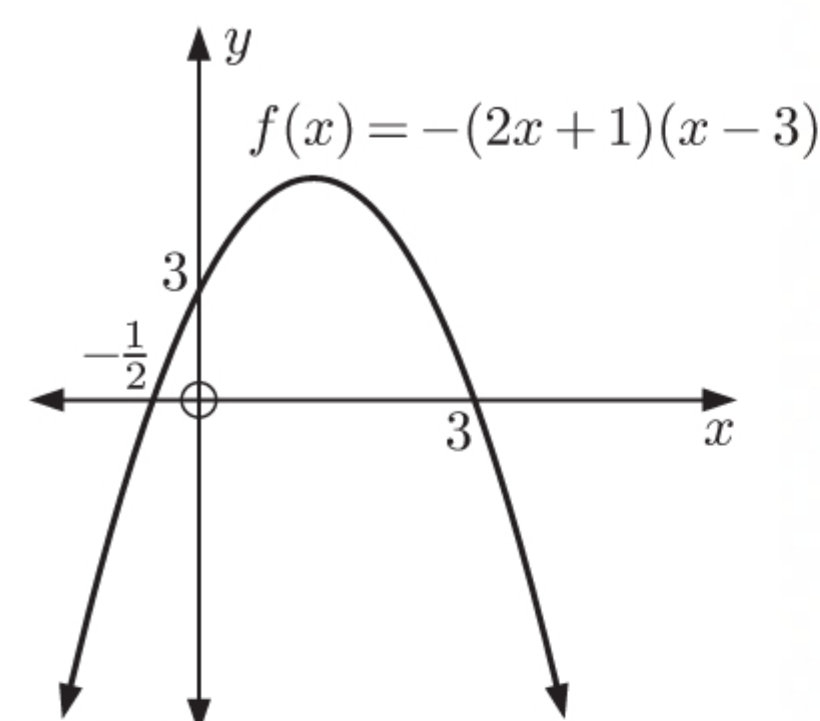
When $f(x) = 0$, $-(2x + 1)(x - 3) = 0$

$\therefore (2x + 1)(x - 3) = 0$

$\therefore 2x + 1 = 0$ or $x - 3 = 0$ {null factor law}

$\therefore x = -\frac{1}{2}$ or 3


\therefore the x -intercepts are $-\frac{1}{2}$ and 3 .



c $y = -\frac{1}{2}(x - 4)^2$

$= -\frac{1}{2}(x^2 - 8x + 16)$

$= -\frac{1}{2}x^2 + 4x - 8$ has $a = -\frac{1}{2}$.

Since $a < 0$, the parabola has shape .

When $x = 0$, $y = -\frac{1}{2}(-4)^2 = -8$

\therefore the y -intercept is -8 .

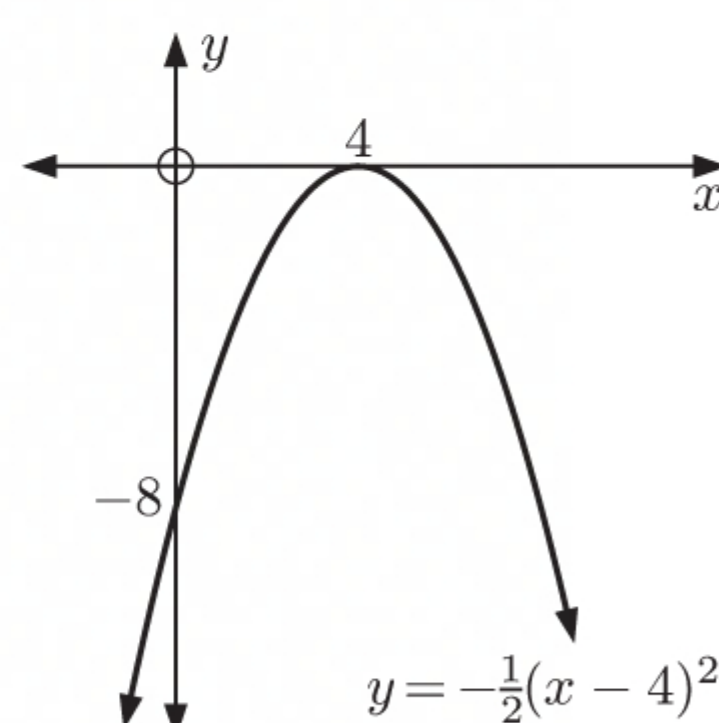
When $y = 0$, $-\frac{1}{2}(x - 4)^2 = 0$

$\therefore (x - 4)^2 = 0$

$\therefore x - 4 = 0$

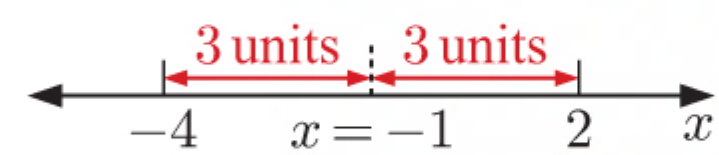
$\therefore x = 4$

\therefore the x -intercept is 4 .



- 16** The axis of symmetry $x = -1$ lies midway between the x -intercepts.

\therefore the other x -intercept is -4 .



- 17 a** $f(x) = 2x^2 + bx - 3$ has $a = 2$, $b = b$, and $c = -3$.

The axis of symmetry is $x = 6$, so $-\frac{b}{2a} = 6$

$$\therefore -\frac{b}{2(2)} = 6$$

$$\therefore b = -24$$

$$\begin{aligned} \mathbf{b} \quad f(6) &= 2(6)^2 - 24(6) - 3 \\ &= 72 - 144 - 3 \\ &= -75 \end{aligned}$$

So, the vertex is $(6, -75)$.

- 18 a** $y = -(x-1)(x+3)$
 $= -(x^2 + 2x - 3)$
 $= -x^2 - 2x + 3$ has $a = -1$, $b = -2$, and $c = 3$.

- i** When $x = 0$, $y = 3$

\therefore the y -intercept is 3.

$$\text{When } y = 0, \quad -(x-1)(x+3) = 0$$

$$\therefore (x-1)(x+3) = 0$$

$$\therefore x-1 = 0 \quad \text{or} \quad x+3 = 0 \quad \{\text{null factor law}\}$$

$$\therefore x = 1 \quad \text{or} \quad -3$$


\therefore the x -intercepts are 1 and -3 .

- ii** The x -intercepts are 1 and -3 .

-1 is halfway between 1 and -3 , so the axis of symmetry is $x = -1$.

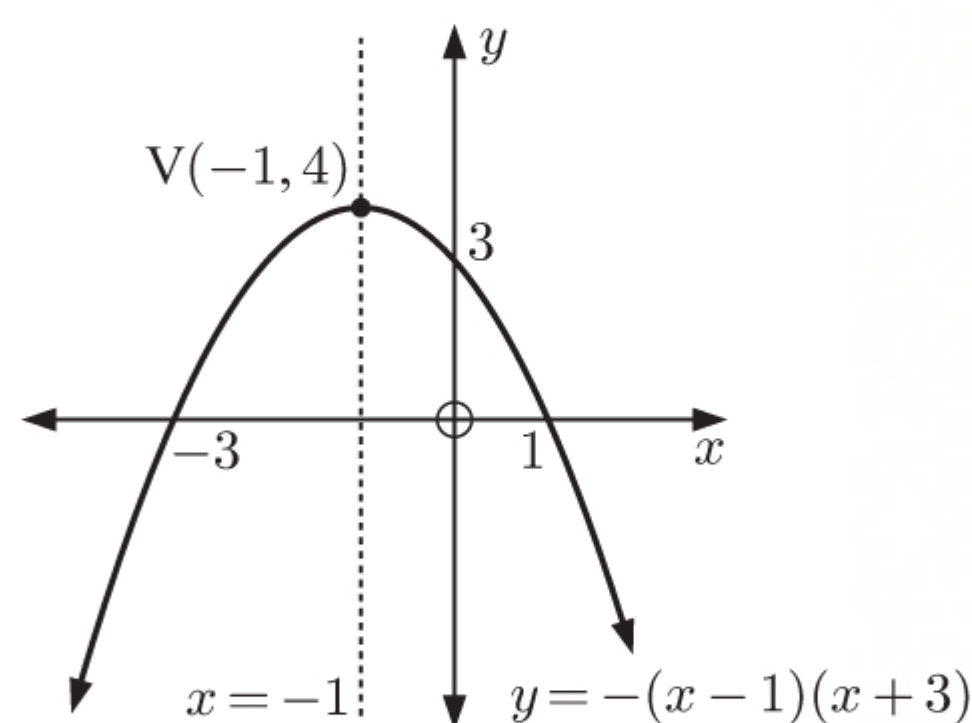
- iii** When $x = -1$, $y = -(-2)(2) = 4$

So, the vertex is $(-1, 4)$.

Since $a < 0$, the parabola has shape .

\therefore the vertex is a maximum turning point.

iv



- v** The domain is $\{x \mid x \in \mathbb{R}\}$.

The range is $\{y \mid y \leq 4\}$.

- b** $y = 2(x+7)(x-2)$
 $= 2(x^2 + 5x - 14)$
 $= 2x^2 + 10x - 28$ has $a = 2$, $b = 10$, and $c = -28$.

- i** When $x = 0$, $y = -28$

\therefore the y -intercept is -28 .

$$\text{When } y = 0, \quad 2(x+7)(x-2) = 0$$

$$\therefore (x+7)(x-2) = 0$$

$$\therefore x+7 = 0 \quad \text{or} \quad x-2 = 0 \quad \{\text{null factor law}\}$$

$$\therefore x = -7 \quad \text{or} \quad 2$$


\therefore the x -intercepts are -7 and 2 .

- ii** The x -intercepts are -7 and 2 .

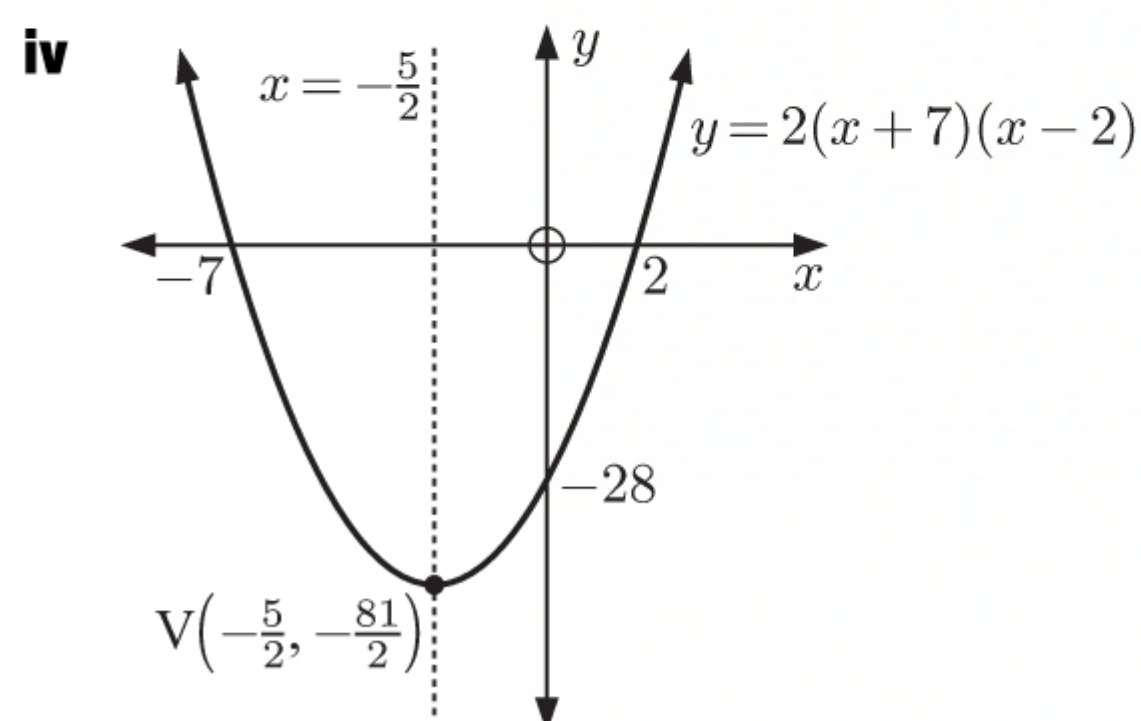
$-\frac{5}{2}$ is halfway between -7 and 2 , so the axis of symmetry is $x = -\frac{5}{2}$.

- iii** When $x = -\frac{5}{2}$, $y = 2(-\frac{5}{2} + 7)(-\frac{5}{2} - 2)$
 $= 2(\frac{9}{2})(-\frac{9}{2})$
 $= -\frac{81}{2}$

So, the vertex is $(-\frac{5}{2}, -\frac{81}{2})$.

Since $a > 0$, the parabola has shape .

\therefore the vertex is a minimum turning point.



- v** The domain is $\{x \mid x \in \mathbb{R}\}$.
The range is $\{y \mid y \geq -\frac{81}{2}\}$.

- 19 a** The x -intercepts are 2 and 5.

\therefore the quadratic has the form $y = a(x - 2)(x - 5)$ where $a > 0$.

But when $x = 6$, $y = 8$

$$\therefore 8 = a(4)(1)$$

$$\therefore a = 2$$

The quadratic is $y = 2(x - 2)(x - 5)$.

- b** The vertex is (3, 6).

\therefore the axis of symmetry is $x = 3$.

The axis of symmetry $x = 3$ lies midway between the x -intercepts.

\therefore the other x -intercept is -3 .

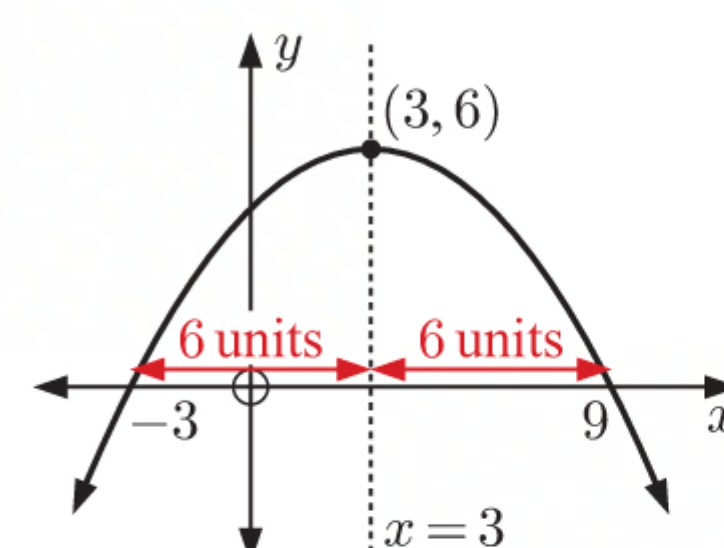
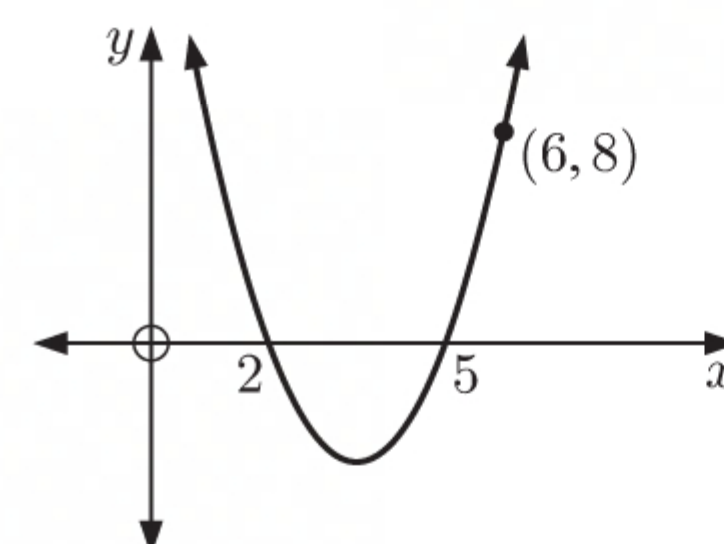
\therefore the quadratic has the form $y = a(x + 3)(x - 9)$ where $a < 0$.

But when $x = 3$, $y = 6$

$$\therefore 6 = a(6)(-6)$$

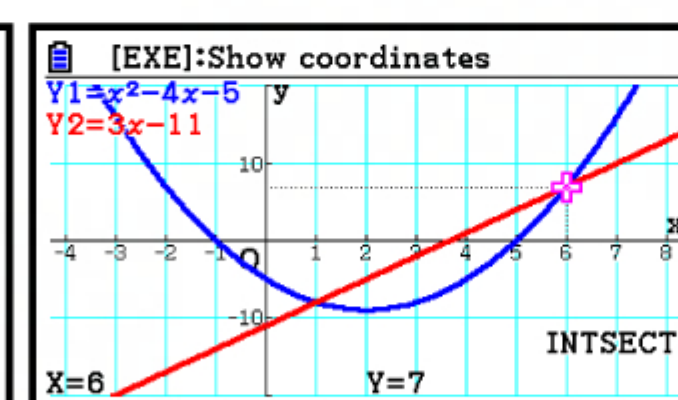
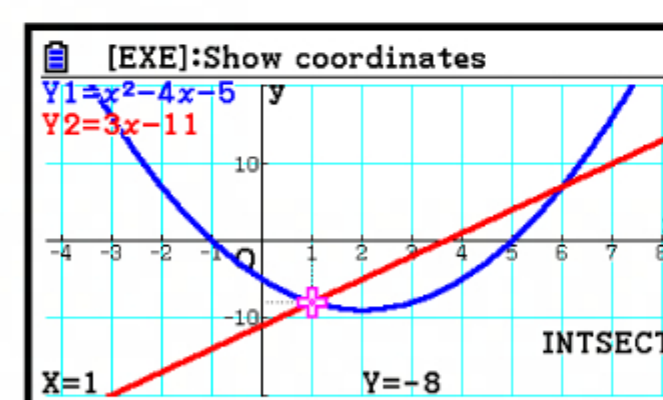
$$\therefore a = -\frac{1}{6}$$

The quadratic is $y = -\frac{1}{6}(x + 3)(x - 9)$.



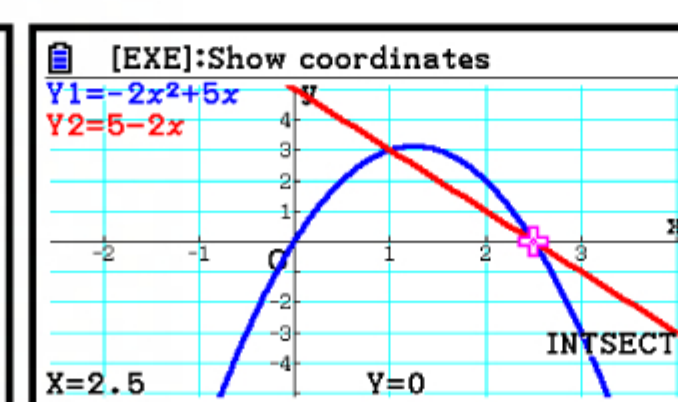
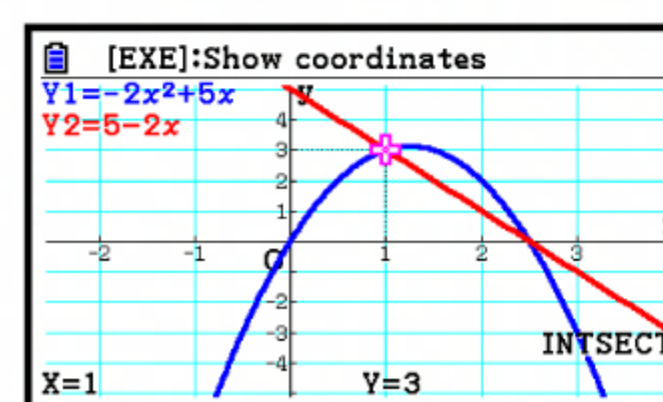
- 20 a** We graph $Y_1 = X^2 - 4X - 5$ and $Y_2 = 3X - 11$ on the same set of axes.

The graphs intersect at (1, -8) and (6, 7).



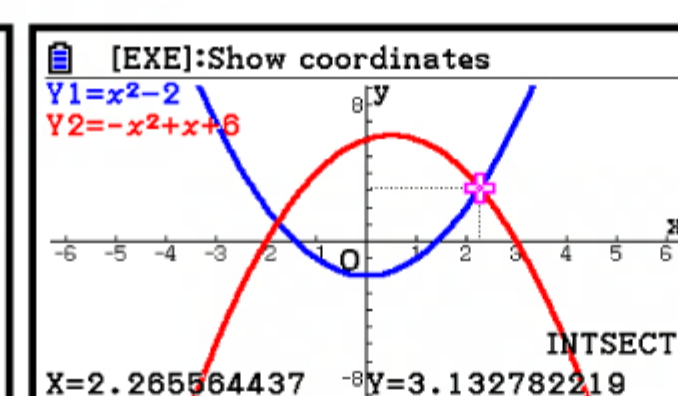
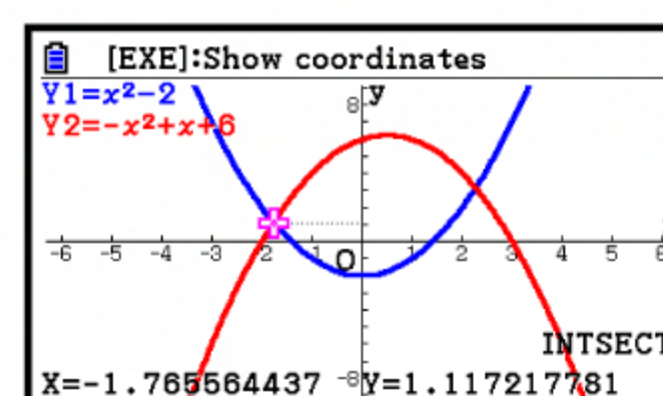
- b** We graph $Y_1 = -2X^2 + 5X$ and $Y_2 = 5 - 2X$ on the same set of axes.

The graphs intersect at (1, 3) and $(\frac{5}{2}, 0)$.



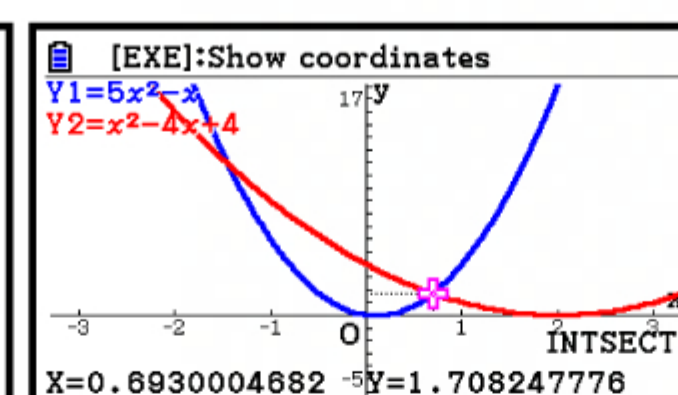
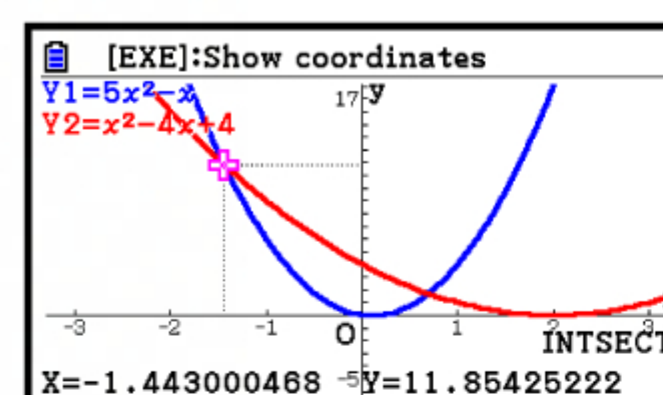
- 21 a** We graph $Y_1 = X^2 - 2$ and $Y_2 = -X^2 + X + 6$ on the same set of axes.

The graphs intersect at $(-1.77, 1.12)$ and $(2.27, 3.13)$.



- b** We graph $Y_1 = 5X^2 - X$ and $Y_2 = X^2 - 4X + 4$ on the same set of axes.

The graphs intersect at $(-1.44, 11.9)$ and $(0.693, 1.71)$.



22 $P = -0.05x^2 + 9x - 60$ dollars

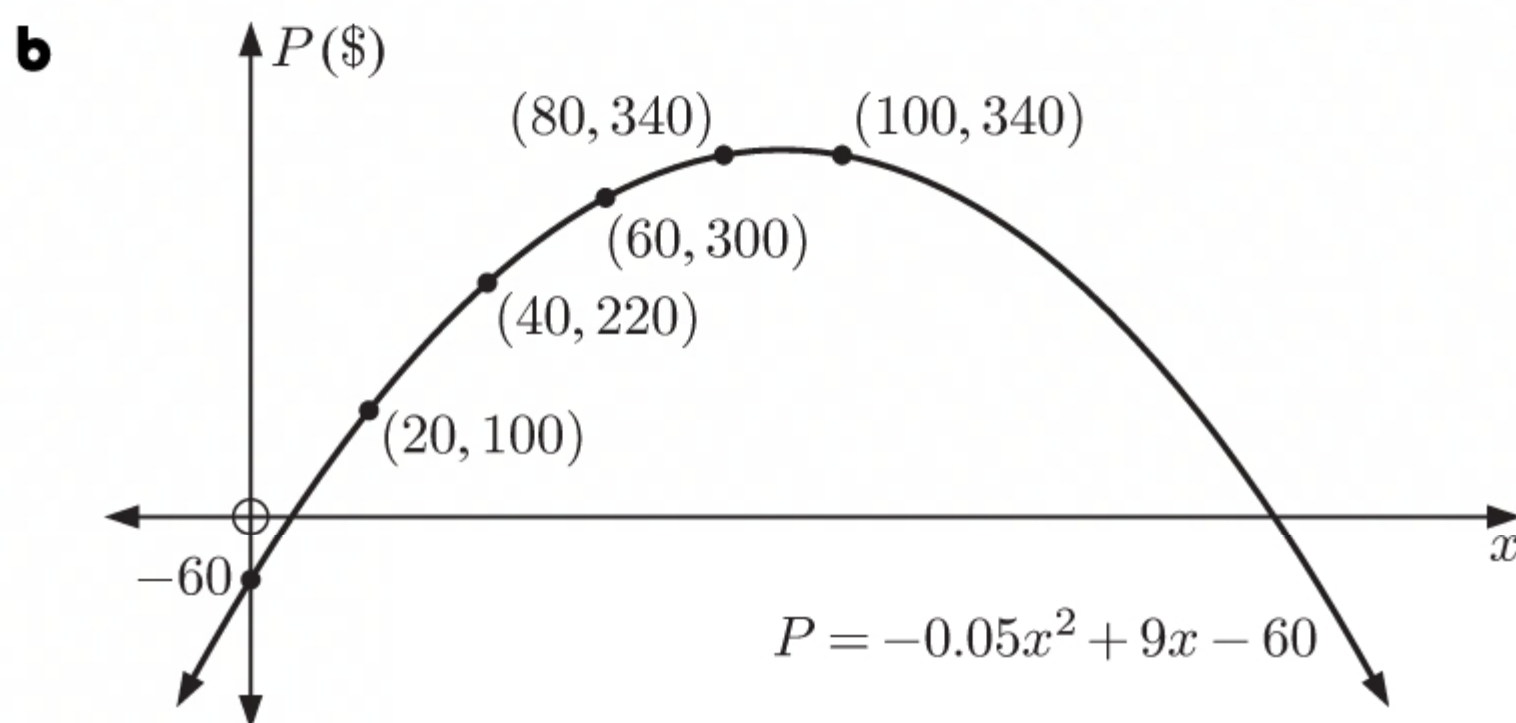
a When $x = 0$, $P = -0.05(0)^2 + 9(0) - 60$
 $= -60$ dollars

When $x = 40$, $P = -0.05(40)^2 + 9(40) - 60$
 $= 220$ dollars

When $x = 80$, $P = -0.05(80)^2 + 9(80) - 60$
 $= 340$ dollars

So,


x	0	20	40	60	80	100
P	-60	100	220	300	340	340



c P has $a = -0.05$, $b = 9$, and $c = -60$

i Now $-\frac{b}{2a} = -\frac{9}{2(-0.05)} = 90$

\therefore the axis of symmetry is $x = 90$.

Since $a < 0$, the parabola has shape .

\therefore the profit is maximised when 90 pies are sold.

iii When $P = 200$,

$$\begin{aligned} -0.05x^2 + 9x - 60 &= 200 \\ \therefore -0.05x^2 + 9x - 260 &= 0 \\ \therefore x &\approx 143.9 \text{ or } 36.1 \\ &\quad \{\text{technology}\} \end{aligned}$$

Now when $x = 36$, $P = 199.2$
 $x = 37$, $P = 204.55$
 $x = 143$, $P = 204.55$
and $x = 144$, $P = 199.2$

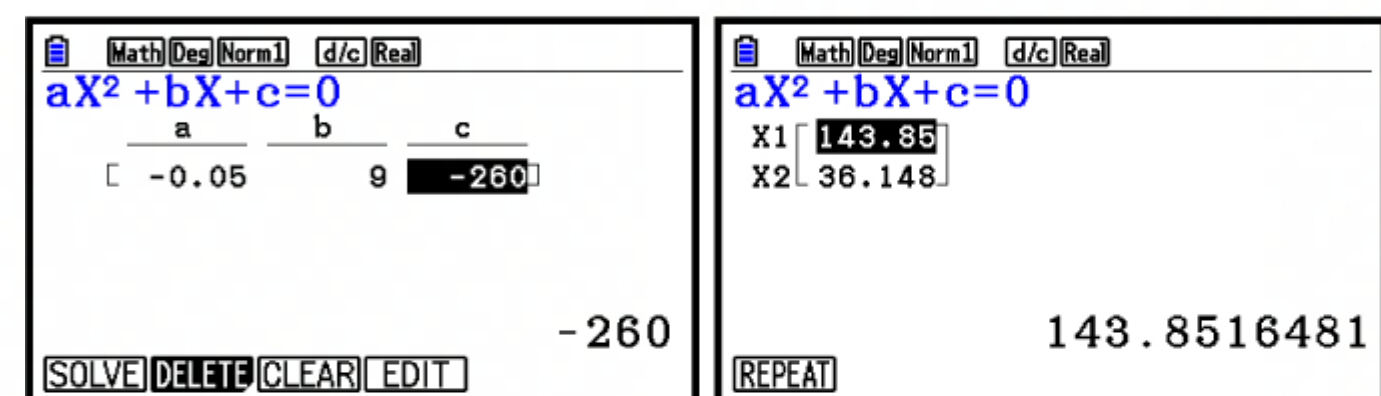
\therefore between 37 and 143 pies must be sold to make a profit of \$200.

iv When $x = 0$, $P = -60$

\therefore the baker loses \$60 if no pies are sold.

ii When $x = 90$, $P = -0.05(90)^2 + 9(90) - 60$
 $= 345$ dollars

\therefore the maximum possible daily profit is \$345.



23 $W(t) = 1000 - 0.5t$ litres

a $W(0) = 1000$

The initial amount of water in the tank was 1000 L.

c The tank is empty when $W(t) = 0$.

This occurs when $0.5t = 1000$
 $\therefore t = 2000$

It will take 2000 hours, or 83 days, 8 hours, for the tank to empty.

b $W(t) = 700$, so $700 = 1000 - 0.5t$

$\therefore 0.5t = 300$

$\therefore t = 600$

After 600 hours, or 25 days, the amount of water in the tank is 700 L.

24 From the graph: Domain is $\{x \mid -6 \leq x \leq 6 \text{ and } x \neq 3\}$

Range is $\{y \mid 0 \leq y \leq 5\}$

a $x = 0$ satisfies $-6 \leq x \leq 6$ and $x \neq 3$.

\therefore “0 is in the domain of f ” is true.

b $y = 0$ satisfies $0 \leq y \leq 5$.

\therefore “0 is in the range of f ” is true.

c $y = 6$ does not satisfy $0 \leq y \leq 5$.

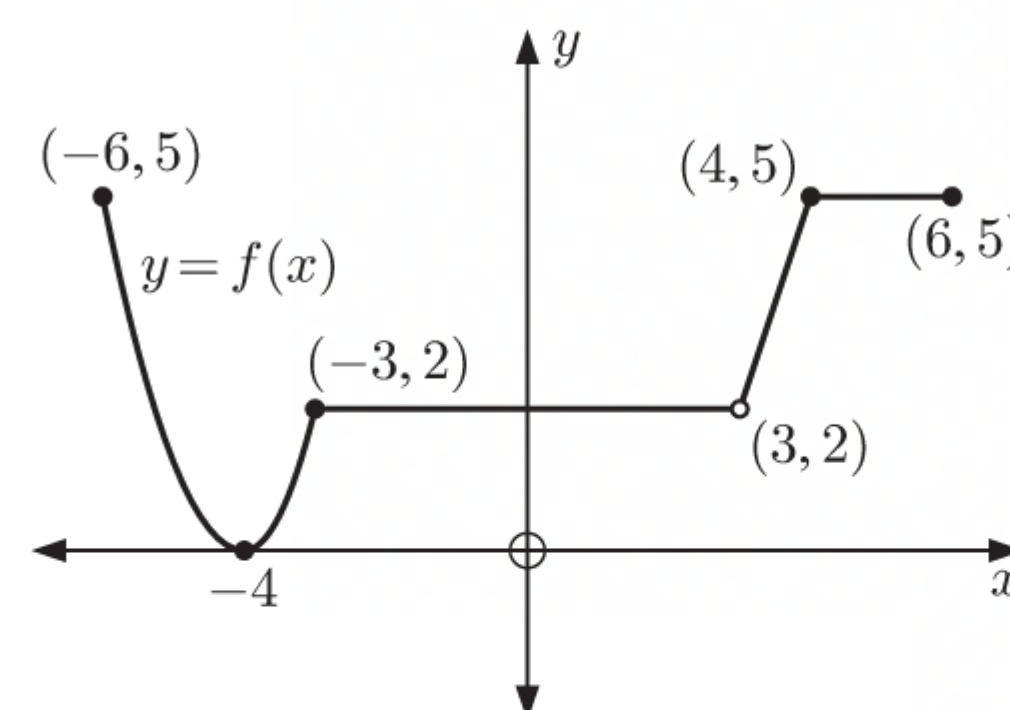
\therefore “6 is in the range of f ” is false.

d $x = 3$ does not satisfy $x \neq 3$.

\therefore “3 is in the domain of f ” is false.

e $y = 2$ satisfies $0 \leq y \leq 5$.

\therefore “2 is in the range of f ” is true.



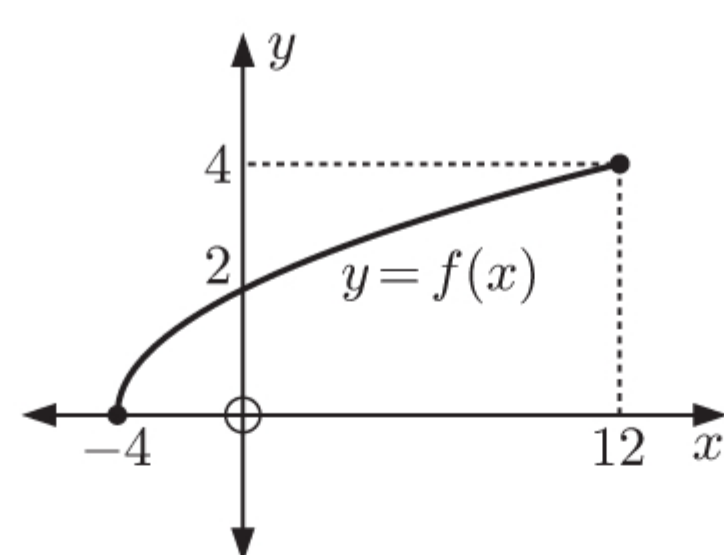
25 $f(x) = \sqrt{x+4}$, $-4 \leq x \leq 12$

a i $f(-4) = \sqrt{0} = 0$

ii $f(0) = \sqrt{4} = 2$

iii $f(12) = \sqrt{16} = 4$

b



c The range is $\{y \mid 0 \leq y \leq 4\}$.

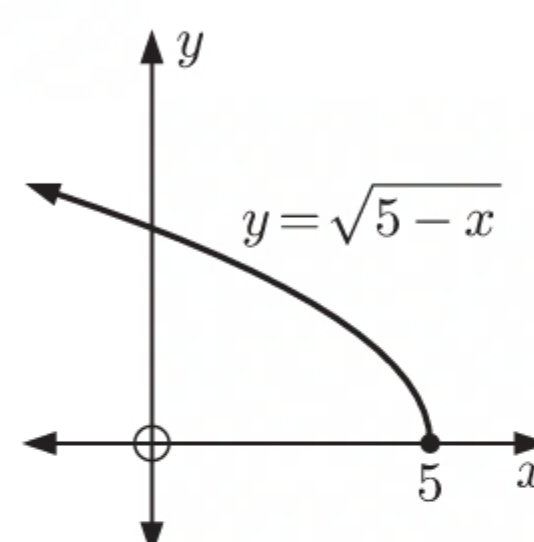
26 a $f(x) = \sqrt{5-x}$

$\sqrt{5-x}$ is defined when $5-x \geq 0$
 $\therefore x \leq 5$

\therefore the domain is $\{x \mid x \leq 5\}$.

A square root cannot be negative.

\therefore the range is $\{y \mid y \geq 0\}$.



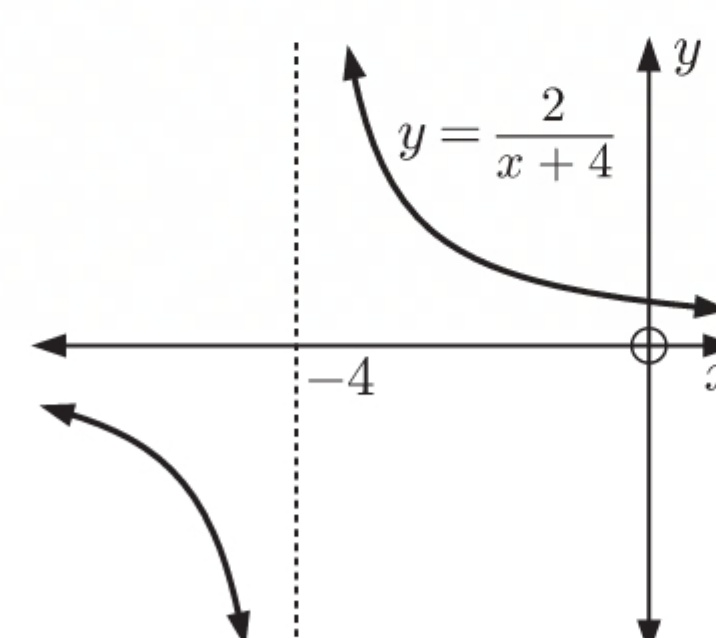
b $f(x) = \frac{2}{x+4}$

$\frac{1}{x+4}$ is defined when $x+4 \neq 0$
 $\therefore x \neq -4$

\therefore the domain is $\{x \mid x \neq -4\}$.

No matter how large or small x is, $y = f(x)$ is never zero.

\therefore the range is $\{y \mid y \neq 0\}$.



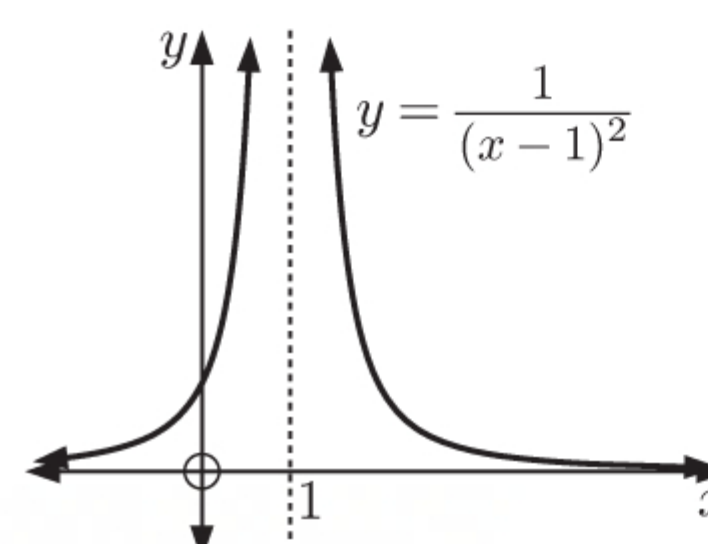
c $f(x) = \frac{1}{(x-1)^2}$

$\frac{1}{(x-1)^2}$ is defined when $(x-1)^2 \neq 0$
 $\therefore x-1 \neq 0$
 $\therefore x \neq 1$

\therefore the domain is $\{x \mid x \neq 1\}$.

$y = f(x)$ is always positive and never zero.

\therefore the range is $\{y \mid y > 0\}$.

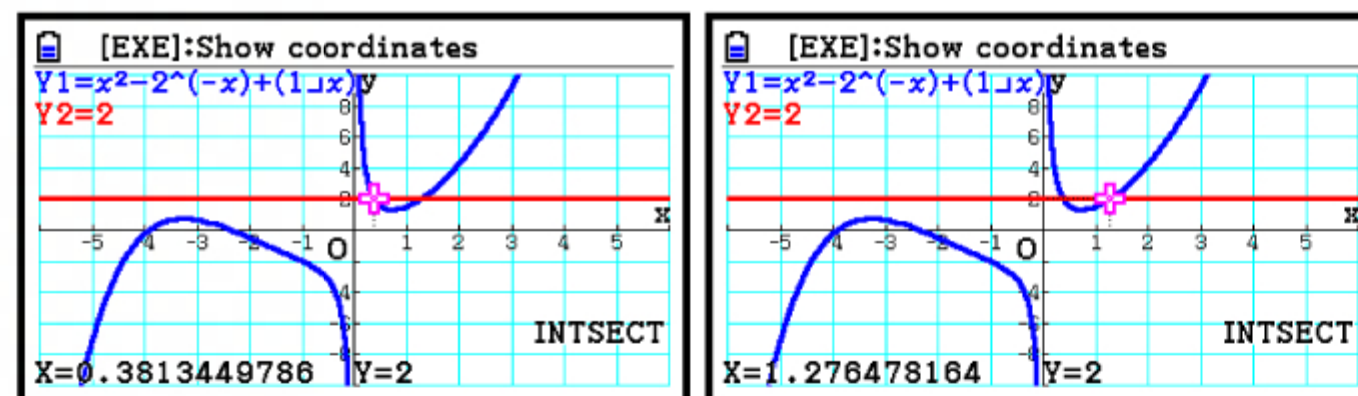


27 $h(x) = x^2 - 2^{-x} + \frac{1}{x}$

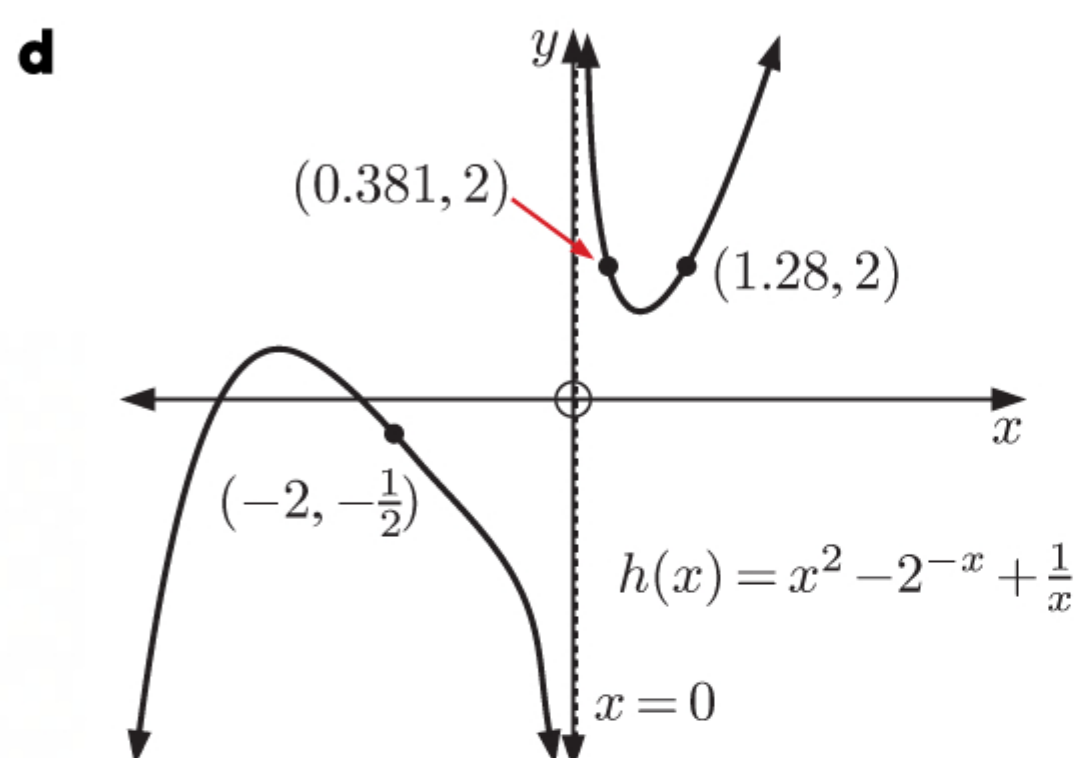
a $h(-2) = (-2)^2 - 2^{-(-2)} + \frac{1}{-2}$
 $= 4 - 4 - \frac{1}{2} = -\frac{1}{2}$

b We graph $Y_1 = X^2 - 2^{-X} + \frac{1}{X}$ and $Y_2 = 2$ on the same set of axes, and find their point of intersection.

The solution to $h(x) = 2$ is $x \approx 0.381$ or 1.28 .



c The vertical asymptote is $x = 0$.

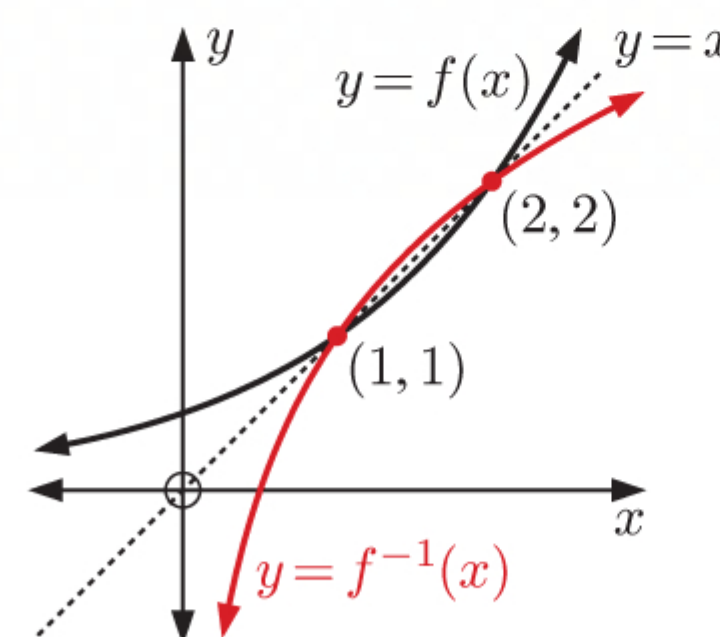


e Using technology, the range is $\{y \mid y < 0.741 \text{ or } y > 1.30, y \in \mathbb{R}\}$.

28 $f(x) = 2^{x-1}$

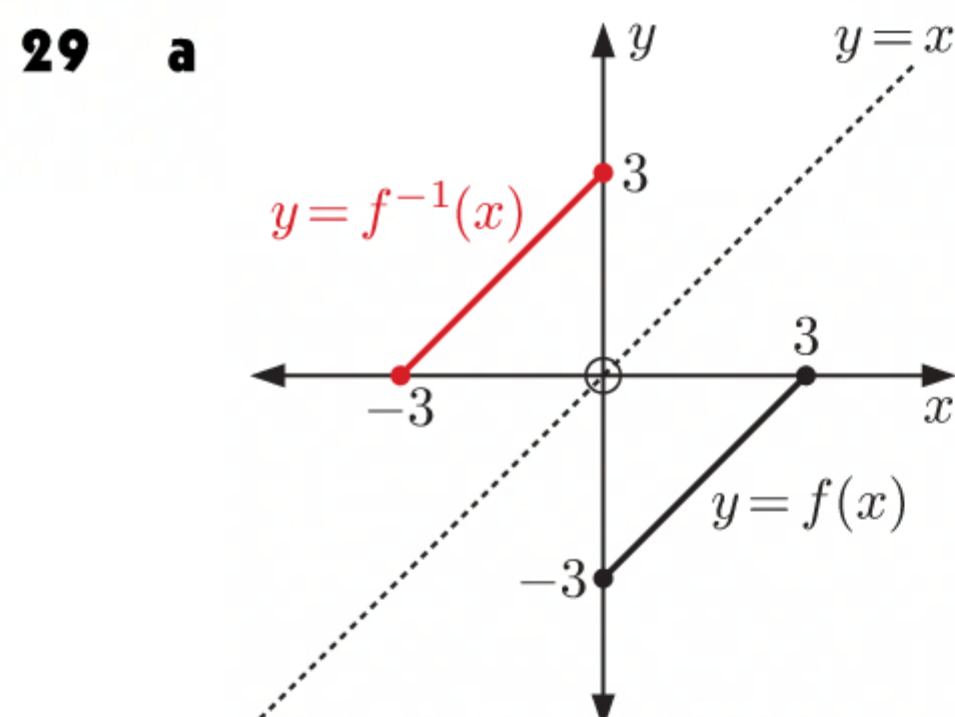
a $f(1) = 2^0 = 1$
 $f(2) = 2^1 = 2$

b The graph of $y = f^{-1}(x)$ is the reflection of the graph $y = f(x)$ in the line $y = x$.



c The domain of $f(x)$ is $\{x \mid x \in \mathbb{R}\}$ and the range of $f(x)$ is $\{y \mid y > 0\}$.

\therefore the domain of $f^{-1}(x)$ is $\{x \mid x > 0\}$ and the range of $f^{-1}(x)$ is $\{y \mid y \in \mathbb{R}\}$.

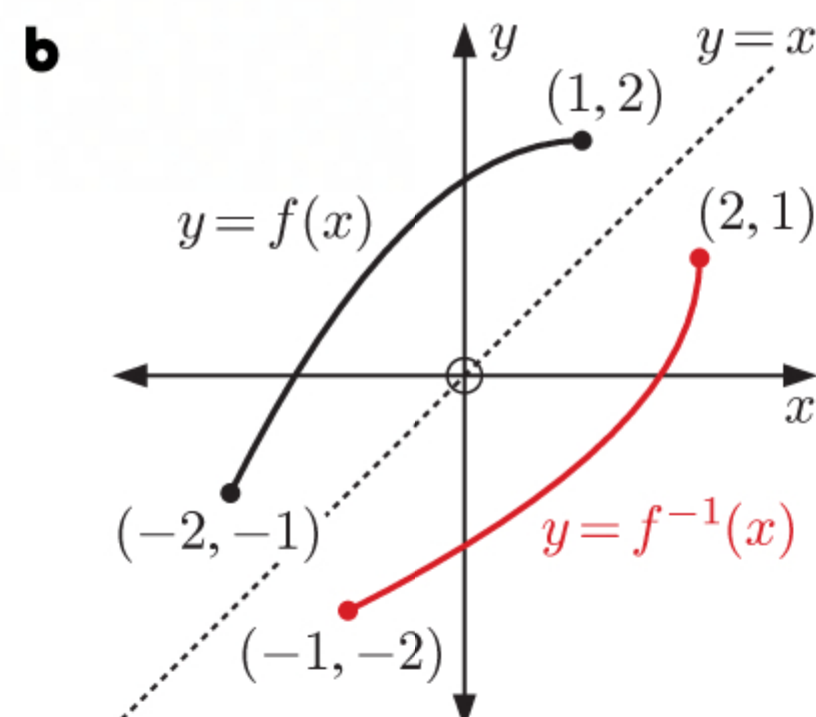


Domain of f : $\{x \mid 0 \leq x \leq 3\}$

Range of f : $\{y \mid -3 \leq y \leq 0\}$

Domain of f^{-1} : $\{x \mid -3 \leq x \leq 0\}$

Range of f^{-1} : $\{y \mid 0 \leq y \leq 3\}$

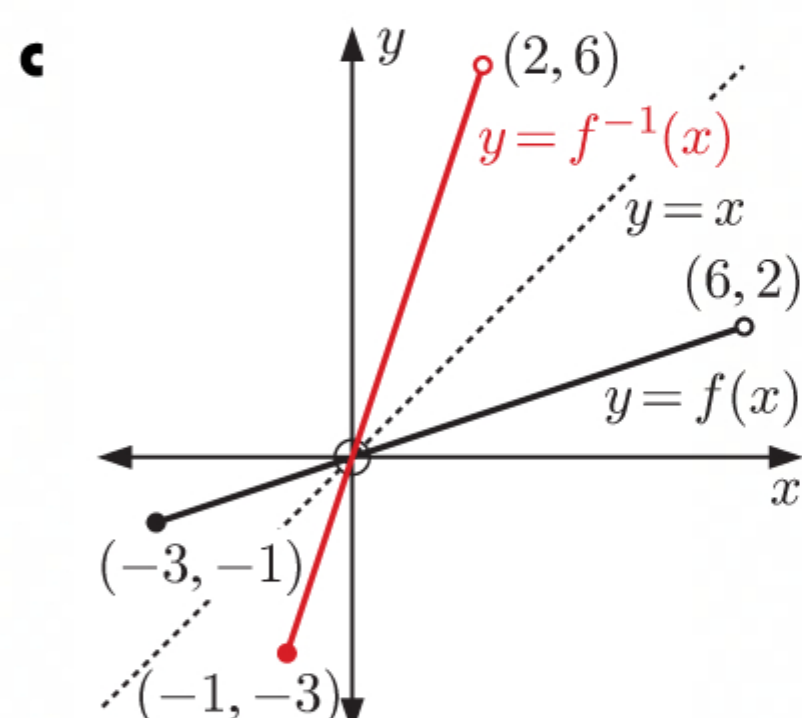


Domain of f : $\{x \mid -2 \leq x \leq 1\}$

Range of f : $\{y \mid -1 \leq y \leq 2\}$

Domain of f^{-1} : $\{x \mid -1 \leq x \leq 2\}$

Range of f^{-1} : $\{y \mid -2 \leq y \leq 1\}$



Domain of f : $\{x \mid -3 \leq x < 6\}$

Range of f : $\{y \mid -1 \leq y < 2\}$

Domain of f^{-1} : $\{x \mid -1 \leq x < 2\}$

Range of f^{-1} : $\{y \mid -3 \leq y < 6\}$

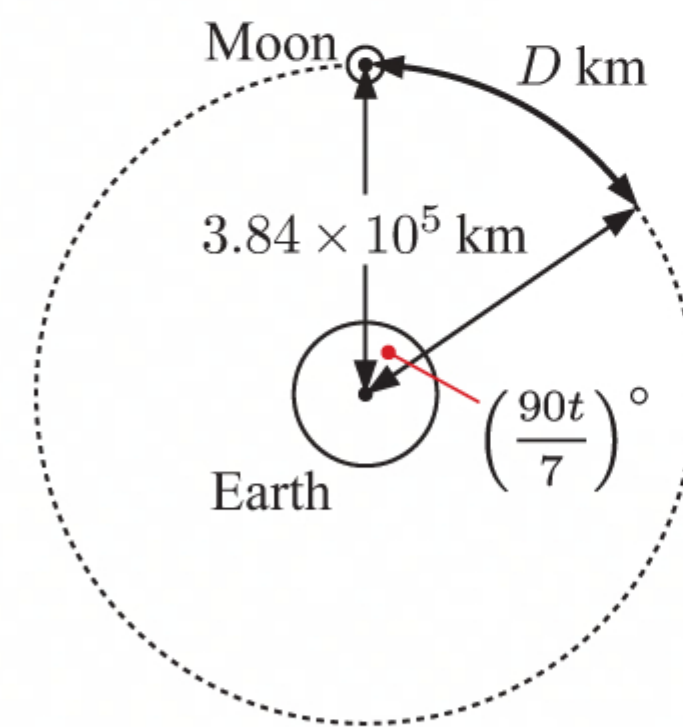
- 30 a** The moon completes one full orbit in about 28 days.

\therefore it completes $\frac{t}{28}$ of an orbit after t days.

So, after t days the moon has travelled through $\frac{t}{28} \times 360^\circ = \left(\frac{90t}{7}\right)^\circ$.

\therefore the distance D km travelled by the moon in t days is given by

$$\begin{aligned} D &= \frac{\frac{90t}{7}}{360} \times 2\pi \times (3.84 \times 10^5) \quad \{\text{arc length formula}\} \\ &= \frac{192\,000\pi}{7} t \end{aligned}$$



- b** We have assumed that:

- the moon has a circular orbit
- the moon moves at a constant speed along its orbit.

- c i** One full orbit is about 28 days.

$$\begin{aligned} \text{When } t = 28, \quad D &= \frac{192\,000\pi}{7} \times 28 \\ &\approx 2.41 \times 10^6 \end{aligned}$$

\therefore the moon travels about 2.41×10^6 km in one full orbit.

$$\begin{aligned} \text{ii When } t = 7, \quad D &= \frac{192\,000\pi}{7} \times 7 \\ &\approx 6.03 \times 10^5 \end{aligned}$$

\therefore the moon travels about 6.03×10^5 km in 7 days.

- d** In our model, the time it takes for the moon to complete one full orbit is an overestimate.

\therefore the fraction of an orbit it has travelled after t days is an underestimate.

\therefore our model is an underestimate of the actual distance travelled by the moon in t days.

So, our answers in **c** are underestimates.

- 31 a i** Isabelita can prepare 600 spring rolls in $\frac{600}{120} = 5$ hours.

- ii** Arturo can prepare 600 spring rolls in $\frac{600}{100} = 6$ hours.

- iii** Isabelita can complete $\frac{120}{600} = \frac{1}{5}$ of the job each hour, and Arturo can complete $\frac{100}{600} = \frac{1}{6}$ of the job each hour.

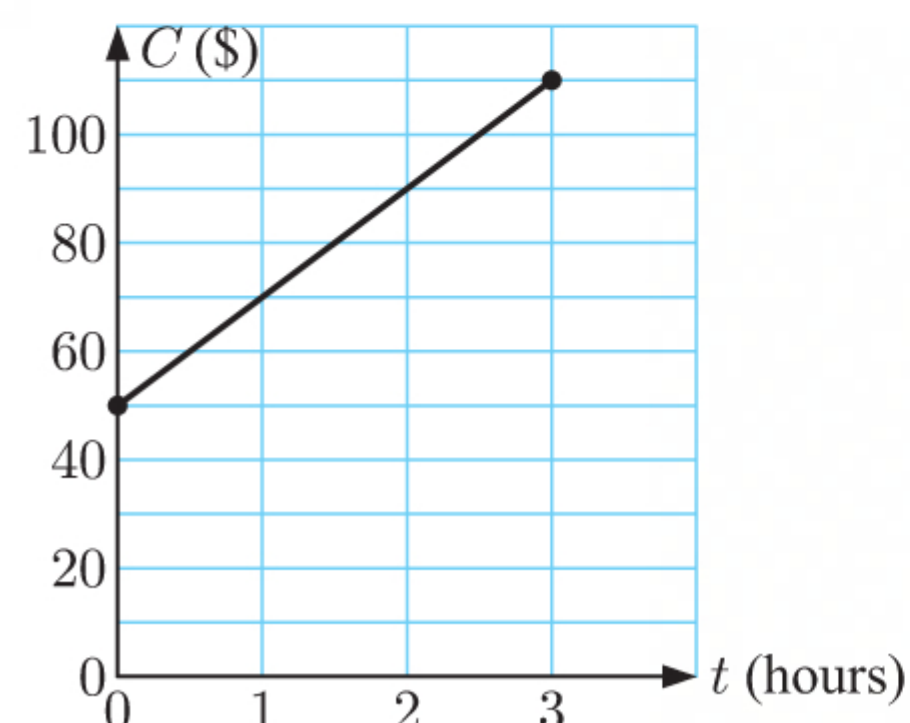
So, working together they can complete $\frac{1}{5} + \frac{1}{6} = \frac{11}{30}$ of the job each hour.

\therefore it would take them $\frac{30}{11} \approx 2.73$ hours ≈ 2 hours 44 minutes to complete the job together.

- b** In **a i** and **a ii**, we assume that Isabelita and Arturo can work at a constant rate over many hours. It is likely that they would slow down or need a rest, so this assumption may not be reasonable.

In **a iii**, we assume that Isabelita and Arturo can work without getting in each other's way. This assumption seems reasonable.

- 32 a**



- b** The C -intercept is 50.

$$\text{The gradient is } \frac{70 - 50}{1 - 0} = 20.$$

$$\therefore C = 20t + 50$$

- c** When $t = 1\frac{1}{2} = \frac{3}{2}$, $C = 20\left(\frac{3}{2}\right) + 50 = 80$

\therefore \$80 is charged for a $1\frac{1}{2}$ hour appointment.

- d** When $C = 118$, $118 = 20t + 50$

$$\therefore 20t = 68$$

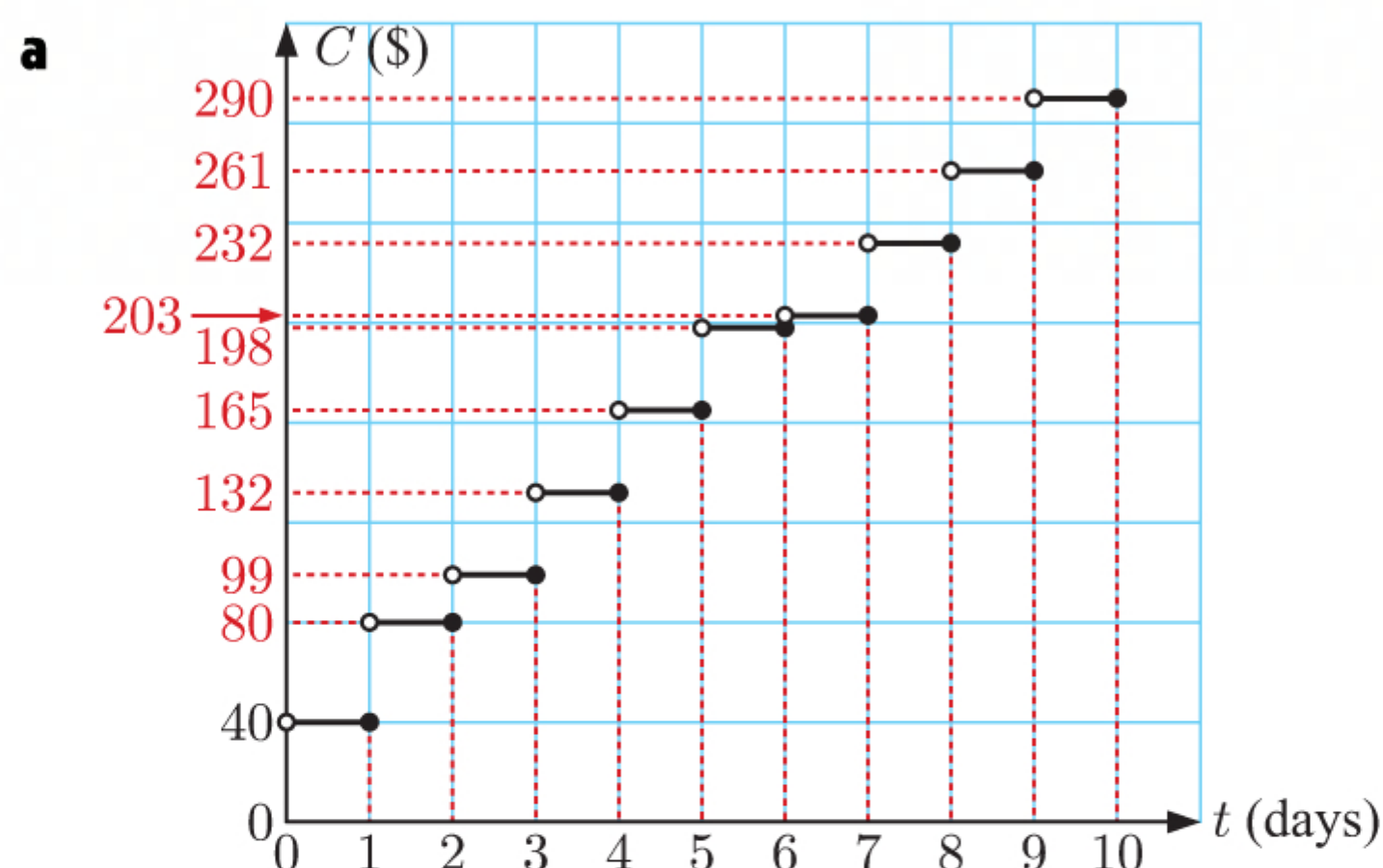
$$\therefore t = 3.4$$

\therefore the appointment lasted 3.4 hours = 3 hours 24 minutes.

12:24 pm is 3 hours and 24 minutes after 9:00 am.

\therefore the technician completed the repairs at 12:24 pm.

Hire period (t days)	Cost
1 - 2	\$40 per day
3 - 6	\$33 per day
7+	\$29 per day



- b**
- i** From the graph, it costs \$80 to hire a car for 2 days.
 - ii** From the graph, it costs \$165 to hire a car for 5 days.
 - iii** From the graph, it costs \$261 to hire a car for 9 days.
- c** If they hire a car for the first 2 days, and the last 3 days, they will spend $\$80 + \$99 = \$179$.
 If they hire a car for the whole 8 days, they will spend \$232.
 \therefore it is cheaper for Georgia and Tim to hire one car for the first 2 days, and a separate car for the last 3 days.

34 a When $t = 0$, $y = 49$
 $\therefore d = 49$

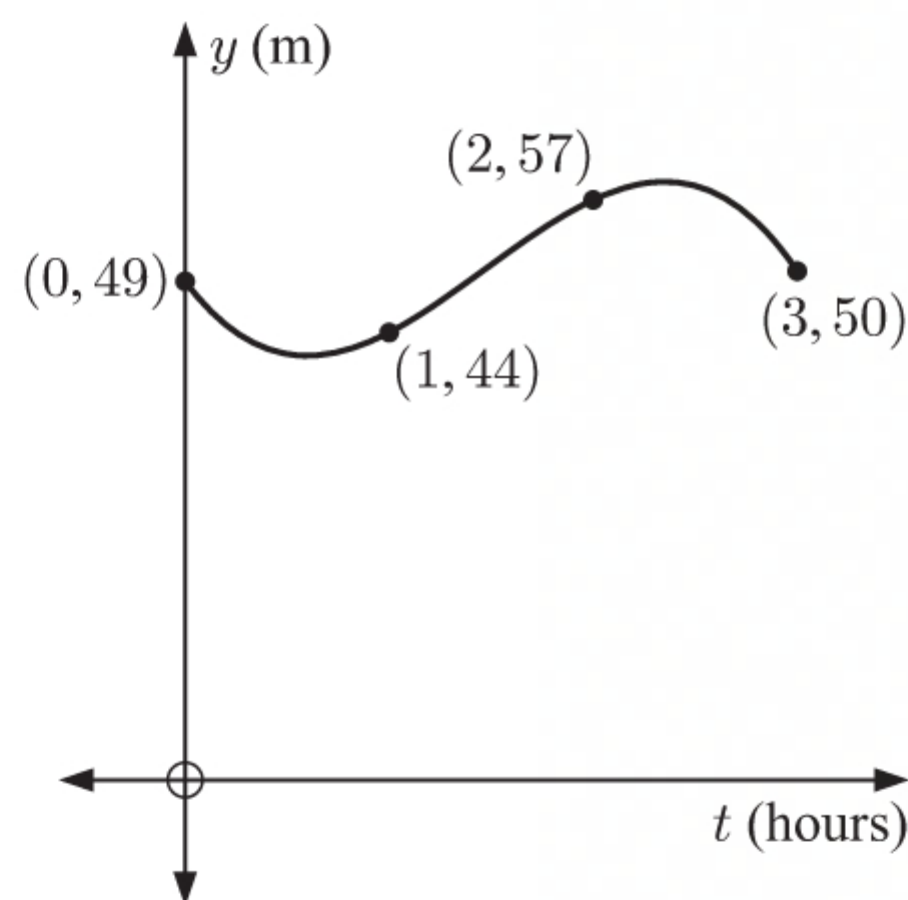
b When $t = 1$, $y = 44$
 $\therefore a(1)^3 + b(1)^2 + c(1) + 49 = 44$
 $\therefore a + b + c = -5$

When $t = 2$, $y = 57$
 $\therefore a(2)^3 + b(2)^2 + c(2) + 49 = 57$
 $\therefore 8a + 4b + 2c = 8$

When $t = 3$, $y = 50$
 $\therefore a(3)^3 + b(3)^2 + c(3) + 49 = 50$
 $\therefore 27a + 9b + 3c = 1$

So, we have the system of equations
$$\begin{cases} a + b + c = -5 \\ 8a + 4b + 2c = 8 \\ 27a + 9b + 3c = 1 \end{cases}$$

Solving these equations simultaneously using technology, we find that $a = -\frac{19}{3}$, $b = 28$, and $c = -\frac{80}{3}$.



	a	b	c	d
1	1	1	1	-5
2	8	4	2	8
3	27	9	3	1

	X	Y	Z
1	-6.333	28	-26.66

c $y = -\frac{19}{3}t^3 + 28t^2 - \frac{80}{3}t + 49$ {from **a** and **b**}

When $t = 2\frac{1}{2} = \frac{5}{2}$,

$y = -\frac{19}{3}\left(\frac{5}{2}\right)^3 + 28\left(\frac{5}{2}\right)^2 - \frac{80}{3}\left(\frac{5}{2}\right) + 49$
 $= 58.375$

\therefore we estimate that Stephen and Hugh's elevation after $2\frac{1}{2}$ hours is 58.375 m.

d Percentage error $= \frac{|V_A - V_E|}{V_E} \times 100\%$
 $= \frac{|58.375 - 60|}{60} \times 100\%$ {from **c**}
 $= \frac{1.625}{60} \times 100\%$
 $\approx 2.71\%$

35 a

z	9	2
w	27	

z is multiplied by $\frac{2}{9}$
 $\therefore w$ is multiplied by $\frac{2}{9}$ {as $w \propto z$ }
 $\therefore w = 27 \times \frac{2}{9} = 6$

b

z	9	
w	27	45

w is multiplied by $\frac{45}{27} = \frac{5}{3}$
 $\therefore z$ is multiplied by $\frac{5}{3}$ {as $w \propto z$ }
 $\therefore z = 9 \times \frac{5}{3} = 15$

36 $V \propto I$, so $V = kI$ where k is the proportionality constant.

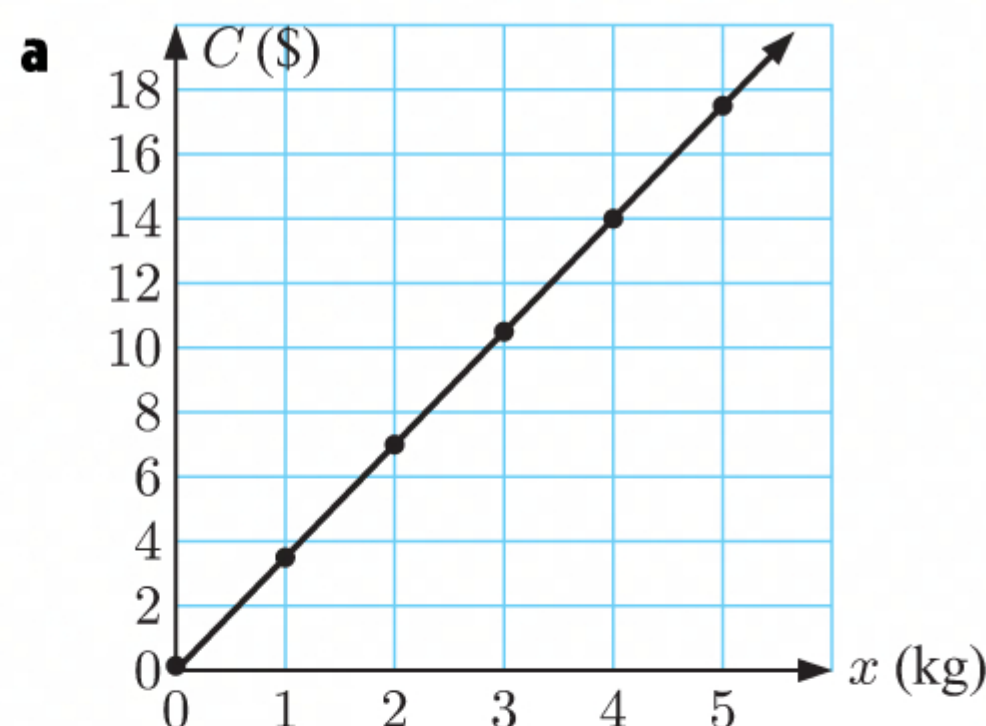
a When $V = 9$ volts, $I = 0.01$ amps, so $9 = k(0.01)$
 $\therefore k = \frac{9}{0.01} = 900$

b When $V = 12$ volts, $12 = 900I$
 $\therefore I \approx 0.0133$ amps

c When $I = 0.018$ amps, $V = 900(0.018)$
 $= 16.2$ volts

37

Weight (x kg)	0	1	2	3	4	5
Cost (\$ C)	0	3.5	7	10.5	14	17.5



b The graph of C against x is a straight line which passes through the origin.
 $\therefore C$ and x are directly proportional.

c i The gradient of the line $= \frac{3.5 - 0}{1 - 0} = 3.5$
 $\therefore C = 3.5x$

ii When $x = 10$, $C = 3.5(10) = 35$
 \therefore purchasing 10 kg of tomatoes costs \$35.

38 The mass m g of an orange is directly proportional to the cube of its diameter d , so $m \propto d^3$.

If d is increased by 14.6%, then

d is multiplied by 1.146
 $\therefore d^3$ is multiplied by $(1.146)^3$
 $\therefore m$ is multiplied by $(1.146)^3 \approx 1.505$ {as $m \propto d^3$ }
 $\therefore m$ is increased by about 50.5%

So, orange A is about 50.5% heavier than orange B.

39 $y \propto \frac{1}{x}$

a If x is tripled, then
 x is multiplied by 3
 $\therefore y$ is multiplied by $\frac{1}{3}$
 $\therefore y$ is divided by 3.

c If x is multiplied by $\frac{5}{3}$, then
 y is multiplied by $\frac{3}{5}$.

b If x is divided by 4, then
 x is multiplied by $\frac{1}{4}$
 $\therefore y$ is multiplied by 4.

d If x is decreased by 60%, then
 x is multiplied by $1 - 0.6 = 0.4$
 $\therefore y$ is multiplied by $\frac{1}{0.4} = 2.5$
 $\therefore y$ is increased by 150%.

- 40** The pressure P is inversely proportional to the area A , so $P = \frac{k}{A}$ where k is a constant.

When $A = 2 \text{ m}^2$, $P = 300 \text{ Pa}$, so $300 = \frac{k}{2}$
 $\therefore k = 600$

So, $P = \frac{600}{k}$.

a When $A = 0.8 \text{ m}^2$, $P = \frac{600}{0.8}$
 $= 750 \text{ Pa}$

- b** If P is reduced by 15%, then

P is multiplied by $1 - 0.15 = 0.85$

$\therefore A$ is multiplied by $\frac{1}{0.85} \approx 1.176$ {as $P \propto \frac{1}{A}$ }

$\therefore A$ is increased by about 17.6%.

41 a

 $\times \frac{5}{2}$

r	2	5
A	7	

r is multiplied by $\frac{5}{2}$

$\therefore r^3$ is multiplied by $(\frac{5}{2})^3 = \frac{125}{8}$

$\therefore A$ is multiplied by $\frac{8}{125}$ {as $A \propto \frac{1}{r^3}$ }

$\therefore A = 7 \times \frac{8}{125} = \frac{56}{125} = 0.448$

b

r	2	
A	7	16

 $\times \frac{16}{7}$

A is multiplied by $\frac{16}{7}$

$\therefore r^3$ is multiplied by $\frac{7}{16}$ {as $A \propto \frac{1}{r^3}$ }

$\therefore r$ is multiplied by $\sqrt[3]{\frac{7}{16}}$

$\therefore r = 2 \times \sqrt[3]{\frac{7}{16}} \approx 1.52$

- 42 a** The volume of a cone $V = \frac{1}{3}\pi r^2 h$

$\therefore h = \frac{3V}{\pi r^2}$

$\therefore h \propto \frac{1}{r^2}$ {as V is a constant}

- b** If the radius of the cone is increased to 3.2 cm, then r is multiplied by $\frac{3.2}{2.8} = \frac{8}{7}$

$\therefore r^2$ is multiplied by $(\frac{8}{7})^2 = \frac{64}{49}$

$\therefore h$ is multiplied by $\frac{49}{64}$ {as $h \propto \frac{1}{r^2}$ }

So, the height of the cone is decreased to $14.3 \times \frac{49}{64} \approx 10.9 \text{ cm}$.

- c** If the height of the cone is decreased to 10.8 cm, then h is multiplied by $\frac{10.8}{14.3} = \frac{108}{143}$

$\therefore r^2$ is multiplied by $\frac{143}{108}$ {as $h \propto \frac{1}{r^2}$ }

$\therefore r$ is multiplied by $\sqrt{\frac{143}{108}}$ {as $r > 0$ }

So, the radius of the cone is increased to $2.8 \times \sqrt{\frac{143}{108}} \approx 3.22 \text{ cm}$.

- d** The cones would otherwise be too thin or too wide to be practical to eat.

43 a

x	1	2	4
y	2	$\frac{1}{4}$	$\frac{1}{32}$
$x^2 y$	2	1	$\frac{1}{2}$
$x^3 y$	2	2	2
$x^4 y$	2	4	8

- b** $x^3 y = 2$ for each point.

$\therefore x^3 y = 2$ or $y = \frac{2}{x^3}$ is the correct model.

c When $x = 5$, $y = \frac{2}{5^3}$
 $= \frac{2}{125}$

44

Diameter (d m)	0.77	1.22	1.69	2.25
Mass (m kg)	0.97	2.44	4.68	8.30

- a** As the diameter of a rug increases, we expect the mass of the rug to increase.

\therefore we expect direct variation between the variables.

- b** The correlation coefficient r is very close to 1, so the fit is excellent.

The power is very close to 2, so it is reasonable to conclude that m is directly proportional to d^2 .

The model is $m \approx 1.64d^2$.

- c** When $d = 1.5$, $m \approx 1.64(1.5)^2$
 ≈ 3.69

So, a rug with diameter 1.5 m has mass ≈ 3.69 kg.

45 a

x	2	3	5	6
y	1.13	8.60	111	275

The correlation coefficient r is very close to 1, so the fit is excellent.

The power is very close to 5, so it is reasonable to conclude that y is directly proportional to x^5 .

The model is $y \approx 0.0353x^5$.

b

x	1	4	5	7	8
y	72.1	4.6	2.9	1.5	1.1

The correlation coefficient r is very close to -1 , so the fit is excellent.

The power is very close to -2 , so it is reasonable to conclude that y is inversely proportional to x^2 .

The model is $y \approx \frac{72.6}{x^2}$.

Des	Norm1	d/c	Real
PowerReg			
a	=	1.63744203	
b	=	2.0017764	
r	=	0.99999982	
r ²	=	0.99999964	
MSe	=	4.4849 × 10 ⁻⁷	
y=a · x^b			
[COPY] [DRAW]			

Des	Norm1	d/c	Real
PowerReg			
a	=	0.035267	
b	=	5.00283583	
r	=	0.99999966	
r ²	=	0.99999932	
MSe	=	6.2892 × 10 ⁻⁶	
y=a · x^b			
[COPY] [DRAW]			

Des	Norm1	d/c	Real
PowerReg			
a	=	72.5921348	
b	=	-2.001783	
r	=	-0.9999416	
r ²	=	0.99988329	
MSe	=	4.3086 × 10 ⁻⁴	
y=a · x^b			
[COPY] [DRAW]			

46 $f(x) = 2 \times 3^{-x}$

a $f(0) = 2 \times 3^0$
 $= 2$

b $f(1) = 2 \times 3^{-1}$
 $= \frac{2}{3}$

c $f(-2) = 2 \times 3^2$
 $= 18$

47 $g(x) = 5^x - 5$

a $g(0) = 5^0 - 5$
 $= -4$

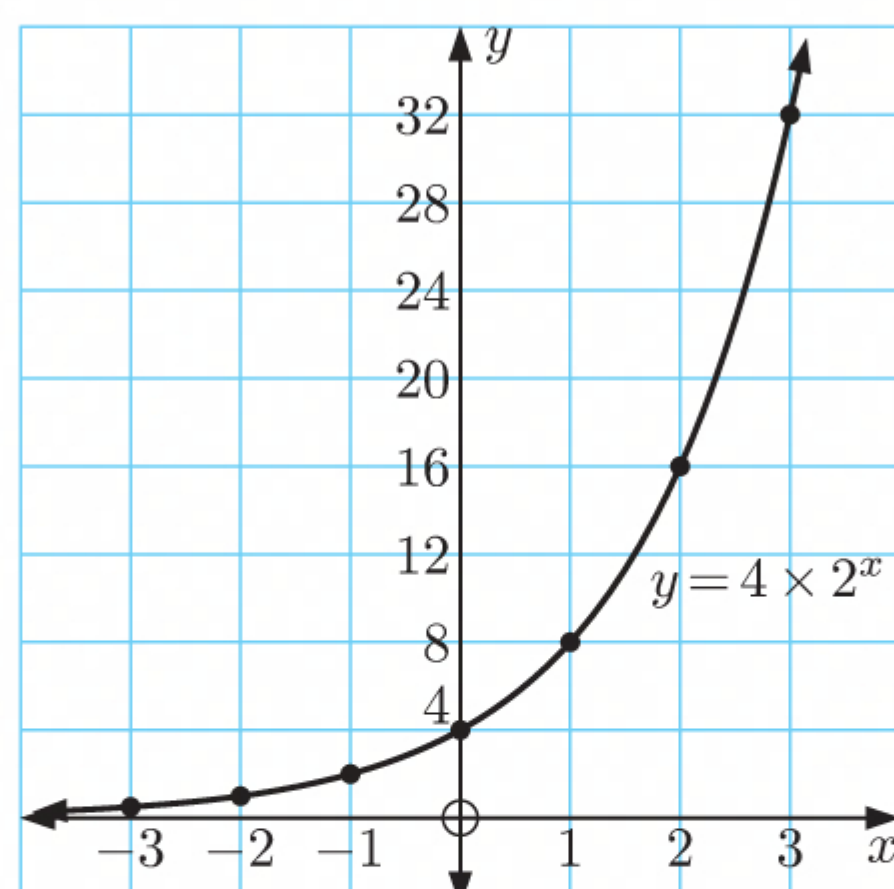
\therefore the y -intercept is -4 .

b $g(1) = 5^1 - 5$
 $= 0$

\therefore the x -intercept is 1.

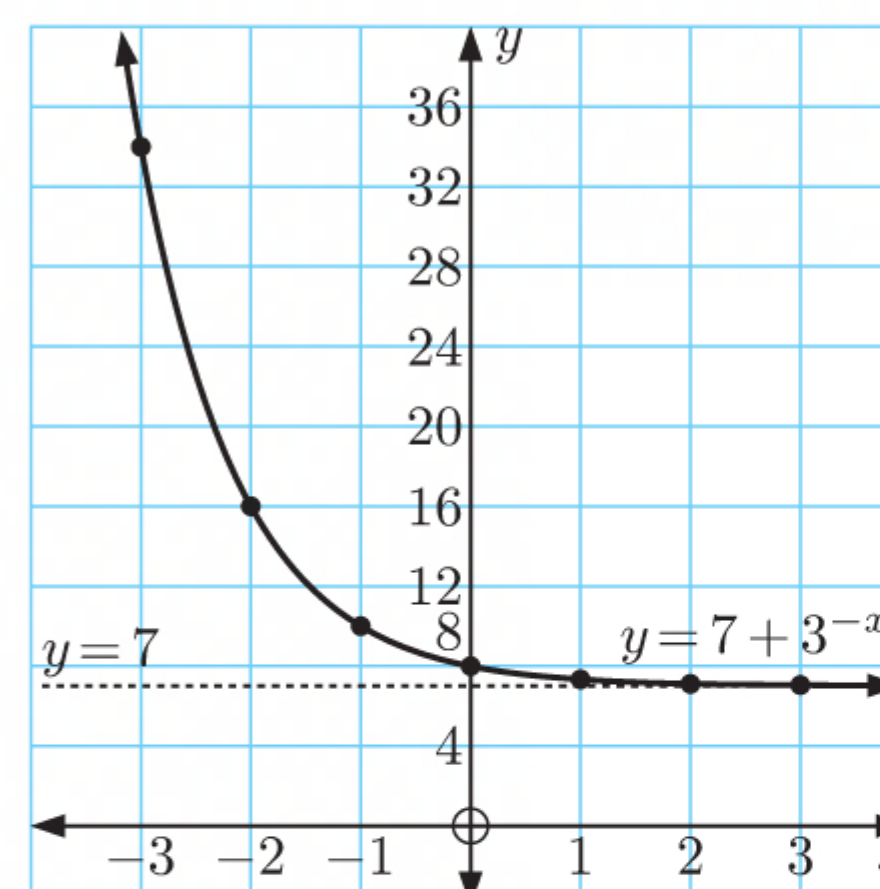
48 a $y = 4 \times 2^x$

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{2}$	1	2	4	8	16	32



b $y = 7 + 3^{-x}$

x	-3	-2	-1	0	1	2	3
y	34	16	10	8	$\frac{22}{3}$	$\frac{64}{9}$	$\frac{190}{27}$



49 $y = -1 + 2^{-x}$

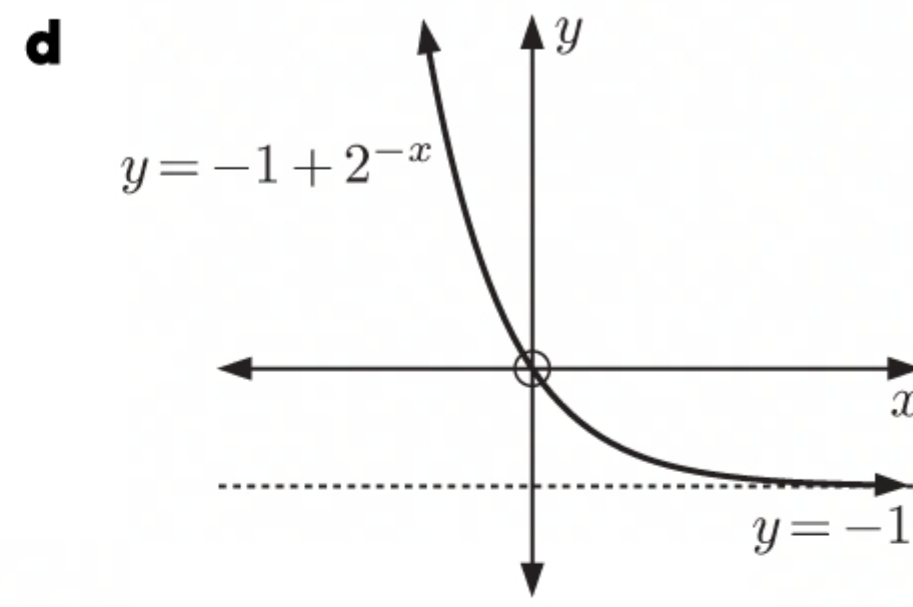
a When $x = 0$, $y = -1 + 1 = 0$

\therefore the y -intercept is 0, and the x -intercept is 0.

b The horizontal asymptote is $y = -1$.

c The domain is $\{x \mid x \in \mathbb{R}\}$.

The range is $\{y \mid y > -1\}$.



50 $y = a \times 2^x + b$

x	0	1	2	3
y	20	p	35	q

a When $x = 0$, $y = 20$

$$\therefore 20 = a \times 2^0 + b$$

$$\therefore a + b = 20 \quad \dots (1)$$

When $x = 2$, $y = 35$

$$\therefore 35 = a \times 2^2 + b$$

$$\therefore 4a + b = 35 \quad \dots (2)$$

c Using **b**, $y = 5 \times 2^x + 15$

When $x = 1$, $y = p$

$$\therefore p = 5 \times 2^1 + 15$$

$$\therefore p = 25$$

When $x = 3$, $y = q$

$$\therefore q = 5 \times 2^3 + 15$$

$$\therefore q = 55$$

b Using (1), $b = 20 - a \quad \dots (3)$

Substituting $b = 20 - a$ into (2) gives

$$4a + 20 - a = 35$$

$$\therefore 3a = 15$$

$$\therefore a = 5$$

Substituting $a = 5$ into (3) gives $b = 20 - 5 = 15$

$$\therefore a = 5 \text{ and } b = 15$$

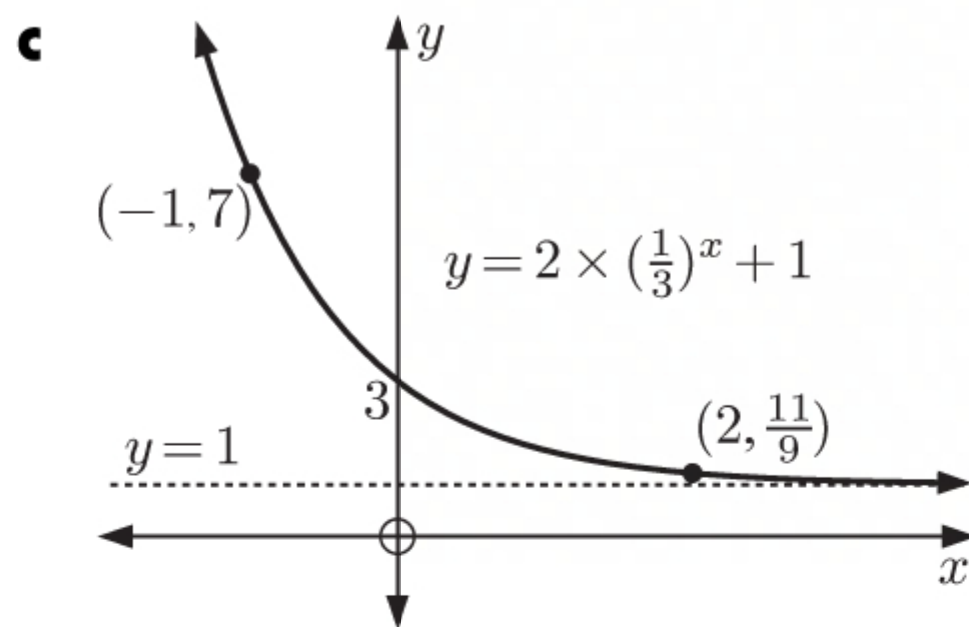
51 $f(x) = 2 \times \left(\frac{1}{3}\right)^x + 1$

a i $f(0) = 2 + 1$
 $= 3$

ii $f(2) = 2 \times \left(\frac{1}{3}\right)^2 + 1$
 $= \frac{2}{9} + 1$
 $= \frac{11}{9}$

iii $f(-1) = 2 \times \left(\frac{1}{3}\right)^{-1} + 1$
 $= 2 \times 3 + 1$
 $= 7$

b The horizontal asymptote is $y = 1$.

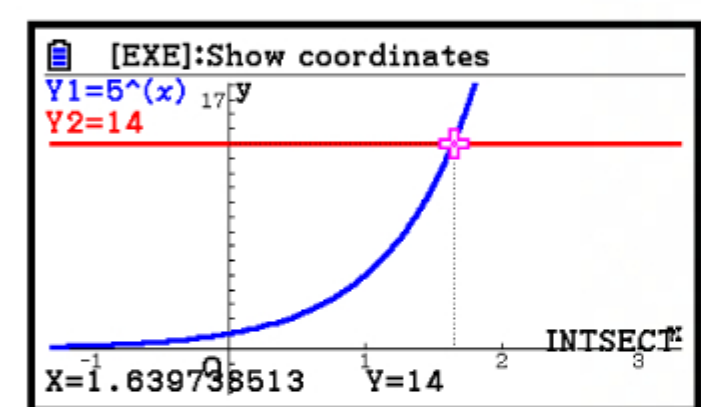


d The domain is $\{x \mid x \in \mathbb{R}\}$.

The range is $\{y \mid y > 1\}$.

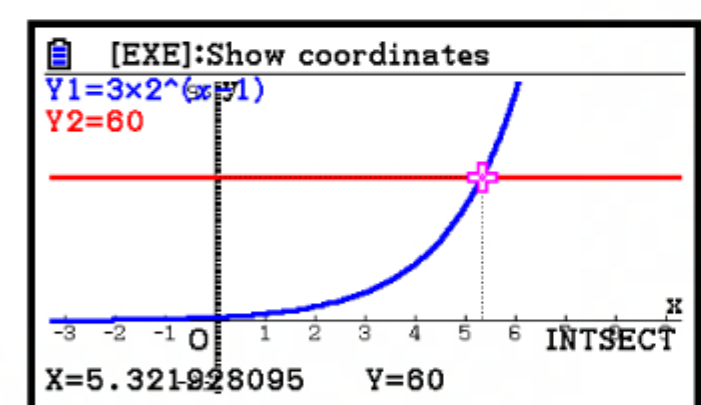
52 a We graph $Y_1 = 5^x$ and $Y_2 = 14$ on the same set of axes, and find their point of intersection.

The solution is $x \approx 1.64$.



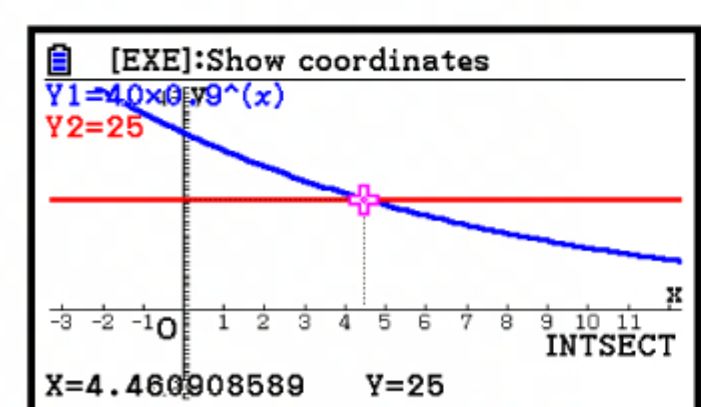
b We graph $Y_1 = 3 \times 2^{x-1}$ and $Y_2 = 60$ on the same set of axes, and find their point of intersection.

The solution is $x \approx 5.32$.



c We graph $Y_1 = 40 \times (0.9)^x$ and $Y_2 = 25$ on the same set of axes, and find their point of intersection.

The solution is $x \approx 4.46$.



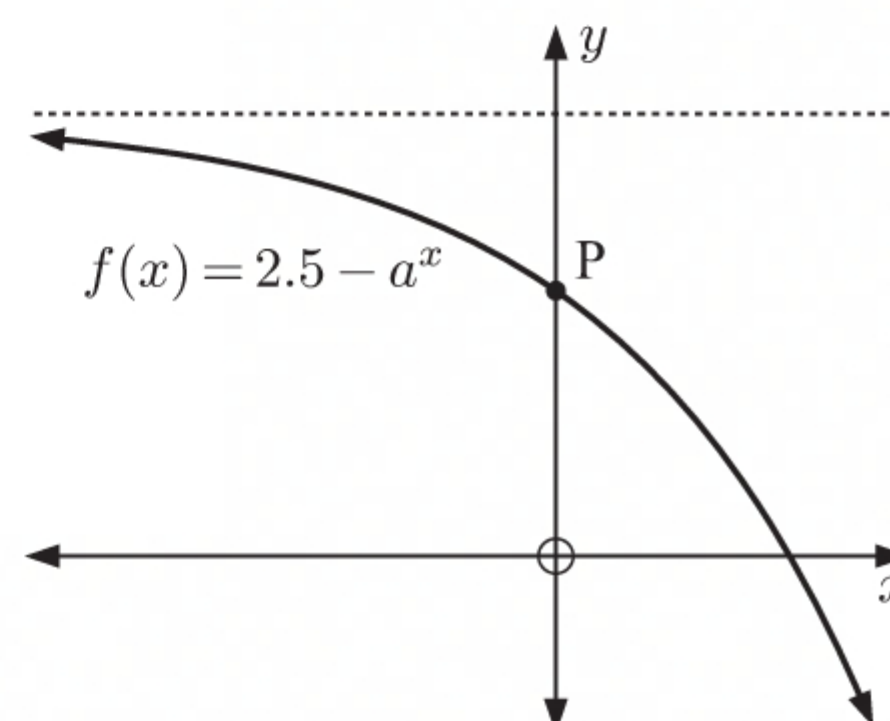
$$\begin{aligned} 53 \quad a \quad f(0) &= 2.5 - a^0 \\ &= 2.5 - 1 \quad \{a > 0\} \\ &= 1.5 \end{aligned}$$

\therefore P has coordinates $(0, 1.5)$.

b The point $(3, -5.5)$ lies on the graph

$$\begin{aligned} \therefore f(3) &= -5.5 \\ \therefore -5.5 &= 2.5 - a^3 \\ \therefore a^3 &= 8 \\ \therefore a &= 2 \end{aligned}$$

c The horizontal asymptote has equation $y = 2.5$.



$$54 \quad a \quad P(t) = a(0.95)^t + b$$

$$\text{Now } P(0) = 2500$$

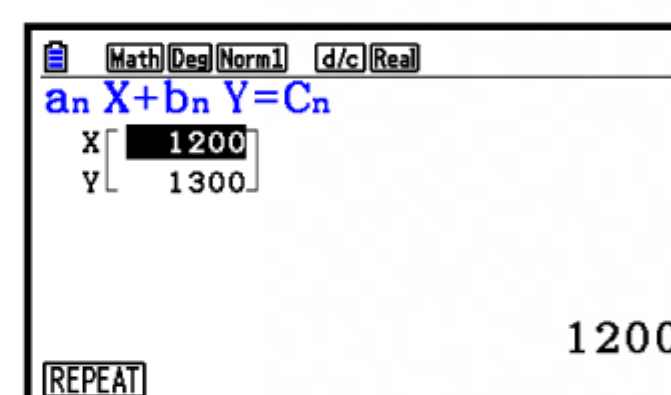
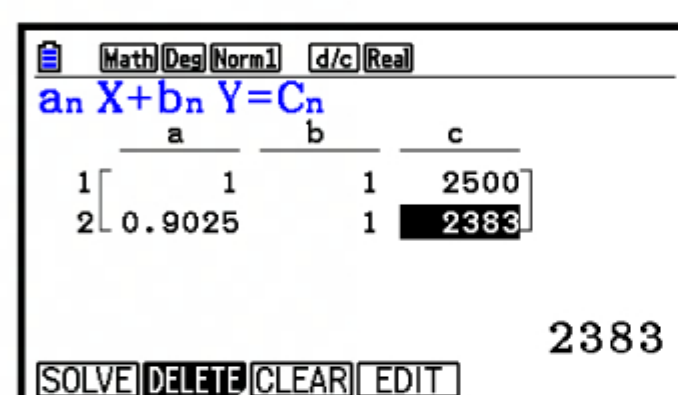
$$\text{and } P(2) = 2383$$

$$\therefore 2500 = a(0.95)^0 + b$$

$$\therefore 2383 = a(0.95)^2 + b$$

$$\therefore a + b = 2500 \quad \dots (1)$$

$$\therefore 0.9025a + b = 2383 \quad \dots (2)$$



Solving (1) and (2) simultaneously, we find:

$$\text{i} \quad a = 1200$$

$$\text{ii} \quad b = 1300$$

b $P(t) = 1200(0.95)^t + 1300$ {using **a**}

$$\begin{aligned} \text{i} \quad P(3) &= 1200(0.95)^3 + 1300 \\ &= 2328.85 \end{aligned}$$

After 3 weeks, there are about 2330 bees.

$$\begin{aligned} \text{ii} \quad P(5) &= 1200(0.95)^5 + 1300 \\ &\approx 2228.54 \end{aligned}$$

After 5 weeks, there are about 2230 bees.

c The horizontal asymptote is $P = 1300$.

As $t \rightarrow \infty$, the population approaches 1300 bees.

$$55 \quad a \quad \text{i} \quad \text{When } t = 4, P \approx 60$$

\therefore there is about 60% of Carbon-14 remaining after 4000 years.

$$\text{ii} \quad \text{When } P = 50, t \approx 5.5$$

\therefore it will take approximately 5500 years for the percentage of Carbon-14 to fall to 50%.

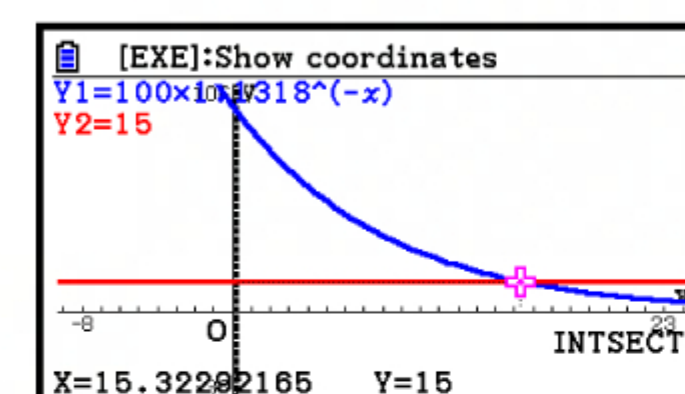
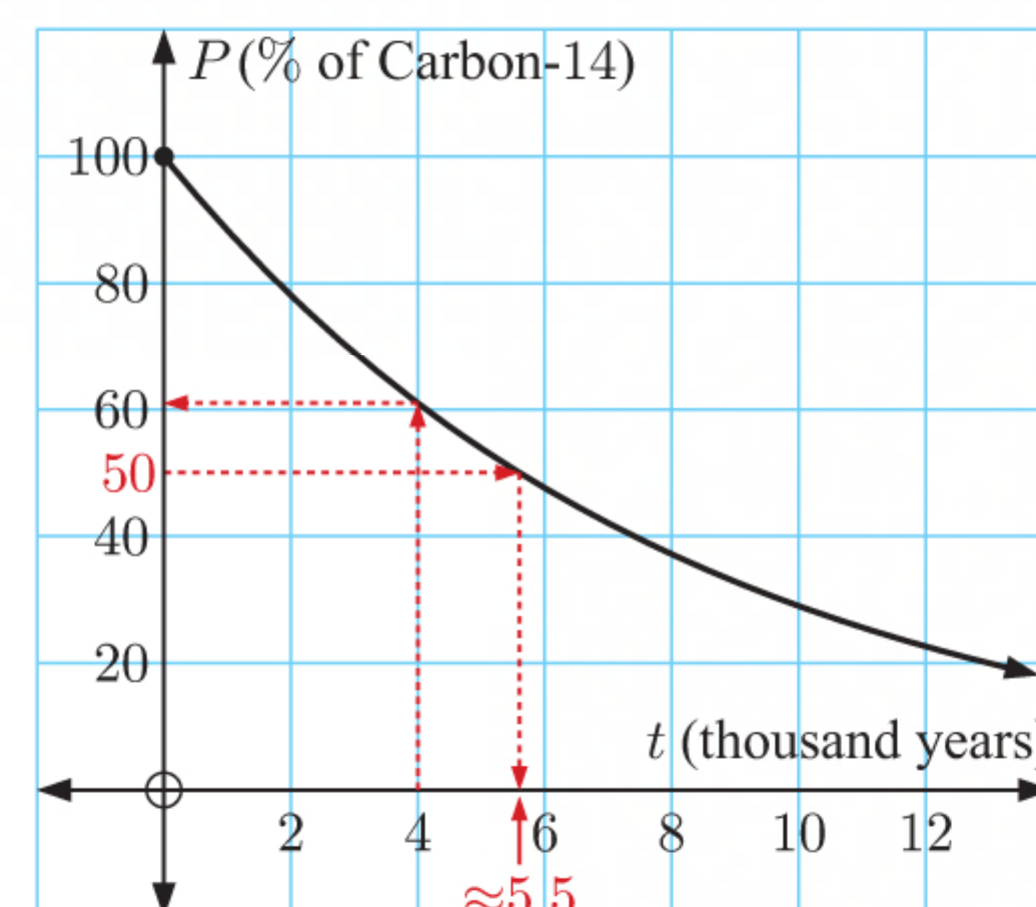
$$\text{b} \quad P = 100 \times (1.1318)^{-t}, t \geq 0$$

$$\begin{aligned} \text{i} \quad \text{When } t = 8, \quad P &= 100 \times (1.1318)^{-8} \\ &\approx 37.1 \end{aligned}$$

\therefore there is about 37.1% of Carbon-14 remaining after 8000 years.

$$\begin{aligned} \text{ii} \quad \text{When } P = 15, \quad 15 &= 100 \times (1.1318)^{-t} \\ \therefore t &\approx 15.3 \quad \{\text{using technology}\} \end{aligned}$$

\therefore it will take approximately 15 300 years for the percentage of Carbon-14 to fall to 15%.



$$56 \quad N = 120 \times (1.04)^t$$

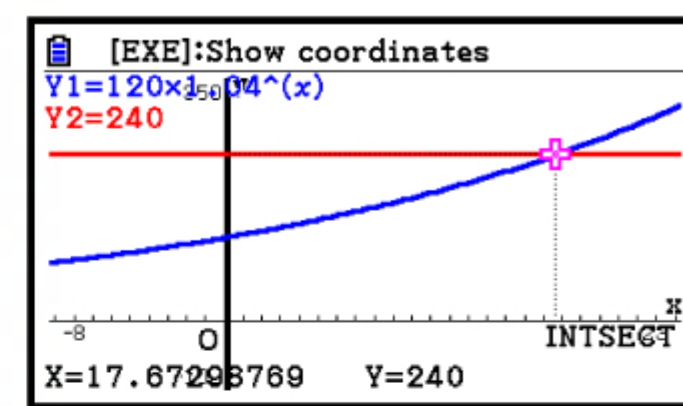
$$\text{a} \quad \text{When } t = 0, N = 120$$

\therefore there were 120 people who started the settlement.

$$\begin{aligned} \text{b} \quad \text{When } t = 4, \quad N &= 120 \times (1.04)^4 \\ &\approx 140 \end{aligned}$$

\therefore there were about 140 people on the island after 4 years.

- c** When $N = 120 \times 2 = 240$, $240 = 120 \times (1.04)^t$
 $\therefore t \approx 17.7$ {using technology}
 \therefore it will take about 17.7 years for the number of people to double.



- 57 a** $W(t) = 100 \times a^t$
 $\therefore W(0) = 100 \times a^0$
 $= 100 \quad \{a > 0\}$
The initial weight of the sample is 100 mg.

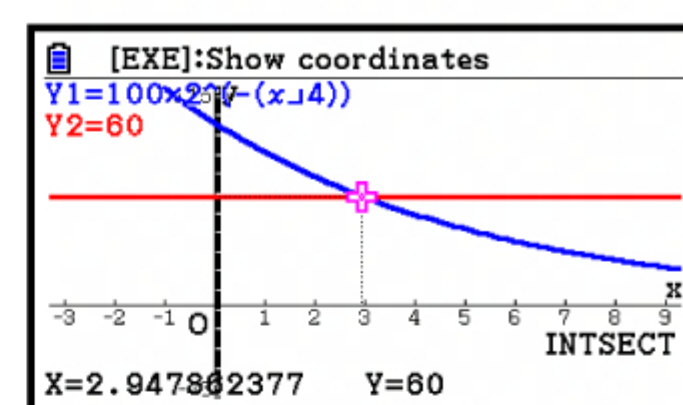
- b** After 4 days, the sample is $\frac{1}{2} \times 100 = 50$ mg.
 $\therefore W(4) = 50$
 $\therefore 50 = 100 \times a^4$
 $\therefore a^4 = \frac{1}{2}$
 $\therefore a = \frac{1}{\sqrt[4]{2}} \quad \{a > 0\}$
 ≈ 0.8409

The weight of the sample on any given day is reduced to about 84.09% of this weight the following day.

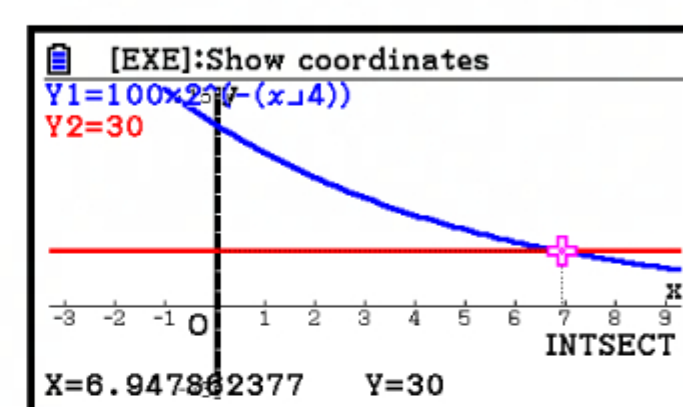
- c** $W(t) = 100 \times \left(\frac{1}{\sqrt[4]{2}}\right)^t$ {from **b**}
 $\therefore W(6) = 100 \times \left(\frac{1}{\sqrt[4]{2}}\right)^6$
 ≈ 35.4

The weight of the sample after 6 days is about 35.4 mg.

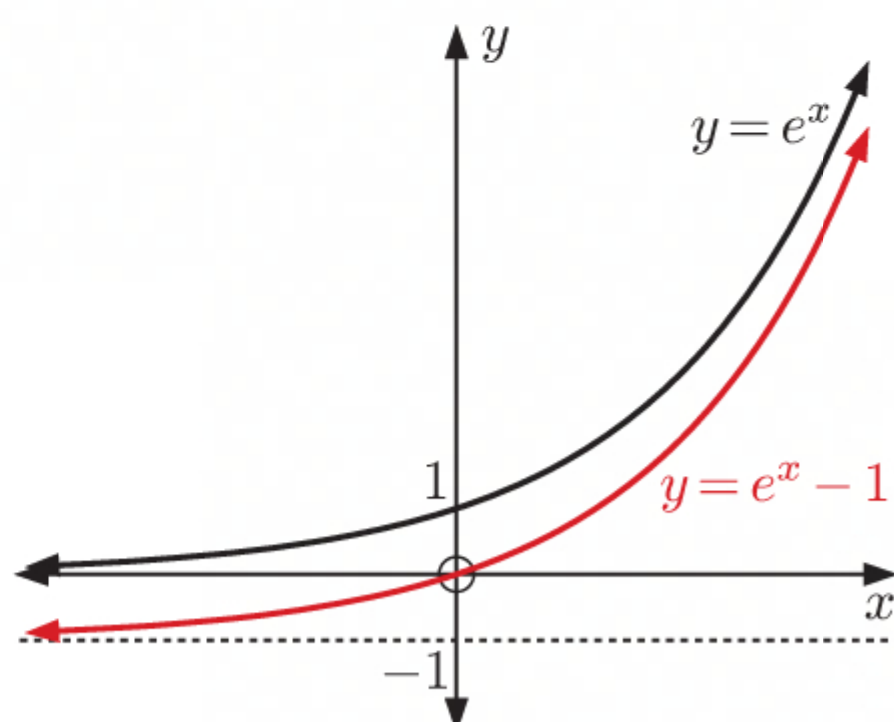
- d i** When $W(t) = 60$, $60 = 100 \times 2^{-\frac{t}{4}}$
 $\therefore t \approx 2.95$ {using technology}
It will take about 2.95 days for the weight to fall to 60 mg.



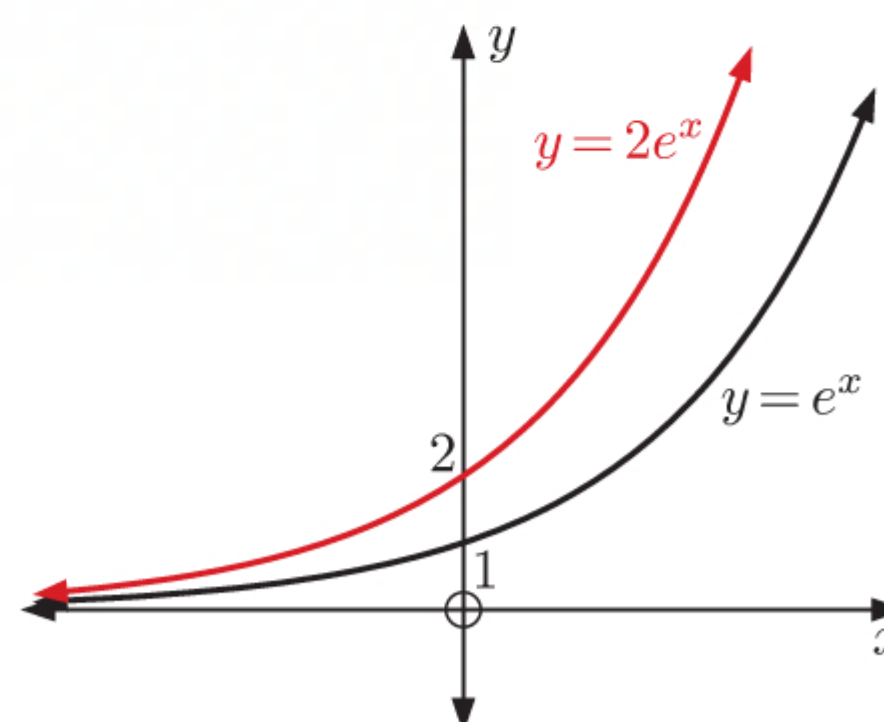
- ii** When $W(t) = 30$, $30 = 100 \times 2^{-\frac{t}{4}}$
 $\therefore t \approx 6.95$ {using technology}
It will take about 6.95 days for the weight to fall to 30 mg.



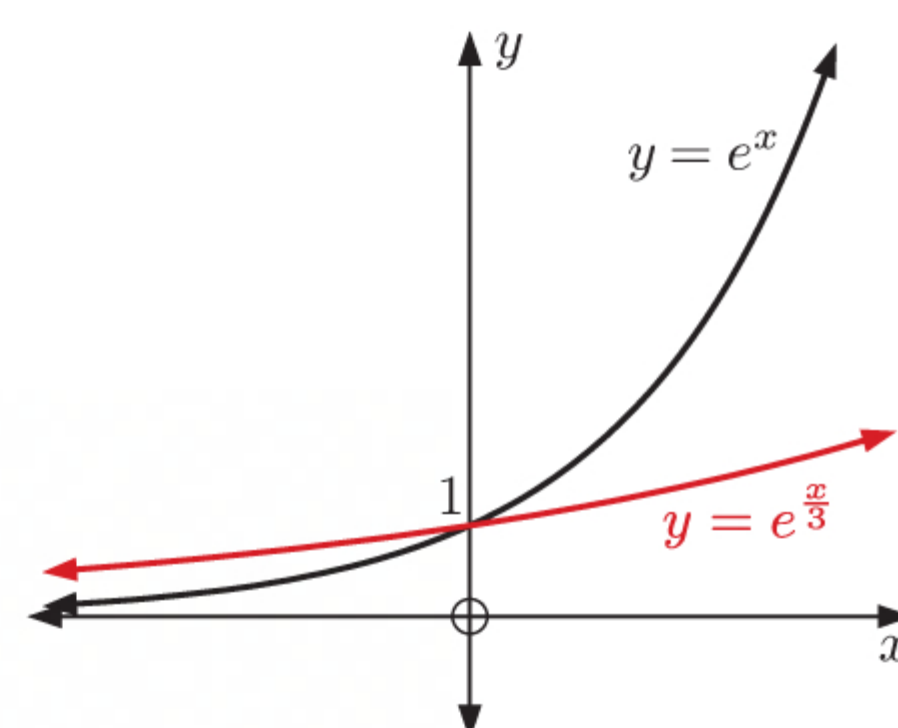
- 58 a** $y = e^x - 1$ is a vertical translation of $y = e^x$ by 1 unit downwards.
The y -intercept is 0, and the horizontal asymptote is $y = -1$.



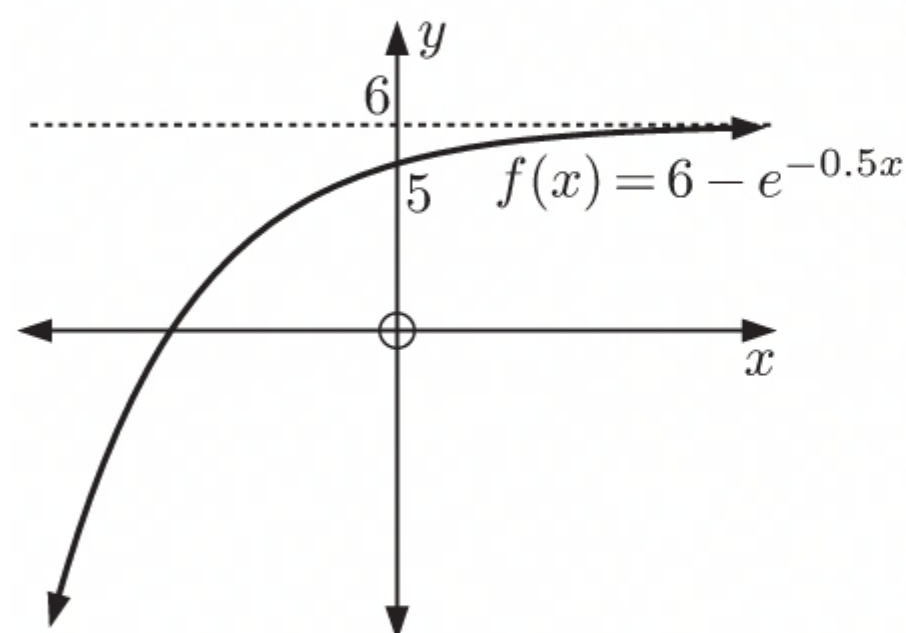
- b** $y = 2e^x$ is a vertical stretch of $y = e^x$ with scale factor 2.
The y -intercept is 2, and the horizontal asymptote is $y = 0$.



- c** $y = e^{\frac{x}{3}}$ is a horizontal stretch of $y = e^x$ with scale factor 3.
The y -intercept is 1, and the horizontal asymptote is $y = 0$.



59 a


 b The domain of $f(x)$ is $\{x \mid x \in \mathbb{R}\}$.

 The range of $f(x)$ is $\{y \mid y < 6\}$.

 c As $x \rightarrow -\infty$, $y = f(x) \rightarrow -\infty$.

 As $x \rightarrow \infty$, $y = f(x) \rightarrow 6$.

d From the graph in a:

 i $f(x) = k$ has one solution if $k < 6$.

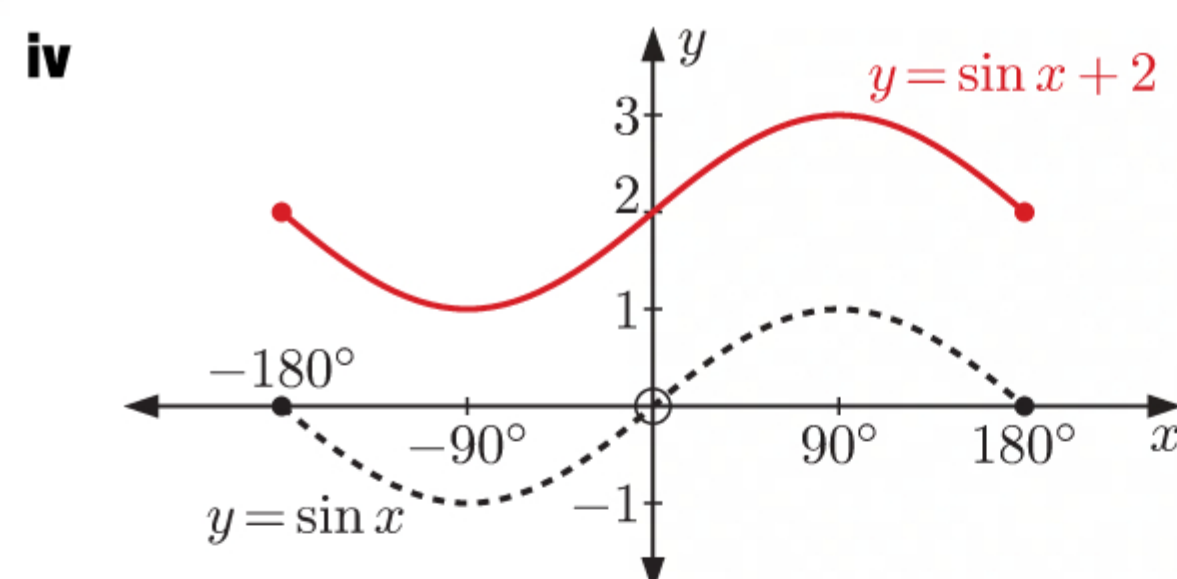
 ii $f(x) = k$ has no solutions if $k \geq 6$.

 60 a For $y = f(x) = \sin 4x$:

- the amplitude is $|1| = 1$
- the principal axis is $y = 0$
- the period is $\frac{360^\circ}{4} = 90^\circ$.

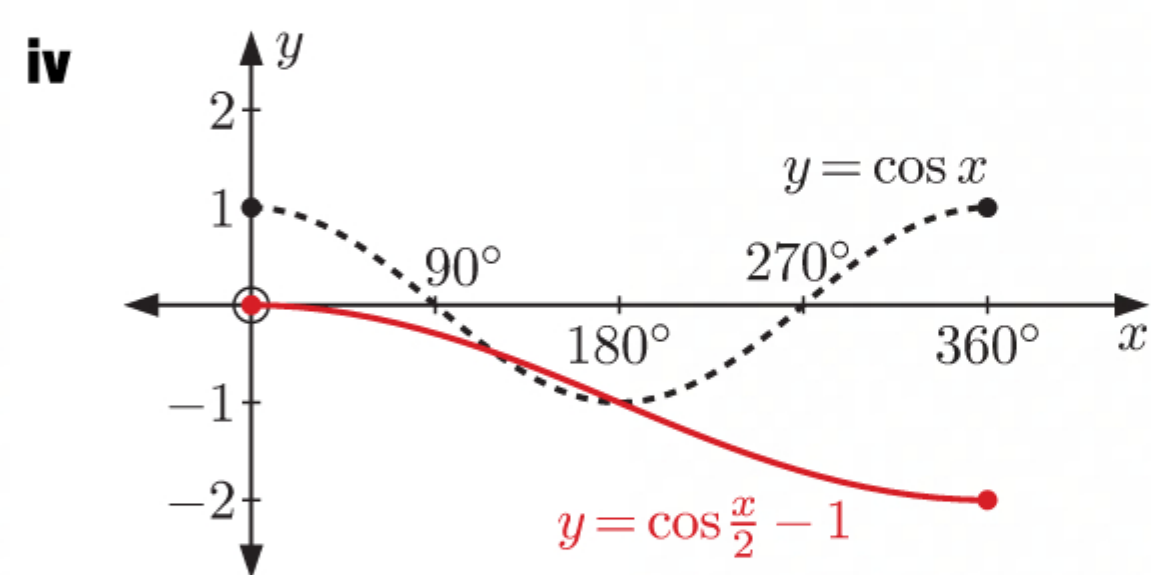
61 a i amplitude = 1

 ii principal axis is $y = 2$

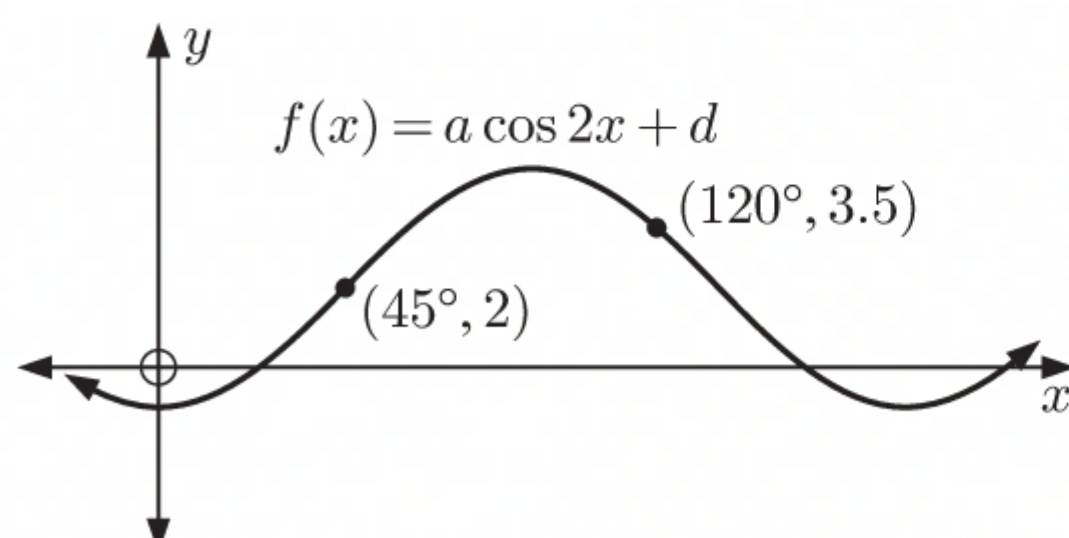
 iii period = $\frac{360^\circ}{1} = 360^\circ$


c i amplitude = 1

 ii principal axis is $y = -1$

 iii period = $\frac{360^\circ}{\frac{1}{2}} = 720^\circ$


62 a



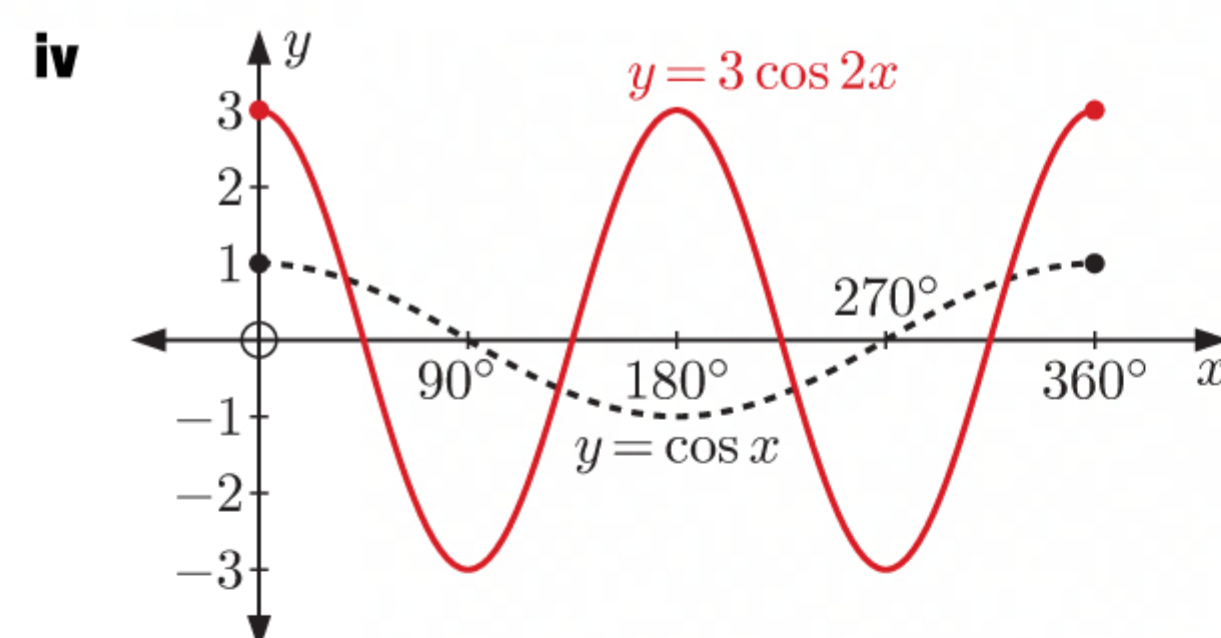
$$\begin{aligned} f(45^\circ) &= 2 & f(120^\circ) &= 3.5 \\ \therefore a \cos 90^\circ + d &= 2 & \therefore a \cos 240^\circ + d &= 3.5 \\ \therefore d &= 2 & \therefore -\frac{1}{2}a + 2 &= 3.5 \\ & & \therefore -\frac{1}{2}a &= 1.5 \\ & & \therefore a &= -3 \end{aligned}$$

 b For $y = f(x) = -2 \sin \frac{x}{2} - 1$:

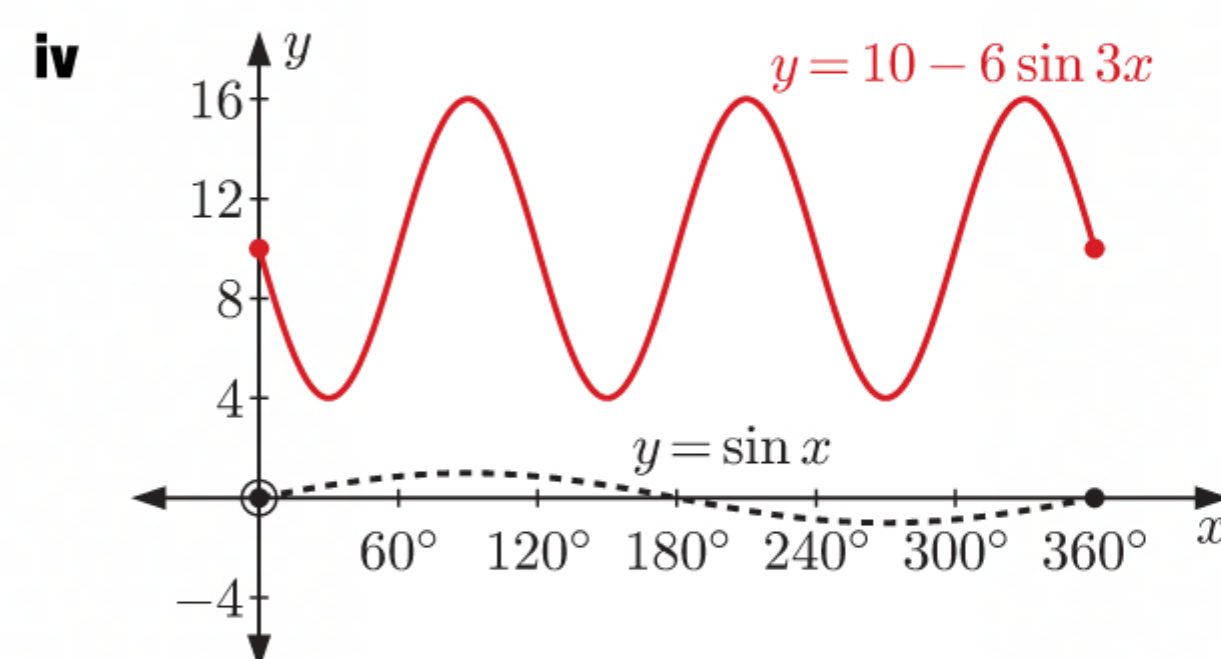
- the amplitude is $|-2| = 2$
- the principal axis is $y = -1$
- the period is $\frac{360^\circ}{\frac{1}{2}} = 720^\circ$.

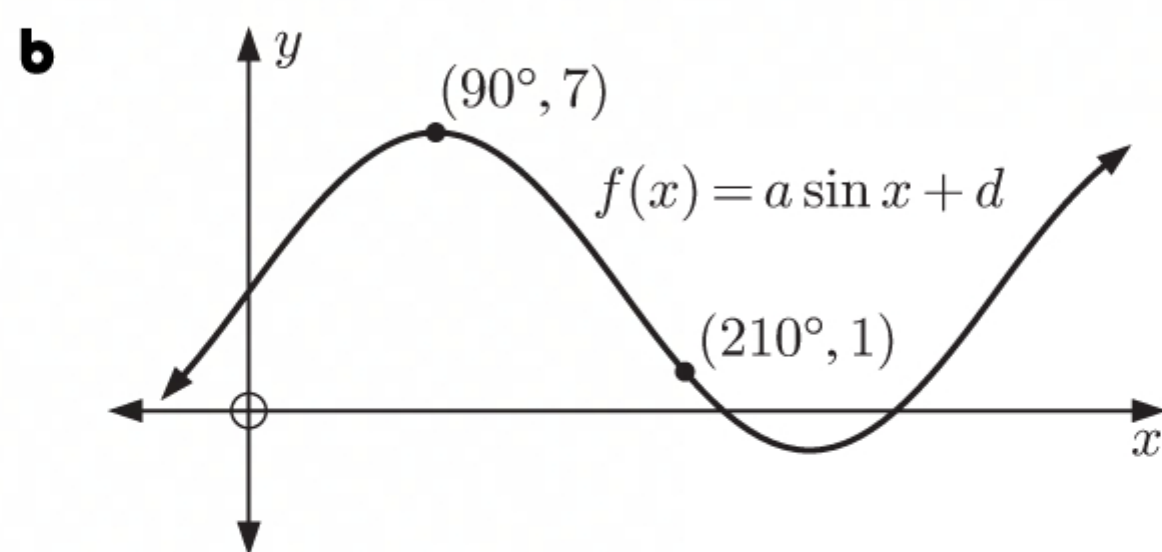
b i amplitude = 3

 ii principal axis is $y = 0$

 iii period = $\frac{360^\circ}{2} = 180^\circ$

 d i amplitude = $|-6| = 6$

 ii principal axis is $y = 10$

 iii period = $\frac{360^\circ}{3} = 120^\circ$




$$\begin{aligned}
 f(90^\circ) &= 7 & f(210^\circ) &= 1 \\
 \therefore a \sin 90^\circ + d &= 7 & \therefore a \sin 210^\circ + d &= 1 \\
 \therefore a + d &= 7 \quad \dots (1) & \therefore -\frac{1}{2}a + d &= 1 \quad \dots (2)
 \end{aligned}$$

Solving (1) and (2) simultaneously using technology gives $a = 4$ and $d = 3$.

- 63 a** High tide is 4.7 metres, and low tide is 2.4 metres below it.

$$\therefore \text{low tide is } 4.7 - 2.4 = 2.3 \text{ metres.}$$

$$\text{So, amplitude} = \frac{4.7 - 2.3}{2}$$

$$\therefore a = 1.2$$

$$\text{period} = 12.3 \text{ hours}$$

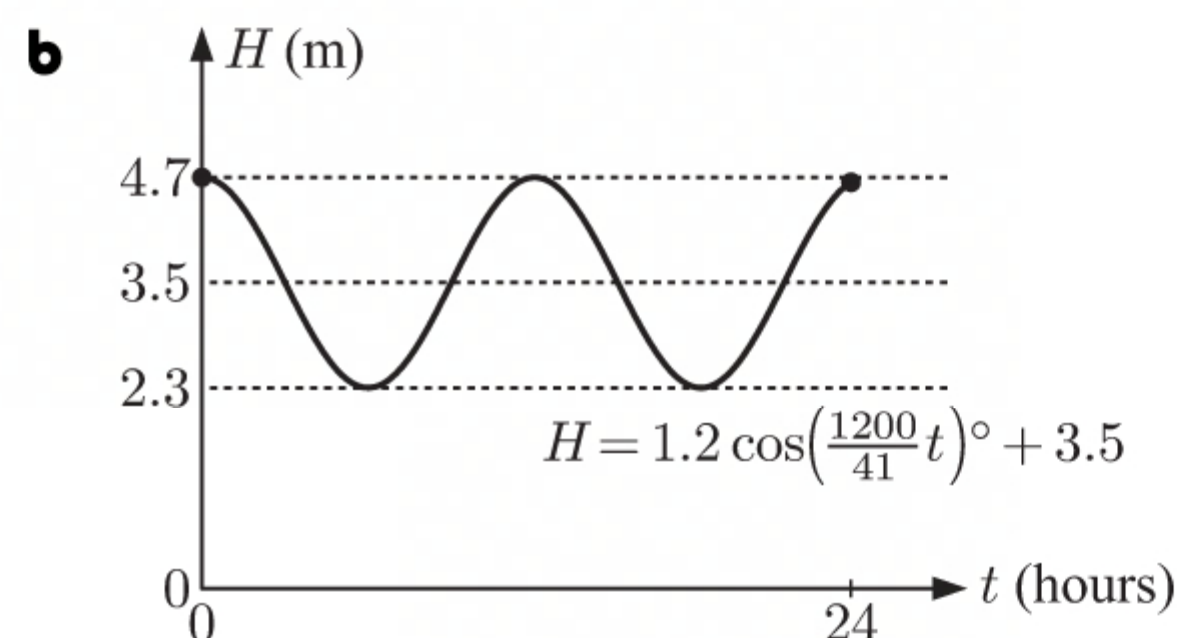
$$\therefore \frac{360}{b} = 12.3$$

$$\therefore b = \frac{1200}{41}$$

$$\text{and the principal axis is } H = \frac{4.7 + 2.3}{2}$$

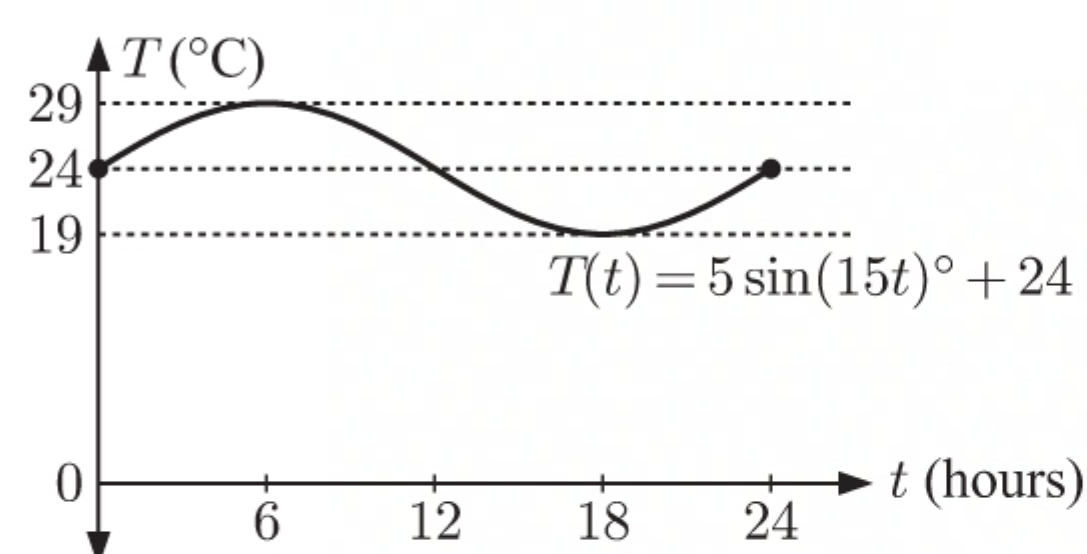
$$\therefore d = 3.5$$

$$\therefore H = 1.2 \cos\left(\frac{1200}{41}t\right)^\circ + 3.5$$



- 64 a** For $T(t) = 5 \sin(15t)^\circ + 24$:

- the amplitude is $|5| = 5$
- the period is $\frac{360}{15} = 24$ hours
- the principal axis is $T = 24$.



- b i** 2 pm is 2 hours after 12 noon.

$$T(2) = 5 \sin(15 \times 2)^\circ + 24$$

$$= 5 \sin 30^\circ + 24$$

$$= 5\left(\frac{1}{2}\right) + 24$$

$$= 26.5$$

\therefore at 2 pm, the temperature inside Pam's caravan is 26.5°C .

- ii** 9 pm is 9 hours after 12 noon.

$$T(9) = 5 \sin(15 \times 9)^\circ + 24$$

$$= 5 \sin 135^\circ + 24$$

$$= 5\left(\frac{1}{\sqrt{2}}\right) + 24$$

$$\approx 27.5$$

\therefore at 9 pm, the temperature inside Pam's caravan is about 27.5°C .

- c** The minimum temperature inside Pam's caravan is $24 - 5 = 19^\circ\text{C}$, which occurs when $t = 18$.

18 hours after 12 noon is 6 am the following day.

So, the minimum temperature inside Pam's caravan occurs at 6 am the following day.

TOPIC 3 SKILL BUILDER QUESTIONS

- 1 a Perimeter = $2 \times$ line segment length + inner arc length + outer arc length

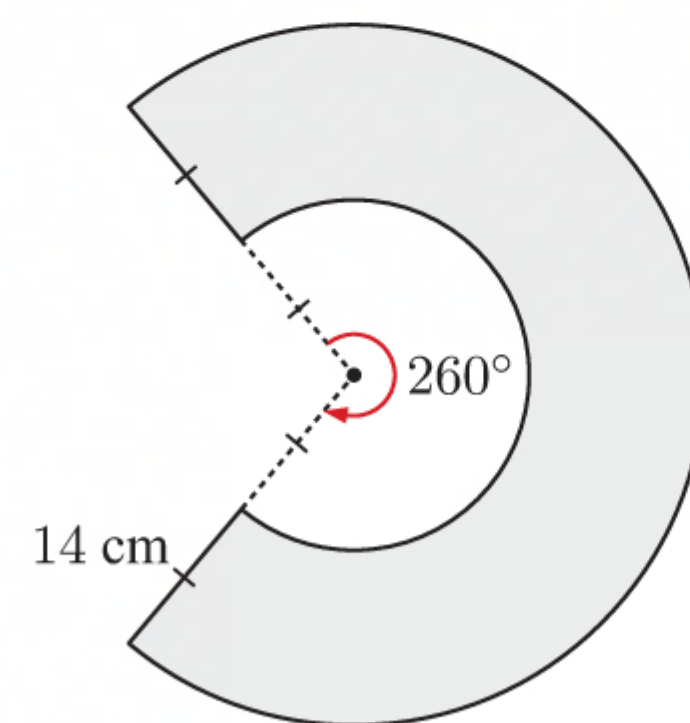
$$= 2 \times 14 + \frac{260}{360} \times 2\pi \times 14 + \frac{260}{360} \times 2\pi \times 28$$

$$\approx 219 \text{ cm}$$

- b Area = outer sector area - inner sector area

$$= \frac{260}{360} \times \pi \times 28^2 - \frac{260}{360} \times \pi \times 14^2$$

$$\approx 1330 \text{ cm}^2$$



- 2 a Let the height of the triangles be h cm.

$$\text{Now } h^2 = 20^2 + 5^2$$

$$\therefore h = \sqrt{20^2 + 5^2} = 5\sqrt{17}$$

Surface area

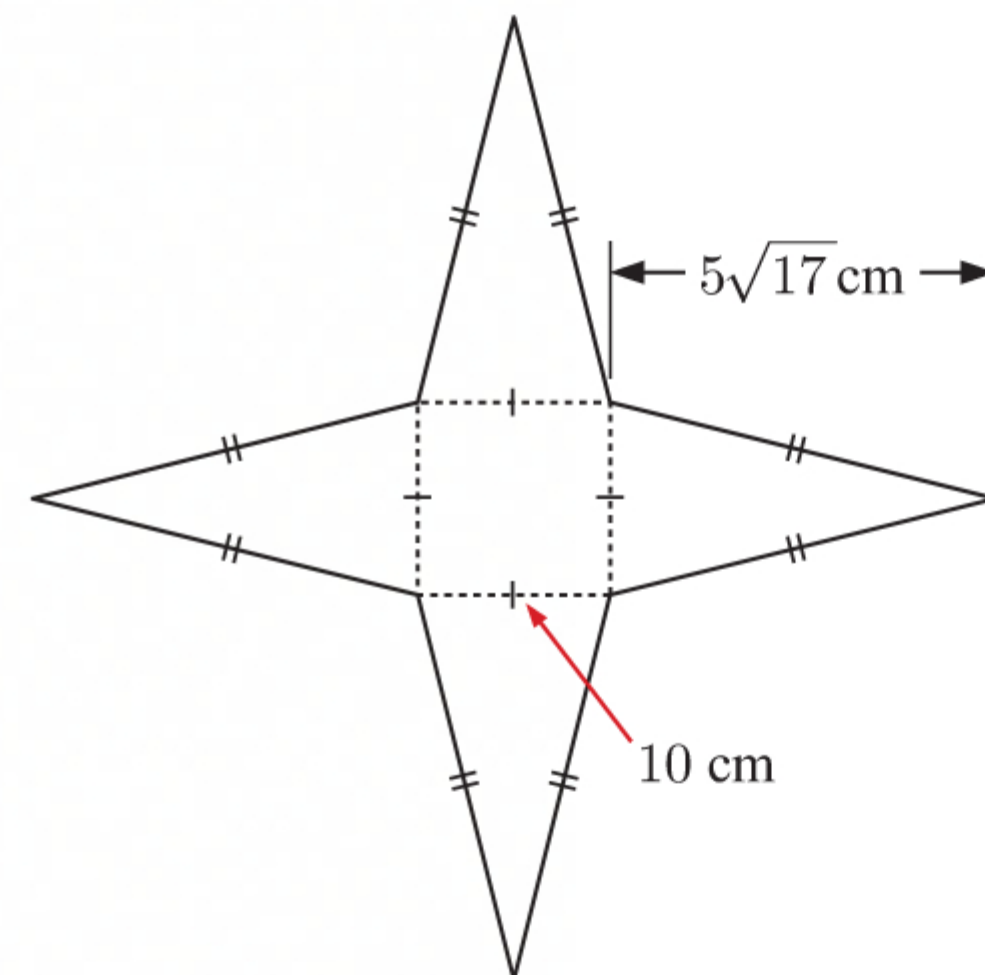
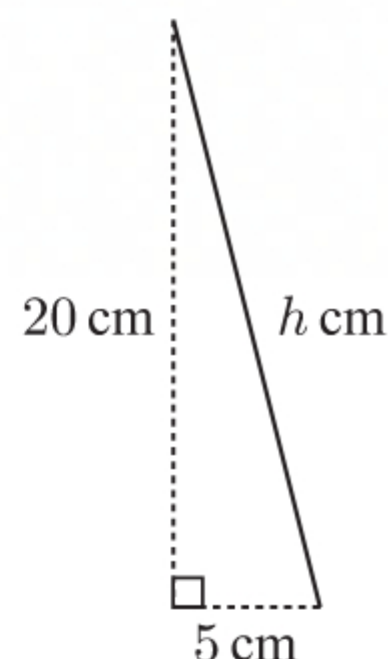
$$= \text{area of square} + 4 \times \text{area of triangle}$$

$$= 10^2 + 4 \left(\frac{1}{2} \times 10 \times 5\sqrt{17} \right)$$

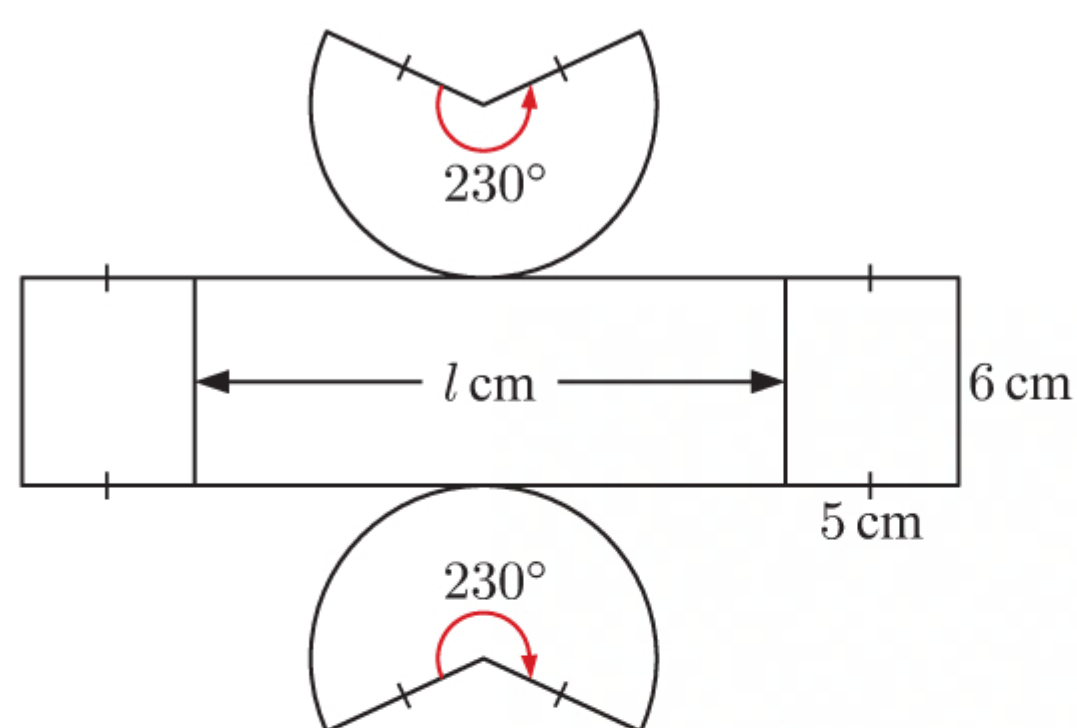
$$= 100 + 4(25\sqrt{17})$$

$$= 100 + 100\sqrt{17} \text{ cm}^2$$

$$\approx 512 \text{ cm}^2$$



- b



$$l = 2\pi r \times \frac{230}{360}$$

$$= 2\pi(5) \times \frac{23}{36}$$

$$= \frac{115\pi}{18}$$

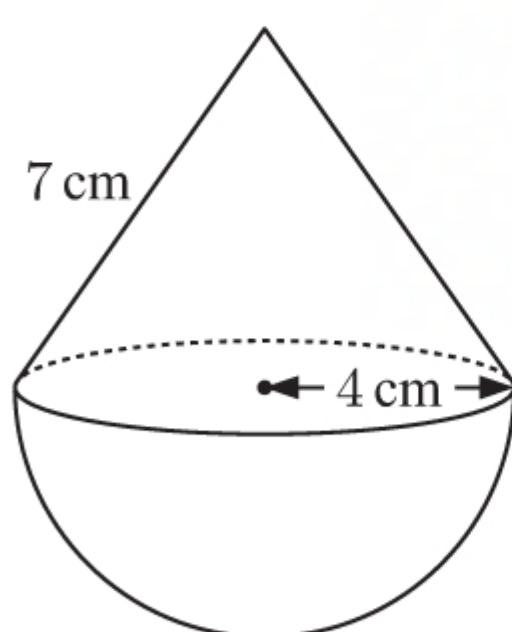
Surface area = $2 \times$ area of sector + area of curved surface
+ $2 \times$ area of rectangle

$$= 2 \times \frac{230}{360} \times \pi(5)^2 + \frac{115\pi}{18} \times 6 + 2 \times 6 \times 5$$

$$= \frac{575\pi}{18} + \frac{115\pi}{3} + 60$$

$$\approx 281 \text{ cm}^2$$

- c



$$\text{Surface area} = \frac{1}{2}4\pi r^2 + \pi rs$$

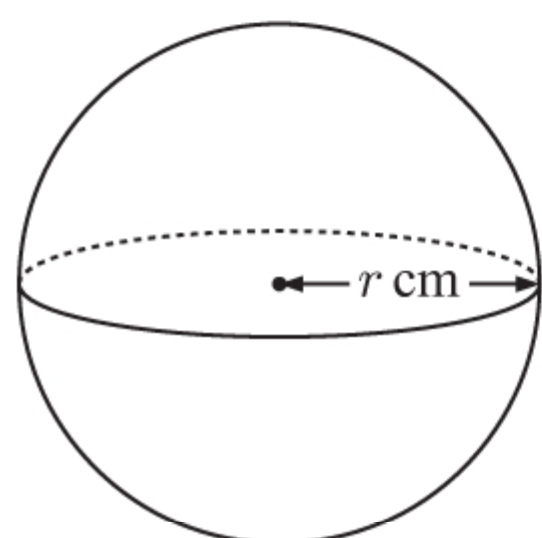
$$= \frac{1}{2} \times 4\pi(4)^2 + \pi(4)(7)$$

$$= 32\pi + 28\pi$$

$$= 60\pi \text{ cm}^2$$

$$\approx 188 \text{ cm}^2$$

- 3



Let the beach ball have radius r cm.

$$\text{Surface area} = 2800 \text{ cm}^2$$

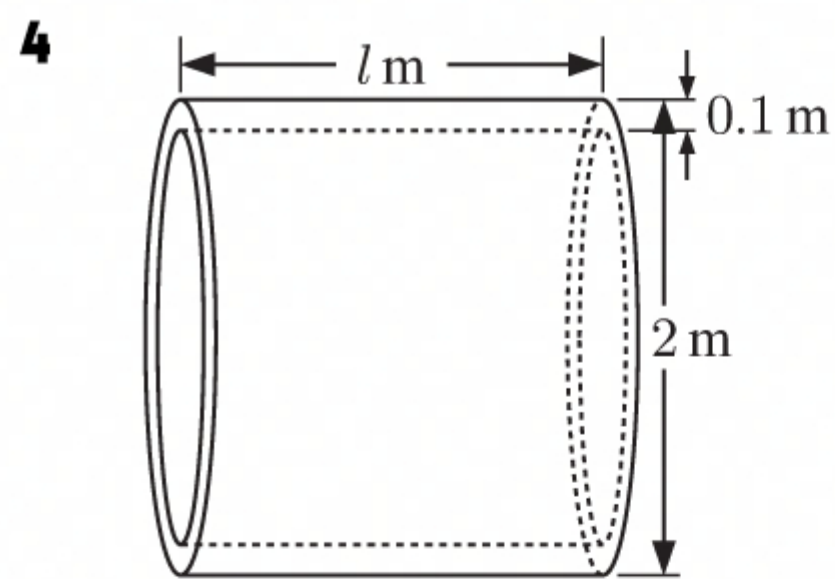
$$\therefore 4\pi r^2 = 2800$$

$$\therefore r^2 = \frac{700}{\pi}$$

$$\therefore r = \sqrt{\frac{700}{\pi}} \quad \{r > 0\}$$

$$\therefore r \approx 14.9$$

\therefore the radius of the beach ball is approximately 14.9 cm.



Let the pipe have length l m.

Now volume of concrete = volume of whole cylinder – volume of hollow section

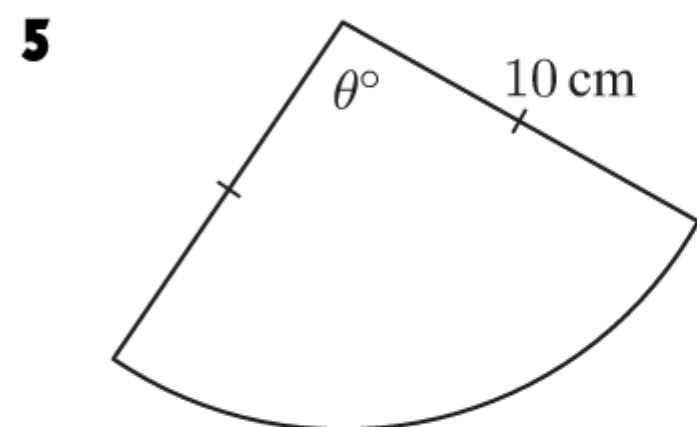
$$\therefore 3 = \pi(1)^2 \times l - \pi(0.9)^2 \times l$$

$$\therefore 3 = \pi l - 0.81\pi l$$

$$\therefore 3 = 0.19\pi l$$

$$\therefore l = \frac{3}{0.19\pi} \approx 5.03$$

\therefore the pipe is approximately 5.03 m long.



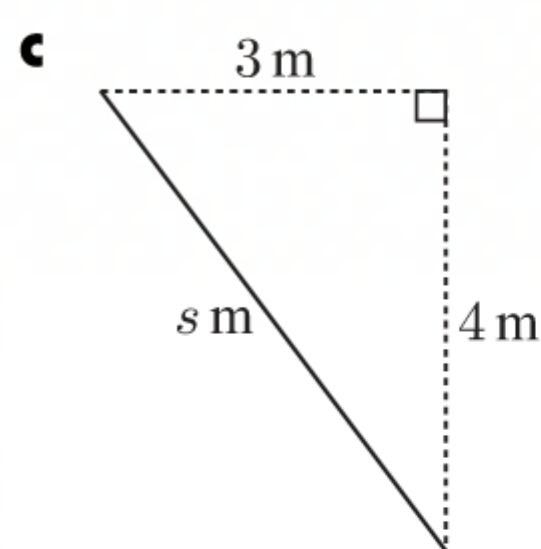
a Perimeter = $2r + \text{arc length}$
 $\therefore 40 = 2(10) + \text{arc length}$
 $\therefore \text{arc length} = 20 \text{ cm}$

b Now, arc length = $\frac{\theta}{360} \times 2\pi r$
 $\therefore 20 = \frac{\theta}{360} \times 2\pi(10) \quad \{\text{from a}\}$
 $\therefore 20 = \frac{\theta\pi}{18}$
 $\therefore \theta = \frac{360}{\pi}$

$$\begin{aligned} \therefore \text{area} &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{\left(\frac{360}{\pi}\right)}{360} \times \pi(10)^2 \\ &= 100 \text{ cm}^2 \end{aligned}$$

6 a Total height = hemisphere radius + cone height
 $\therefore 7 = \text{hemisphere radius} + 4$
 $\therefore \text{hemisphere radius} = 3 \text{ m}$
 $\therefore \text{cone radius} = 3 \text{ m} \quad \{\text{hemisphere radius} = \text{cone radius}\}$

b Volume = volume of hemisphere + volume of cone
 $= \frac{1}{2} \times \frac{4}{3}\pi r^3 + \frac{1}{3} \times \pi r^2 \times h$
 $= \frac{1}{2} \times \frac{4}{3}\pi(3)^3 + \frac{1}{3} \times \pi(3)^2 \times 4$
 $= 18\pi + 12\pi$
 $= 30\pi \approx 94.2 \text{ m}^3$



Let the slant height of the cone be s m.

$$\therefore s^2 = 4^2 + 3^2 \quad \{\text{Pythagoras}\}$$

$$\therefore s^2 = 16 + 9$$

$$\therefore s^2 = 25$$

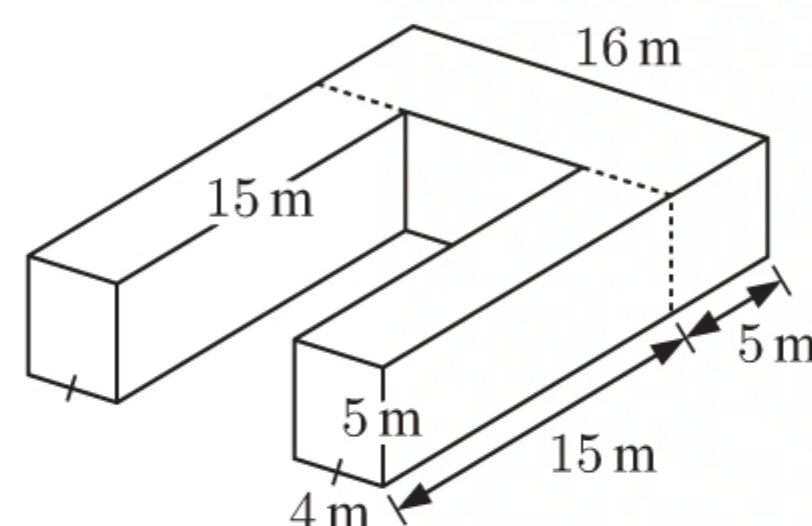
$$\therefore s = 5 \quad \{s > 0\}$$

\therefore the slant height of the cone is 5 m.

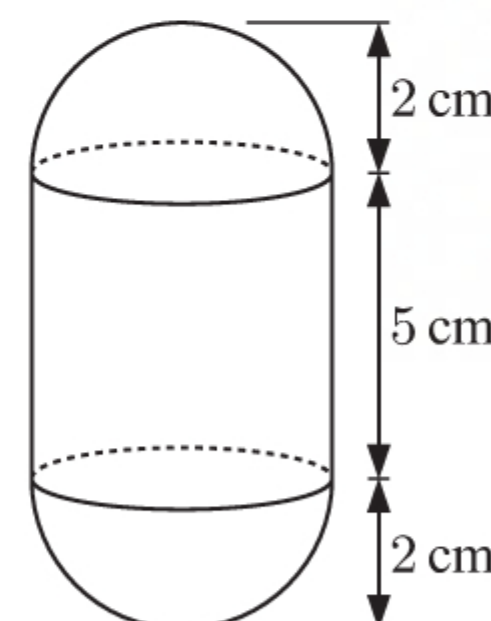
d Surface area = $\frac{1}{2} \times 4\pi r^2 + \pi r s$
 $= 2\pi(3)^2 + \pi(3)(5)$
 $= 18\pi + 15\pi$
 $= 33\pi \approx 104 \text{ m}^2$

e Weight = surface area \times weight of polymer per m^2
 $= 33\pi \times 1.23$
 $\approx 128 \text{ kg}$

7 a Volume = $2 \times (15 \times 5 \times 4) + (16 \times 5 \times 5)$
 $= 1000 \text{ m}^3$



b Volume = volume of sphere + volume of cylinder
 $= \frac{4}{3}\pi(2)^3 + \pi(2)^2 \times 5$
 $= \frac{32\pi}{3} + 20\pi$
 $\approx 96.3 \text{ cm}^3$



c Let the height of the triangular cross-section be h cm.

$$\therefore h^2 + \left(\frac{15}{2}\right)^2 = 20^2 \quad \{\text{Pythagoras}\}$$

$$\therefore h^2 + \frac{225}{4} = 400$$

$$\therefore h^2 = \frac{1375}{4}$$

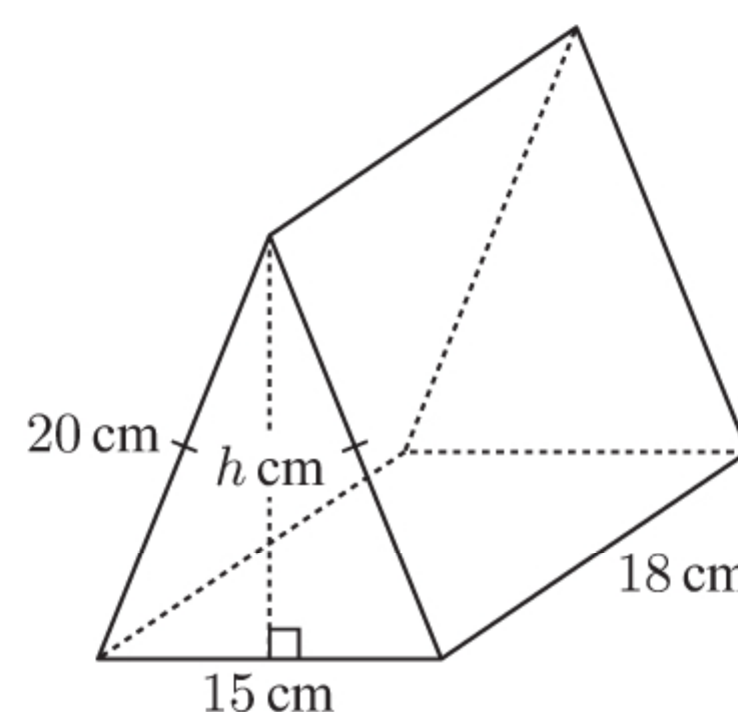
$$\therefore h = \frac{\sqrt{1375}}{2} \quad \{h > 0\}$$

\therefore volume = cross-sectional area \times length

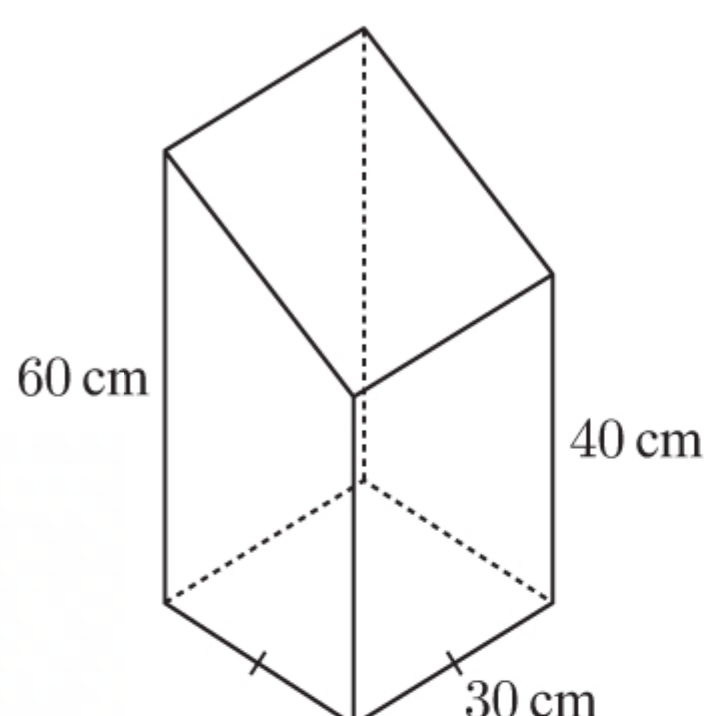
$$= \left(\frac{1}{2} \times h \times 15\right) \times 18$$

$$= \frac{1}{2} \times \frac{\sqrt{1375}}{2} \times 15 \times 18$$

$$\approx 2500 \text{ cm}^3$$



8



Volume = cross-sectional area \times length

$$= \left(\frac{60 + 40}{2}\right) \times 30 \times 30$$

$$= 50 \times 30 \times 30$$

$$= 45\,000 \text{ cm}^3$$

$$\therefore \text{capacity} = 45\,000 \text{ mL} = 45 \text{ L}$$

$$\therefore \text{number of filled containers} = \frac{300 \text{ L}}{45 \text{ L}} = 6\frac{2}{3}$$

\therefore 6 petrol containers can be completely filled with 300 L of petrol.

9 a A(2, 4, 1) and B(4, 0, 7)

i $AB = \sqrt{(4-2)^2 + (0-4)^2 + (7-1)^2}$

$$= \sqrt{2^2 + (-4)^2 + 6^2}$$

$$= \sqrt{4 + 16 + 36}$$

$$= \sqrt{56} \text{ units}$$

ii The midpoint is $\left(\frac{2+4}{2}, \frac{4+0}{2}, \frac{1+7}{2}\right)$,
 which is (3, 2, 4).

b A(3, -5, 2) and B(-1, 2, -3)

i $AB = \sqrt{(-1-3)^2 + (2-(-5))^2 + (-3-2)^2}$

$$= \sqrt{(-4)^2 + 7^2 + (-5)^2}$$

$$= \sqrt{16 + 49 + 25}$$

$$= \sqrt{90} \text{ units}$$

ii The midpoint is $\left(\frac{3+(-1)}{2}, \frac{-5+2}{2}, \frac{2+(-3)}{2}\right)$,
 which is $\left(1, -\frac{3}{2}, -\frac{1}{2}\right)$.

c A(-6, 0, 5) and B(-3, -3, 1)

i $AB = \sqrt{(-3-(-6))^2 + (-3-0)^2 + (1-5)^2}$

$$= \sqrt{3^2 + (-3)^2 + (-4)^2}$$

$$= \sqrt{9 + 9 + 16}$$

$$= \sqrt{34} \text{ units}$$

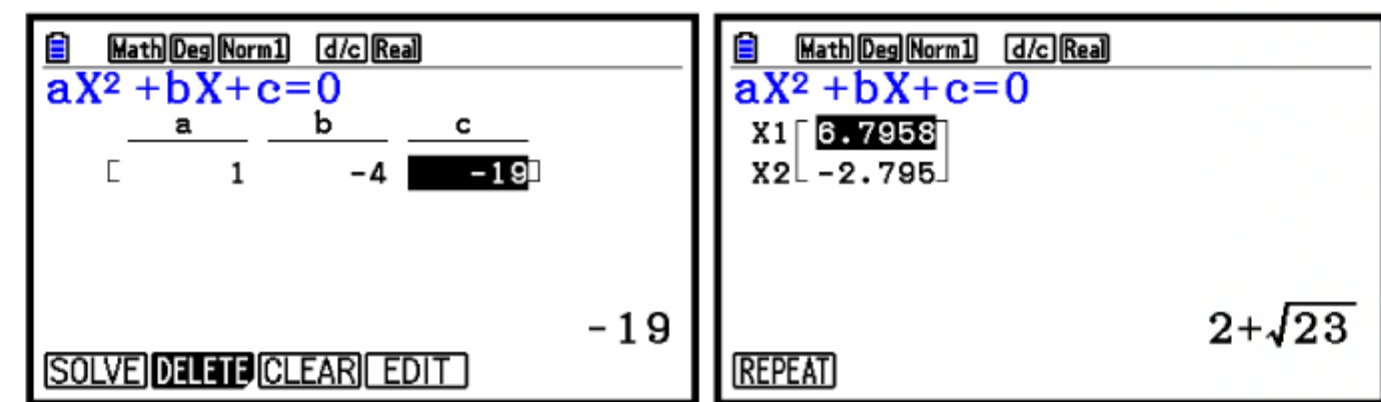
ii The midpoint is $\left(\frac{-6+(-3)}{2}, \frac{0+(-3)}{2}, \frac{5+1}{2}\right)$,
 which is $\left(-\frac{9}{2}, -\frac{3}{2}, 3\right)$.

- 10** $P(k, 6, -5)$ and $Q(2, -1, -8)$

$$\begin{aligned} PQ &= \sqrt{(2-k)^2 + (-1-6)^2 + (-8-(-5))^2} \\ &= \sqrt{(2-k)^2 + (-7)^2 + (-3)^2} \\ &= \sqrt{4-4k+k^2+49+9} \\ &= \sqrt{k^2-4k+62} \end{aligned}$$

Now $PQ = 9$ units

$$\begin{aligned} \therefore \sqrt{k^2-4k+62} &= 9 \\ \therefore k^2-4k+62 &= 81 \\ \therefore k^2-4k-19 &= 0 \\ \therefore k &\approx -2.80 \text{ or } 6.80 \quad \{\text{technology}\} \end{aligned}$$



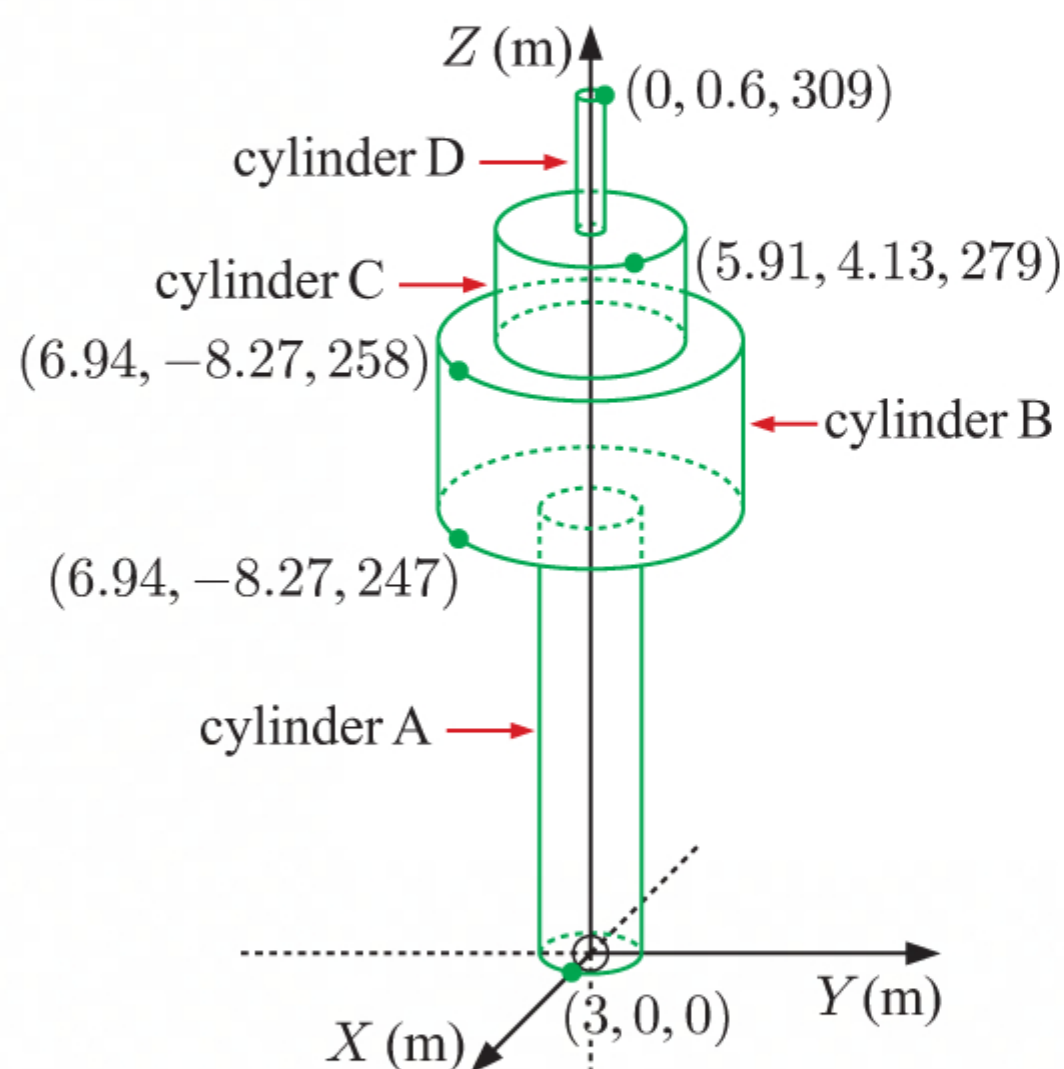
- 11** A is $(3, 4, -6)$ and M is $(-\frac{1}{2}, 9, -7)$.

Let B have coordinates (x, y, z) .

$$\begin{aligned} \therefore \left(\frac{x+3}{2}, \frac{y+4}{2}, \frac{z-6}{2} \right) &= \left(-\frac{1}{2}, 9, -7 \right) \\ \therefore \frac{x+3}{2} &= -\frac{1}{2}, \quad \frac{y+4}{2} = 9, \quad \text{and} \quad \frac{z-6}{2} = -7 \\ \therefore x+3 &= -1, \quad y+4 = 18, \quad \text{and} \quad z-6 = -14 \\ \therefore x &= -4, \quad y = 14, \quad \text{and} \quad z = -8 \end{aligned}$$

So, B has coordinates $(-4, 14, -8)$.

12



Height of cylinder A = 247 m

Radius of cylinder A = 3 m

Height of cylinder B = $258 - 247 = 11$ m

Radius of cylinder B = $\sqrt{6.94^2 + (-8.27)^2}$
 $= \sqrt{116.5565}$ m

Height of cylinder C = $279 - 258 = 21$ m

Radius of cylinder C = $\sqrt{5.91^2 + 4.13^2}$
 $= \sqrt{51.985}$ m

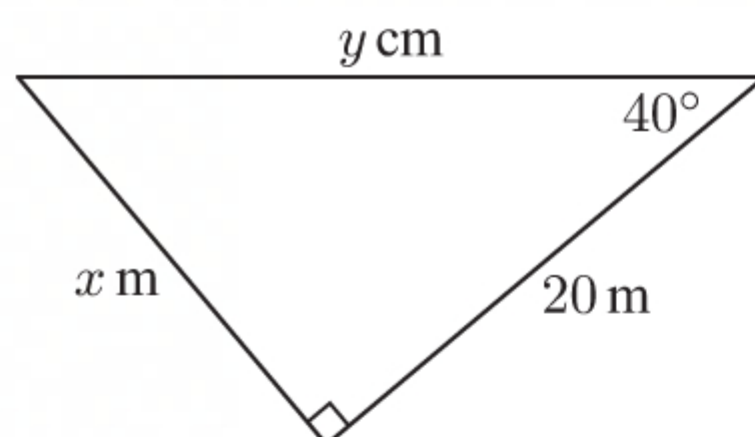
Height of cylinder D = $309 - 279 = 30$ m

Radius of cylinder D = 0.6 m

Volume of tower = volume of cylinder A + volume of cylinder B + volume of cylinder C + volume of cylinder D

$$\begin{aligned} &= \pi(3)^2 \times 247 + \pi(\sqrt{116.5565})^2 \times 11 + \pi(\sqrt{51.985})^2 \times 21 + \pi(0.6)^2 \times 30 \\ &= 2223\pi + 1282.1215\pi + 1091.685\pi + 10.8\pi \\ &= 4607.6065\pi \\ &\approx 14\,475.22 \\ &\approx 14\,500 \text{ m}^3 \end{aligned}$$

13 a



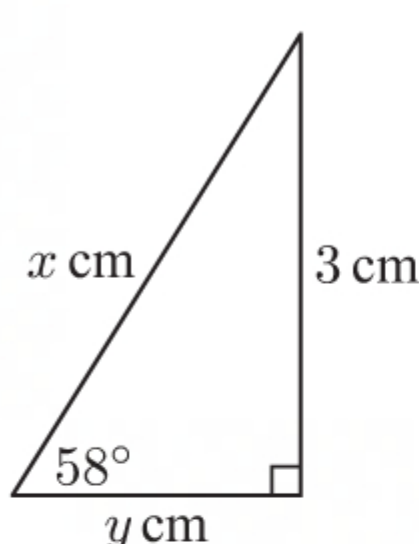
$$\tan 40^\circ = \frac{x}{20}$$

$$\therefore x = 20 \tan 40^\circ \approx 16.8$$

$$\cos 40^\circ = \frac{20}{y}$$

$$\therefore y = \frac{20}{\cos 40^\circ} \approx 26.1$$

b



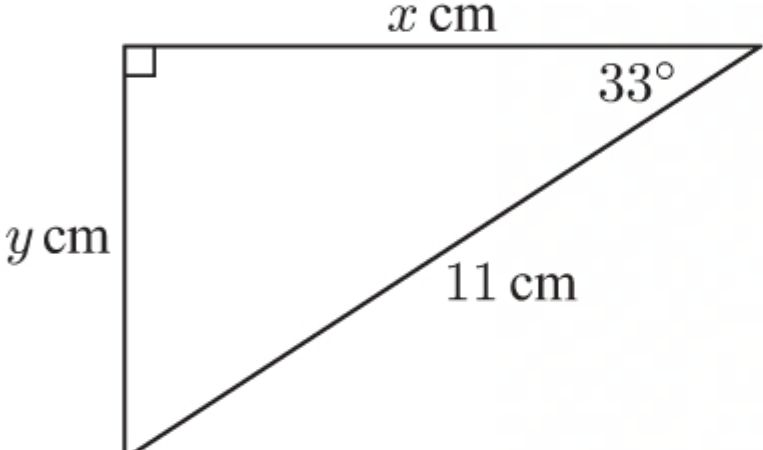
$$\sin 58^\circ = \frac{3}{x}$$

$$\therefore x = \frac{3}{\sin 58^\circ} \approx 3.54$$

$$\tan 58^\circ = \frac{3}{y}$$

$$\therefore y = \frac{3}{\tan 58^\circ} \approx 1.87$$

c

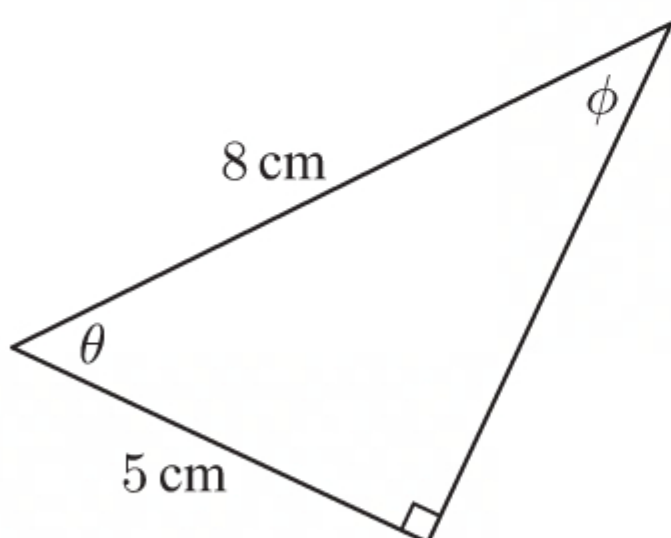


$$\cos 33^\circ = \frac{x}{11} \quad \sin 33^\circ = \frac{y}{11}$$

$$\therefore x = 11 \cos 33^\circ \quad \therefore y = 11 \sin 33^\circ$$

$$\approx 9.23 \quad \approx 5.99$$

14 a

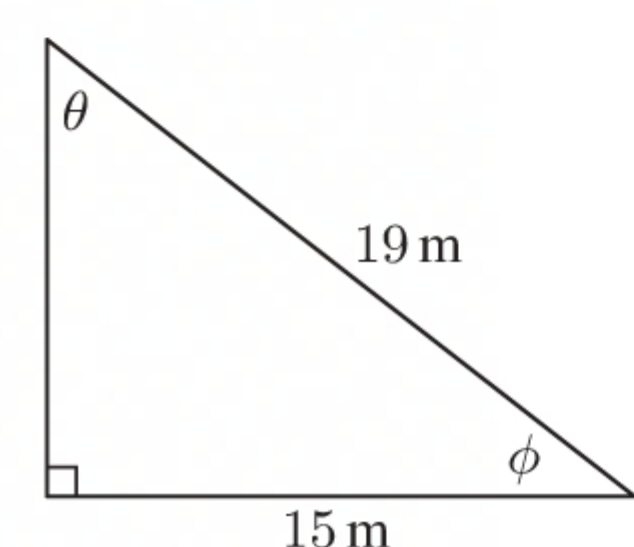


$$\cos \theta = \frac{5}{11} \quad \sin \phi = \frac{8}{11}$$

$$\therefore \theta = \cos^{-1}\left(\frac{5}{11}\right) \quad \therefore \phi = \sin^{-1}\left(\frac{8}{11}\right)$$

$$\approx 51.3^\circ \quad \approx 38.7^\circ$$

b

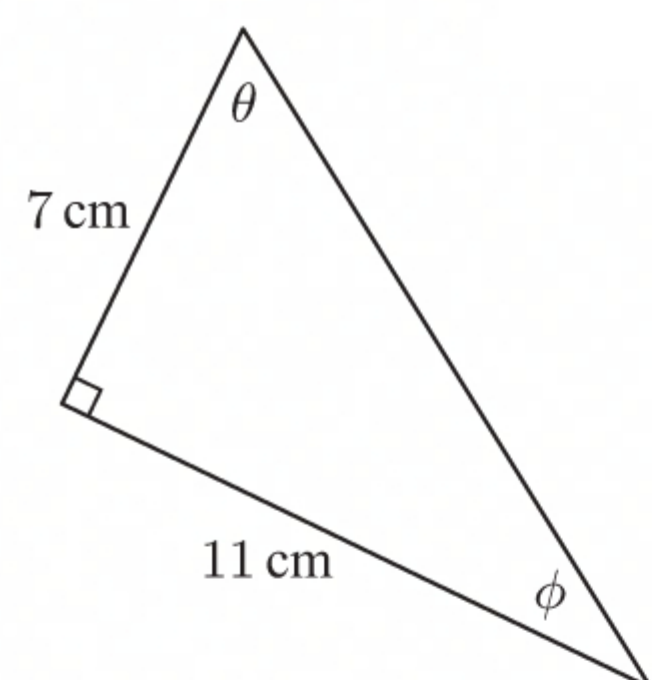


$$\sin \theta = \frac{15}{19} \quad \cos \phi = \frac{15}{19}$$

$$\therefore \theta = \sin^{-1}\left(\frac{15}{19}\right) \quad \therefore \phi = \cos^{-1}\left(\frac{15}{19}\right)$$

$$\approx 52.1^\circ \quad \approx 37.9^\circ$$

c

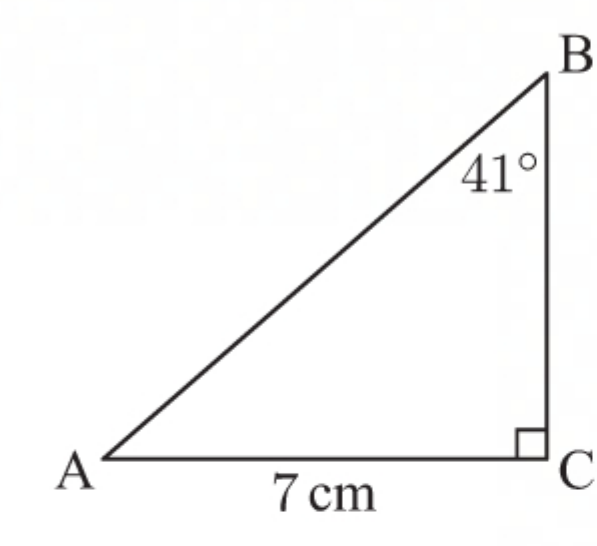


$$\tan \theta = \frac{7}{11} \quad \tan \phi = \frac{7}{11}$$

$$\therefore \theta = \tan^{-1}\left(\frac{7}{11}\right) \quad \therefore \phi = \tan^{-1}\left(\frac{7}{11}\right)$$

$$\approx 32.5^\circ \quad \approx 32.5^\circ$$

15 a



$$\sin 41^\circ = \frac{7}{AB}$$

$$\therefore AB = \frac{7}{\sin 41^\circ}$$

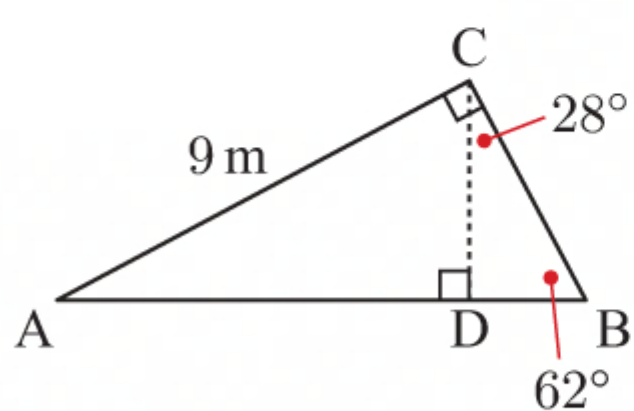
and $\tan 41^\circ = \frac{7}{BC}$

$$\therefore BC = \frac{7}{\tan 41^\circ}$$

$$\begin{aligned} \text{Perimeter} &= AB + BC + AC \\ &= \frac{7}{\sin 41^\circ} + \frac{7}{\tan 41^\circ} + 7 \\ &\approx 25.7 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times AC \times BC \\ &= \frac{1}{2} \times 7 \times \frac{7}{\tan 41^\circ} \\ &\approx 28.2 \text{ cm}^2 \end{aligned}$$

b



$$\widehat{ABC} = 90^\circ - 28^\circ = 62^\circ \quad \{\text{angles in triangle BCD}\}$$

Now in triangle ABC:

$$\tan 62^\circ = \frac{9}{BC}$$

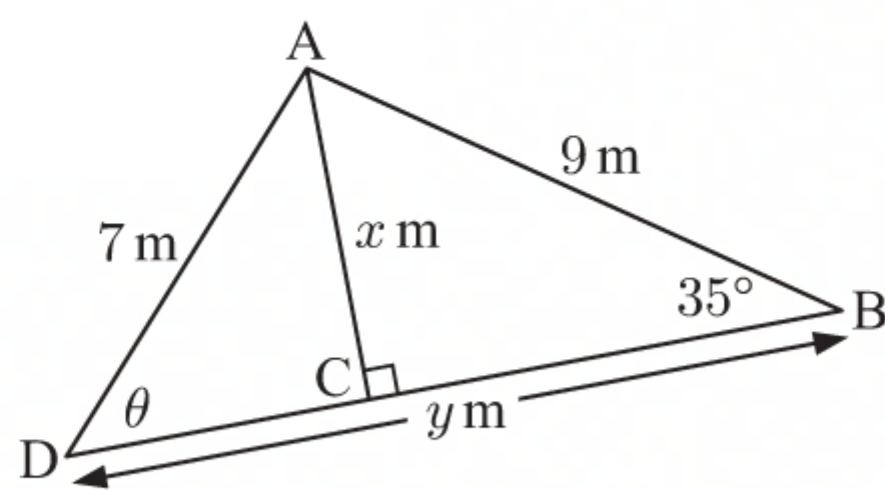
$$\therefore BC = \frac{9}{\tan 62^\circ}$$

and $\sin 62^\circ = \frac{9}{AB}$

$$\therefore AB = \frac{9}{\sin 62^\circ}$$

$$\begin{aligned} \text{Perimeter} &= AB + BC + AC \\ &= \frac{9}{\sin 62^\circ} + \frac{9}{\tan 62^\circ} + 9 \\ &\approx 24.0 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times AC \times BC \\ &= \frac{1}{2} \times 9 \times \frac{9}{\tan 62^\circ} \\ &\approx 21.5 \text{ m}^2 \end{aligned}$$

16 a

$$\text{In triangle ABC, } \sin 35^\circ = \frac{x}{9}$$

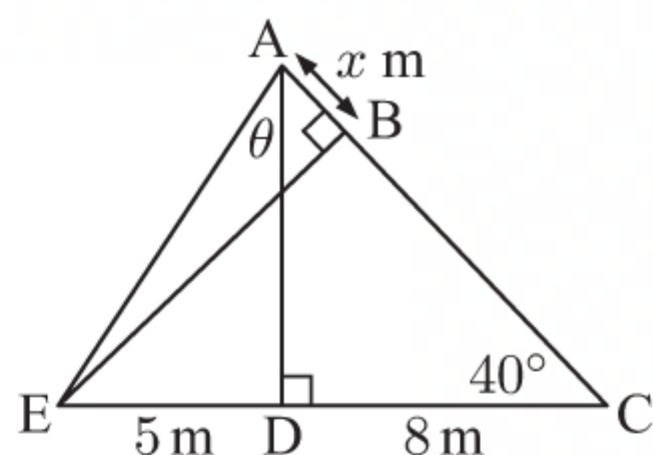
$$\therefore x = 9 \sin 35^\circ \approx 5.16$$

$$\begin{aligned} \text{In triangle ACD, } \sin \theta &= \frac{x}{7} \\ &= \frac{9 \sin 35^\circ}{7} \end{aligned}$$

$$\therefore \theta = \sin^{-1}\left(\frac{9 \sin 35^\circ}{7}\right) \approx 47.5^\circ$$

$$\begin{aligned} \widehat{DAB} &\approx 180^\circ - 35^\circ - 47.5^\circ \quad \{\text{angles in triangle DAB}\} \\ &\approx 97.5^\circ \end{aligned}$$

$$\text{Using the cosine rule in triangle DAB, } y \approx \sqrt{7^2 + 9^2 - 2(7)(9) \cos 97.5^\circ} \approx 12.1$$

b

$$\text{In triangle ACD, } \tan 40^\circ = \frac{AD}{8}$$

$$\therefore AD = 8 \tan 40^\circ$$

$$\begin{aligned} \text{In triangle ADE, } \tan \theta &= \frac{5}{AD} \\ &= \frac{5}{8 \tan 40^\circ} \end{aligned}$$

$$\therefore \theta = \tan^{-1}\left(\frac{5}{8 \tan 40^\circ}\right)$$

$$\approx 36.68^\circ$$

$$\approx 36.7^\circ$$

$$\text{and } \sin 36.68^\circ \approx \frac{5}{AE}$$

$$\therefore AE \approx \frac{5}{\sin 36.68^\circ}$$

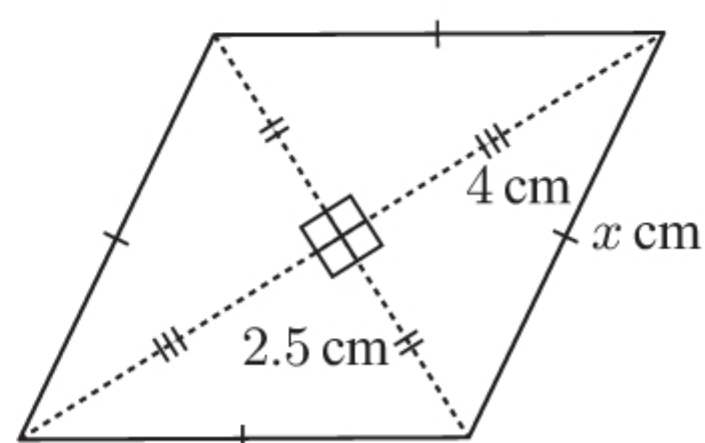
$$\begin{aligned} \text{Now } \widehat{DAC} &= 90^\circ - 40^\circ \quad \{\text{angles in triangle ACD}\} \\ &= 50^\circ \end{aligned}$$

$$\therefore \widehat{EAB} \approx 50^\circ + 36.68^\circ \approx 86.68^\circ$$

$$\text{In triangle ABE, } \cos \widehat{EAB} = \frac{x}{AE}$$

$$\therefore \cos 86.68^\circ \approx \frac{x}{\left(\frac{5}{\sin 36.68^\circ}\right)}$$

$$\therefore x \approx \frac{5 \cos 86.68^\circ}{\sin 36.68^\circ} \approx 0.485$$

17 a**b** Let x cm be the side length of the rhombus.

$$\therefore x^2 = 4^2 + (2.5)^2 \quad \{\text{Pythagoras}\}$$

$$\therefore x^2 = 22.25$$

$$\therefore x = \sqrt{22.25} \quad \{x > 0\}$$

$$\therefore x \approx 4.72$$

\therefore the length of the rhombus' sides are approximately 4.72 cm.

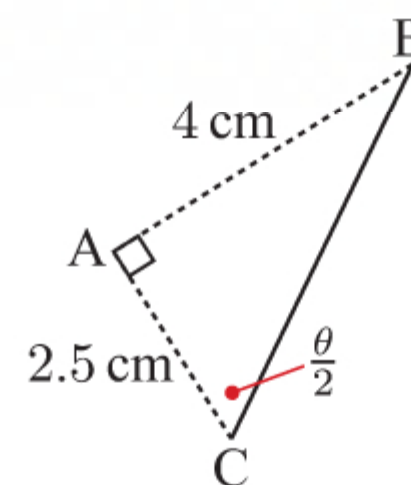
c Let θ be the larger angle in the rhombus.

$$\therefore \widehat{ACB} = \frac{\theta}{2} \quad \{\text{diagonals of a rhombus bisect its angles}\}$$

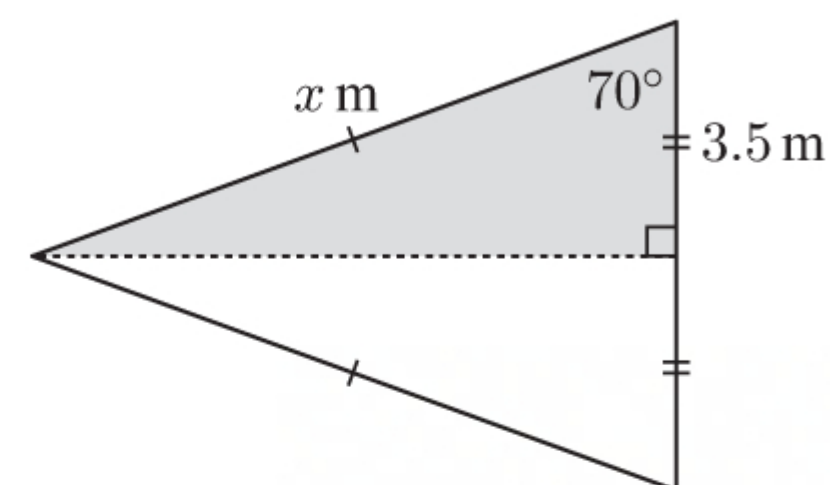
$$\therefore \tan \frac{\theta}{2} = \frac{4}{2.5}$$

$$\therefore \frac{\theta}{2} = \tan^{-1}\left(\frac{4}{2.5}\right)$$

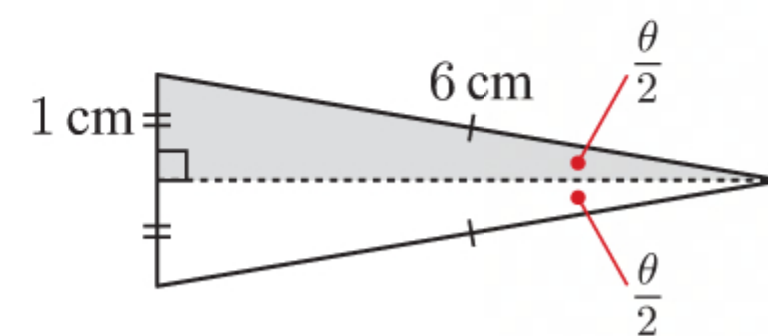
$$\therefore \theta = 2 \tan^{-1}\left(\frac{4}{2.5}\right) \approx 116^\circ$$

**18 a** In the shaded right angled triangle, $\cos 70^\circ = \frac{3.5}{x}$

$$\therefore x = \frac{3.5}{\cos 70^\circ} \approx 10.2$$

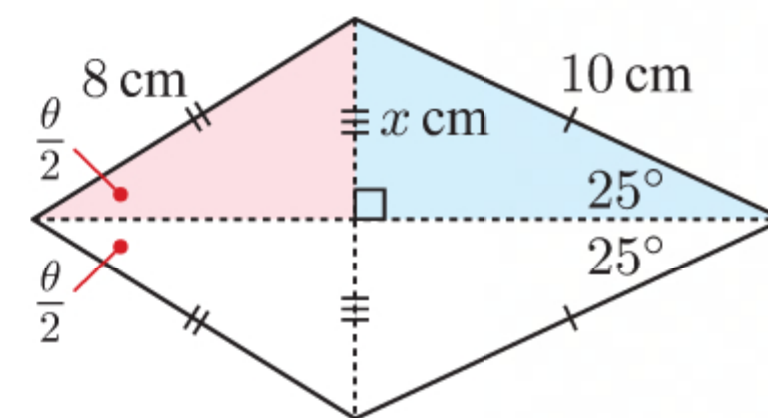


- b** In the shaded right angled triangle, $\sin \frac{\theta}{2} = \frac{1}{6}$
 $\therefore \frac{\theta}{2} = \sin^{-1}\left(\frac{1}{6}\right)$
 $\therefore \theta = 2 \sin^{-1}\left(\frac{1}{6}\right) \approx 19.2^\circ$

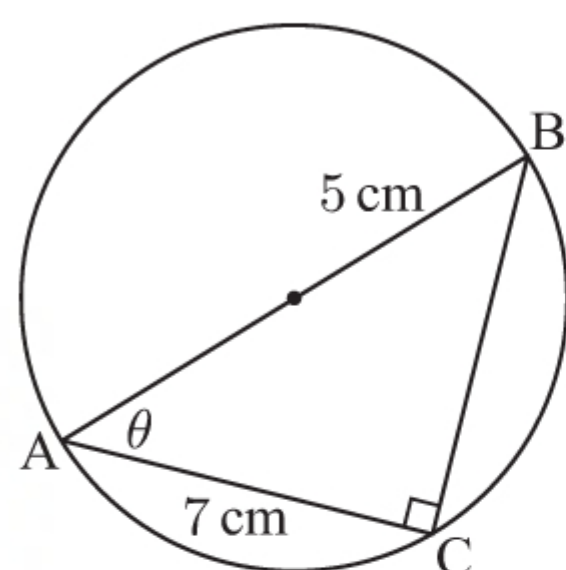


- c** In the blue right angled triangle, $\sin 25^\circ = \frac{x}{10}$
 $\therefore x = 10 \sin 25^\circ$

- In the pink right angled triangle, $\sin \frac{\theta}{2} = \frac{x}{8}$
 $\therefore \sin \frac{\theta}{2} = \frac{10 \sin 25^\circ}{8}$
 $\therefore \frac{\theta}{2} = \sin^{-1}\left(\frac{10 \sin 25^\circ}{8}\right)$
 $\therefore \theta = 2 \sin^{-1}\left(\frac{10 \sin 25^\circ}{8}\right) \approx 63.8^\circ$

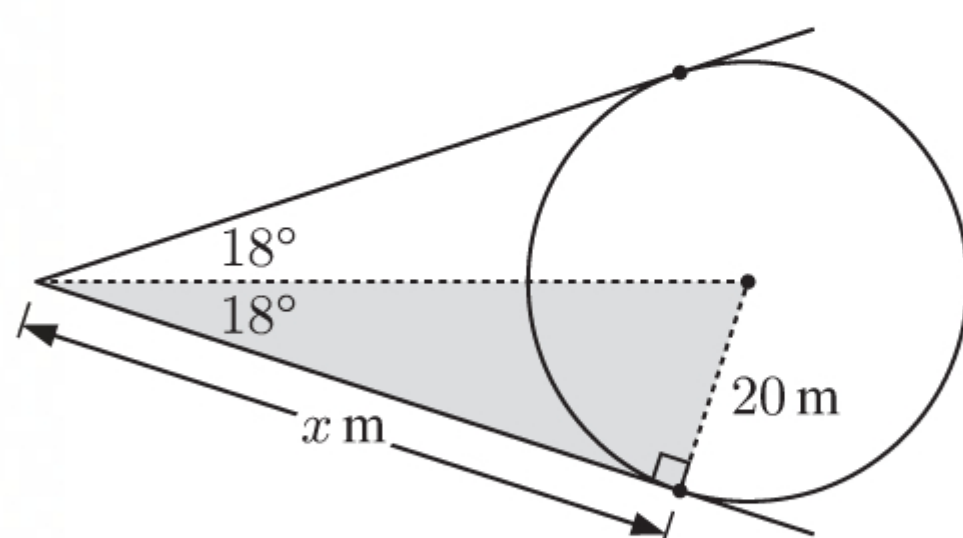


19 a



- $\widehat{ACB} = 90^\circ$ {angle in a semi-circle}
 $\therefore \triangle ABC$ is right angled at C.
 $\therefore \cos \theta = \frac{7}{AB}$
 $\therefore \cos \theta = \frac{7}{10}$
 $\therefore \theta = \cos^{-1}\left(\frac{7}{10}\right) \approx 45.6^\circ$

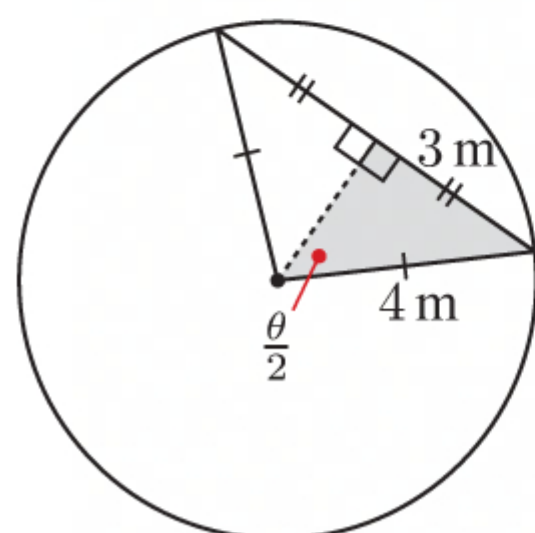
b



We construct the right angled triangle as shown.

- For the shaded triangle, $\tan 18^\circ = \frac{20}{x}$
 $\therefore x = \frac{20}{\tan 18^\circ}$
 $\therefore x \approx 61.6$

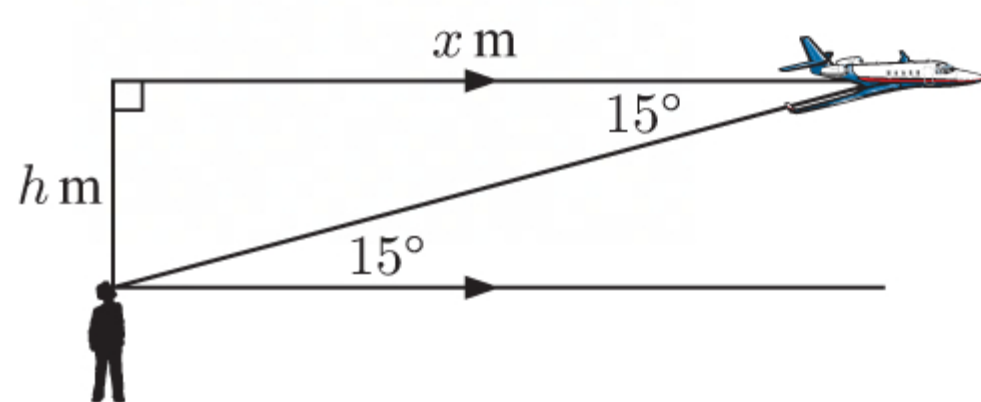
c



We construct the altitude as shown.

- For the shaded triangle, $\sin \frac{\theta}{2} = \frac{3}{4}$
 $\therefore \frac{\theta}{2} = \sin^{-1}\left(\frac{3}{4}\right)$
 $\therefore \theta = 2 \sin^{-1}\left(\frac{3}{4}\right)$
 $\therefore \theta \approx 97.2^\circ$

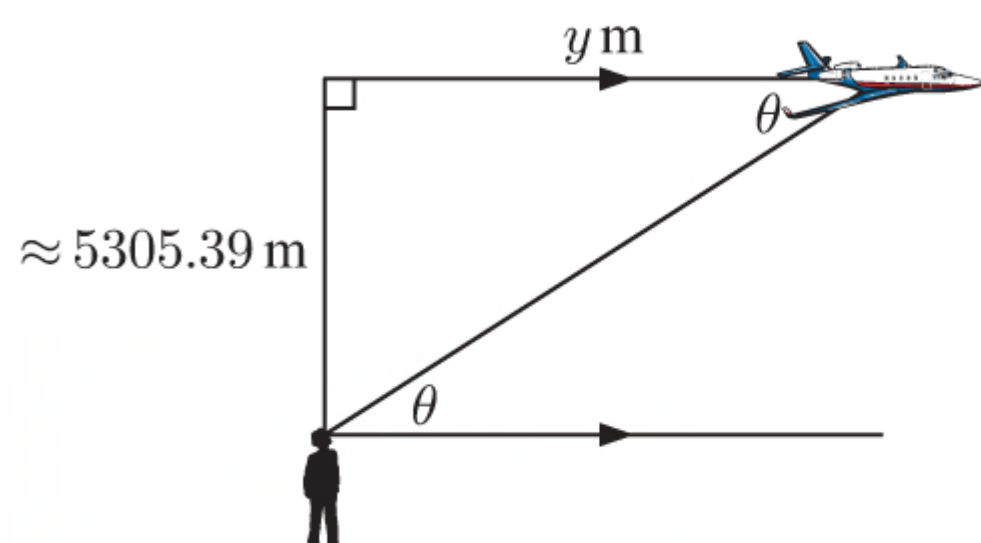
20 a



- $x = \text{speed} \times \text{time}$
 $= 110 \times 180$ {3 minutes = 180 seconds}
 $= 19800$
 $\therefore \tan 15^\circ = \frac{h}{19800}$
 $\therefore h = 19800 \tan 15^\circ$
 $\therefore h \approx 5305.39 \approx 5310$

\therefore the plane is approximately 5310 m \approx 5.31 km above the ground.

b



- $y = \text{speed} \times \text{time}$
 $= 110 \times 420$ {7 minutes = 420 seconds}
 $= 46200$
 $\therefore \tan \theta \approx \frac{5305.39}{46200}$
 $\therefore \theta \approx \tan^{-1}\left(\frac{5305.39}{46200}\right)$
 $\therefore \theta \approx 6.55^\circ$

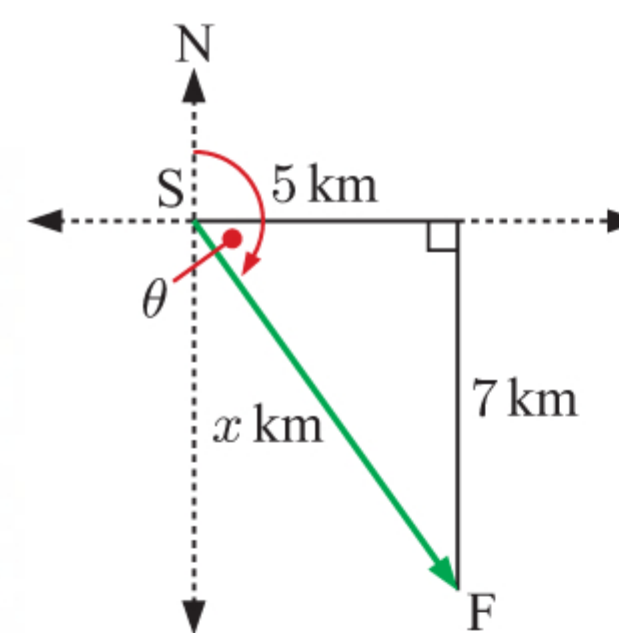
\therefore the angle of elevation of the plane at 2:42 pm is approximately 6.55°.

- 21 a** Suppose the helicopter starts at S and lands at F.

$$\begin{aligned}\text{Now } x^2 &= 7^2 + 5^2 \quad \{\text{Pythagoras}\} \\ &= 74 \\ \therefore x &= \sqrt{74} \quad \{x > 0\} \\ &\approx 8.60\end{aligned}$$

\therefore the helicopter is about 8.60 km from its starting point.

b $\tan \theta = \frac{7}{5}$
 $\therefore \theta = \tan^{-1}\left(\frac{7}{5}\right) \approx 54.5^\circ$
 So, the bearing $\approx 90^\circ + 54.5^\circ$
 $\approx 144.5^\circ$

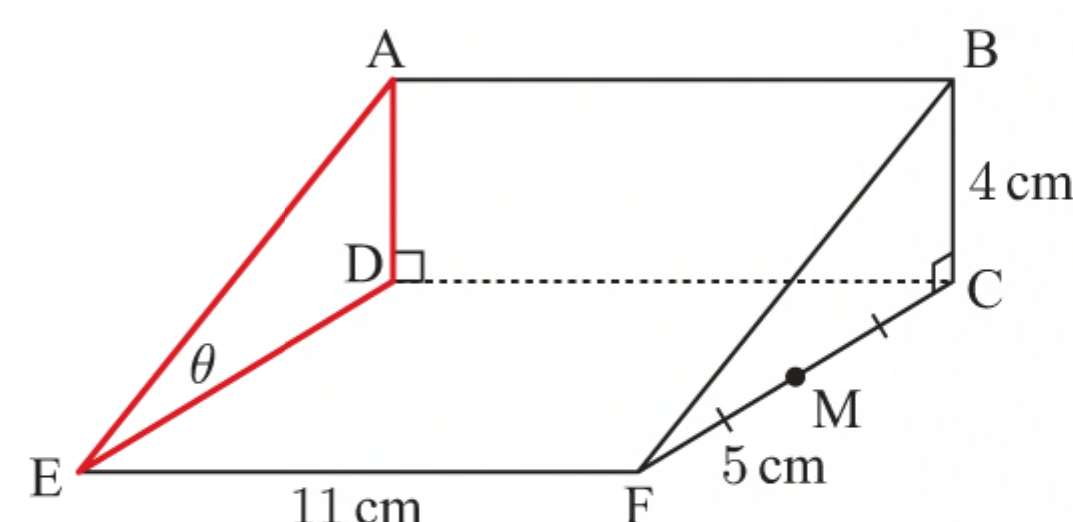


- 22 a** The projection of [AE] onto the base plane is [DE].

\therefore the required angle is \widehat{AED} .

$$\begin{aligned}\tan \theta &= \frac{4}{10} \\ \therefore \theta &= \tan^{-1}\left(\frac{4}{10}\right) \approx 21.8^\circ\end{aligned}$$

The angle is about 21.8° .

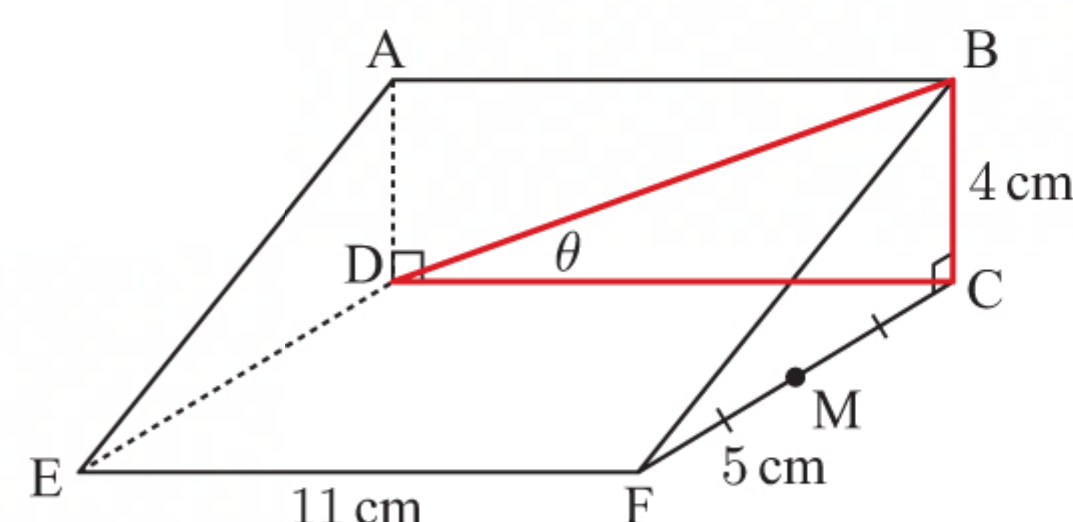


- b** The projection of [BD] onto the base plane is [CD].

\therefore the required angle is \widehat{BDC} .

$$\begin{aligned}\tan \theta &= \frac{4}{11} \\ \therefore \theta &= \tan^{-1}\left(\frac{4}{11}\right) \approx 20.0^\circ\end{aligned}$$

The angle is about 20.0° .



- c** The projection of [BE] onto the base plane is [CE].

\therefore the required angle is \widehat{BEC} .

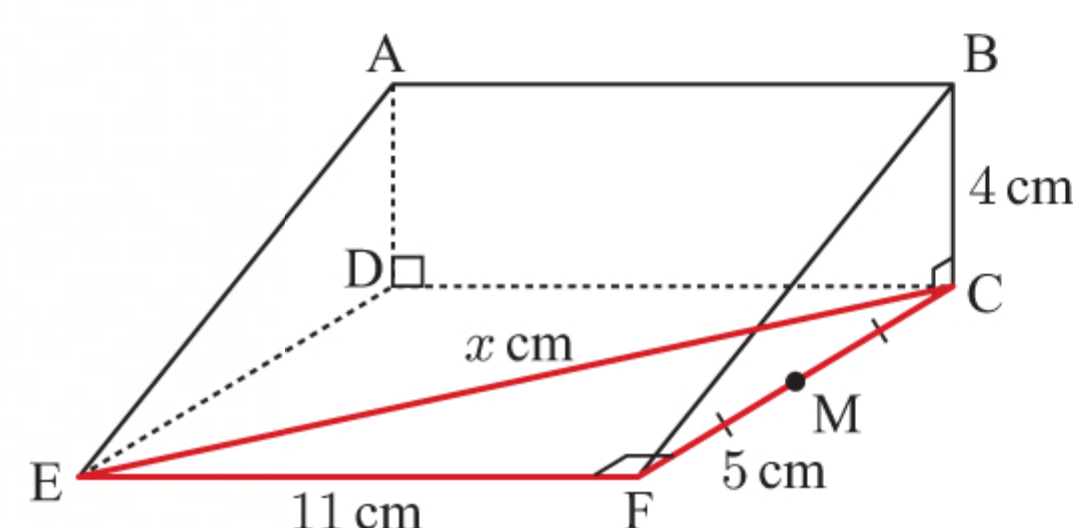
Let CE be x cm.

$$\begin{aligned}\text{Using Pythagoras in } \triangle ECF, \quad x^2 &= 10^2 + 11^2 \\ \therefore x^2 &= 221 \\ \therefore x &= \sqrt{221} \quad \{x > 0\}\end{aligned}$$

Let \widehat{BEC} be α .

$$\begin{aligned}\therefore \tan \alpha &= \frac{4}{\sqrt{221}} \\ \therefore \alpha &= \tan^{-1}\left(\frac{4}{\sqrt{221}}\right) \approx 15.1^\circ\end{aligned}$$

The angle is about 15.1° .



- d** The projection of [AM] onto the base plane is [DM].

\therefore the required angle is \widehat{AMD} .

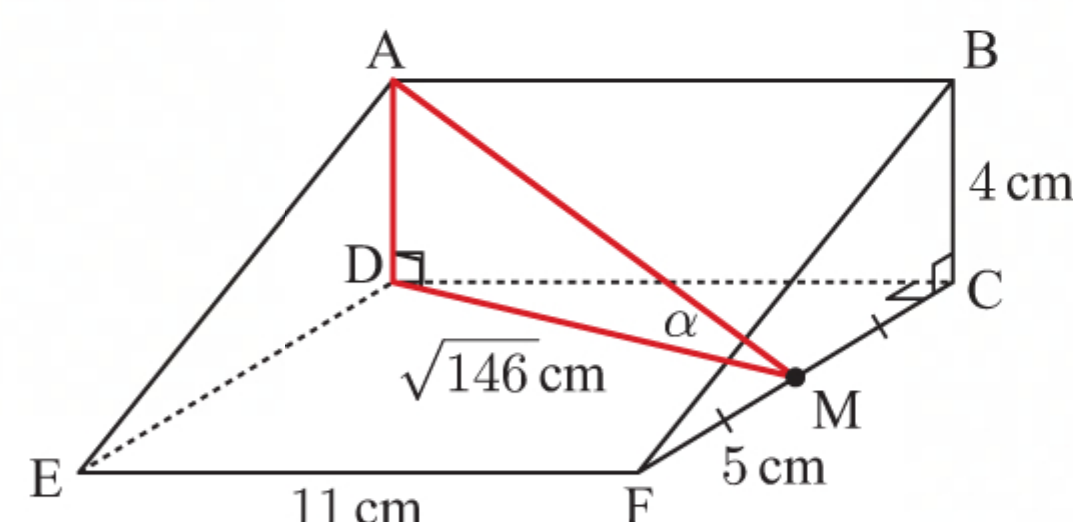
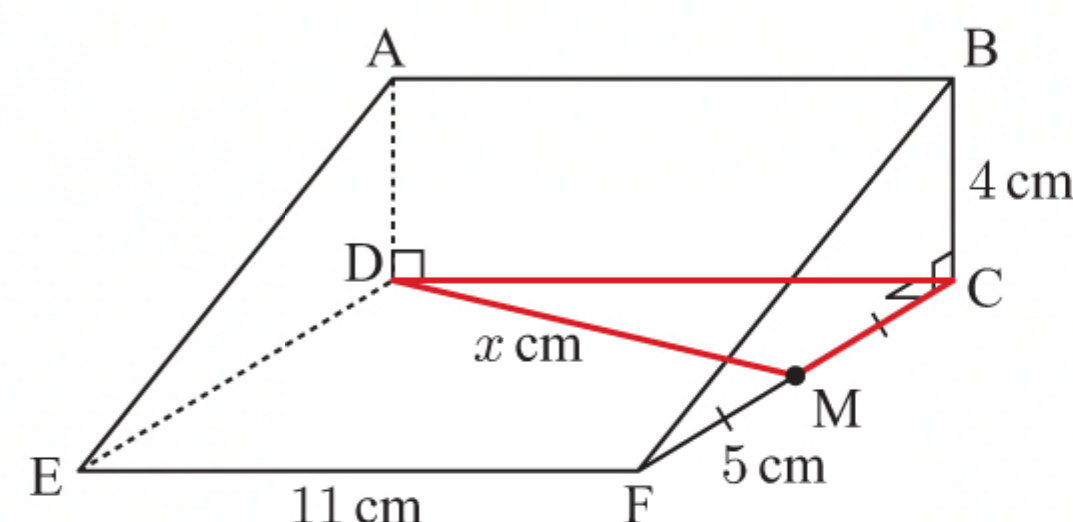
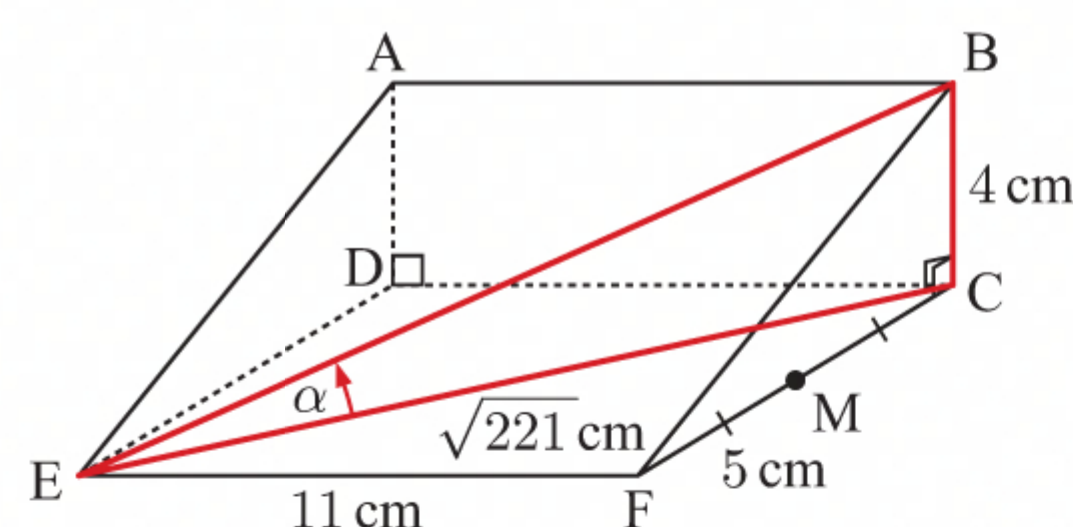
Let DM be x cm.

$$\begin{aligned}\text{Using Pythagoras in } \triangle DCM, \quad x^2 &= 11^2 + 5^2 \\ \therefore x^2 &= 146 \\ \therefore x &= \sqrt{146} \quad \{x > 0\}\end{aligned}$$

Let \widehat{AMD} be α .

$$\begin{aligned}\therefore \tan \alpha &= \frac{4}{\sqrt{146}} \\ \therefore \alpha &= \tan^{-1}\left(\frac{4}{\sqrt{146}}\right) \approx 18.3^\circ\end{aligned}$$

The angle is about 18.3° .



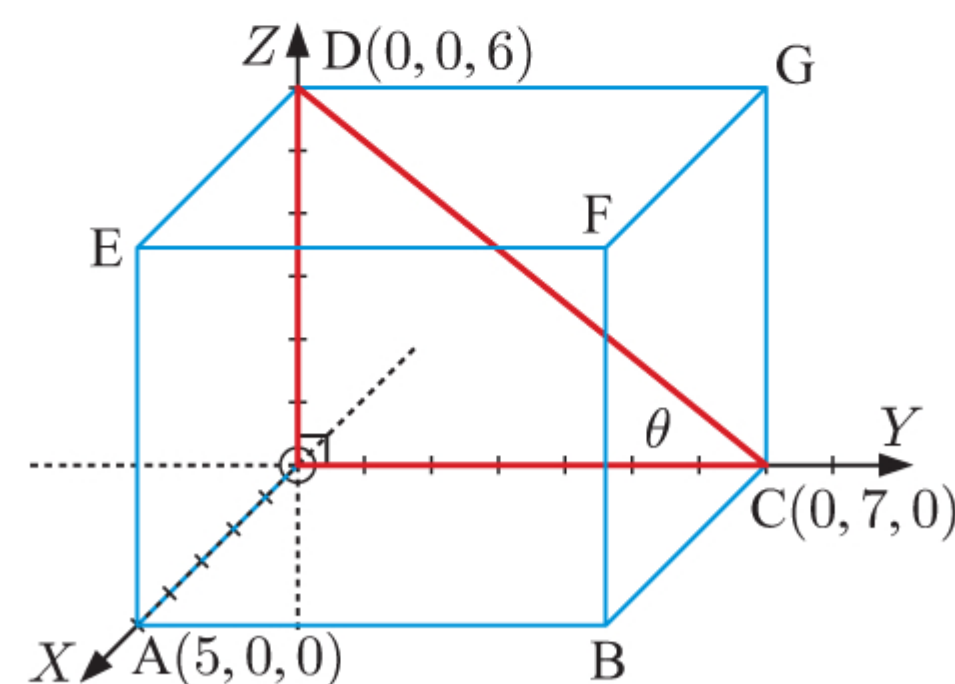
- 23 a** The projection of $[CD]$ onto the base plane is $[CO]$.

\therefore the required angle is \widehat{OCD} .

$$\tan \theta = \frac{6}{7}$$

$$\therefore \theta = \tan^{-1}\left(\frac{6}{7}\right) \approx 40.6^\circ$$

The angle is about 40.6° .



- b** The projection of $[OF]$ onto the base plane is $[OB]$.

\therefore the required angle is \widehat{BOF} .

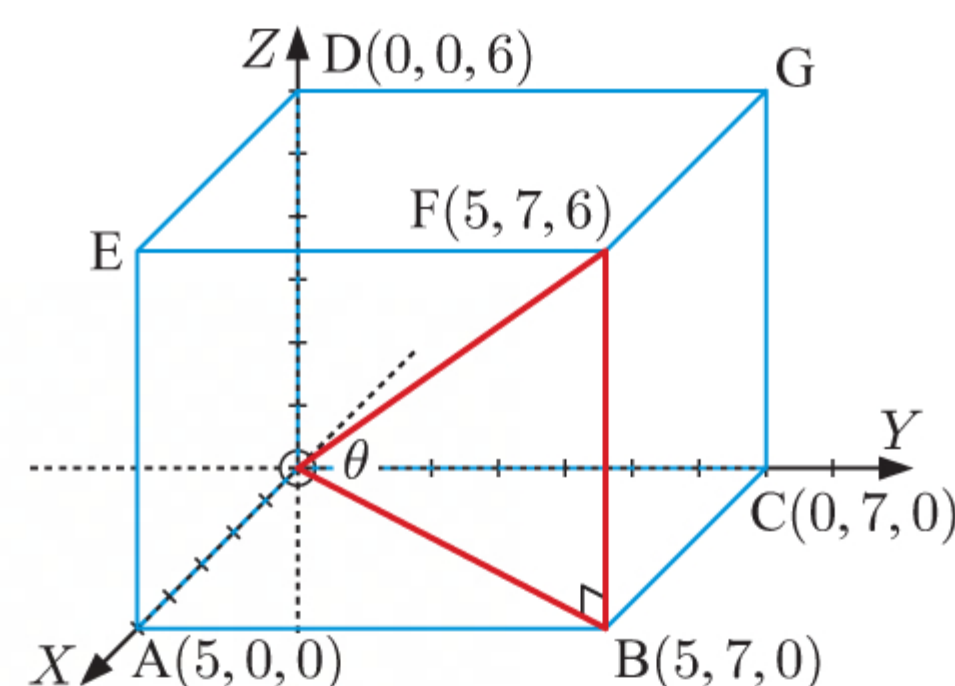
Now $FB = 6$ units

$$\begin{aligned} \text{and } OB &= \sqrt{(5-0)^2 + (7-0)^2 + (0-0)^2} \\ &= \sqrt{25 + 49} \\ &= \sqrt{74} \text{ units} \end{aligned}$$

$$\therefore \tan \theta = \frac{6}{\sqrt{74}}$$

$$\therefore \theta = \tan^{-1}\left(\frac{6}{\sqrt{74}}\right) \approx 34.9^\circ$$

The angle is about 34.9° .



- c** The projection of $[AG]$ onto the base plane is $[AC]$.

\therefore the required angle is \widehat{GAC} .

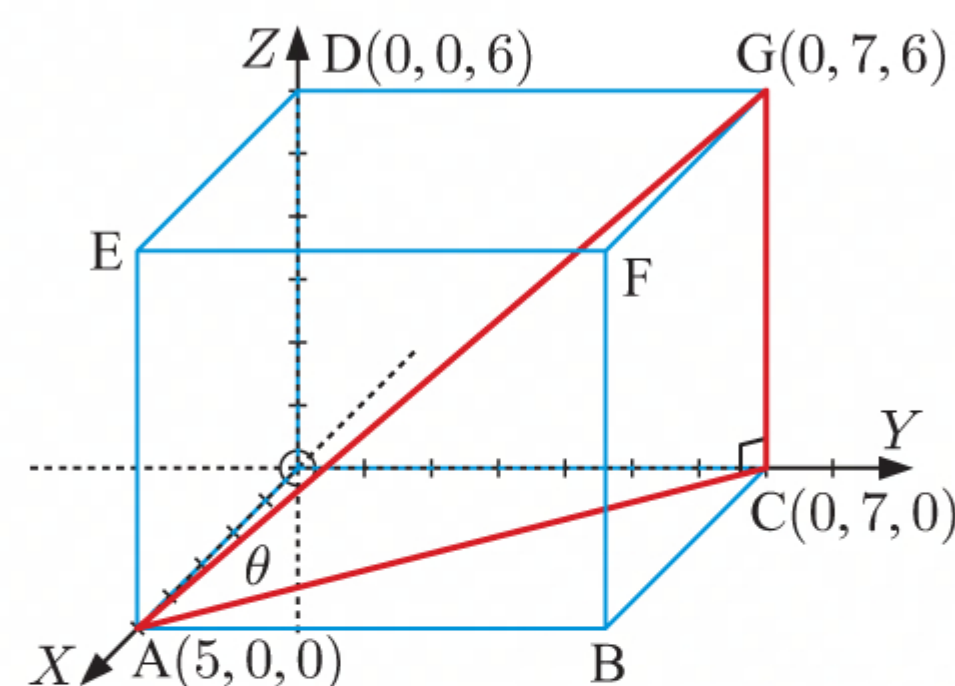
Now $GC = 6$ units

$$\begin{aligned} \text{and } AC &= \sqrt{(0-5)^2 + (7-0)^2 + (0-0)^2} \\ &= \sqrt{25 + 49} \\ &= \sqrt{74} \text{ units} \end{aligned}$$

$$\therefore \tan \theta = \frac{6}{\sqrt{74}}$$

$$\therefore \theta = \tan^{-1}\left(\frac{6}{\sqrt{74}}\right) \approx 34.9^\circ$$

The angle is about 34.9° .



- 24 a** M is the midpoint of $[AD]$ which is $\left(\frac{0+(-10)}{2}, \frac{4+4}{2}, \frac{6+6}{2}\right)$ or $(-5, 4, 6)$.

- b** Let \widehat{CMD} be θ .

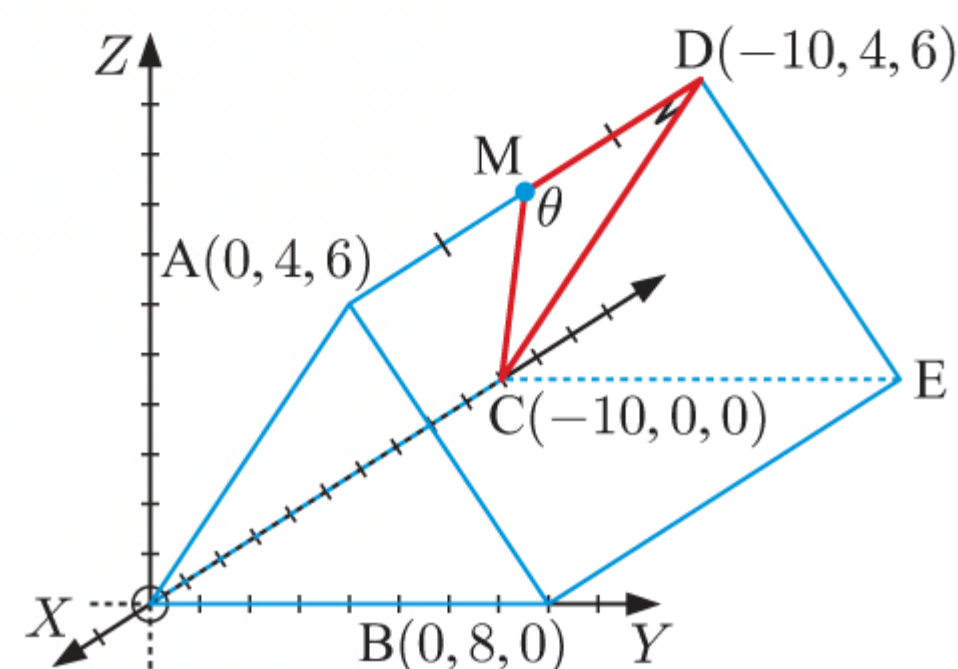
Now $DM = 5$ units

$$\begin{aligned} \text{and } CD &= \sqrt{(-10-(-10))^2 + (4-0)^2 + (6-0)^2} \\ &= \sqrt{0^2 + 4^2 + 6^2} \\ &= \sqrt{52} \text{ units} \end{aligned}$$

$$\therefore \tan \theta = \frac{\sqrt{52}}{5}$$

$$\therefore \theta = \tan^{-1}\left(\frac{\sqrt{52}}{5}\right) \approx 55.3^\circ$$

$$\therefore \widehat{CMD} \approx 55.3^\circ$$



- c i** The required angle is \widehat{DON} , where N has coordinates $(-10, 4, 0)$.

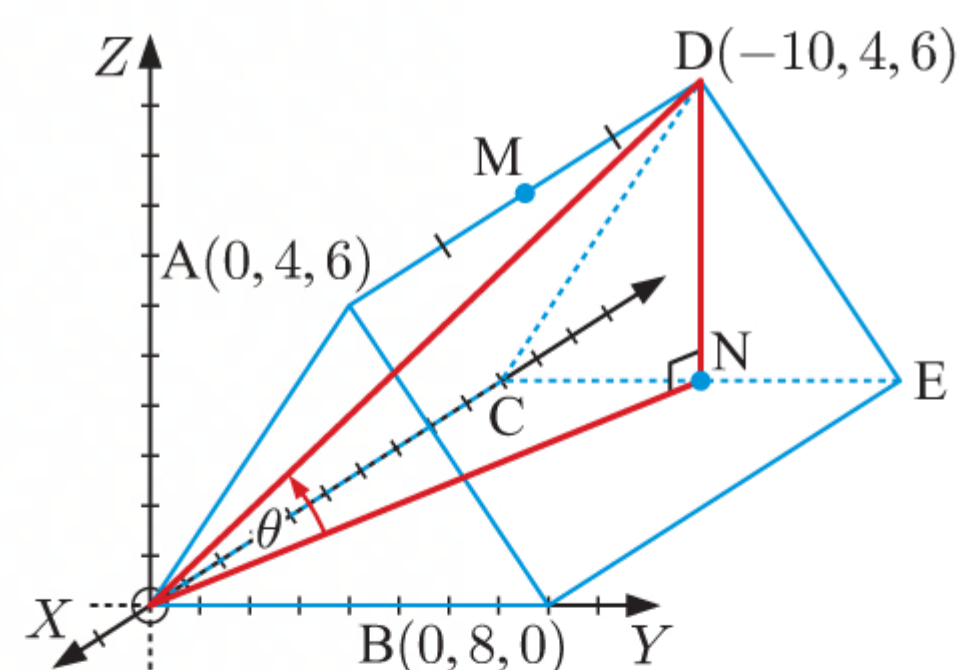
Now $DN = 6$ units

$$\begin{aligned} \text{and } NO &= \sqrt{(-10-0)^2 + (4-0)^2 + (0-0)^2} \\ &= \sqrt{100 + 16 + 0} \\ &= \sqrt{116} \text{ units} \end{aligned}$$

$$\therefore \tan \theta = \frac{6}{\sqrt{116}}$$

$$\therefore \theta = \tan^{-1}\left(\frac{6}{\sqrt{116}}\right) \approx 29.1^\circ$$

The angle is about 29.1° .



- ii The required angle is \widehat{MEP} , where P has coordinates $(-5, 4, 0)$.

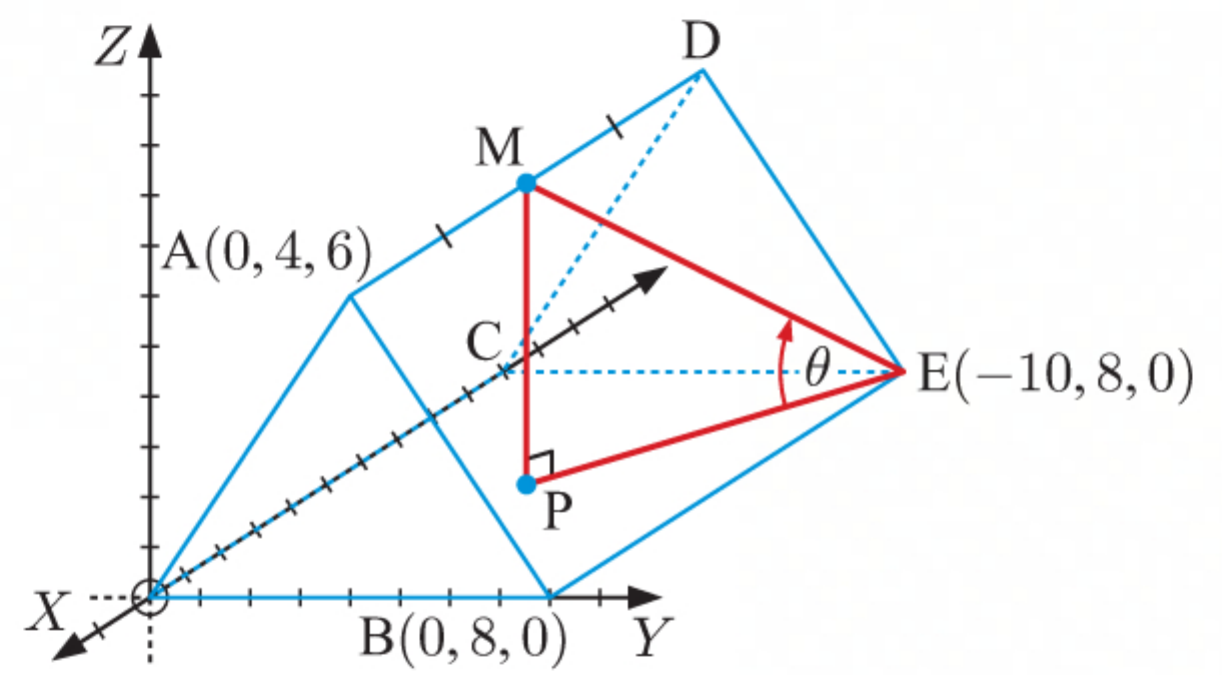
Now $MP = 6$ units

$$\begin{aligned}\text{and } PE &= \sqrt{(-10 - (-5))^2 + (8 - 4)^2 + (0 - 0)^2} \\ &= \sqrt{(-5)^2 + 4^2 + 0^2} \\ &= \sqrt{41} \text{ units}\end{aligned}$$

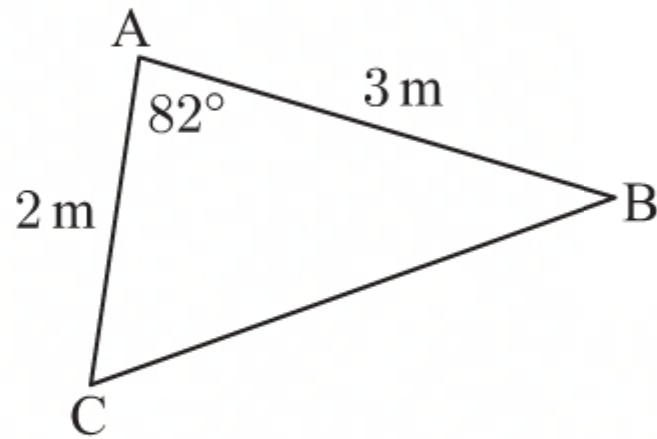
$$\therefore \tan \theta = \frac{6}{\sqrt{41}}$$

$$\therefore \theta = \tan^{-1}\left(\frac{6}{\sqrt{41}}\right) \approx 43.1^\circ$$

The angle is about 43.1° .

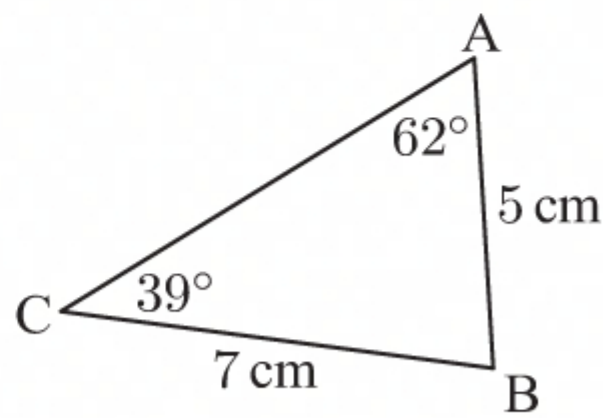


25 a



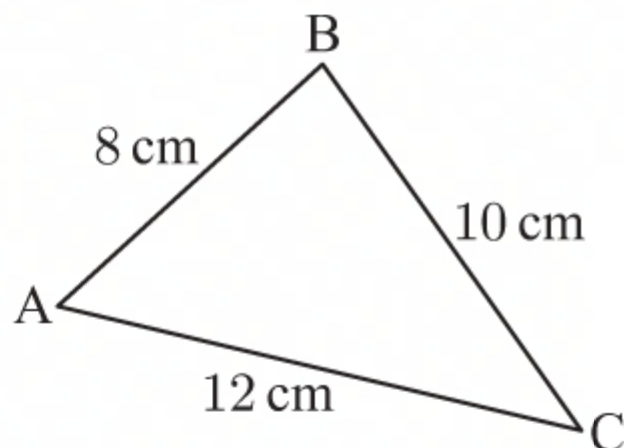
$$\begin{aligned}\text{Area} &= \frac{1}{2}bc \sin A \\ &= \frac{1}{2} \times 2 \times 3 \times \sin 82^\circ \\ &\approx 2.97 \text{ m}^2\end{aligned}$$

b



$$\begin{aligned}\widehat{ABC} &= 180^\circ - 62^\circ - 39^\circ \quad \{\text{angles in a triangle}\} \\ &= 79^\circ \\ \therefore \text{area} &= \frac{1}{2}ac \sin B \\ &= \frac{1}{2} \times 7 \times 5 \times \sin 79^\circ \\ &\approx 17.2 \text{ cm}^2\end{aligned}$$

26 a



- b The smallest angle in triangle ABC is opposite the shortest side.

$\therefore \widehat{BCA}$ is the smallest angle.

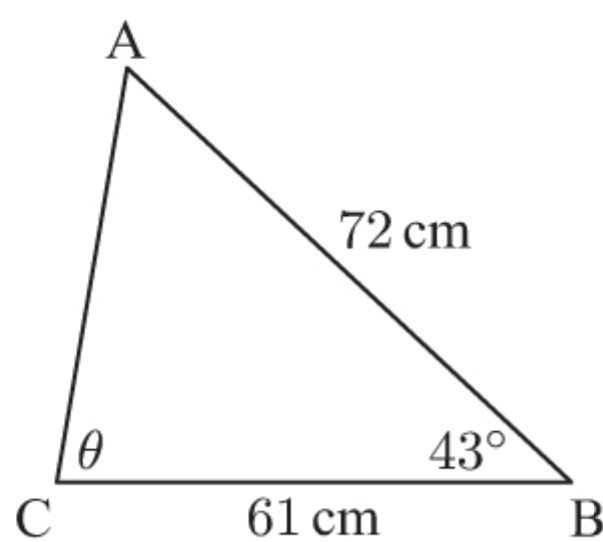
By the cosine rule:

$$\begin{aligned}\cos \widehat{BCA} &= \frac{12^2 + 10^2 - 8^2}{2 \times 12 \times 10} \\ \therefore \cos \widehat{BCA} &= \frac{180}{240} = \frac{3}{4} \\ \therefore \widehat{BCA} &= \cos^{-1}\left(\frac{3}{4}\right) \approx 41.4^\circ\end{aligned}$$

c

$$\begin{aligned}\text{Area} &= \frac{1}{2}ab \sin C \\ &\approx \frac{1}{2} \times 10 \times 12 \times \sin 41.4^\circ \\ &\approx 39.7 \text{ cm}^2\end{aligned}$$

27 a

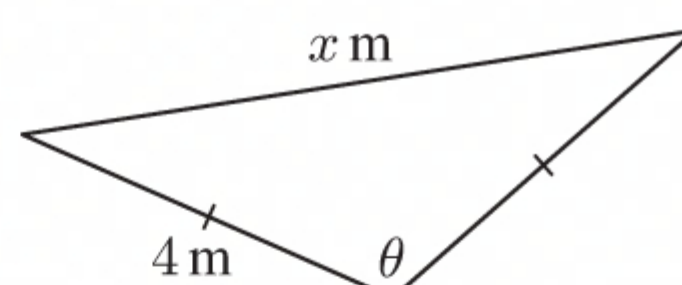


$$\begin{aligned}\text{Using the cosine rule, } AC^2 &= 61^2 + 72^2 - 2 \times 61 \times 72 \times \cos 43^\circ \\ &\approx 2480.8 \\ \therefore AC &\approx 49.808 \\ &\approx 49.8 \text{ cm} \quad \{\text{as } AC > 0\}\end{aligned}$$

- b Let $\widehat{ACB} = \theta$

$$\begin{aligned}\text{Using the sine rule, } \frac{\sin \theta}{72} &= \frac{\sin 43^\circ}{AC} \\ \therefore \sin \theta &\approx \frac{72 \sin 43^\circ}{49.808} \\ &\approx 0.9859 \\ \therefore \theta &\approx 80.4^\circ \\ \text{and so } \widehat{ACB} &\approx 80.4^\circ\end{aligned}$$

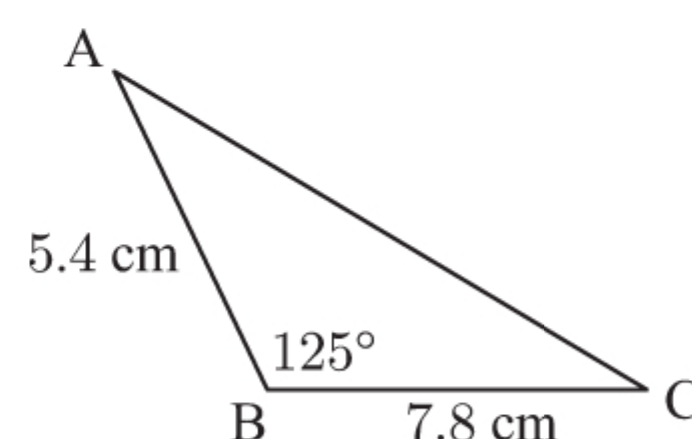
28 a Area = $\frac{1}{2}ab \sin \theta$
 $= \frac{1}{2} \times 4 \times 4 \times \sin \theta$
 $\therefore 4 = 8 \sin \theta$
 $\therefore \sin \theta = \frac{1}{2}$
 $\therefore \theta = 150^\circ$ {since θ is obtuse}



b Using the cosine rule, $x^2 = 4^2 + 4^2 - 2 \times 4 \times 4 \times \cos 150^\circ$
 $\therefore x = \sqrt{4^2 + 4^2 - 2 \times 4 \times 4 \times \cos 150^\circ}$
 $\therefore x \approx 7.73$

29 a Area = $\frac{1}{2} \times 5.4 \times 7.8 \times \sin 125^\circ$
 $\approx 17.3 \text{ cm}^2$

b Using the cosine rule,
 $AC^2 = 5.4^2 + 7.8^2 - 2 \times 5.4 \times 7.8 \times \cos 125^\circ$
 $\therefore AC = \sqrt{5.4^2 + 7.8^2 - 2 \times 5.4 \times 7.8 \times \cos 125^\circ}$
 $\therefore AC \approx 11.8 \text{ cm}$

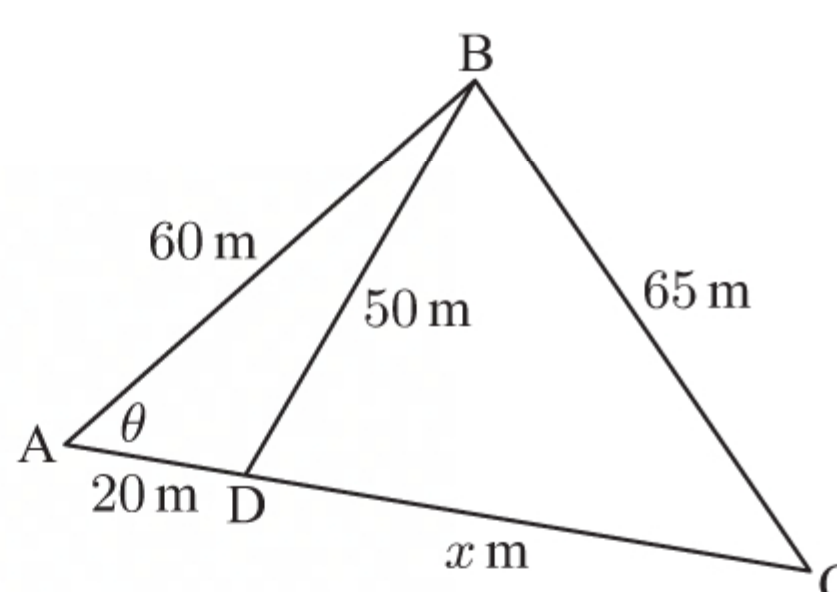


30 a By the cosine rule in $\triangle BAD$:

$$\cos \theta = \frac{60^2 + 20^2 - 50^2}{2 \times 60 \times 20}$$

$$\therefore \cos \theta = \frac{1500}{2400}$$

$$\therefore \cos \theta = \frac{5}{8}$$



b By the cosine rule in $\triangle ABC$:

$$65^2 = 60^2 + (20 + x)^2 - 2(60)(20 + x) \cos \theta$$

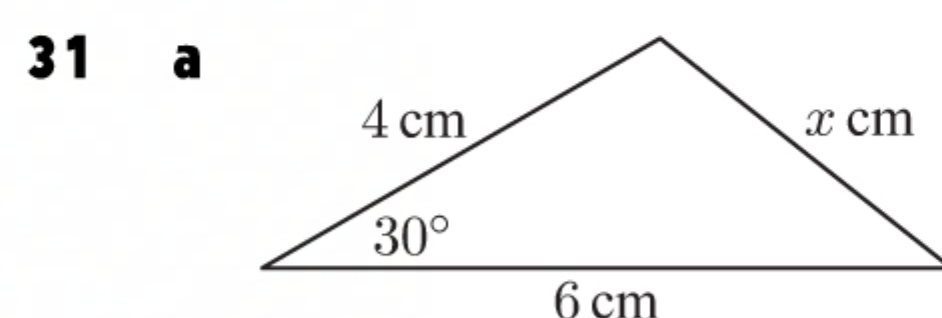
$$\therefore 65^2 = 60^2 + (20 + x)^2 - 2(60)(20 + x) \left(\frac{5}{8}\right) \quad \{\text{using a}\}$$

$$\therefore 4225 = 3600 + 400 + 40x + x^2 - 1500 - 75x$$

$$\therefore x^2 - 35x - 1725 = 0$$

$$\therefore x \approx -27.7 \text{ or } 62.6 \quad \{\text{technology}\}$$

$$\therefore x \approx 62.6 \quad \{x > 0\}$$



By the cosine rule: $x^2 = 4^2 + 6^2 - 2(4)(6) \cos 30^\circ$
 $\therefore x^2 = 16 + 36 - 48 \left(\frac{\sqrt{3}}{2}\right)$
 $\therefore x^2 = 52 - 24\sqrt{3}$
 $\therefore x = \sqrt{52 - 24\sqrt{3}} \quad \{x > 0\}$
 $\therefore x \approx 3.23$

b By the cosine rule:

$$9^2 = x^2 + 7^2 - 2(x)(7) \cos 120^\circ$$

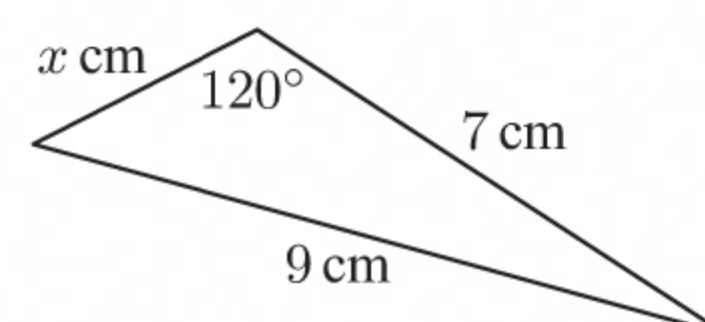
$$\therefore 81 = x^2 + 49 - 14 \left(-\frac{1}{2}\right)x$$

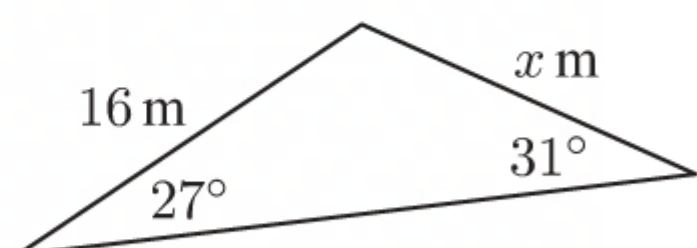
$$\therefore 81 = x^2 + 49 + 7x$$

$$\therefore x^2 + 7x - 32 = 0$$

$$\therefore x \approx -10.2 \text{ or } 3.15 \quad \{\text{technology}\}$$

$$\therefore x \approx 3.15 \quad \{x > 0\}$$



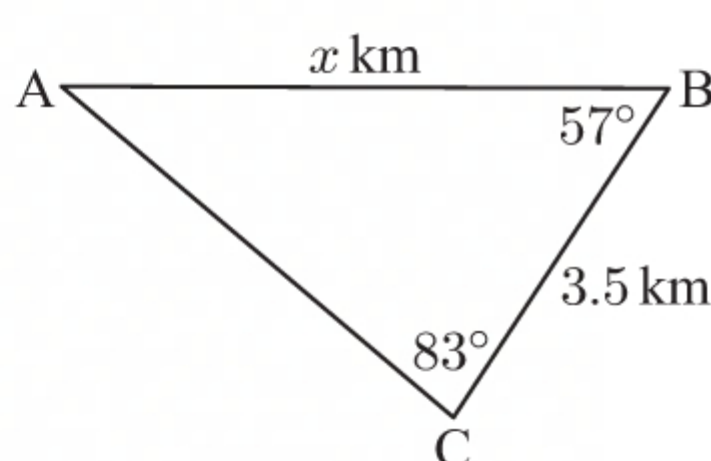
32 a

By the sine rule:

$$\frac{x}{\sin 27^\circ} = \frac{16}{\sin 31^\circ}$$

$$\therefore x = \frac{16 \sin 27^\circ}{\sin 31^\circ}$$

$$\therefore x \approx 14.1$$

b

$$\widehat{BAC} = 180^\circ - 57^\circ - 83^\circ \quad \{\text{angles in a triangle}\}$$

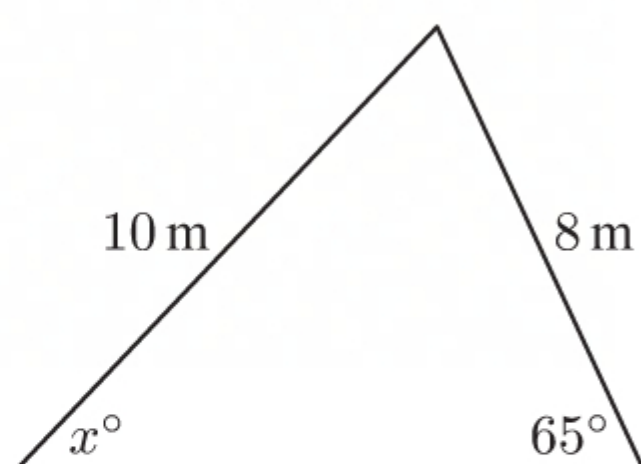
$$= 40^\circ$$

Now by the sine rule:

$$\frac{x}{\sin 83^\circ} = \frac{3.5}{\sin 40^\circ}$$

$$\therefore x = \frac{3.5 \sin 83^\circ}{\sin 40^\circ}$$

$$\therefore x \approx 5.40$$

33 a

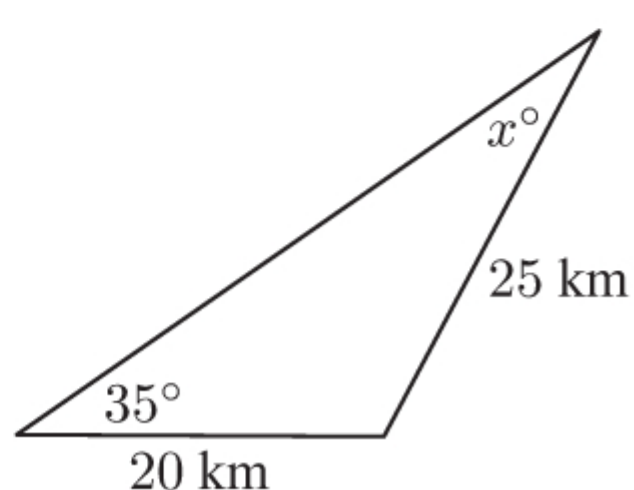
By the sine rule:

$$\frac{\sin x^\circ}{8} = \frac{\sin 65^\circ}{10}$$

$$\therefore \sin x^\circ = \frac{8 \sin 65^\circ}{10}$$

$$\therefore x = \sin^{-1}\left(\frac{8 \sin 65^\circ}{10}\right)$$

$$\therefore x \approx 46.5$$

b

By the sine rule:

$$\frac{\sin x^\circ}{20} = \frac{\sin 35^\circ}{25}$$

$$\therefore \sin x^\circ = \frac{20 \sin 35^\circ}{25}$$

$$\therefore x = \sin^{-1}\left(\frac{20 \sin 35^\circ}{25}\right)$$

$$\therefore x \approx 27.3$$

34 By the cosine rule in $\triangle ABD$:

$$\cos \theta = \frac{8^2 + 4^2 - 5^2}{2 \times 8 \times 4}$$

$$\therefore \cos \theta = \frac{55}{64}$$

$$\therefore \theta = \cos^{-1}\left(\frac{55}{64}\right) \approx 30.8^\circ$$

By the cosine rule in $\triangle DBC$:

$$\cos \phi = \frac{8^2 + 6^2 - 6^2}{2 \times 8 \times 6}$$

$$\therefore \cos \phi = \frac{64}{96} = \frac{2}{3}$$

$$\therefore \phi = \cos^{-1}\left(\frac{2}{3}\right) \approx 48.2^\circ$$

$$\text{Now } \widehat{ABC} = \theta + \phi$$

$$\approx 30.8^\circ + 48.2^\circ$$

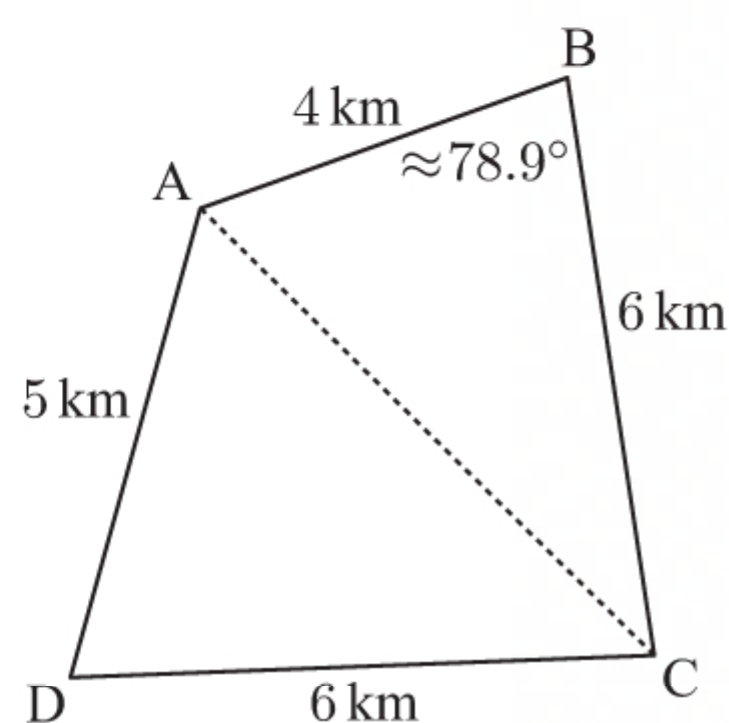
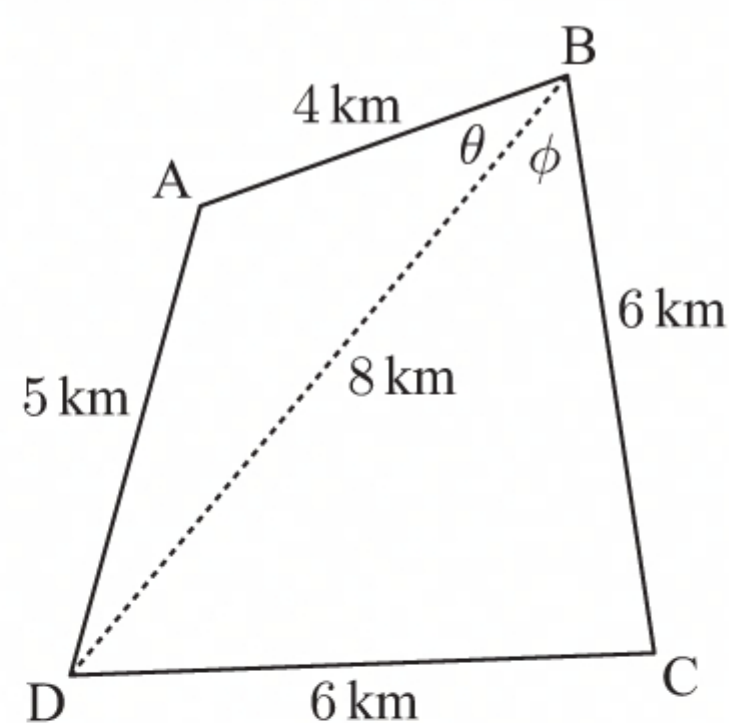
$$\approx 78.9^\circ$$

By the cosine rule in $\triangle ABC$:

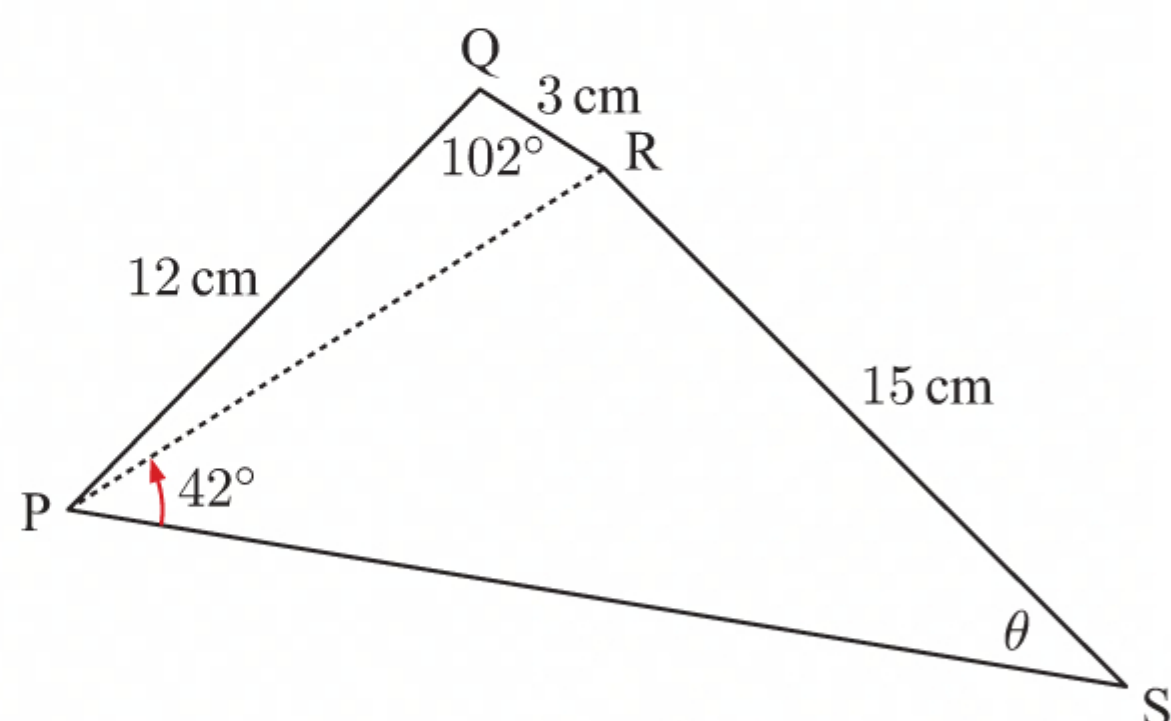
$$AC^2 \approx 4^2 + 6^2 - 2 \times 4 \times 6 \times \cos 78.9^\circ$$

$$\therefore AC \approx \sqrt{4^2 + 6^2 - 2 \times 4 \times 6 \times \cos 78.9^\circ}$$

$$\therefore AC \approx 6.54 \text{ cm}$$



35 a

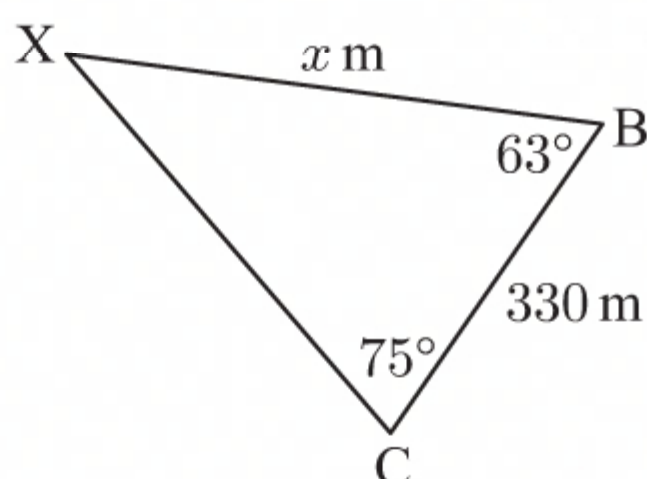

 By the cosine rule in $\triangle PQR$:

$$\begin{aligned} PR^2 &= 3^2 + 12^2 - 2(3)(12) \cos 102^\circ \\ \therefore PR &= \sqrt{3^2 + 12^2 - 2(3)(12) \cos 102^\circ} \\ \therefore PR &\approx 12.96 \text{ cm} \approx 13.0 \text{ cm} \end{aligned}$$

 b By the sine rule in $\triangle PRS$:

$$\begin{aligned} \frac{\sin \theta}{PR} &= \frac{\sin 42^\circ}{15} \\ \therefore \sin \theta &= \frac{PR \sin 42^\circ}{15} \\ \therefore \sin \theta &\approx \frac{12.96 \sin 42^\circ}{15} \quad \{\text{from a}\} \\ \therefore \theta &\approx \sin^{-1} \left(\frac{12.96 \sin 42^\circ}{15} \right) \approx 35.3^\circ \end{aligned}$$

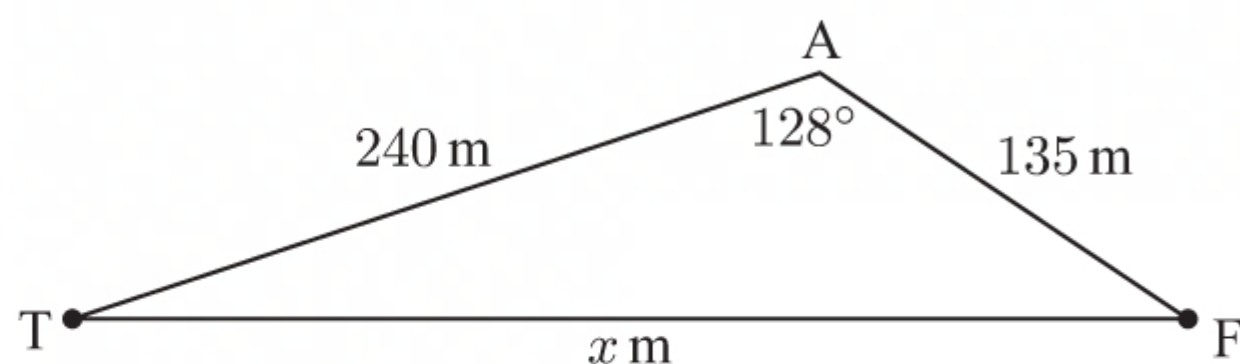
36 a


 b Let XB be x m.

$$\begin{aligned} \widehat{CXB} &= 180^\circ - 63^\circ - 75^\circ \quad \{\text{angles in a triangle}\} \\ &= 42^\circ \\ \therefore \frac{x}{\sin 75^\circ} &= \frac{330}{\sin 42^\circ} \quad \{\text{sine rule}\} \\ \therefore x &= \frac{330 \sin 75^\circ}{\sin 42^\circ} \approx 476 \end{aligned}$$

The distance between the monument and B is about 476 m.

37


 a Let the distance from T to F be x m.

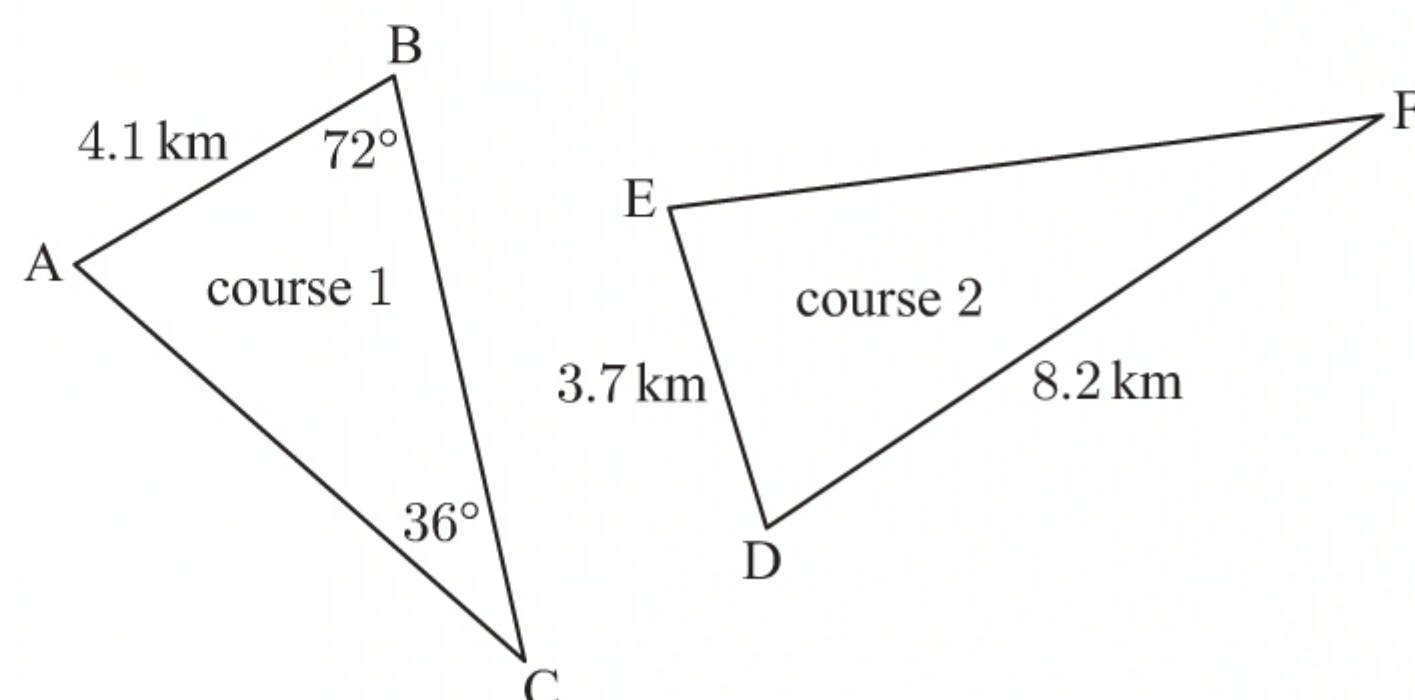
Using the cosine rule,

$$\begin{aligned} x^2 &= 240^2 + 135^2 - 2 \times 240 \times 135 \cos 128^\circ \\ \therefore x &\approx 340.18 \end{aligned}$$

So, the distance is about 340 m.

$$\begin{aligned} \text{b Using the sine rule, } \frac{\sin \widehat{ATF}}{135} &= \frac{\sin 128^\circ}{340.18} \\ \therefore \widehat{ATF} &\approx \sin^{-1} \left(\frac{135 \sin 128^\circ}{340.18} \right) \approx 18.2^\circ \end{aligned}$$

38


 a By the sine rule in $\triangle ABC$:

$$\begin{aligned} \frac{AC}{\sin 72^\circ} &= \frac{4.1}{\sin 36^\circ} \\ \therefore AC &= \frac{4.1 \sin 72^\circ}{\sin 36^\circ} \end{aligned}$$

Now [EF] is 20% longer than [AC].

$$\begin{aligned} \therefore EF &= 1.2 \times AC \\ &= 1.2 \times \frac{4.1 \sin 72^\circ}{\sin 36^\circ} \\ &\approx 7.96 \text{ km} \end{aligned}$$

 b By the cosine rule in $\triangle DEF$:

$$\begin{aligned} \cos \widehat{DEF} &\approx \frac{3.7^2 + 7.96^2 - 8.2^2}{2 \times 3.7 \times 7.96} \\ \therefore \widehat{DEF} &\approx \cos^{-1} \left(\frac{3.7^2 + 7.96^2 - 8.2^2}{2 \times 3.7 \times 7.96} \right) \\ \therefore \widehat{DEF} &\approx 80.4^\circ \end{aligned}$$

c Area of course 2 $\approx \frac{1}{2} \times 3.7 \times 7.96 \times \sin 80.4^\circ$
 $\approx 14.5 \text{ km}^2$

d From **a**, $AC = \frac{4.1 \sin 72^\circ}{\sin 36^\circ} \approx 6.63 \text{ km}$

$$\begin{aligned}\widehat{BAC} &= 180^\circ - 72^\circ - 36^\circ \quad \{\text{angles in a triangle}\} \\ &= 72^\circ \\ &= \widehat{ABC}\end{aligned}$$

$\therefore \triangle ABC$ is an isosceles triangle.

$\therefore BC = AC$

$\therefore \text{total length of course 1} \approx 2 \times 6.63 + 4.1$
 $\approx 17.4 \text{ km}$

39 a The midpoint M of [AB] is $\left(\frac{3+7}{2}, \frac{5+3}{2}\right)$ or (5, 4).

The gradient of [AB] is $\frac{3-5}{7-3} = \frac{-2}{4} = -\frac{1}{2}$

\therefore the gradient of the perpendicular bisector is 2.

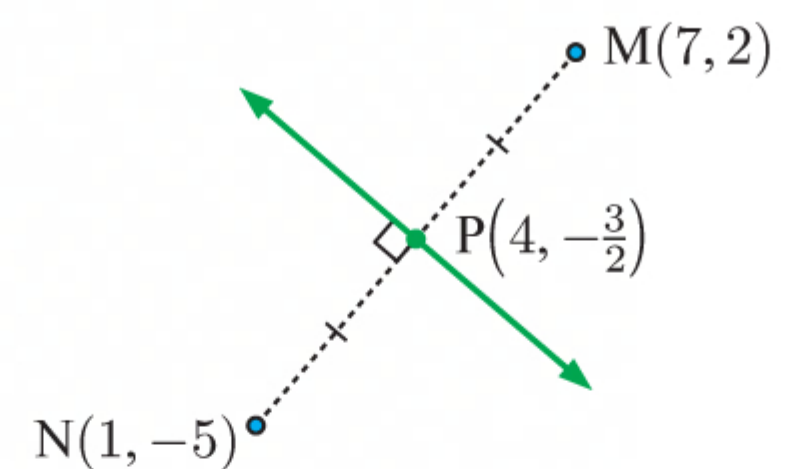
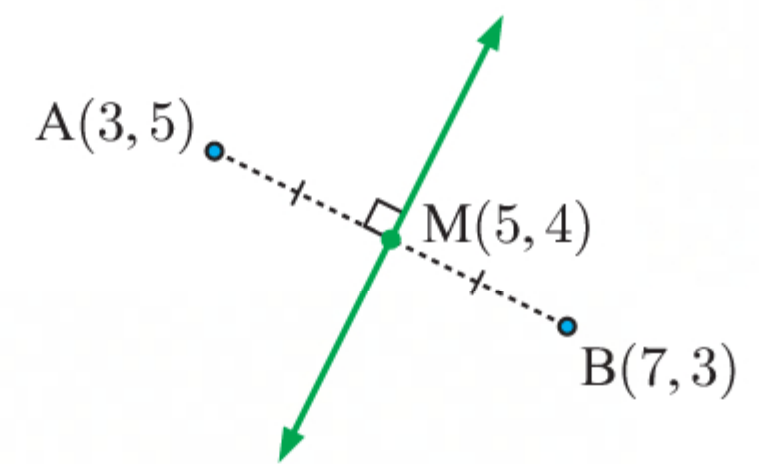
\therefore the equation of the perpendicular bisector is $2x - y = 2(5) - 4$
 which is $2x - y = 6$.

b The midpoint P of [MN] is $\left(\frac{7+1}{2}, \frac{2+(-5)}{2}\right)$ or $\left(4, -\frac{3}{2}\right)$.

The gradient of [MN] is $\frac{-5-2}{1-7} = \frac{-7}{-6} = \frac{7}{6}$.

\therefore the gradient of the perpendicular bisector is $-\frac{6}{7}$.

\therefore the equation of the perpendicular bisector is $6x + 7y = 6(4) + 7(-\frac{3}{2})$
 which is $6x + 7y = \frac{27}{2}$.



40 a i $4x - 3y + 2 = 0$

$\therefore 3y = 4x + 2$

$\therefore y = \frac{4}{3}x + \frac{2}{3}$ has gradient $\frac{4}{3}$

ii The perpendicular bisector has gradient $-\frac{3}{4}$.

b The equation of the perpendicular bisector is $3x + 4y = 3(4) + 4(6)$
 which is $3x + 4y = 36$
 or $3x + 4y - 36 = 0$

41 a i (0, -2) lies in cell H, so the nearest bus stop is H.

ii (3, 2) lies in cell C, so the nearest bus stop is C.

iii (-5, 5) lies in cell A, so the nearest bus stop is A.

iv (-1, -4) lies in cell E, so the nearest bus stop is E.

b i Jerome must live at the vertex adjacent to cells B, C, and G.

\therefore Jerome lives at (2, 3).

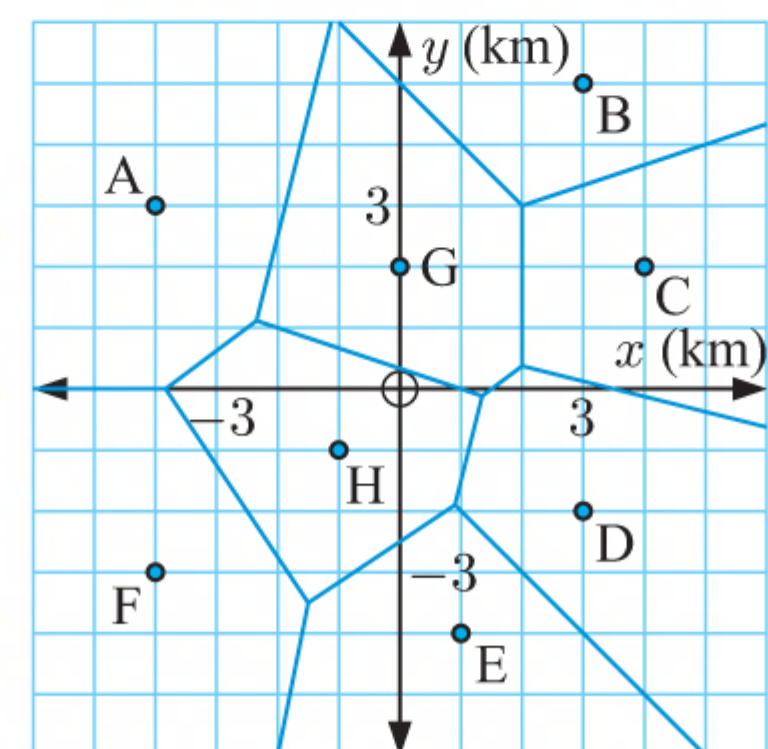
ii Distance Jerome needs to walk = distance between vertex and B

$$= \sqrt{(2-3)^2 + (3-5)^2}$$

$$= \sqrt{(-1)^2 + (-2)^2}$$

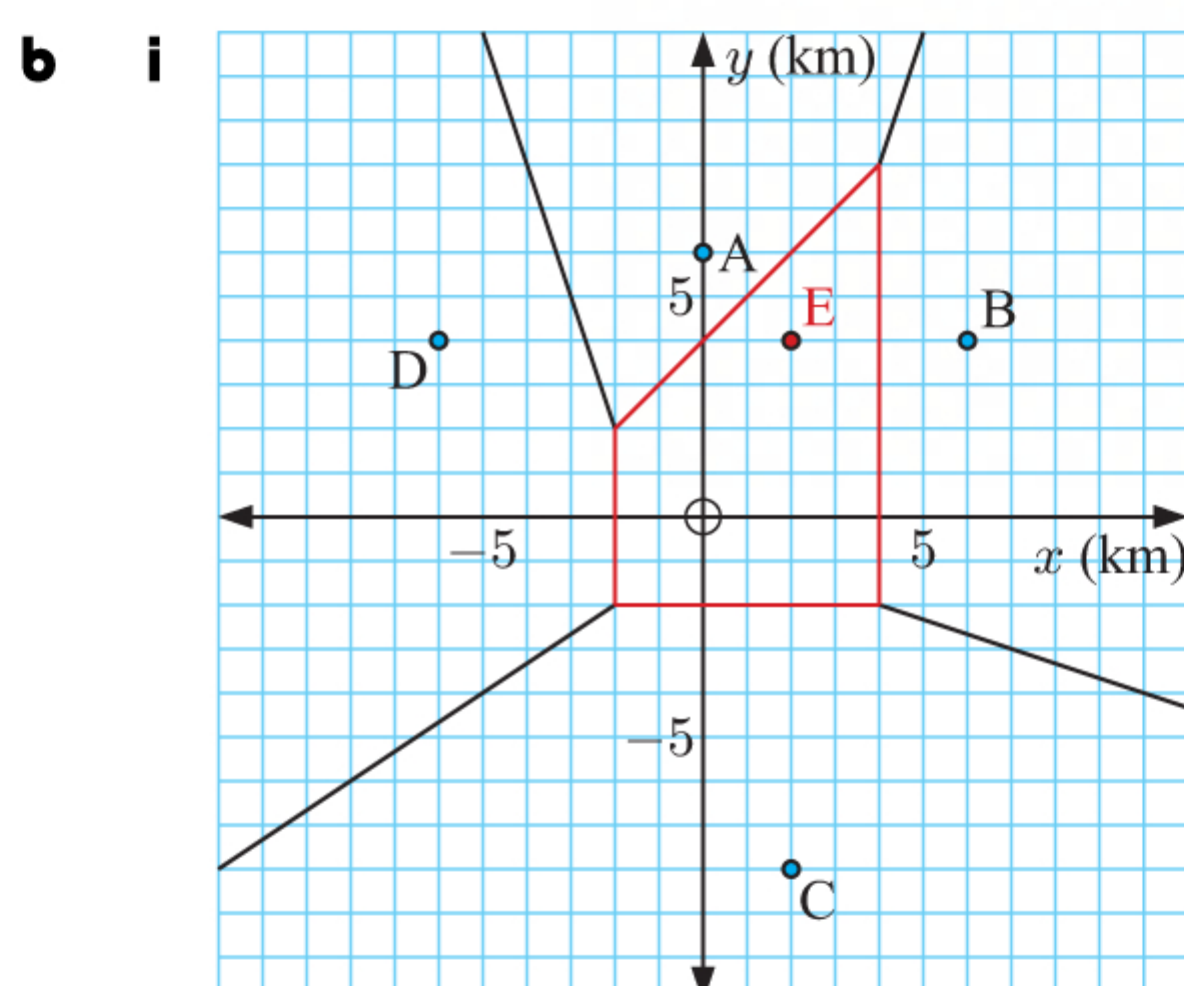
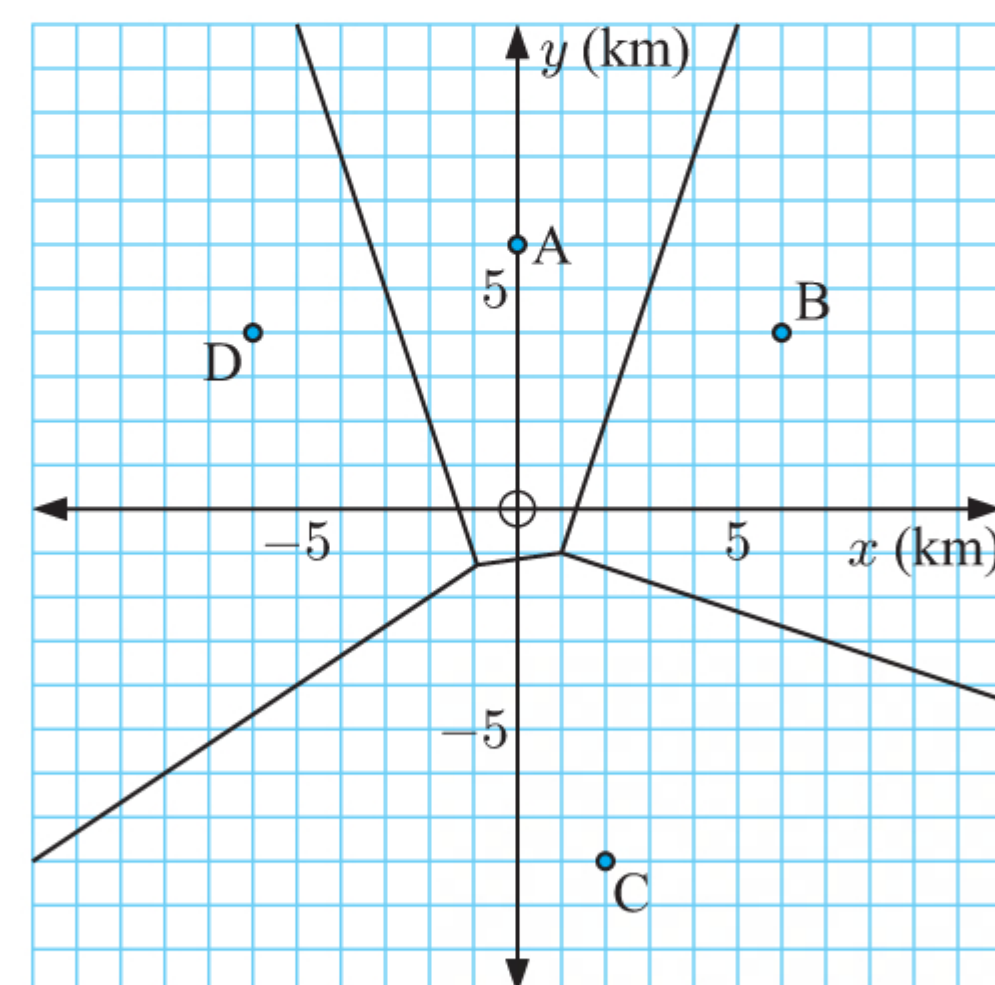
$$= \sqrt{5}$$

$$\approx 2.24 \text{ km}$$



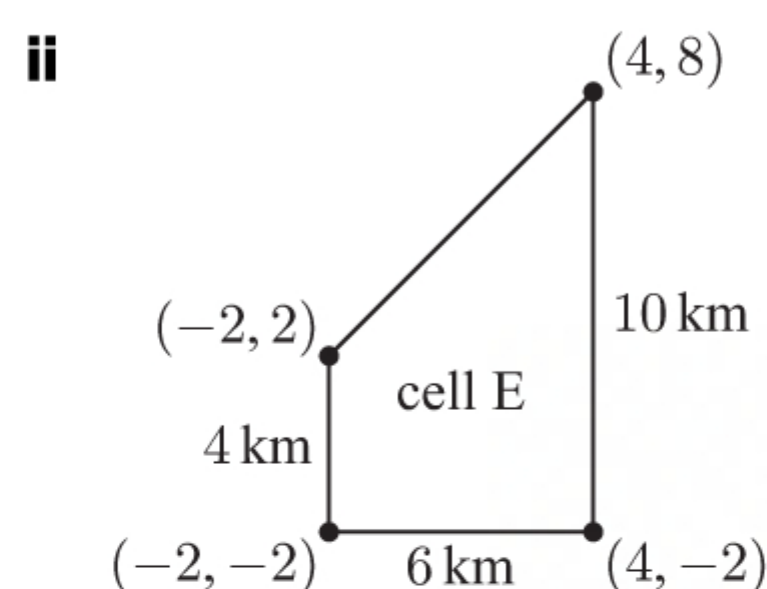
So, Jerome needs to walk about 2.24 km to get to any one of stops B, C, and G.

- 42 a**
- i** $(-2, 8)$ is in cell A, so the closest restaurant is A.
 - ii** $(5, -5)$ is in cell C, so the closest restaurant is C.
 - iii** $(-9, -6)$ is in cell D, so the closest restaurant is D.

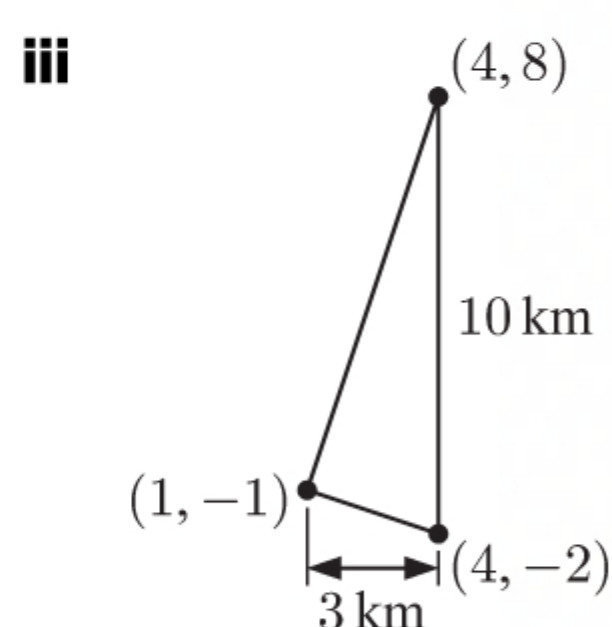


We construct $PB(A, E)$, $PB(B, E)$, $PB(C, E)$, and $PB(D, E)$ within cells A, B, C, and D respectively.

We then remove the segments of edges which now lie within cell E, giving us the Voronoi diagram which includes site E.



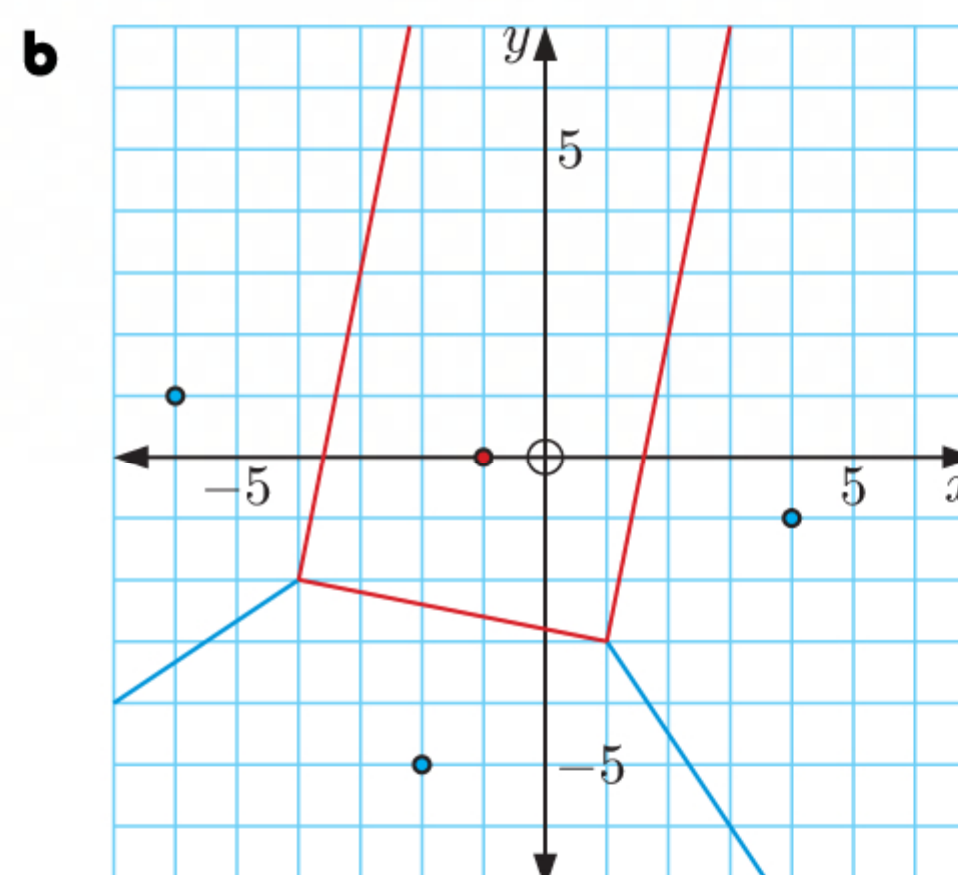
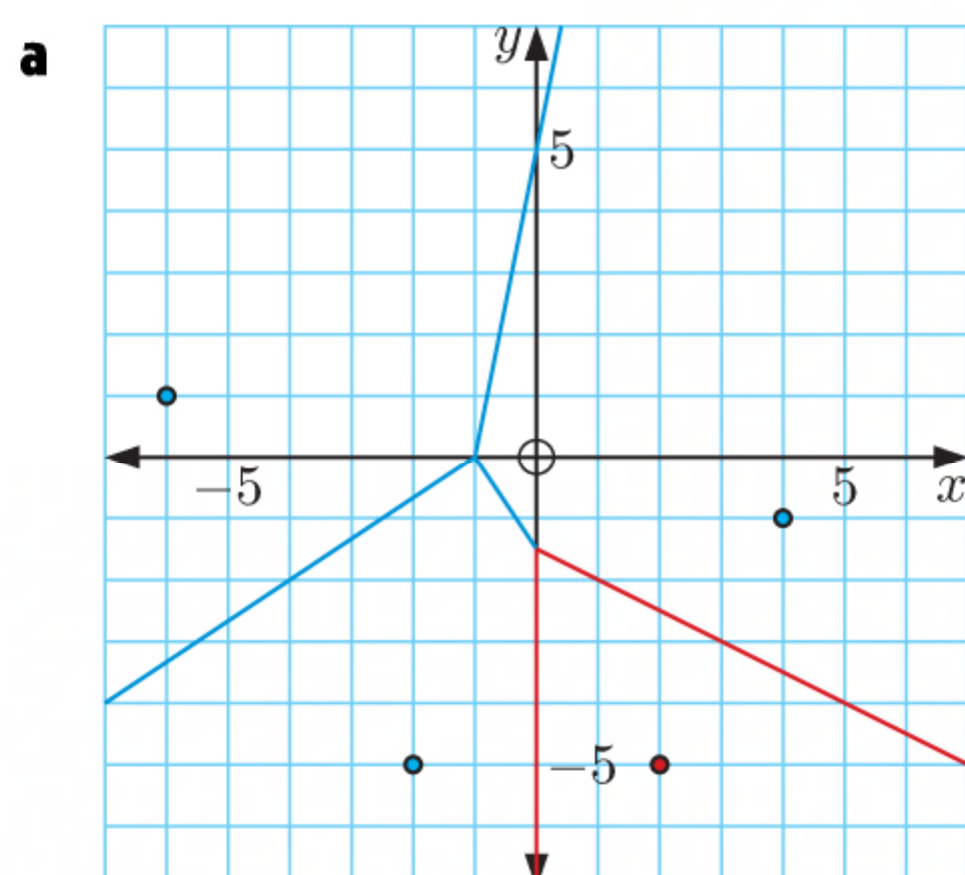
$$\begin{aligned} \text{Area of region} &= \text{area of cell E} \\ &= \left(\frac{10+4}{2} \right) \times 6 \\ &= 42 \text{ km}^2 \end{aligned}$$



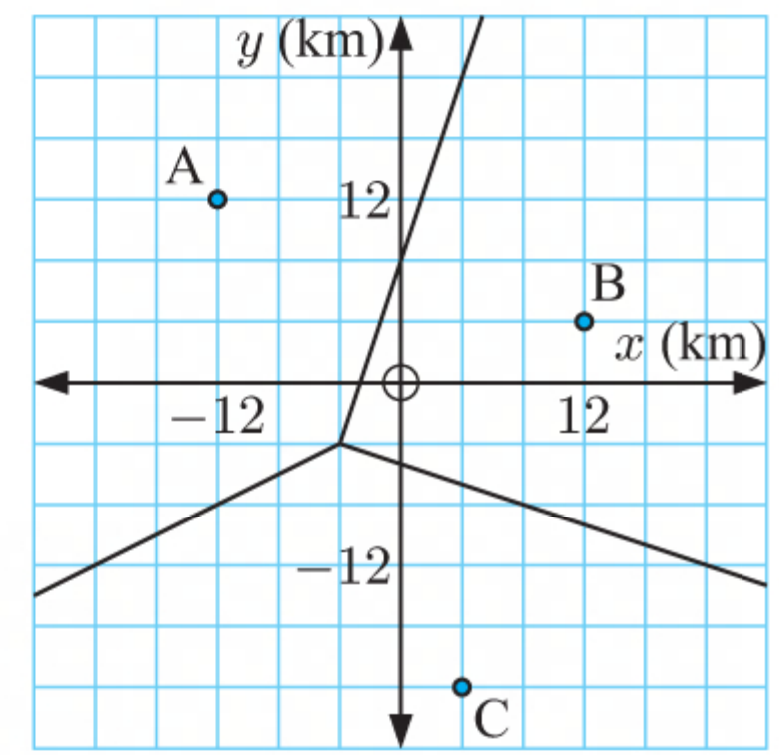
$$\begin{aligned} \text{Area of cell E that was cell B} &= \frac{1}{2} \times 3 \times 10 \\ &= 15 \text{ km}^2 \end{aligned}$$

Assuming that the density of residents is constant throughout the city, the proportion of residents now closest to E who were originally closest to B is $\frac{15}{42} \approx 0.357$.

- 43** We construct the perpendicular bisectors between the new site and each of the original ones, within their respective cells. We then remove the segments of edges which lie within the new cell, giving us the Voronoi diagram which includes the new site.



- 44 a** $(12, 16)$ is closest to B, so we estimate the internet speed at 6 pm to be 42.7 Mbps at $(12, 16)$.
- b** $(-20, -12)$ is on the edge equidistant from A and C, so we estimate the internet speed at 6 pm to be $\frac{44.3 + 45.9}{2} = 45.1$ Mbps at $(-20, -12)$.
- c** $(-4, 4)$ is the vertex equidistant from A, B, and C, so we estimate the internet speed at 6 pm to be $\frac{44.3 + 42.7 + 45.9}{3} = 44.3$ Mbps at $(-4, -4)$.

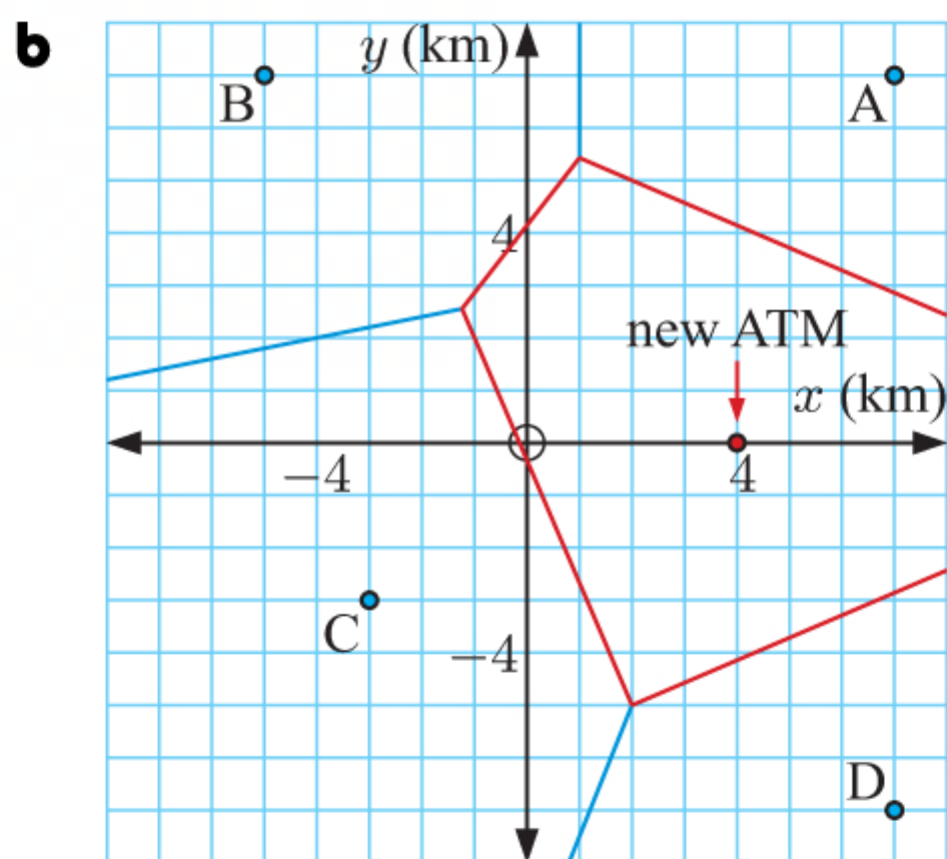
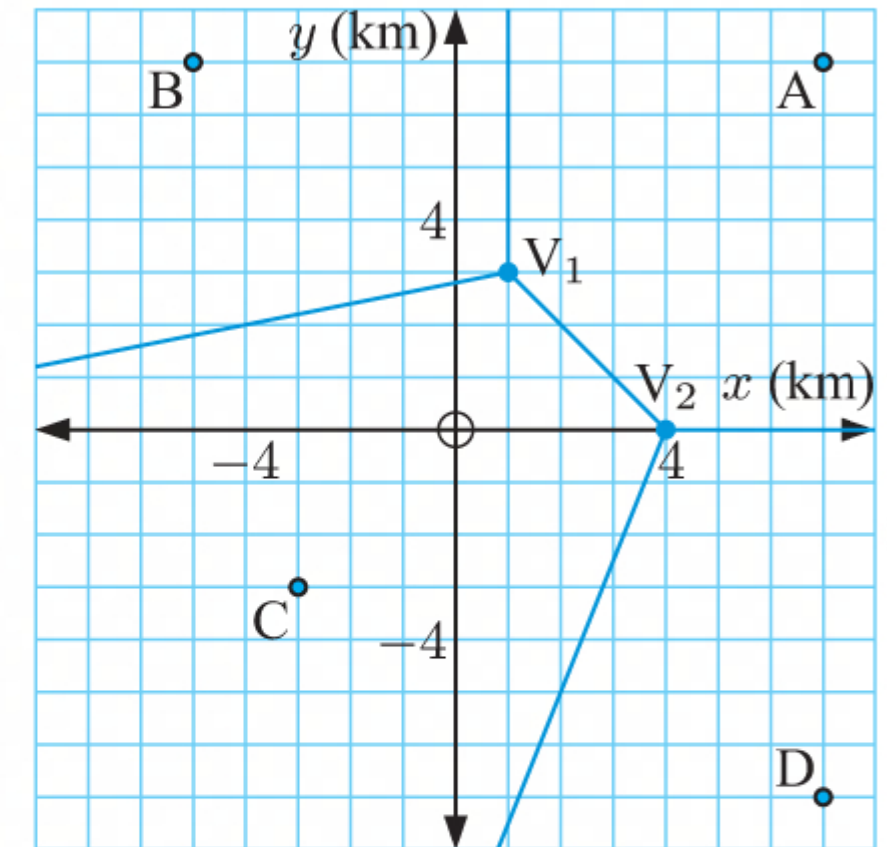


- 45 a** The new ATM needs to be placed at the centre of the largest empty circle, which will be centred at one of the vertices $V_1(1, 3)$ or $V_2(4, 0)$.

$$\begin{aligned} V_1A &= \sqrt{(7-1)^2 + (7-3)^2} \\ &= \sqrt{6^2 + 4^2} \\ &= \sqrt{52} \text{ km} \\ V_2A &= \sqrt{(7-4)^2 + (7-0)^2} \\ &= \sqrt{3^2 + 7^2} \\ &= \sqrt{58} \text{ km} \end{aligned}$$

So, the largest empty circle has centre $V_2(4, 0)$.

\therefore the new ATM should be placed at $(4, 0)$.



We construct the perpendicular bisectors between the site of the new ATM and each of the original ones, within their respective cells.

We then remove the segments of edges which lie within the new cell, giving us the Voronoi diagram which includes the new ATM at $(4, 0)$.

- c** $(-1, 1)$ is in cell C, so Allan is closest to ATM C.

$$\begin{aligned} \therefore \text{distance Allan needs to walk} &= \sqrt{(-1 - (-3))^2 + (1 - (-3))^2} \\ &= \sqrt{2^2 + 4^2} \\ &= \sqrt{20} \\ &\approx 4.47 \text{ km} \end{aligned}$$

- d** In **c**, we assume that the path Allan has to walk to get to the nearest ATM is a straight line.

TOPIC 4 SKILL BUILDER QUESTIONS

- 1 a** The sample size of only 10 students from a total of 500 is far too small, so this approach may produce a coverage error.
- b** The tape measure only allows Gerard to *estimate* the exact height of each student. With only 10 students being measured, any errors will significantly impact the results, so this approach may produce a measurement error.

- 2 a i** A survey of 50 students can be completed in a reasonable time frame. It is also large enough that the results can be representative of the whole student body.
- ii** Surveying all students is time-consuming and often impractical. A non-response error may be produced if students are absent.

	Boys	Girls
Year 8	135	140
Year 9	130	145
Year 10	125	130

- b** Total number of students = $135 + 140 + 130 + 145 + 125 + 130 = 805$

i For the survey, $\frac{135}{805} \times 50 \approx 8$ Year 8 boys will be selected.

ii For the survey, $\frac{140 + 145 + 130}{805} \times 50 \approx 26$ girls will be selected.

- c** A stratified sample is better than a random sample in this case as a stratified sample will fairly represent each year level and gender. A random sample cannot guarantee such a fair representation.
- 3 a** This is a convenience sample because it is more convenient for Marie to sample the first 10 people to visit her office than to sample 10 random people from the whole building for example.
- b** The preferences of the first 10 people to visit Marie's office are likely to come from people who work with her. This may not be representative of the preferences of all people in the building, and so the sample may be biased.
- c** Marie could use a stratified sample where the subgroups may correspond to departments, floor number, and so on. In this way, a fair representation of preferences is more likely to be obtained.

- 4 a** The ticket inspector selects passengers at regular intervals, so the sampling method used is systematic sampling.

- b** The first passenger to be checked is the 8th passenger.

So, the next 6 passengers to be checked are the 28th, 48th, 68th, 88th, 108th, and 128th passengers.

- c** 5000 passengers left the terminal, and every 20th passenger was checked.

$$\therefore \frac{5000}{20} = 250 \text{ passengers were checked.}$$

- 5 a** The number of houses on a particular street can be counted, so it is a discrete variable.

- b** The number of hours spent travelling on an airplane can be measured and can take any value between certain limits, so it is a continuous variable.

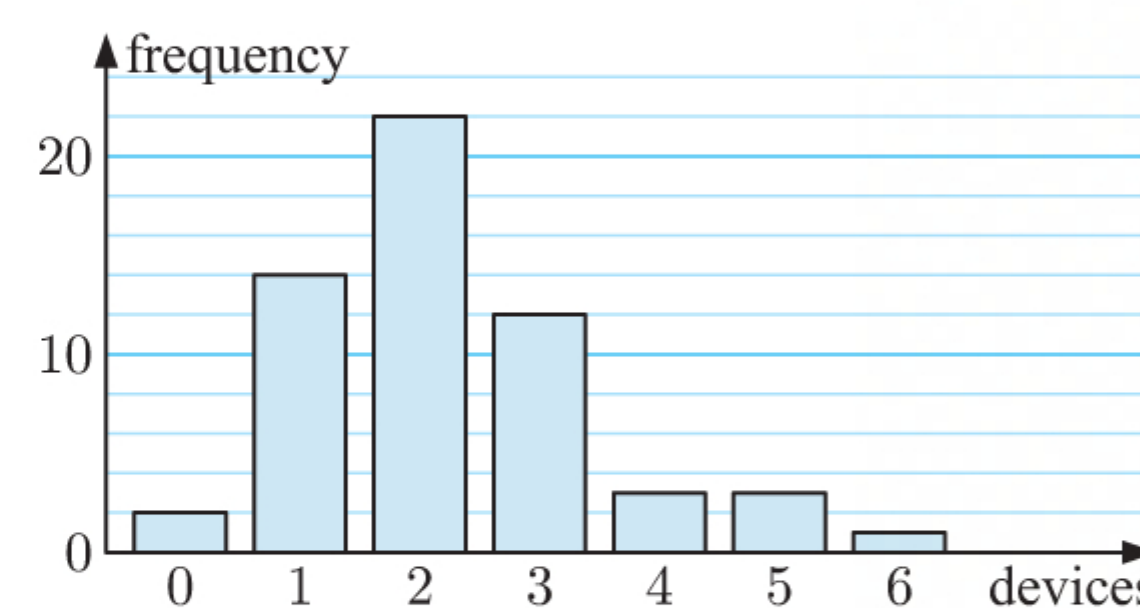
- c** The brand of laptop someone uses is a categorical variable.

- 6 a** $2 + 14 + 22 + 12 + 3 + 3 + 1 = 57$ people were surveyed.

- b** The mode of the data is 2 devices.

- c** $\frac{14 + 22}{57} \times 100\% \approx 63.2\%$ of people browsed the internet using 1 or 2 devices.

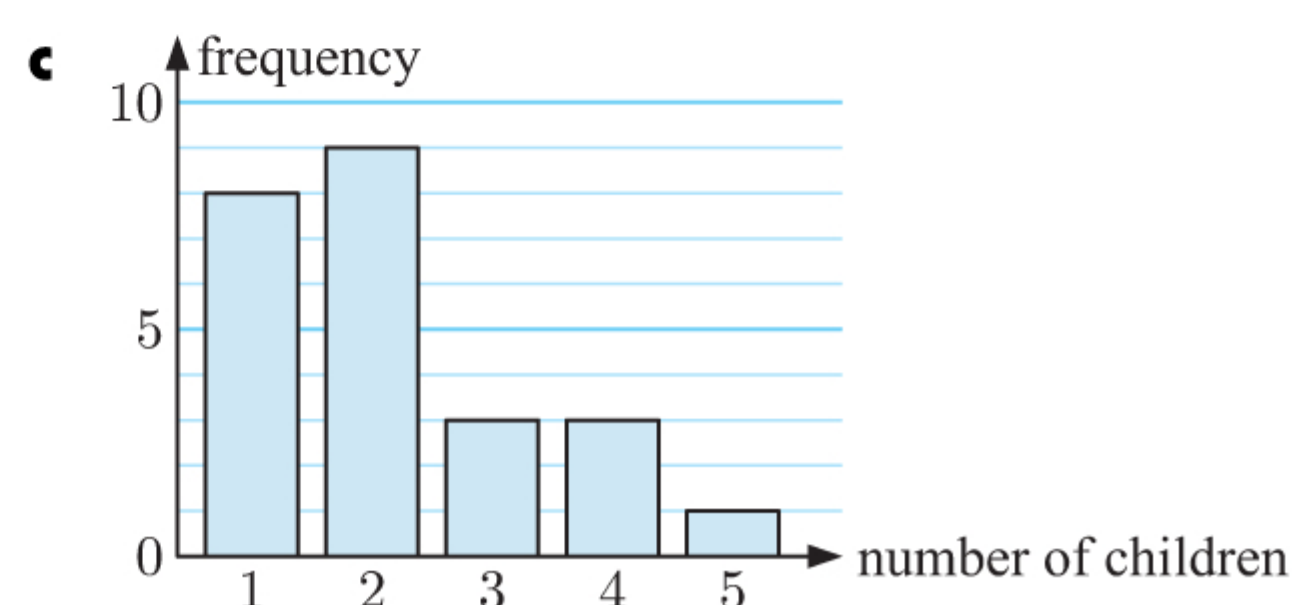
- d** The data is positively skewed with no outliers.



- 7 a** The number of children in each family can be counted and takes exact number values.

\therefore the data is discrete.

Number of children	Frequency
1	8
2	9
3	3
4	3
5	1
Total	24

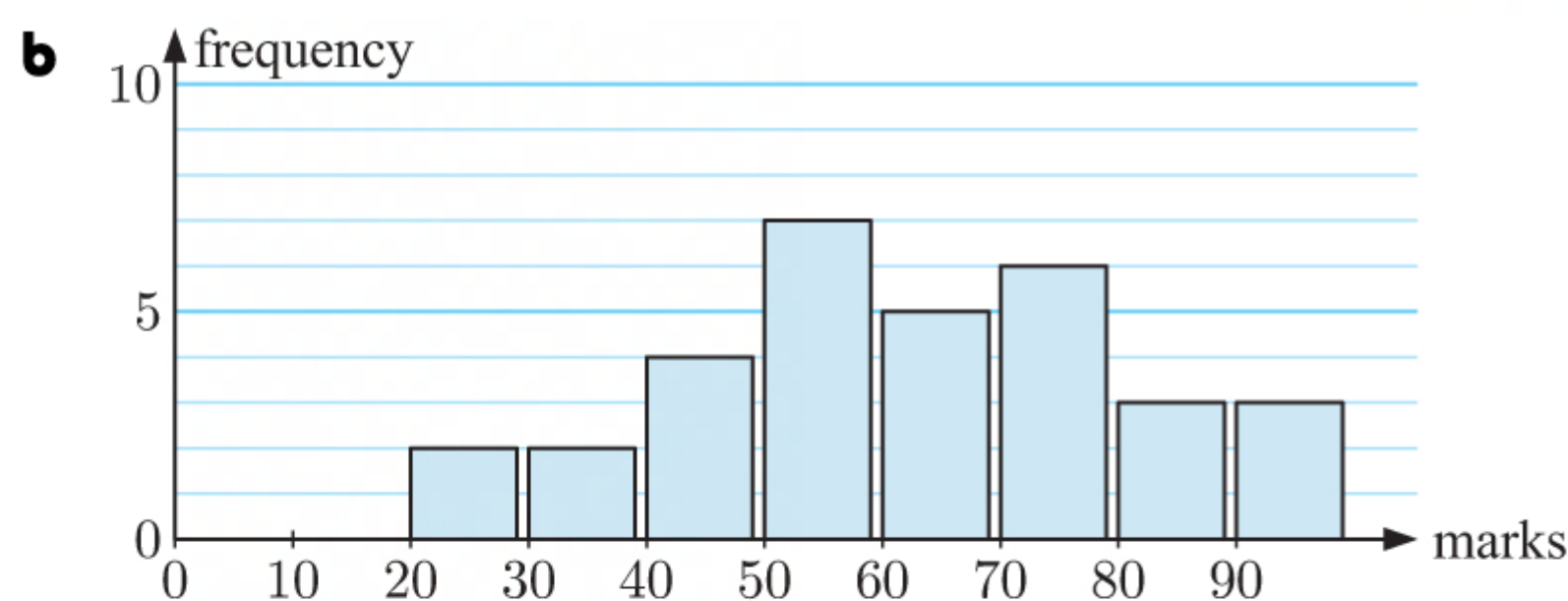


d The data is positively skewed. There are no outliers.

e $\frac{3 + 3 + 1}{24} \times 100\% \approx 29.2\%$ of families have 3 or more children.

8 a

Marks	Tally	Frequency
0 - 9		0
10 - 19		0
20 - 29		2
30 - 39		2
40 - 49		4
50 - 59		7
60 - 69		5
70 - 79		6
80 - 89		2
90 - 99		2
Total		30



c The modal class is 50 - 59 marks.

d $\frac{7 + 5 + 6 + 2 + 2}{30} \times 100\% \approx 73.3\%$ of students passed the examination.

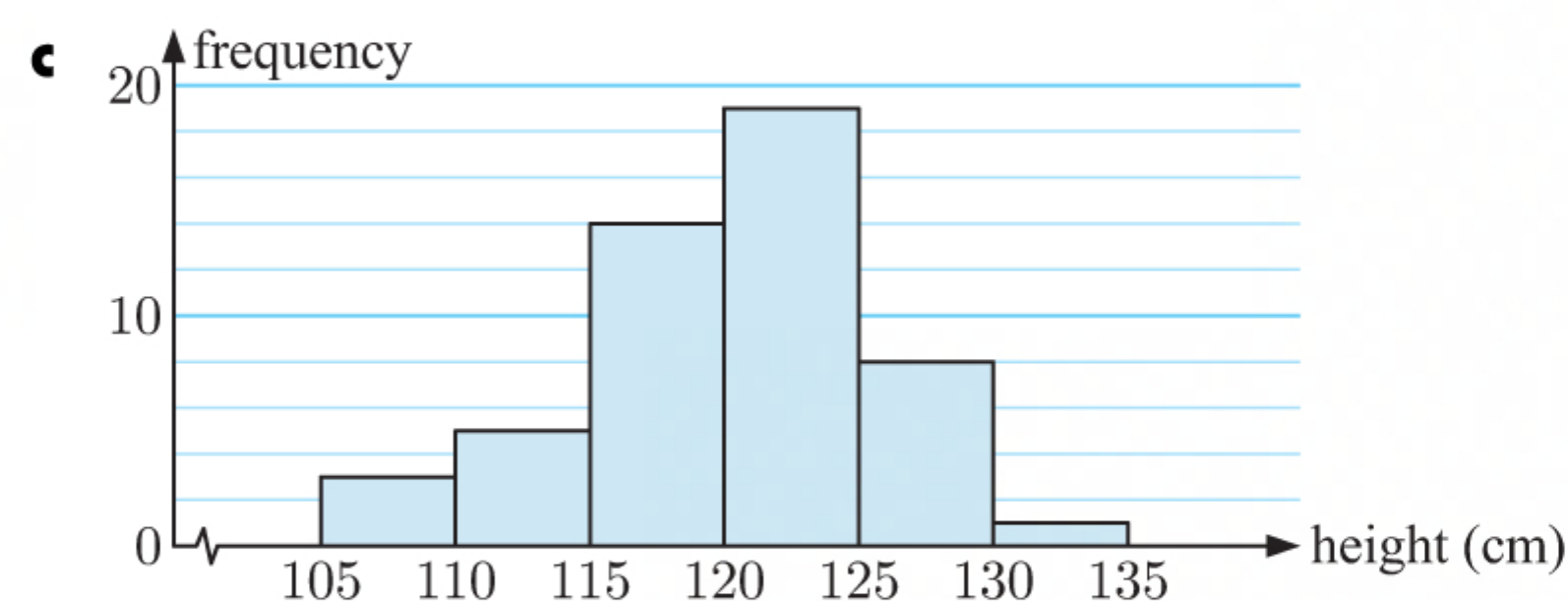
9

Height (h cm)	Frequency
$105 \leq h < 110$	3
$110 \leq h < 115$	5
$115 \leq h < 120$	14
$120 \leq h < 125$	19
$125 \leq h < 130$	8
$130 \leq h < 135$	1

a The *height* of an emperor penguin is a numerical variable which can be measured.

\therefore *height* is a continuous variable.

b $3 + 5 + 14 + 19 + 8 + 1 = 50$ emperor penguins were measured.



d The data appears to be approximately symmetrical.

e The modal class is $120 \leq h < 125$.

More emperor penguins have lengths in this interval than in any other interval.

10 a 26 is the data value which occurs most often, so the mode is 26 customers.

b As $n = 9$, $\frac{n+1}{2} = 5$

The ordered data set is: ~~14~~ ~~16~~ ~~18~~ ~~23~~ **24** ~~25~~ ~~26~~ ~~26~~ ~~34~~

↑
5th value

\therefore median = 24 customers.

c mean = $\frac{14 + 23 + \dots + 16 + 25}{9}$ ← sum of all the data values
← 9 data values
 $= \frac{206}{9}$
 ≈ 22.9 customers

11 a i mean number of visitors for exhibit A

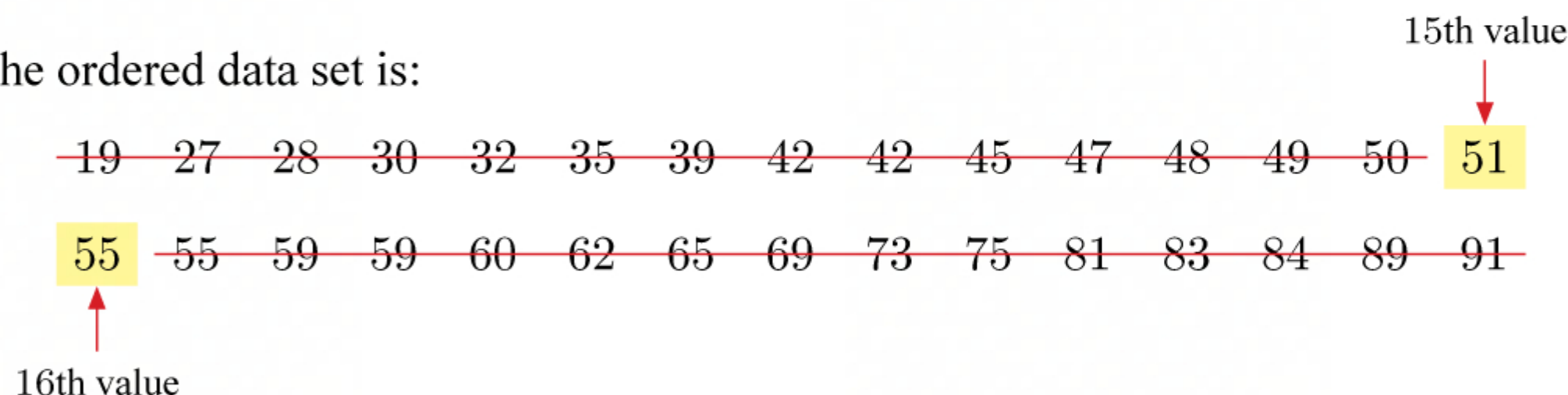
$$\begin{aligned}
 &= \frac{42 + 49 + 55 + 48 + \dots + 32}{30} \\
 &= \frac{1644}{30} \\
 &= 54.8 \text{ visitors}
 \end{aligned}$$

mean number of visitors for exhibit B

$$\begin{aligned}
 &= \frac{59 + 51 + 60 + 44 + \dots + 46}{30} \\
 &= \frac{1711}{30} \\
 &\approx 57.0 \text{ visitors}
 \end{aligned}$$

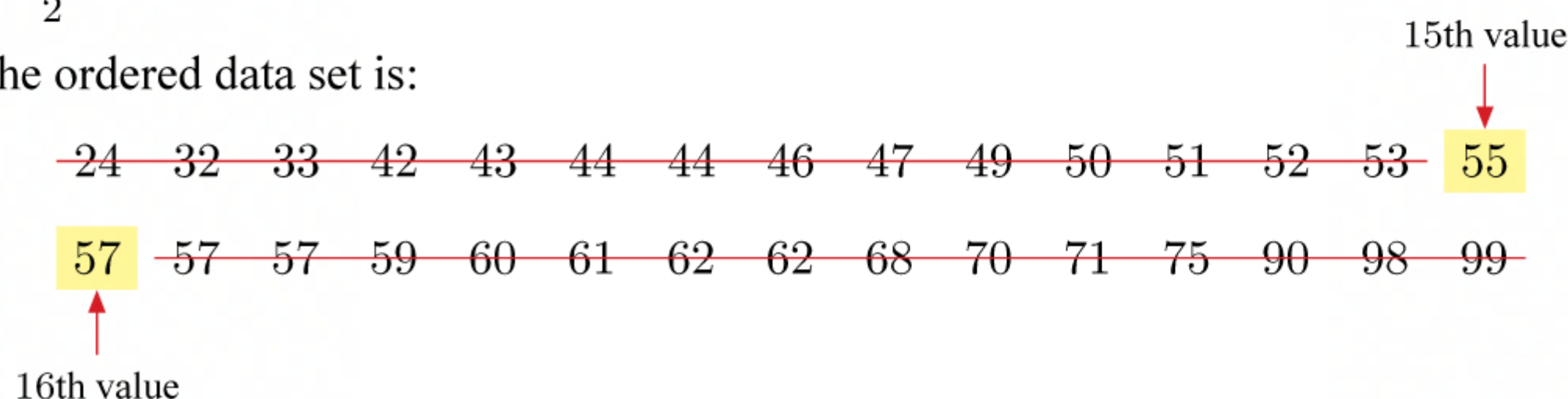
ii As $n = 30$, and $\frac{n+1}{2} = 15.5$, for both data sets, the median is the average of the 15th and 16th ordered data values.

For exhibit A, the ordered data set is:



$$\therefore \text{median} = \frac{51 + 55}{2} = 53 \text{ visitors}$$

For exhibit B, the ordered data set is:



$$\therefore \text{median} = \frac{55 + 57}{2} = 56 \text{ visitors}$$

b Exhibit B was more popular as the mean and median are both higher for exhibit B than for exhibit A.

12 a

$$\begin{aligned}
 \frac{9 + 10 + a + 13 + b + 16 + 21}{7} &= 14 \\
 \therefore \frac{69 + a + b}{7} &= 14 \\
 \therefore 69 + a + b &= 98 \\
 \therefore a + b &= 29
 \end{aligned}$$

Now, a and b are integers such that $10 \leq a \leq 13$ and $13 \leq b \leq 16$.

\therefore the only possible solution is $a = 13$ and $b = 16$.

b Since $n = 6$, $\frac{n+1}{2} = 3.5$

So the median is the average of the 3rd and 4th ordered data values.

The ordered data set is: 1, 5, 9, 11, 16, p
two middle data values

$$\therefore \text{median} = \frac{9 + 11}{2} = 10$$

$$\text{Now, } \frac{1 + 5 + 9 + 11 + 16 + p}{6} = 10$$

$$\therefore \frac{42 + p}{6} = 10$$

$$\therefore 42 + p = 60$$

$$\therefore p = 18$$

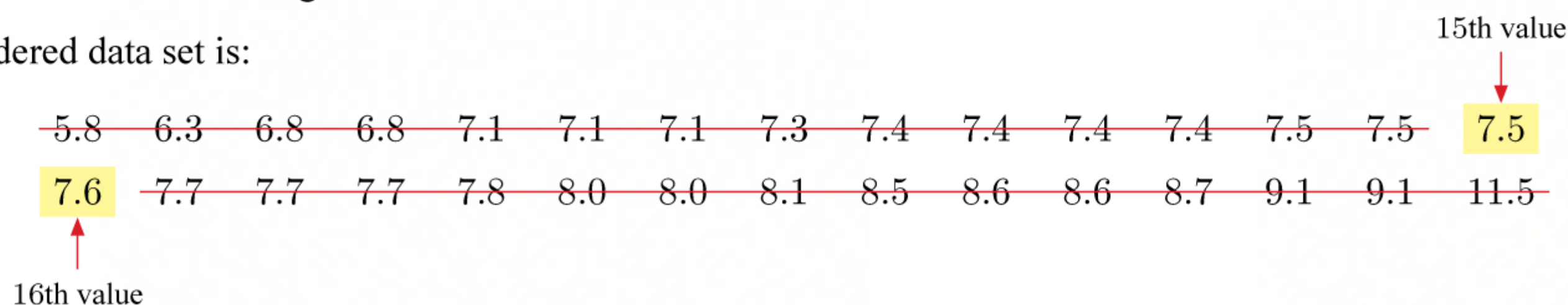
13 a

$$\begin{aligned}
 \text{mean} &= \frac{7.5 + 6.8 + \dots + 8.5}{30} \\
 &= \frac{233.1}{30} \\
 &= 7.77 \text{ hours}
 \end{aligned}$$

$$\text{As } n = 30, \frac{n+1}{2} = 15.5$$

So the median is the average of the 15th and 16th data values.

The ordered data set is:



$$\therefore \text{median} = \frac{7.5 + 7.6}{2} = 7.55 \text{ hours}$$

b The outlier is 11.5 hours.

$$\begin{aligned}
 \text{c i mean} &= \frac{7.5 + 6.8 + \dots + 8.5}{29} \\
 &= \frac{221.6}{29} \\
 &\approx 7.64 \text{ hours}
 \end{aligned}$$

As $n = 29$ with the outlier removed, $\frac{n+1}{2} = 15$.

The ordered data set is:

15th value
↓

5.8	6.3	6.8	6.8	7.1	7.1	7.1	7.3	7.4	7.4	7.4	7.4	7.5	7.5	7.5
7.6	7.7	7.7	7.7	7.8	8.0	8.0	8.1	8.5	8.6	8.6	8.7	9.1	9.1	

\therefore median = 7.5 hours

- ii The measure of centre which is most affected by extreme values is the mean. So, the mean is most affected if the outlier is removed.

$$\begin{aligned}
 \text{14 a mean} &= \frac{10\,613 + 5453 + \dots + 8888}{14} \\
 &= \frac{66\,866}{14} \\
 &\approx 4776 \text{ steps}
 \end{aligned}$$

As $n = 14$, $\frac{n+1}{2} = 7.5$

So the median is the average of the 7th and 8th data values.

The ordered data set is:

7th value
↓

261	1468	2939	3417	3504	3526	4093
4253	5453	5570	6321	6560	8888	10\,613

↑
8th value

$$\therefore \text{median} = \frac{4093 + 4253}{2} = 4173 \text{ steps.}$$

- b The median is not affected by extreme values.

So, the median is the most suitable to determine the typical number of steps taken by Janice each day.

15

Number of touchdowns (x)	Frequency (f)	Product (xf)	Cumulative frequency
0	2	0	2
1	10	10	12
2	7	14	19
3	6	18	25
4	4	16	29
5	2	10	31
6	1	6	32
Total	$\sum f = 32$	$\sum xf = 74$	

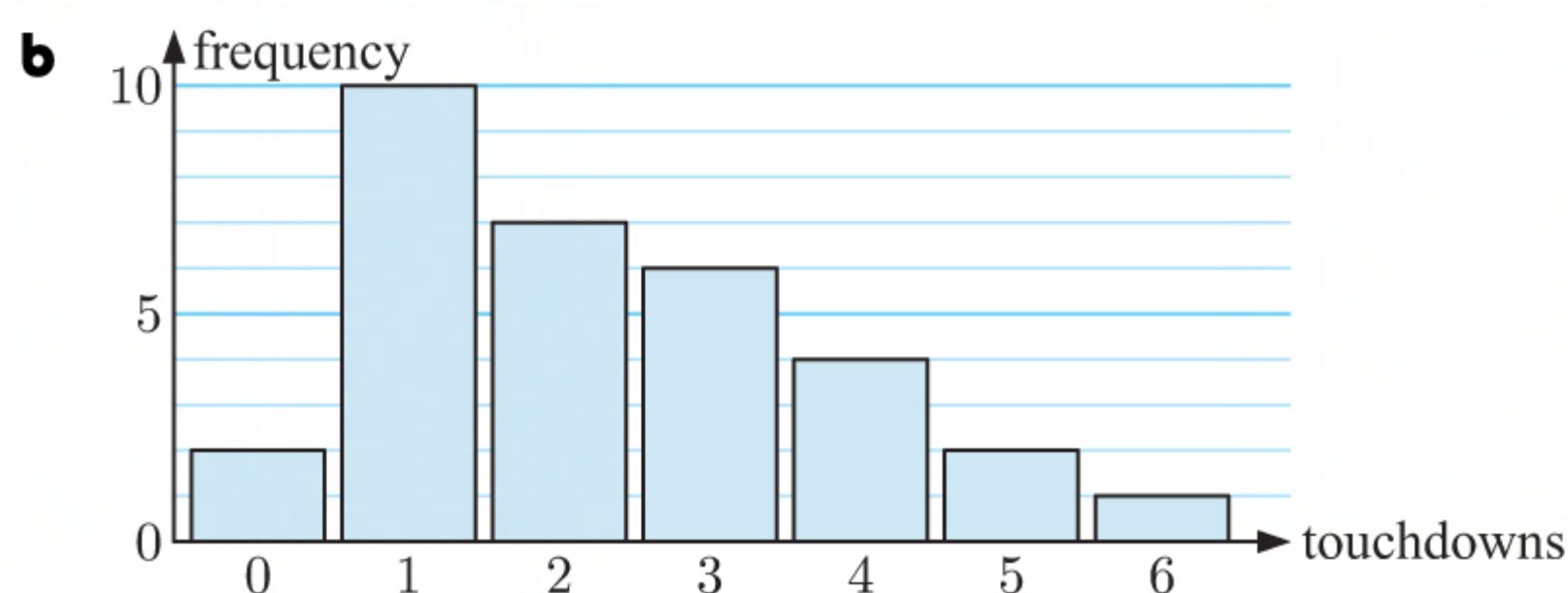
$$\begin{aligned}
 \text{a i } \bar{x} &= \frac{\sum xf}{\sum f} \\
 &= \frac{74}{32} \\
 &= 2.3125 \text{ touchdowns}
 \end{aligned}$$

- ii There are 32 data values, so $n = 32$. $\frac{n+1}{2} = 16.5$, so the median is the average of the 16th and 17th ordered data values. From the cumulative frequency column, the 13th to 19th ordered data values are 2 touchdowns.

\therefore the 16th and 17th ordered data values are 2 touchdowns.

\therefore the median is 2 touchdowns.

- iii Looking down the frequency column, the highest frequency is 10. This corresponds to 1 touchdown, so the mode is 1 touchdown.



c The data appears to be positively skewed.

d The data appears to be positively skewed, so the median is not a suitable measure of centre for this data.

The mean is affected by extreme values, so the mean is not a suitable measure of centre for this data.

The mode gives the most frequent number of touchdowns scored, and so is the most suitable measure of centre for this data.

16 a

Number of cars	Frequency	Cumulative frequency
0	78	78
1	117	195
2	69	264
3	18	282
4	2	284
Total	284	

b i
$$\text{mean} = \frac{0 \times 78 + 1 \times 117 + 2 \times 69 + 3 \times 18 + 4 \times 2}{284}$$

$$= \frac{317}{284}$$

$$\approx 1.12 \text{ cars}$$

ii There are 284 data values, so $n = 284$. $\frac{n+1}{2} = 142.5$, so the median is the average of the 142nd and 143rd ordered data values.

From the cumulative frequency column, the 79th to 195th ordered data values are 1 car.

\therefore the 142nd and 143rd data values are 1 car.

$\therefore \text{median} = \frac{1+1}{2} = 1 \text{ car}$

iii Looking down the frequency column, the highest frequency is 117. This corresponds to 1 car, so the mode is 1 car.

17

Score	7	9	a	13	16
Frequency	1	2	1	2	1

a
$$\bar{x} = \frac{\sum xf}{\sum f}$$

$$\therefore 11 = \frac{7 \times 1 + 9 \times 2 + a \times 1 + 13 \times 2 + 16 \times 1}{1 + 2 + 1 + 2 + 1}$$

$$\therefore 11 = \frac{a + 67}{7}$$

$$\therefore a + 67 = 77$$

$$\therefore a = 10$$

b Let k be the number of goals that Kai will need to score in the next game.

Since she averaged 11 goals in her first 7 games, her average after the next game $= \frac{7 \times 11 + k}{8} = \frac{k + 77}{8}$.

For her overall average to improve to 12, we require $\frac{k + 77}{8} = 12$

$\therefore k + 77 = 96$

$\therefore k = 19$

So, Kai will need to score 19 goals in the next game to improve her overall average to 12.

Weekly rent (€r)	Frequency (f)	Midpoint (x)	Product (xf)
$80 \leq r < 100$	3	90	270
$100 \leq r < 120$	15	110	1650
$120 \leq r < 140$	26	130	3380
$140 \leq r < 160$	30	150	4500
$160 \leq r < 180$	14	170	2380
$180 \leq r < 200$	1	190	190
Total	$\sum f = 89$		$\sum xf = 12\,370$

$$\begin{aligned} \text{a } \bar{x} &= \frac{\sum xf}{\sum f} \\ &= \frac{12\,370}{89} \\ &\approx 139 \end{aligned}$$

$$\begin{aligned} \text{b } P(r \geq 140) &= \frac{30 + 14 + 1}{89} \\ &\approx 0.506 \end{aligned}$$

\therefore the mean weekly rent was about €139.

19 a The ordered data set is: 7 8 8 10 11 13 14 14 15 18 21 (11 data values)

i Since $n = 11$, $\frac{n+1}{2} = 6$ \therefore the median is the 6th data value.

~~7 8 8 10 11 13 14 14 15 18 21~~

\therefore median = 13

ii Since the median is a data value we now ignore it and split the remaining data into two:

$\overbrace{7 \ 8 \ 8 \ 10 \ 11}^{\text{lower half}} \quad \overbrace{14 \ 14 \ 15 \ 18 \ 21}^{\text{upper half}}$

$Q_1 = \text{median of lower half} = 8$

$Q_3 = \text{median of upper half} = 15$

iii Range = maximum – minimum
 $= 21 - 7$
 $= 14$

iv $IQR = Q_3 - Q_1$
 $= 15 - 8 \quad \{\text{from ii}\}$
 $= 7$

b The ordered data set is: 18 19 22 26 32 35 41 43 (8 data values)

i Since $n = 8$, $\frac{n+1}{2} = 4.5$

\therefore the median is the average of the 4th and 5th data values.

~~18 19 22 26 32 35 41 43~~

\therefore median = $\frac{26 + 32}{2} = 29$

ii We split the data into two:
 $\overbrace{18 \ 19 \ 22}^{\text{lower half}} \quad \overbrace{32 \ 35 \ 41}^{\text{upper half}}$

$Q_1 = \text{median of lower half} = \frac{19 + 22}{2} = 20.5$

$Q_3 = \text{median of upper half} = \frac{35 + 41}{2} = 38$

iii Range = maximum – minimum
 $= 43 - 18$
 $= 25$

iv $IQR = Q_3 - Q_1$
 $= 38 - 20.5 \quad \{\text{from ii}\}$
 $= 17.5$

c The ordered data set is: 14 16 18 20 25 27 29 37 38 39 46 52 (12 data values)

i Since $n = 12$, $\frac{n+1}{2} = 6.5$

\therefore the median is the average of the 6th and 7th data values.

~~14 16 18 20 25 27 29 37 38 39 46 52~~

\therefore median = $\frac{27 + 29}{2} = 28$

- ii We split the data into two:
- | lower half | | | | | upper half | | | | | | |
|------------|----|----|----|----|------------|----|----|----|----|----|----|
| 14 | 16 | 18 | 20 | 25 | 27 | 29 | 37 | 38 | 39 | 46 | 52 |
- $$Q_1 = \text{median of lower half} = \frac{18 + 20}{2} = 19$$
- $$Q_3 = \text{median of upper half} = \frac{38 + 39}{2} = 38.5$$
- iii Range = maximum – minimum
- $$= 52 - 14$$
- $$= 38$$
- iv IQR = $Q_3 - Q_1$
- $$= 38.5 - 19 \quad \{\text{from ii}\}$$
- $$= 19.5$$

20 The ordered data sets are:

Cailan: 79 80 81 83 84 85 86 87 90 92 (10 data values)

Miles: 82 82 83 84 84 85 87 88 90 91 (10 data values)

a *Cailan:*

$$\begin{aligned} \text{range} &= \text{maximum} - \text{minimum} \\ &= 92 - 79 \\ &= 13 \end{aligned}$$

We have an even number of data values, so we include all data values when we split the data set into two:

lower half					upper half				
79	80	81	83	84	85	86	87	90	92

$$Q_1 = \text{median of lower half} = 81$$

$$Q_3 = \text{median of upper half} = 87$$

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 87 - 81 \\ &= 6 \end{aligned}$$

Miles:

$$\begin{aligned} \text{range} &= \text{maximum} - \text{minimum} \\ &= 91 - 82 \\ &= 9 \end{aligned}$$

We have an even number of data values, so we include all data values when we split the data set into two:

lower half					upper half				
82	82	83	84	84	85	87	88	90	91

$$Q_1 = \text{median of lower half} = 83$$

$$Q_3 = \text{median of upper half} = 88$$

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 88 - 83 \\ &= 5 \end{aligned}$$

b i Miles' data has the lower range.

ii Miles' data has the lower interquartile range.

c The interquartile range is more appropriate than the range for determining who is generally the more consistent golfer as it is less affected by outliers.

21 a The ordered data set is:

4	9	10	12	12	14	14	15	16	16	16	17	18	18	18	20	23	26	31	{19 data values}
				↓					↓					↓					
				$Q_1 = 12$					$\text{median} = 16$					$Q_3 = 18$					

So the five-number summary is:

{	minimum = 4	$Q_1 = 12$
	median = 16	$Q_3 = 18$
	maximum = 31	

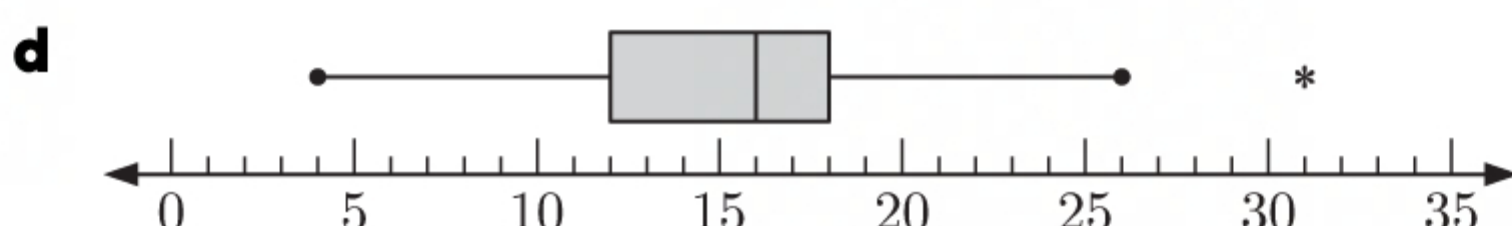
b $\text{IQR} = Q_3 - Q_1$

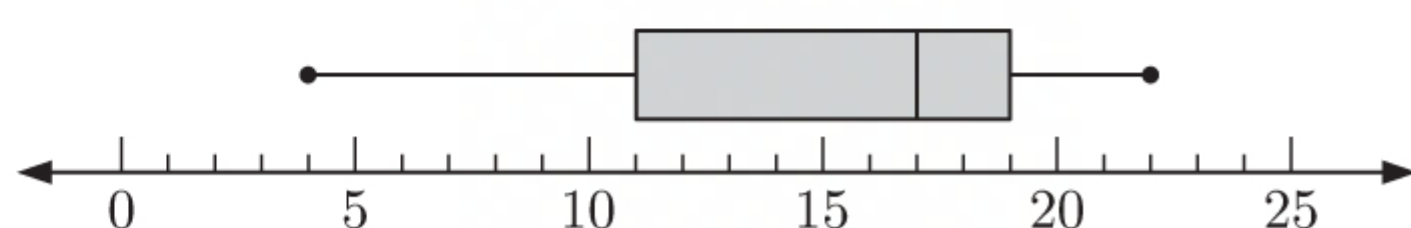
$$= 18 - 12$$

$$= 6$$

c upper boundary	lower boundary
= upper quartile + $1.5 \times \text{IQR}$	= lower quartile - $1.5 \times \text{IQR}$
= $18 + 1.5 \times 6$	= $12 - 1.5 \times 6$
= 27	= 3

31 is above the upper boundary, so it is an outlier.



22**a** minimum value = 4 cm**c** median = 17 cm**e** lower quartile = 11 cm

f range = maximum – minimum
 $= 22 - 4$
 $= 18 \text{ cm}$

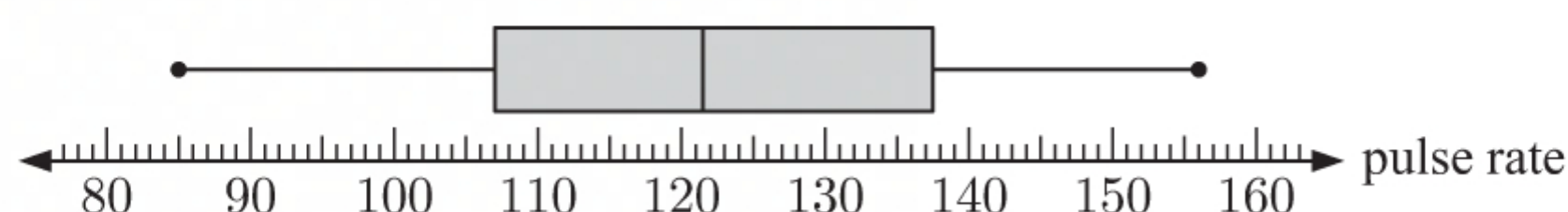
b maximum value = 22 cm**d** upper quartile = 19 cm

g $\text{IQR} = Q_3 - Q_1$
 $= 19 - 11$
 $= 8 \text{ cm}$

23 a The ordered data set is:

85 96 98 100 105 106 108 108 112 112 118 120 123 125 126 128 133 135 140 144 144 148 148 156
↓ ↓ ↓
 $Q_1 = 107$ $\text{median} = 121.5$ $Q_3 = 137.5$ {24 data values}

So the five-number summary is: $\begin{cases} \text{minimum} = 85 & Q_1 = 107 \\ \text{median} = 121.5 & Q_3 = 137.5 \\ \text{maximum} = 156 \end{cases}$

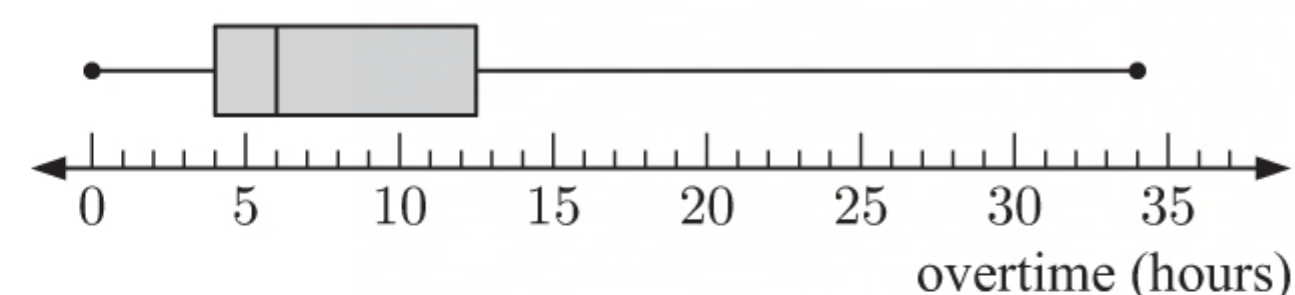
b

c Range = maximum – minimum
 $= 156 - 85$
 $= 71$

$\text{IQR} = Q_3 - Q_1$
 $= 137.5 - 107$
 $= 30.5$

24 a The ordered data set is:

0 0 1 2 4 4 4 5 5 5 5 6 6 7 9 9 11 11 12 13 15 18 22 26 34
↓ ↓ ↓
i $Q_1 = 4$ **ii** median = 6 **iii** $Q_3 = 12.5$ {25 data values}

b**c** The data is not symmetric, but rather is positively skewed.

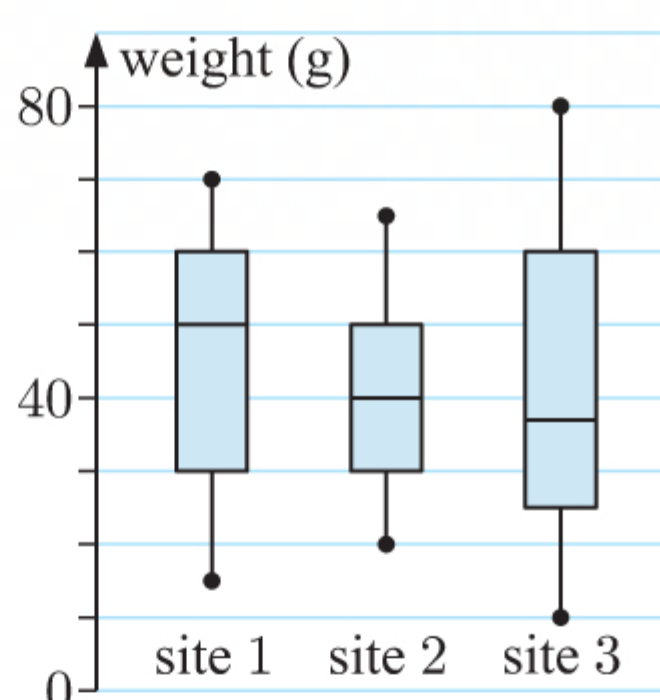
The median value is 2 hours above the first quartile, but 6.5 hours below the upper quartile. Additionally, the median is 6 hours above the minimum, but 28 hours below the maximum.

d i Range = $34 - 0$
 $= 34 \text{ hours}$

ii $\text{IQR} = Q_3 - Q_1$
 $= 12.5 - 4$
 $= 8.5 \text{ hours}$

e i “The central 50% of employees work between 4 and 12.5 hours of overtime.”

ii “75% of employees work a maximum of approximately 12.5 hours overtime.”

25**a** The five-number summary for site 1 is:

$\begin{cases} \text{minimum} = 15 & Q_1 = 30 \\ \text{median} = 50 & Q_3 = 60 \\ \text{maximum} = 70 \end{cases}$

b Site 3 has the greatest range of weights.**c** The weights of fungi have the least variation at site 2.**d** Site 1 has the highest median weight of fungi.**e** Site 1 has the highest proportion of weights above 40 grams.

26 a Old recipe:

1-Variable	
n	=12
minX	=6
Q1	=7
Med	=7.75
Q3	=8
maxX	=9

The five number summaries are:

Old recipe: minimum = 6

$Q_1 = 7$

median = 7.75

$Q_3 = 8$

maximum = 9

New recipe:

1-Variable	
n	=12
minX	=4
Q1	=6.25
Med	=7.25
Q3	=8
maxX	=9

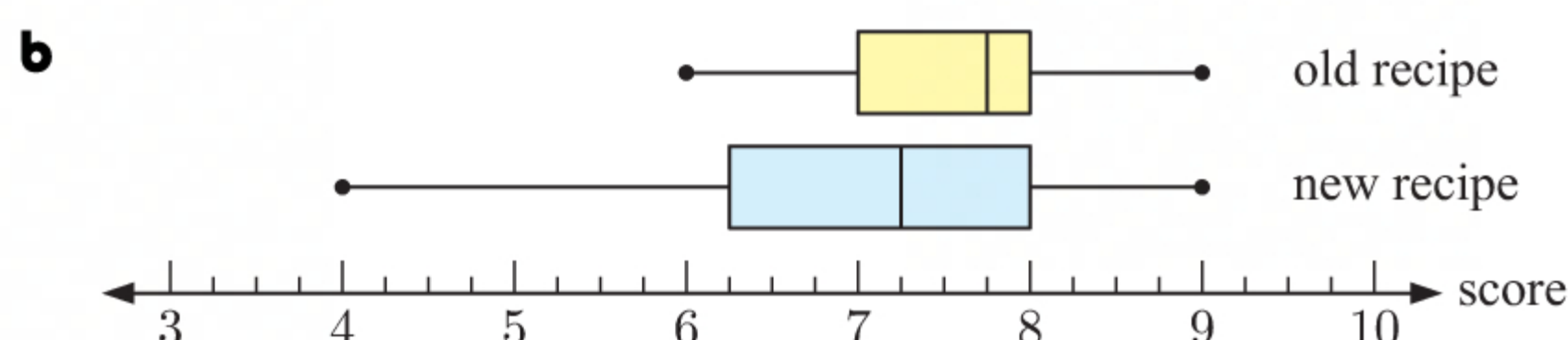
New recipe: minimum = 4

$Q_1 = 6.25$

median = 7.25

$Q_3 = 8$

maximum = 9

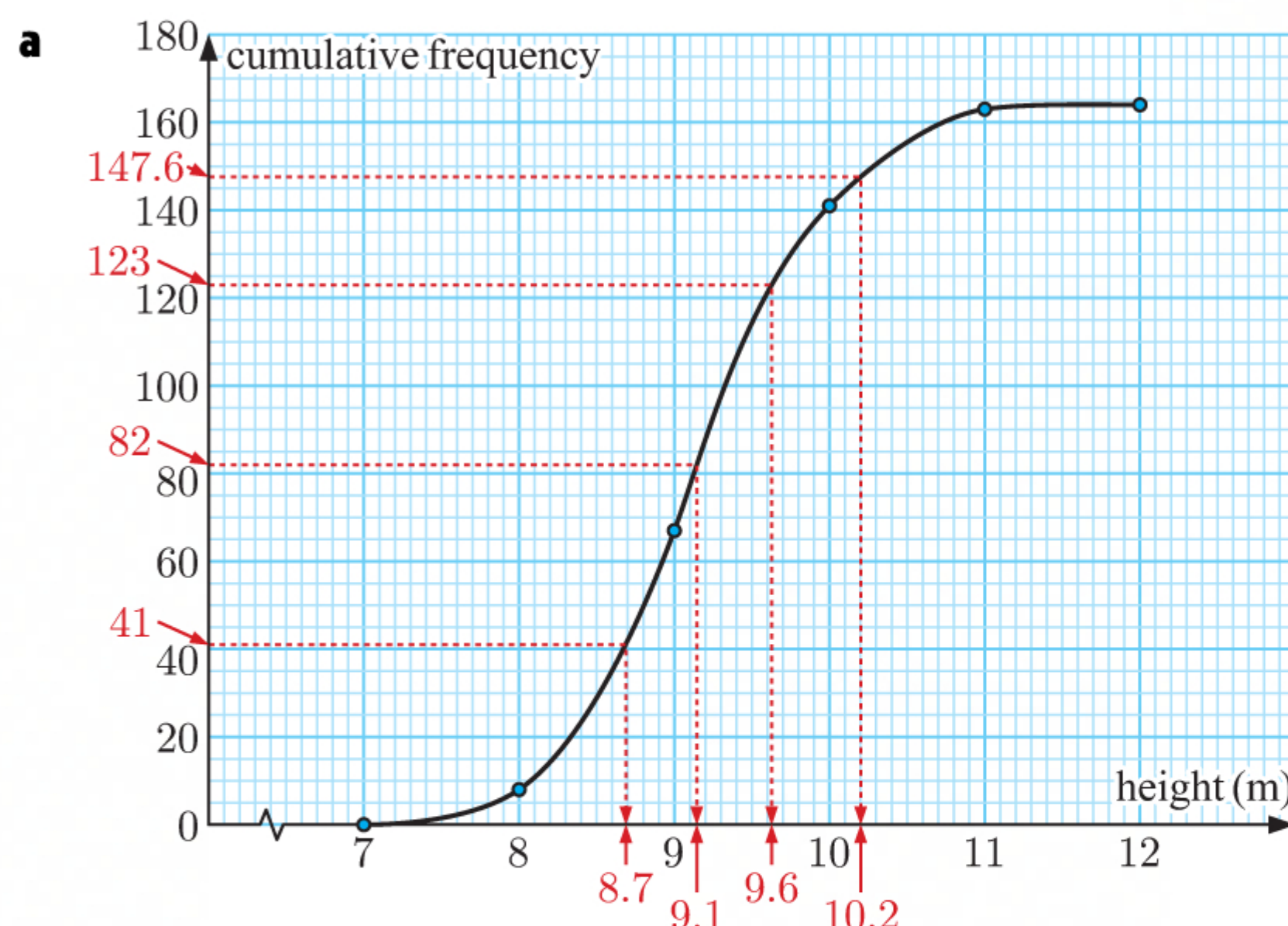


- c The minimum, Q_1 , and the median are higher for the old recipe, and Q_3 and the maximum are the same for both recipes.

The group generally preferred the old recipe over the new recipe, so the distributor should not adopt this new recipe for their drink.

27

Height (h m)	Frequency	Cumulative frequency
$7 \leq h < 8$	8	8
$8 \leq h < 9$	59	67
$9 \leq h < 10$	74	141
$10 \leq h < 11$	22	163
$11 \leq h < 12$	1	164



- b The median is the 50th percentile. As 50% of 164 is 82, we start with the cumulative frequency 82 and find the corresponding height.

From the graph, the median ≈ 9.1 m.

- c Q_1 is the 25th percentile. As 25% of 164 is 41, we start with the cumulative frequency 41 and find the corresponding height.

From the graph, $Q_1 \approx 8.7$ m

Q_3 is the 75th percentile. As 75% of 164 is 123, we start with the cumulative frequency 123 and find the corresponding height.

From the graph, $Q_3 \approx 9.6$ m

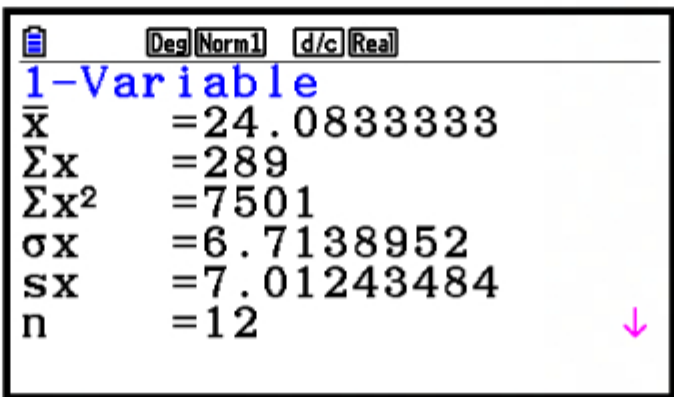
$$\text{IQR} = Q_3 - Q_1$$

$$\approx 9.6 - 8.7$$

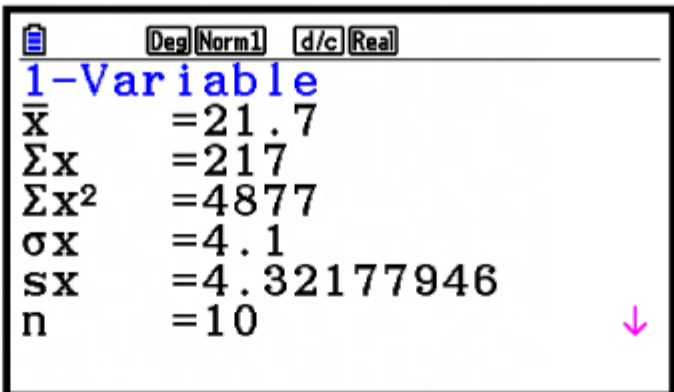
$$\approx 0.9 \text{ m}$$

- d As 90% of 164 is 147.6, we start with the cumulative frequency 147.6 and find the corresponding height.

The 90th percentile ≈ 10.2 m which means that 90% of trees are shorter than about 10.2 m.

28 a  mean ≈ 24.1 , standard deviation ≈ 6.71 {using technology}

b Contestants with times greater than $\approx 24.1 + 6.71 \approx 30.81$ minutes will be eliminated.
So, the contestants who took 32 minutes and 40 minutes are immediately eliminated.

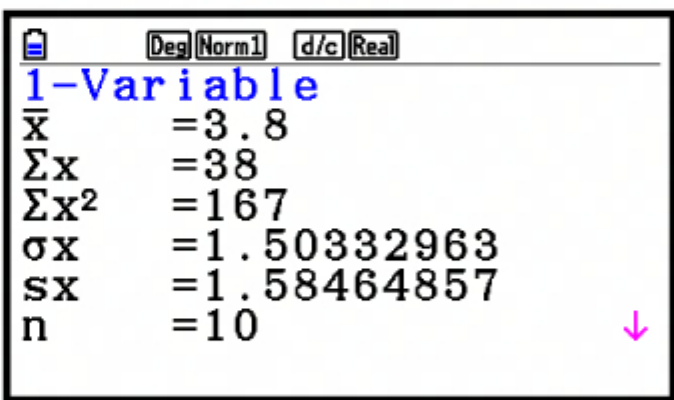
c  mean = 21.7, standard deviation = 4.1 {using technology}

d Contestants with times between $21.7 - 4.1 = 17.6$ minutes and $21.7 + 4.1 = 25.8$ minutes will participate in the next game.
 \therefore 6 contestants (those taking 20, 25, 19, 21, 25, 22 minutes) will participate in the next game.

29 Anthony: $1\frac{1}{2}, 2, 2\frac{1}{2}, 4, 4\frac{1}{2}, 3, 3\frac{1}{2}, 5, 6, 6$
Katherine: $3, 3\frac{1}{2}, 4, 3, 3, 3\frac{1}{2}, 4, 4, 4\frac{1}{2}, 4$

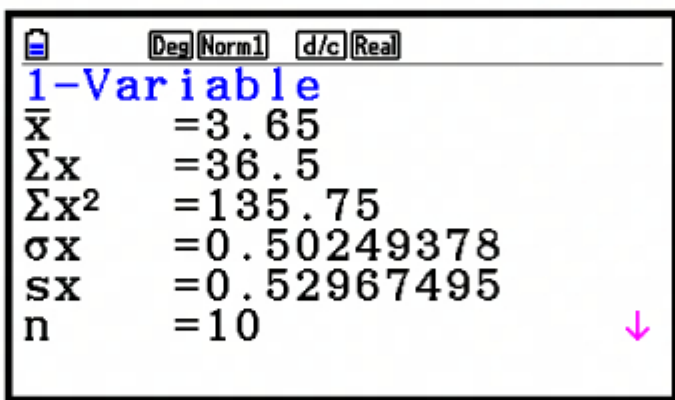
a Using technology:

Anthony:



The mean $\mu = 3.8$ hours and the standard deviation $\sigma \approx 1.50$ hours.

Katherine:



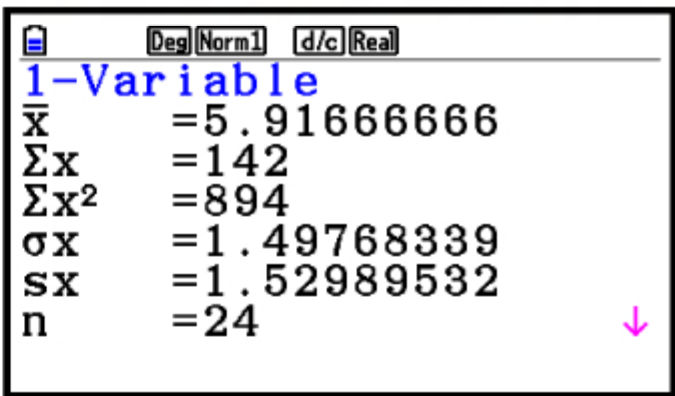
The mean $\mu = 3.65$ hours and the standard deviation $\sigma \approx 0.502$ hours.

- b Anthony's mean is higher than Katherine's, so Anthony generally practised for longer.
c Katherine's standard deviation is lower than Anthony's, so there is less deviation from the mean for her data set. Katherine therefore practised more consistently than Anthony.

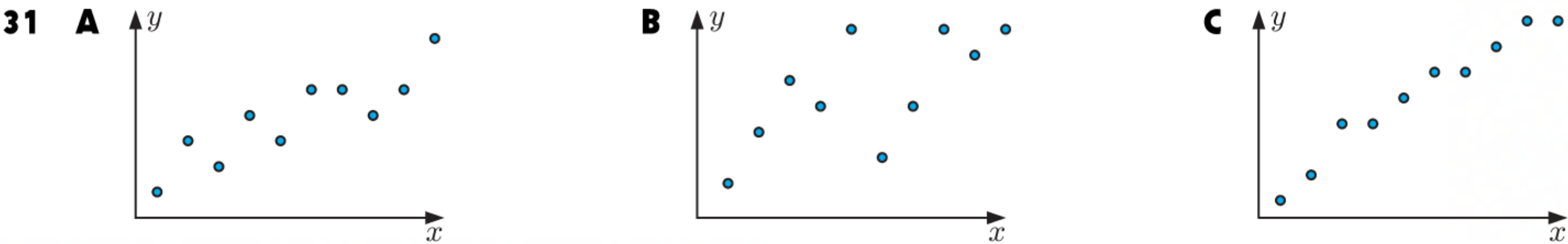
30

Mark	3	4	5	6	7	8	9	10
Frequency	1	3	5	8	4	2	0	1

Using technology:



The mean test score $\mu \approx 5.92$, and the population standard deviation $\sigma \approx 1.50$.



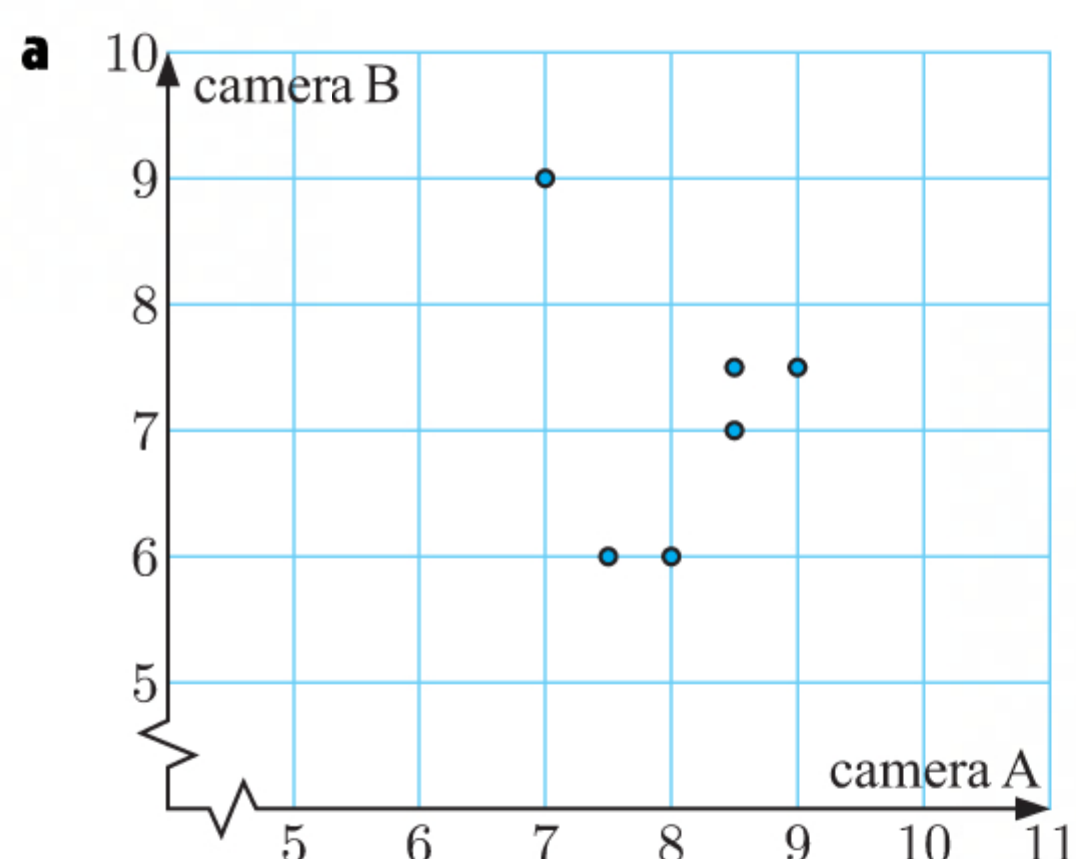
a In each scatter diagram, as x increases y generally increases.
 \therefore each scatter diagram shows a positive association between x and y .

b

Strength of correlation	Scatter diagram
Weak	B
Moderate	A
Strong	C

32

Camera A	8.5	8	9	7	8.5	7.5
Camera B	7	6	7.5	9	7.5	6



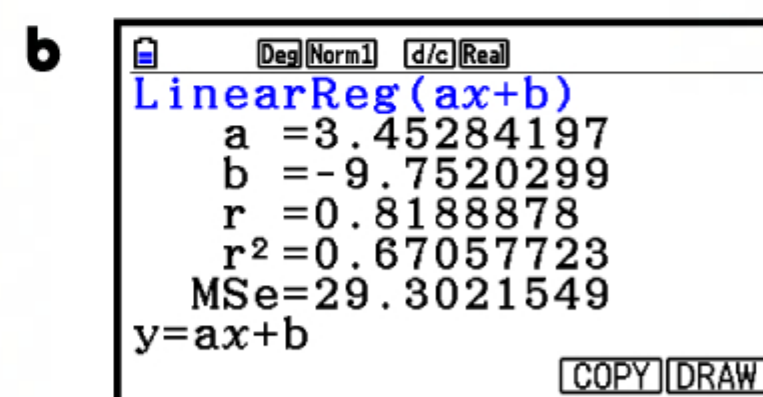
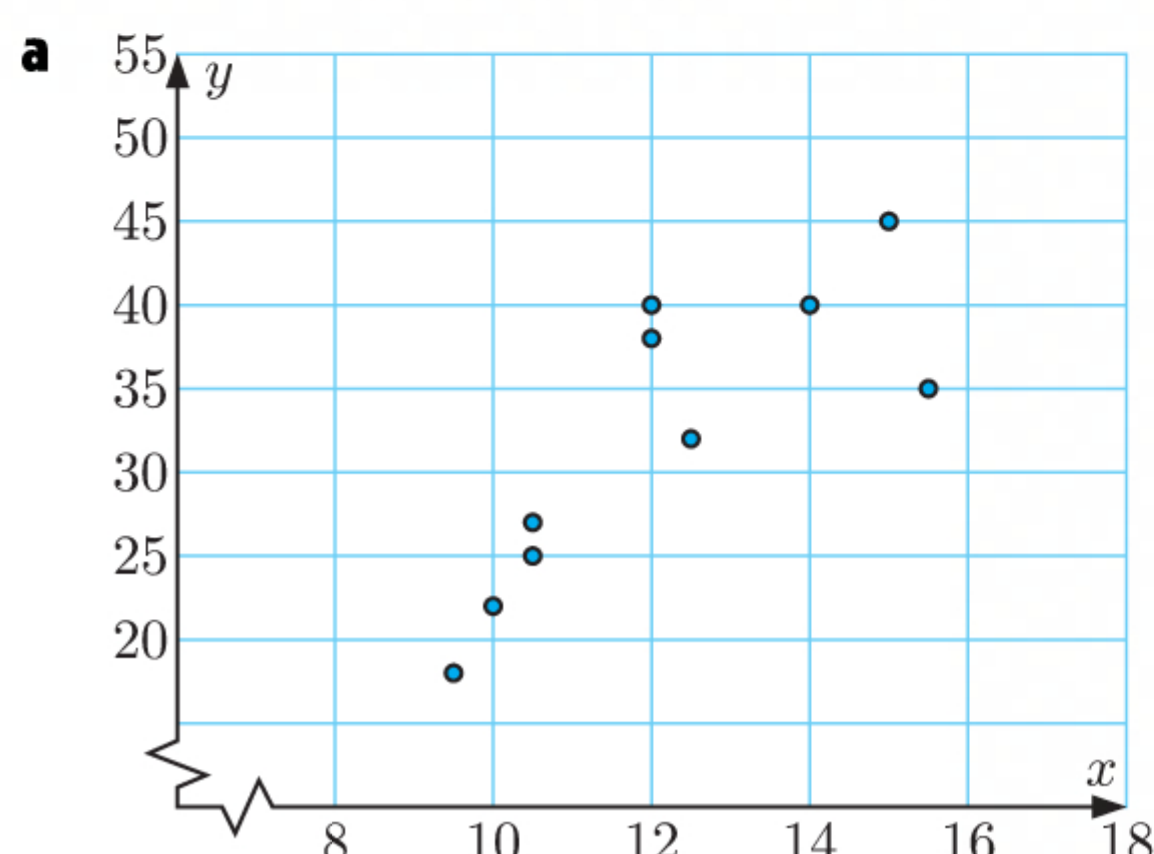
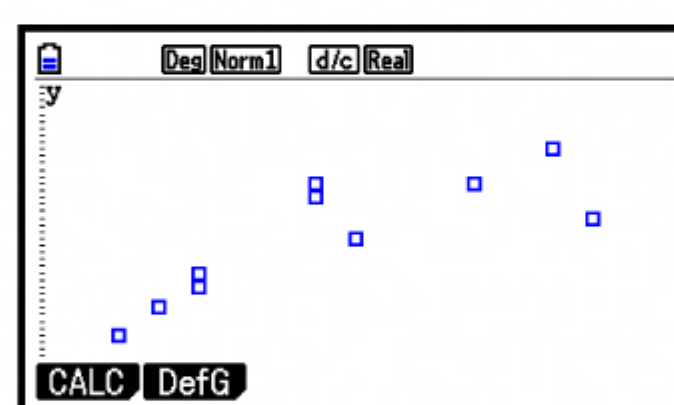
b The point (7, 9) appears to be an outlier. This corresponds to the review scoring camera A a 7, and camera B a 9.

- c**
- With the outlier removed, there appears to be a strong, positive, linear correlation between camera A's scores and camera B's scores.
 - No, an increase in camera A's scores is not likely to cause an increase in camera B's scores. It is more likely that both scores are related to the preferences of each reviewer.

33

Language (x)	12.5	15.0	10.5	12.0	9.5	10.5	15.5	10.0	14.0	12.0
Mathematics (y)	32	45	27	38	18	25	35	22	40	40

	List 1	List 2	List 3	List 4
SUB				
1	12.5	32		
2	15	45		
3	10.5	27		
4	12	38		

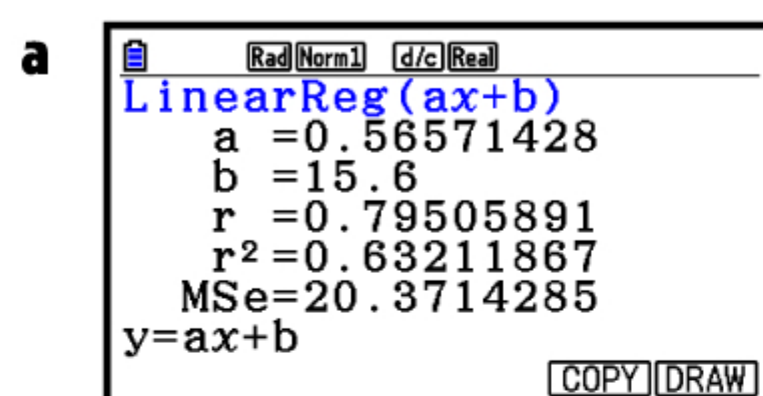


So, $r \approx 0.819$.

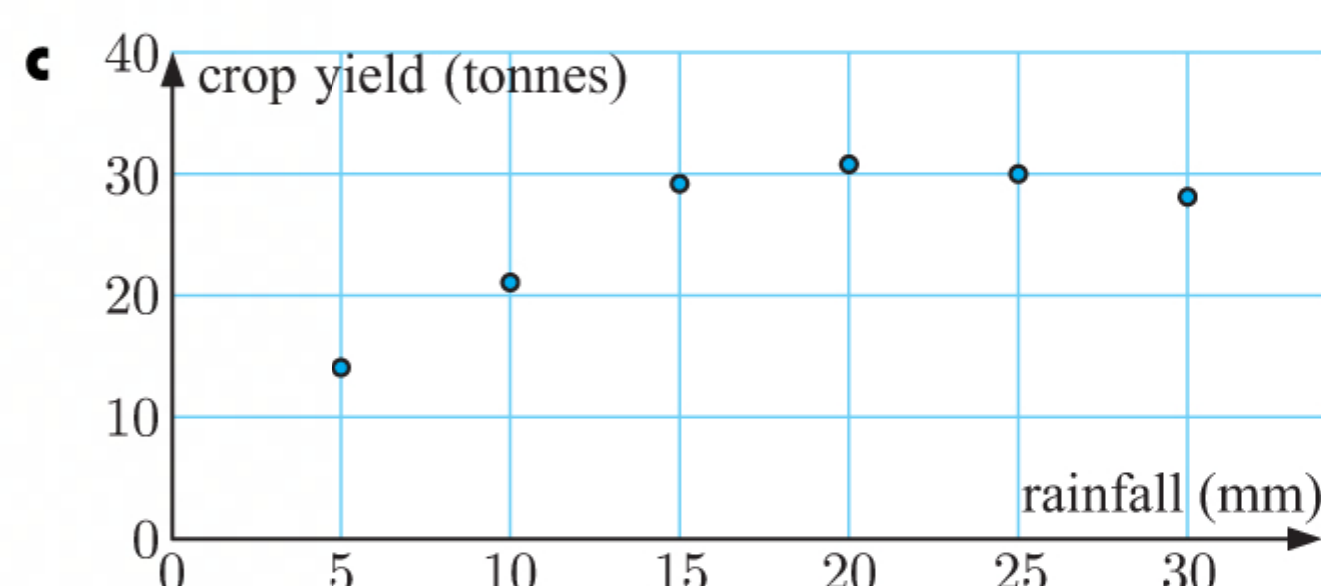
- c** The data suggests that there is a moderate, positive, linear correlation between the students' language scores and their mathematics scores. So, "Those who do well in languages also do well in mathematics." is a moderately reasonable statement.

34

Monthly rainfall (mm)	5	10	15	20	25	30
Crop yield (tonnes)	14	21	29	31	30	28



So, $r \approx 0.795$.

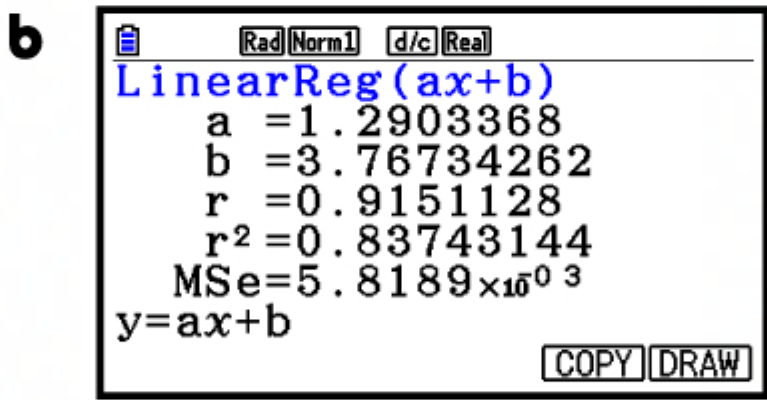
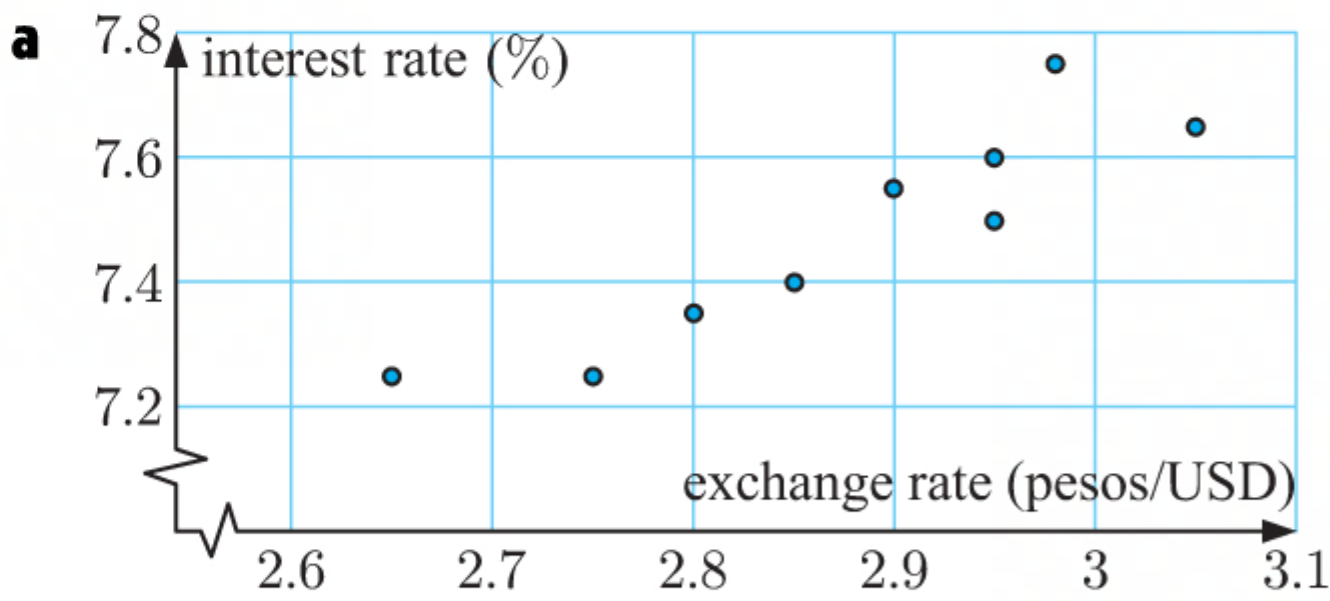


- b** A moderate, positive relationship may exist between crop yield and rainfall.

- d** The relationship between rainfall and crop yield does not appear to be linear and so Pearson's correlation coefficient may not be appropriate for this data.

35

Exchange rate (pesos/USD)	2.85	2.95	2.90	2.75	2.65	2.80	3.05	2.98	2.95
Interest rate (%)	7.40	7.50	7.55	7.25	7.25	7.35	7.65	7.75	7.60

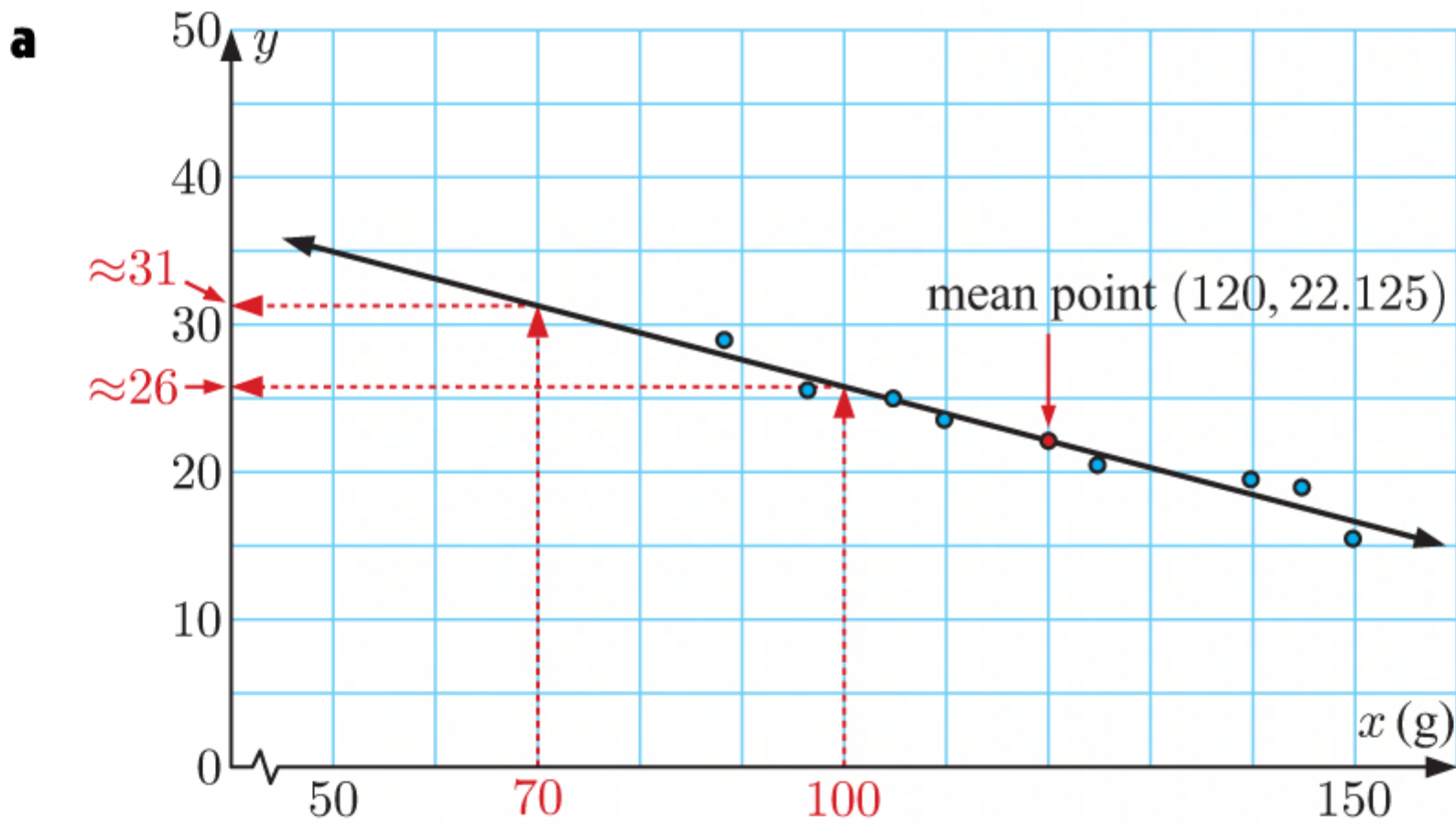


So, $r \approx 0.915$.

c There is a strong, positive, linear relationship between the exchange rate and interest rate.

36

Median weight (x g)	88	97	105	110	125	140	145	150
Number in bag (y)	28	26	26	23	21	19	18	16



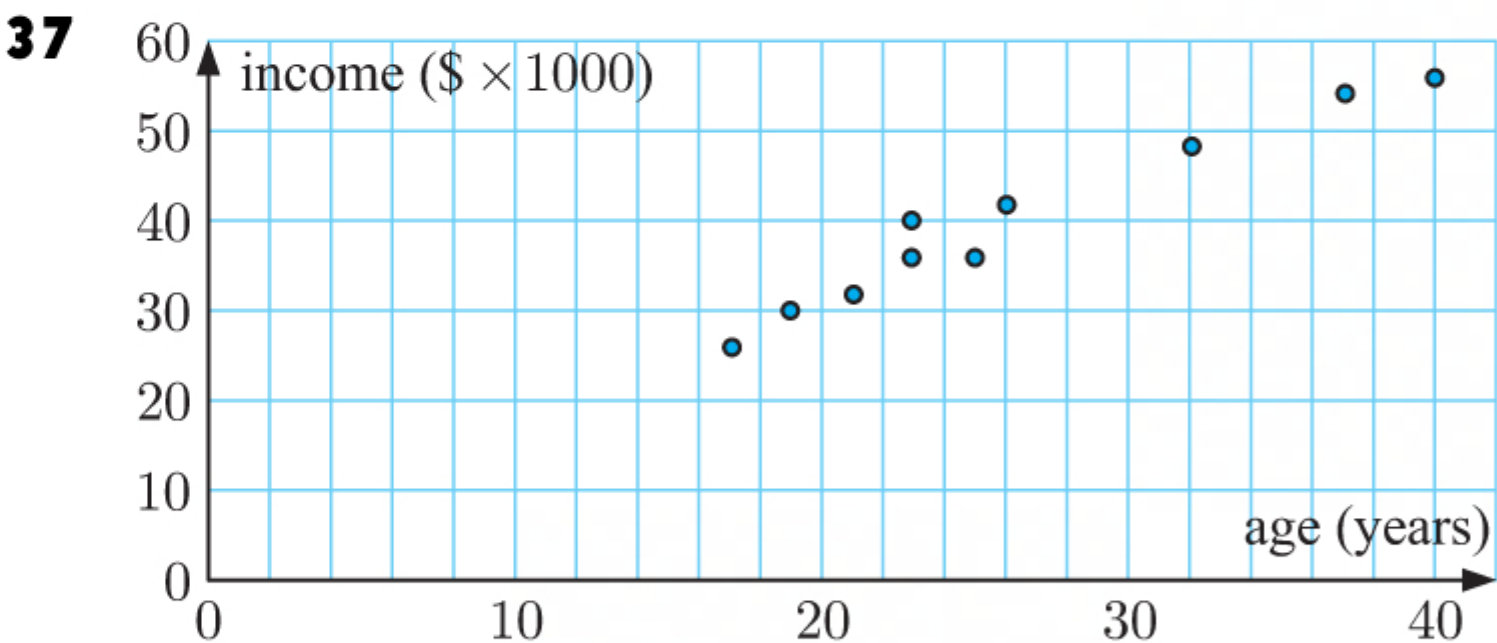
b i When $x = 100$, $y \approx 26$.

If the median weight is 100 grams, there are about 26 potatoes in a bag.

ii When $x = 70$, $y \approx 31$.

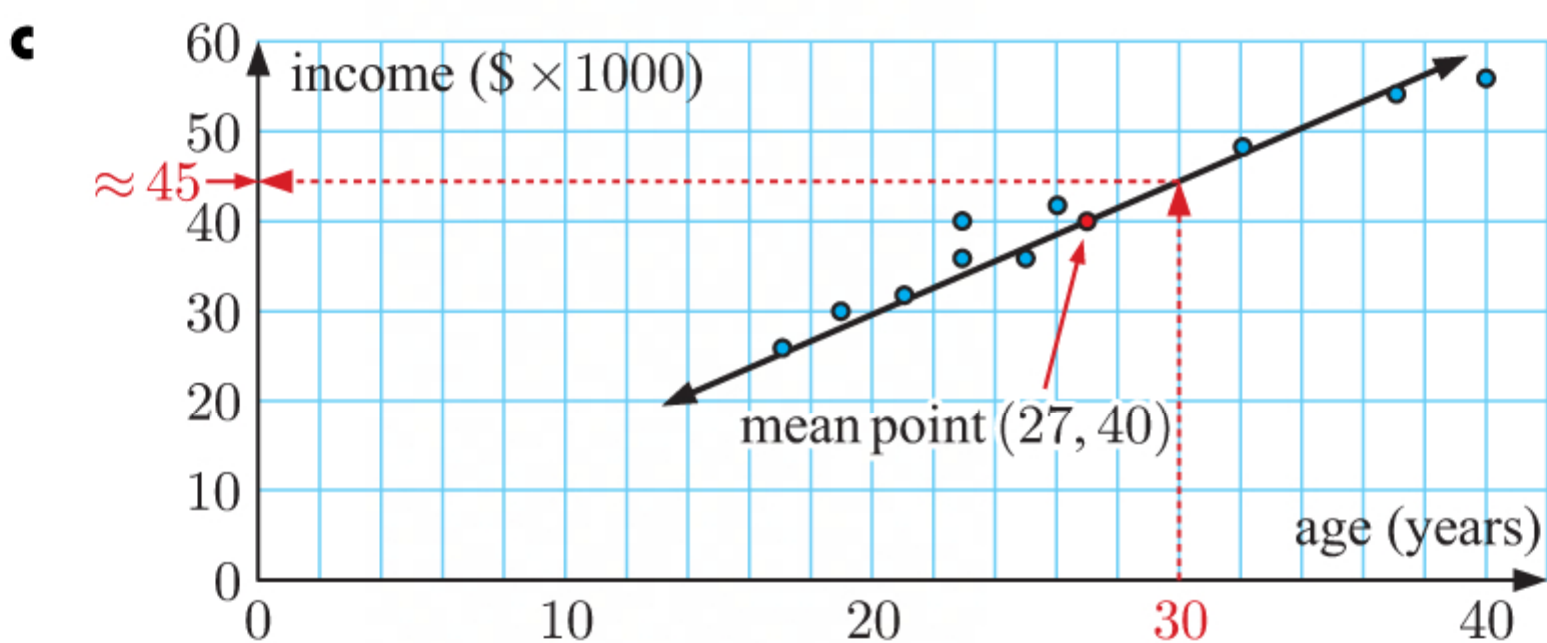
If the median weight is 70 grams, there are about 31 potatoes in a bag.

c The estimate in **b i** is an interpolation, and the estimate in **b ii** is an extrapolation.
So, the estimate in **b i** is likely to be more reliable.



a There is a strong, positive correlation between the age of an individual and their annual income.

b No, the relationship is more likely dependent on the amount of professional experience or qualifications an individual has.



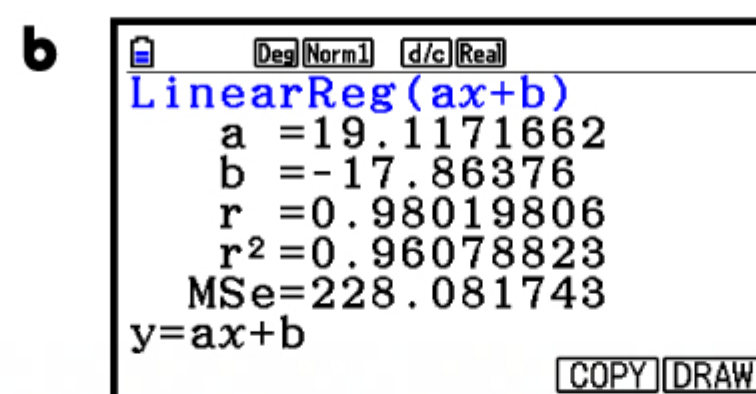
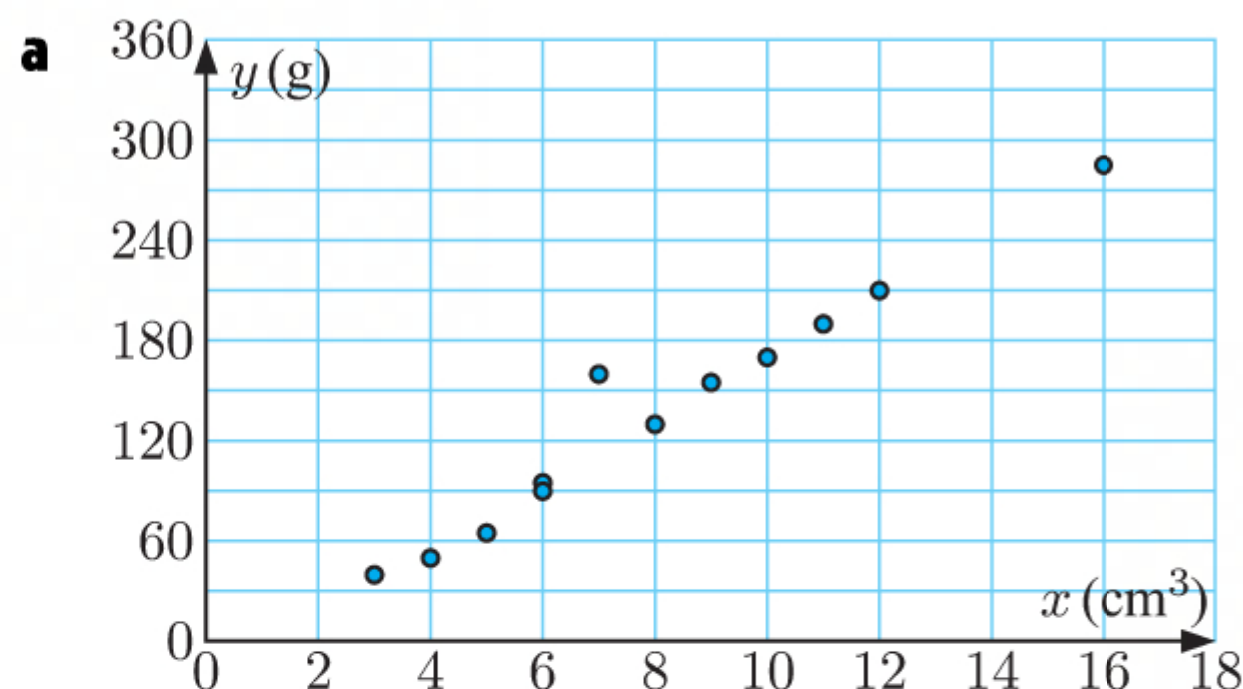
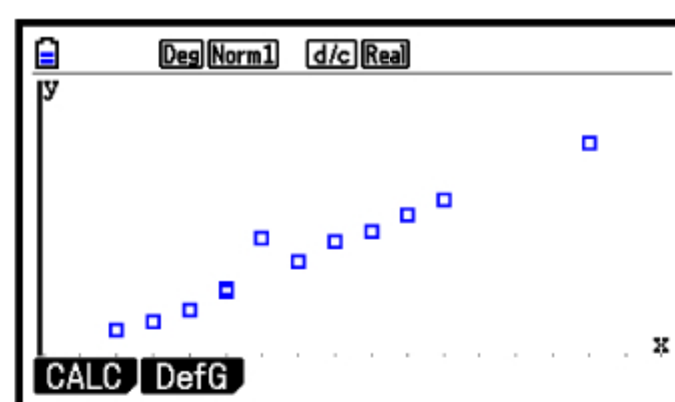
d When $x = 30$, $y \approx 45$.

The annual income of someone who is 30 years old is approximately \$45 000. This is an interpolation, so the estimate is reliable.

38

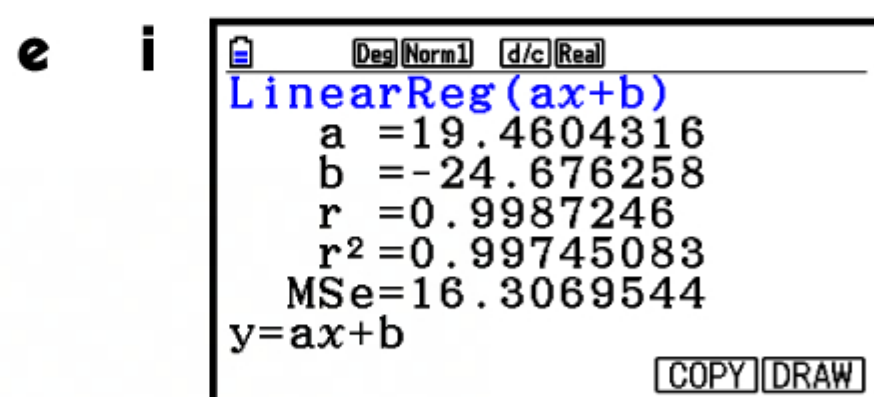
Sample	A	B	C	D	E	F	G	H	I	J	K	L
Volume ($x \text{ cm}^3$)	3	6	4	7	16	8	5	12	9	6	10	11
Mass ($y \text{ g}$)	40	95	50	160	285	130	65	210	155	90	170	190

	List 1	List 2	List 3	List 4
SUB				
1	3	40		
2	6	95		
3	4	50		
4	7	160		


 So, $r \approx 0.980$.

c There appears to be a strong, positive correlation between the *volume* of a sample of silver and its *mass*.

d The data point (7, 160) which corresponds to sample D appears to be an outlier. We therefore agree with the jeweller that there is a fake sample.



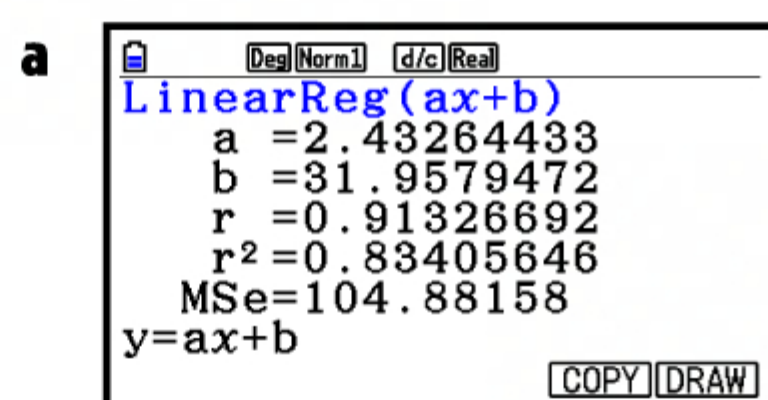
Using technology, the regression line is $y \approx 19.5x - 24.7$.

ii When $x = 7$, $y \approx 19.5(7) - 24.7 \approx 112$

So, a sample of silver with volume 7 cm^3 would weigh approximately 112 g.

39

Study time ($x \text{ h}$)	7	6	3	16	15	11	18	32	20
Result ($y \%$)	56	42	25	80	65	60	85	96	90



Using technology, the least squares regression line is $y \approx 2.43x + 32.0$.

b From a, $r \approx 0.913$.

So, there is a strong, positive correlation between the number of hours that a student studies and their examination result.

c Yes, this is a causal relationship as spending more time studying for the examination is likely to cause a better result.

d When $y = 70$, $70 \approx 2.43x + 32.0$

$$\therefore 38 \approx 2.43x$$

$$\therefore x \approx 15.6$$

So, Tony studied for approximately 15.6 hours.

e The y -intercept of the line of best fit ≈ 32.0 . This indicates that if a student did not spend any time studying, they would obtain a result of 32% on average.

The gradient of the line of best fit ≈ 2.43 . This indicates that for every additional hour of study, the result obtained increases by an average of 2.43%.

40

Time (t days)	0	1	2	3	4	5	6	7	8	9
Height (h mm)	5	5.7	5.7	6.2	6.8	7.1	8	8.3	9	9.3

a

Deg Norm1 d/c Real LinearReg(ax+b) a = 0.48787878 b = 4.91454545 r = 0.99265002 r ² = 0.98535406 MSe = 0.03648484 y = ax + b [COPY] [DRAW]

So, $r \approx 0.993$.

b r is very close to 1 which indicates a very strong correlation between the variables.

The sign of r is positive which indicates that the variables are positively correlated. An increase in one variable results in an increase in the other.

c $h \approx 0.4879t + 4.9145$

i When $t = 14$, $h \approx 0.4879(14) + 4.9145$
 ≈ 11.7

\therefore after 14 days, the grass is about 11.7 mm high.

ii When $h = 20$, $20 \approx 0.4879t + 4.9145$

$$\therefore 15.0855 \approx 0.4879t$$

$$\therefore t \approx 30.9$$

\therefore the grass reaches a height of 20 mm after about 30.9 days.

41

Age (x years)	28	40	21	38	30	26	18	32	25	29	20	24
Time (y min)	20	32	15	40	26	25	19	28	21	25	16	22

a

Rad Norm1 d/c Real LinearReg(ax+b) a = 0.92969136 b = -1.5606535 r = 0.89822155 r ² = 0.80680195 MSe = 10.4504043 y = ax + b [COPY] [DRAW]

So, $r \approx 0.898$.

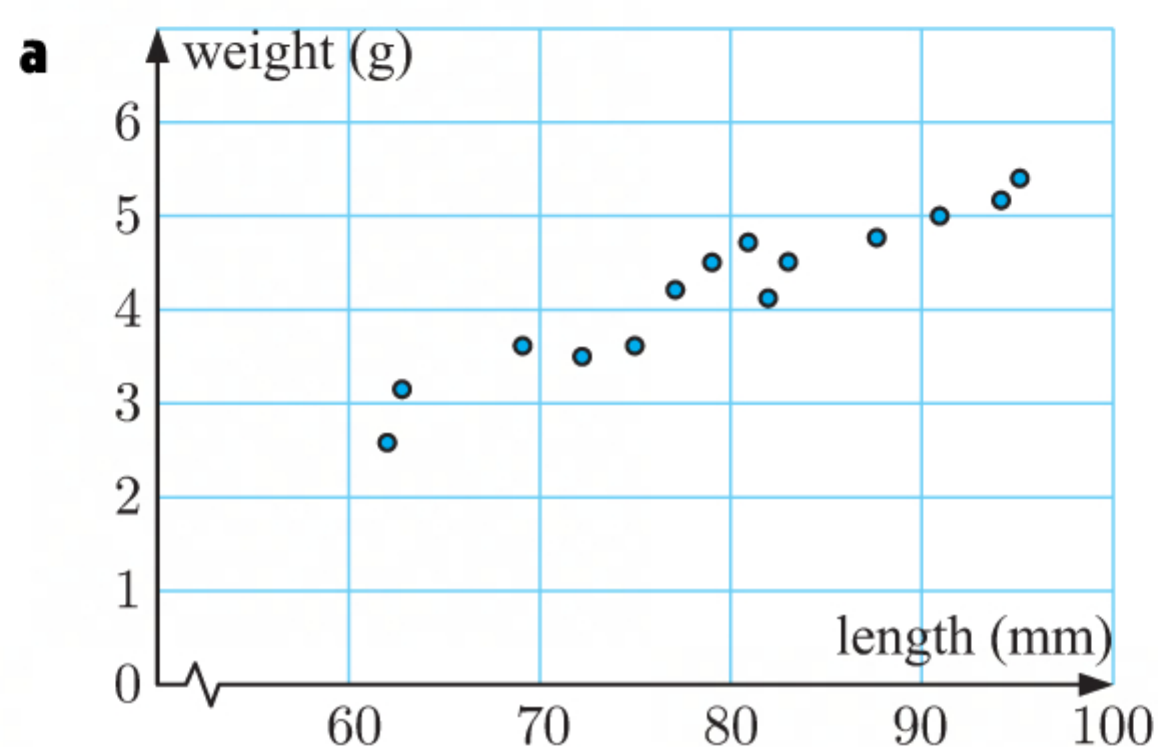
There is a strong, positive correlation between the age of contestants and the time taken to complete the task.

b i From the screenshot in **a**, the linear regression line is $y \approx 0.930x - 1.56$.

ii The gradient ≈ 0.930 . This means that an increase of one year in age will add about 0.93 minutes to the time to complete the task.

42

Length (mm)	95	83	91	82	75	62	79	63	81	69	94	88	72	77
Weight (g)	5.4	4.5	5.0	4.1	3.7	2.6	4.5	3.1	4.7	3.7	5.1	4.8	3.6	4.2



b

Rad Norm1 d/c Real LinearReg(ax+b) a = 0.07291841 b = -1.5723113 r = 0.96187039 r ² = 0.92519465 MSe = 0.05184722 y = ax + b [COPY] [DRAW]

So, $r \approx 0.962$.

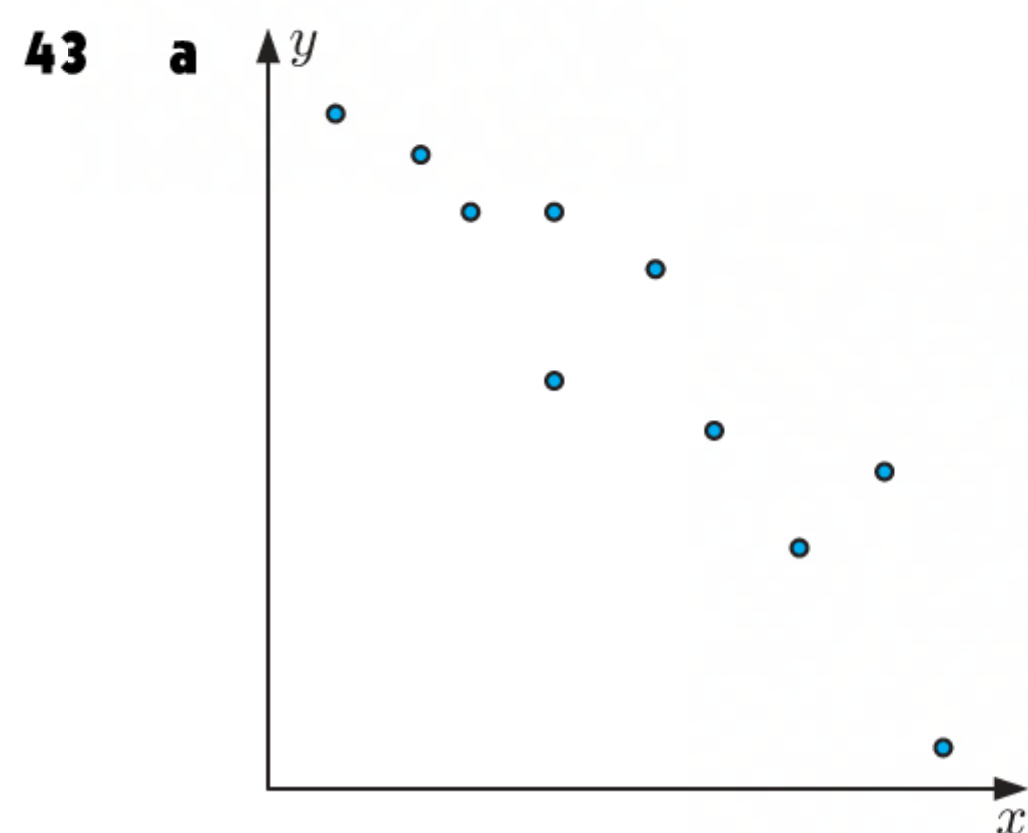
c There is very strong, positive, linear correlation between *length* and *weight*.

d From the screenshot in **b**, the equation of the least squares regression line is $y \approx 0.0729x - 1.57$.

e i When $x = 110$ mm,
 $y \approx 0.0729 \times 110 - 1.57$
 ≈ 6.45 g

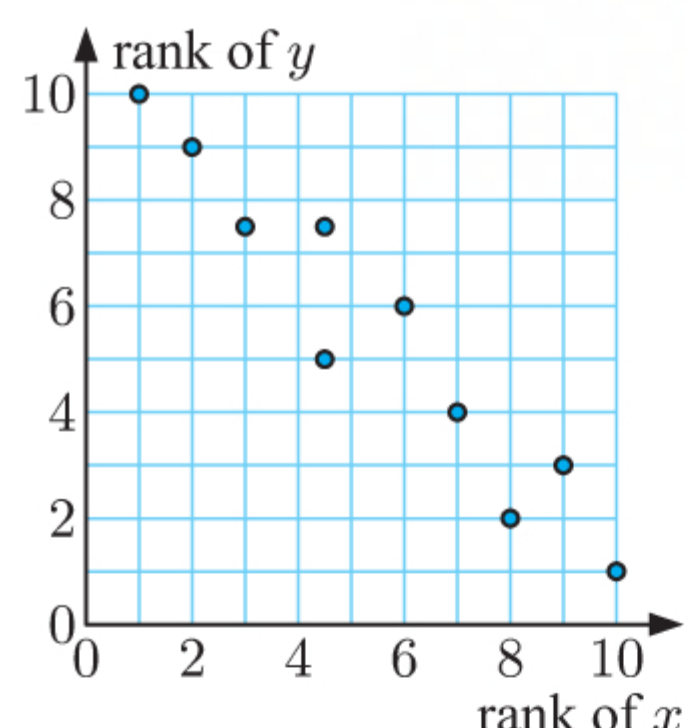
ii When $x = 70$ mm,
 $y \approx 0.0729 \times 70 - 1.57$
 ≈ 3.53 g

f The prediction in **e ii** is more likely to be reliable, as it is an interpolation.



As x increases, y generally decreases.

So as the rank of x increases, the rank of y generally decreases.

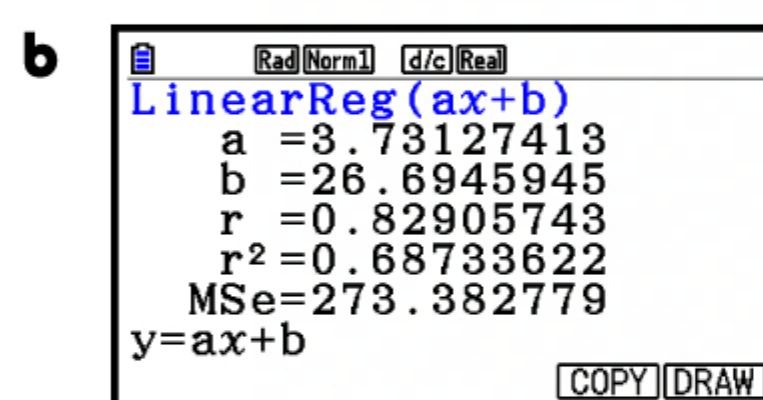
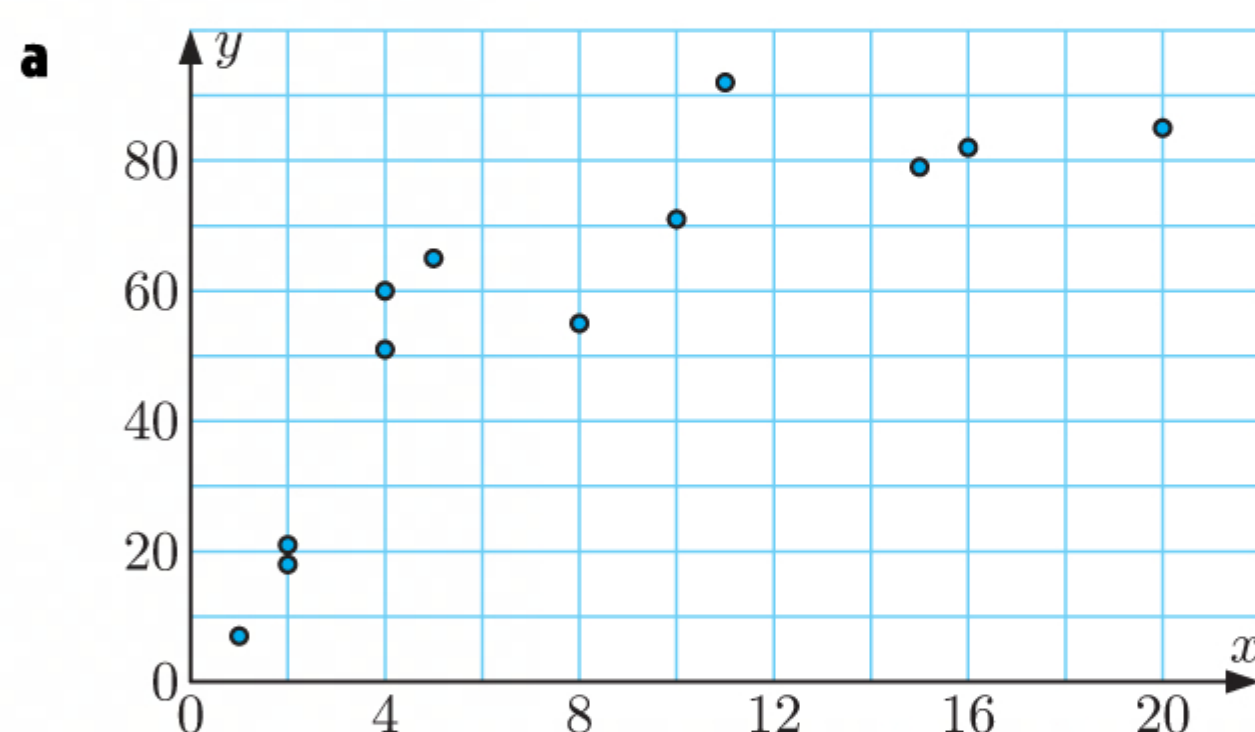


Looking at the first few points, we see that **B** is the correct rank scatter diagram.

- b** The rank scatter diagram has a strong, negative linear correlation, so the correct value of Spearman's rank correlation coefficient is $r_s \approx -0.960$ (**C**).

44

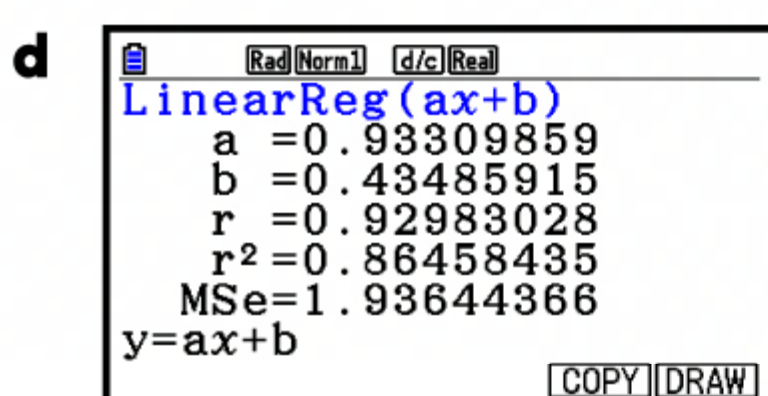
<i>Number of matches played (x)</i>	11	5	10	16	2	1	8	20	15	2	4	4
<i>Highest score (y)</i>	92	65	71	82	21	7	55	85	79	18	60	51



So, $r_p \approx 0.829$.

c

<i>Number of matches played (x)</i>	11	5	10	16	2	1	8	20	15	2	4	4
<i>rank of x</i>	9	6	8	11	2.5	1	7	12	10	2.5	4.5	4.5
<i>Highest score (y)</i>	92	65	71	82	21	7	55	85	79	18	60	51
<i>rank of y</i>	12	7	8	10	3	1	5	11	9	2	6	4



So, $r_s \approx 0.930$.

- e** The scatter diagram in **a** shows a non-linear trend.

Using **d**, there appears to be a strong, positive, non-linear correlation between *number of matches played* and *highest score*.

45

<i>Time (min)</i>	<i>Frequency</i>
35 - 39	10
40 - 44	46
45 - 49	43
50+	15
<i>Total</i>	114

a $P(40 \text{ to } 44 \text{ minutes}) \approx \frac{46}{114}$
 ≈ 0.404

b $P(\text{at least } 50 \text{ minutes}) \approx \frac{15}{114}$
 ≈ 0.132

c $P(\text{between } 35 \text{ and } 49 \text{ minutes inclusive}) \approx \frac{10 + 46 + 43}{114}$
 ≈ 0.868

46

Division	2017	2018	2019
1	4	5	5
2	6	7	8
3	13	12	14
4	18	10	14
5	20	17	16
Total	61	51	57

a $P(\text{player in the 2017 tournament played in division 1}) \approx \frac{4}{61}$ ← number of division 1 players in 2017 tournament
← total number of players in 2017 tournament
 ≈ 0.0656

b There were $13 + 12 + 14 = 39$ division 3 players in total,
and $61 + 51 + 57 = 169$ players in total.

$\therefore P(\text{player in any of the past tournaments played in division 3}) \approx \frac{39}{169}$
 ≈ 0.231

c In the 2019 tournament, 8 players played in division 2 and 14 players played in division 4. So, $57 - 8 - 14 = 35$ players in the 2019 tournament did *not* play in division 2 or 4.

$\therefore P(\text{player in the 2019 tournament did not play in division 2 or 4}) \approx \frac{35}{57}$
 ≈ 0.614

47 a

	< 40	40 - 59	≥ 60	Total
Male	56	127	419	602
Female	75	113	230	418
Total	131	240	649	1020

b i 602 out of the 1020 patients were male.

$\therefore P(\text{male}) \approx \frac{602}{1020} \approx 0.590$

ii 75 out of the 1020 patients were female and younger than 40.

$\therefore P(\text{female and younger than 40}) \approx \frac{75}{1020} \approx 0.0735$

iii 230 out of the 418 female patients were 60 or older.

$\therefore P(60 \text{ or older, given they were female}) \approx \frac{230}{418} \approx 0.550$

iv $127 + 419 = 546$ out of the $240 + 649 = 889$ patients who were 40 or older were male.

$\therefore P(\text{male, given they were 40 or older}) \approx \frac{546}{889} \approx 0.614$

48 a $P(\text{card is black}) = \frac{26}{52} = \frac{1}{2}$

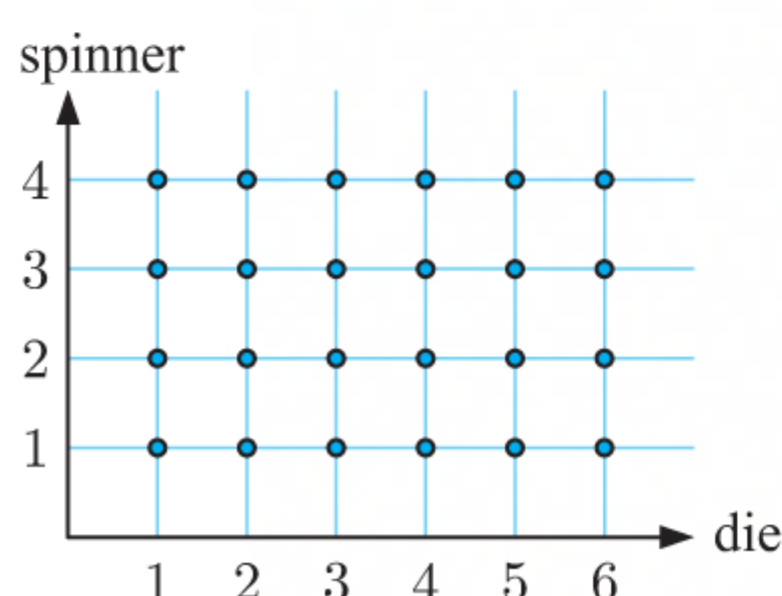
b $P(\text{card is a club}) = \frac{13}{52} = \frac{1}{4}$

c $P(\text{card is a red number between 3 and 6 inclusive}) = \frac{2 \times 4}{52} = \frac{2}{13}$

d $P(\text{card is not a number}) = \frac{4 \times 3}{52} = \frac{3}{13}$

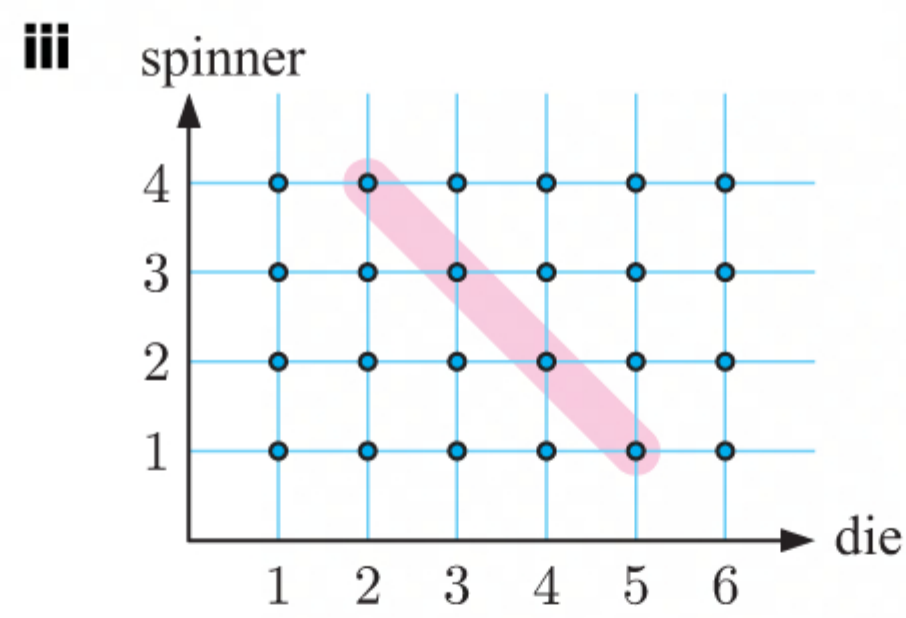
e $P(\text{card is ace of spades}) = \frac{1}{52}$

49 a

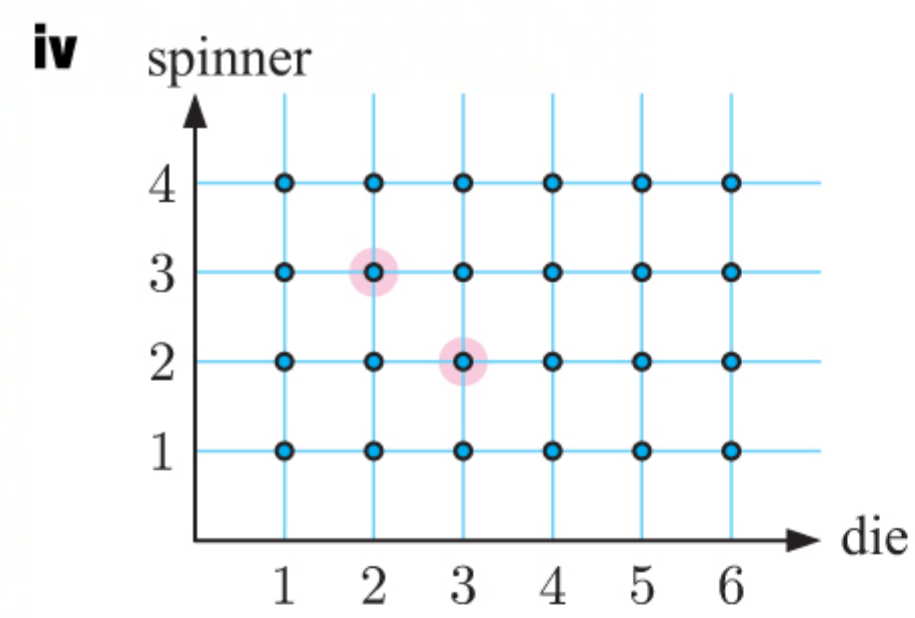


b i $P(\text{two 1s}) = \frac{1}{24}$

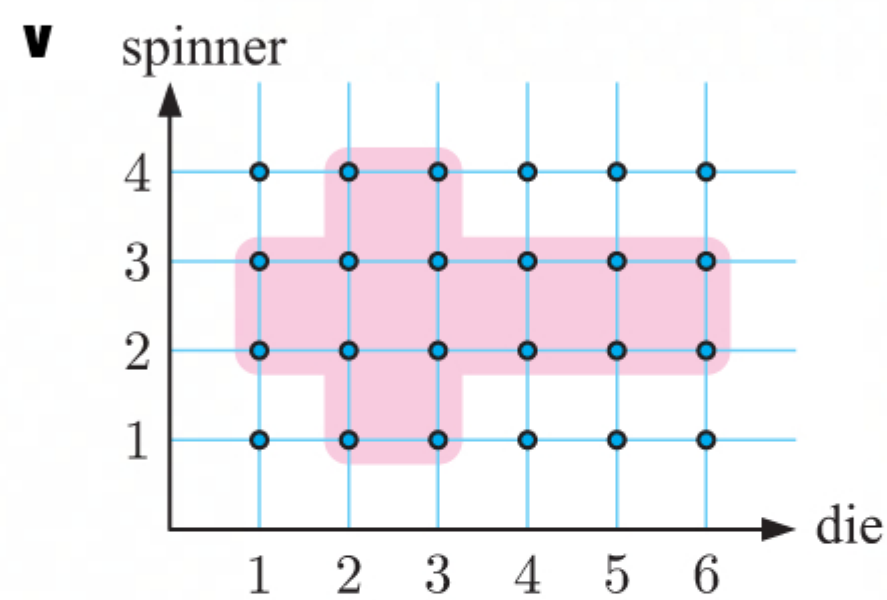
ii $P(\text{two 5s}) = 0$ {the spinner does not have a 5}



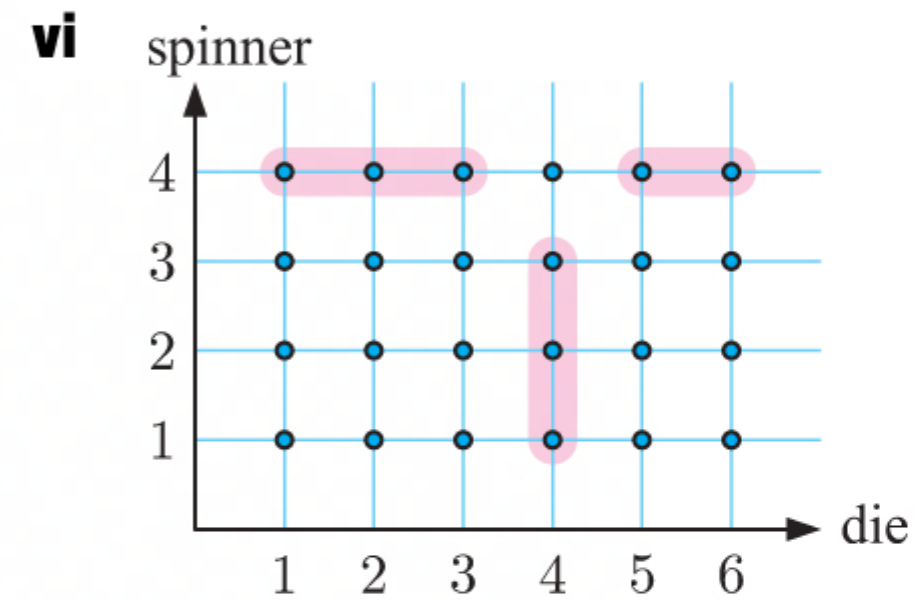
$$\begin{aligned} P(\text{a sum of 6}) &= \frac{4}{24} \\ &= \frac{1}{6} \end{aligned}$$



$$\begin{aligned} P(\text{a 2 and a 3}) &= \frac{2}{24} \\ &= \frac{1}{12} \end{aligned}$$



$$\begin{aligned} P(\text{a 2 or a 3 (or both)}) &= \frac{16}{24} \\ &= \frac{2}{3} \end{aligned}$$



$$\begin{aligned} P(\text{exactly one 4}) &= \frac{8}{24} \\ &= \frac{1}{3} \end{aligned}$$

50 a

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ \therefore 0.78 &= 0.37 + 0.41 - P(A \cap B) \\ \therefore P(A \cap B) &= 0 \end{aligned}$$

b Since $P(A \cap B) = 0$, A and B are mutually exclusive events.

51 A and B are mutually exclusive $\therefore P(A \cap B) = 0$

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

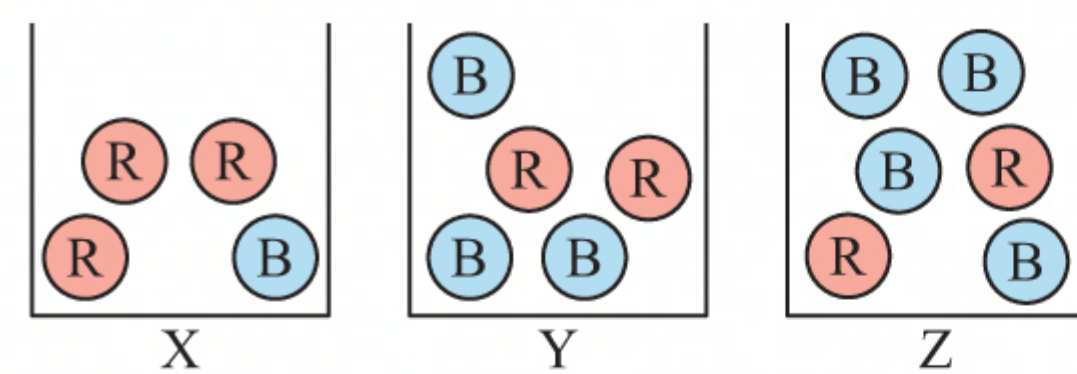
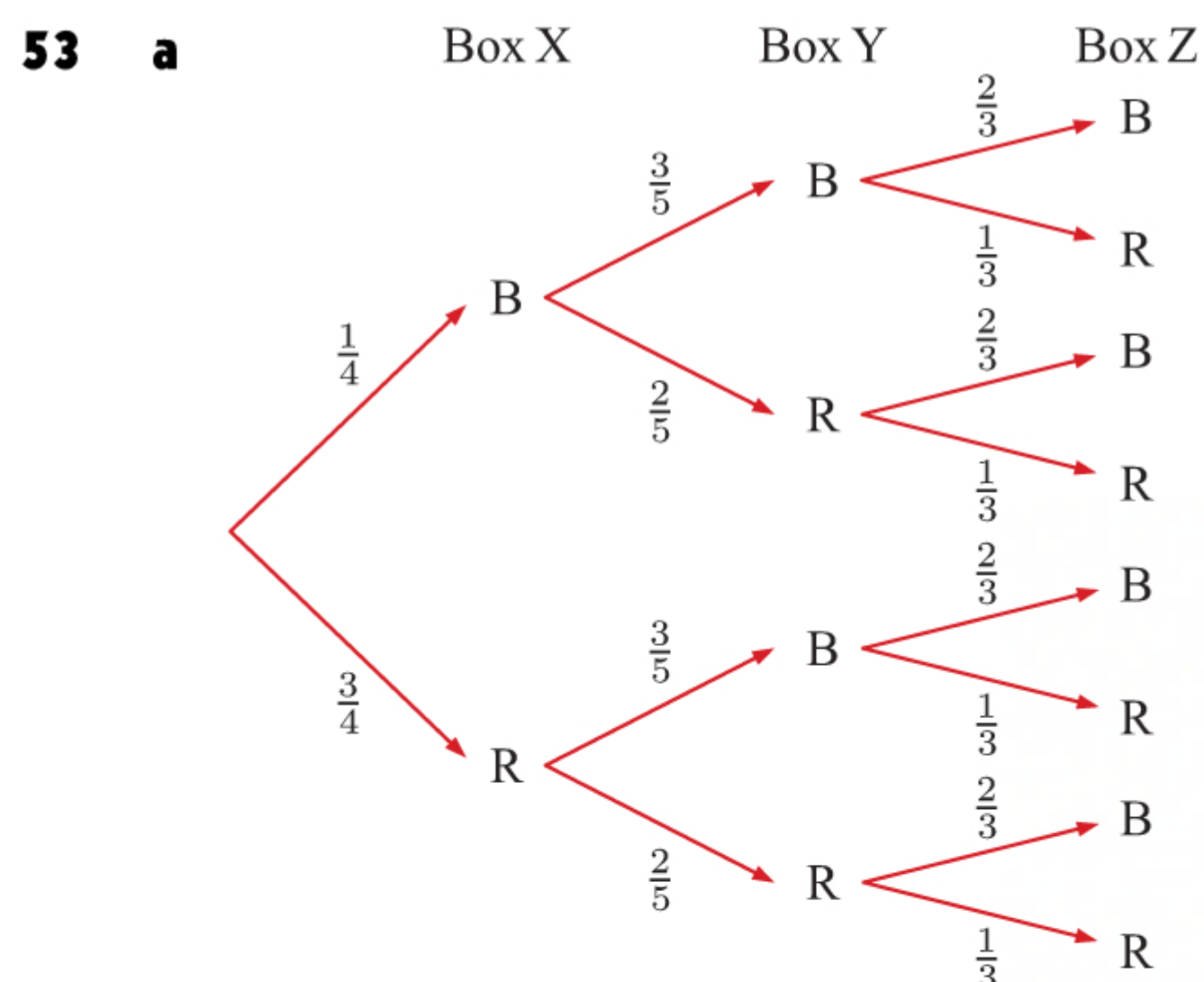
$$\begin{aligned} &= P(A) + P(B) \\ \therefore 0.55 &= P(A) + 0.3 \\ \therefore P(A) &= 0.25 \end{aligned}$$

52 $P(A \cup B) = 1 - P((A \cup B)')$

$$\begin{aligned} &= 1 - \frac{1}{12} \\ &= \frac{11}{12} \end{aligned}$$

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\begin{aligned} \therefore \frac{11}{12} &= \frac{23}{50} + \frac{5}{7} - P(A \cap B) \\ \therefore P(A \cap B) &= \frac{541}{2100} \end{aligned}$$



b i $P(\text{exactly 2 red balls are drawn})$

$$\begin{aligned} &= P(RRB) + P(RBR) + P(BRR) \\ &= \frac{3}{4} \times \frac{2}{5} \times \frac{2}{3} + \frac{3}{4} \times \frac{3}{5} \times \frac{1}{3} + \frac{1}{4} \times \frac{2}{5} \times \frac{1}{3} \\ &= \frac{1}{5} + \frac{3}{20} + \frac{1}{30} \\ &= \frac{23}{60} \end{aligned}$$

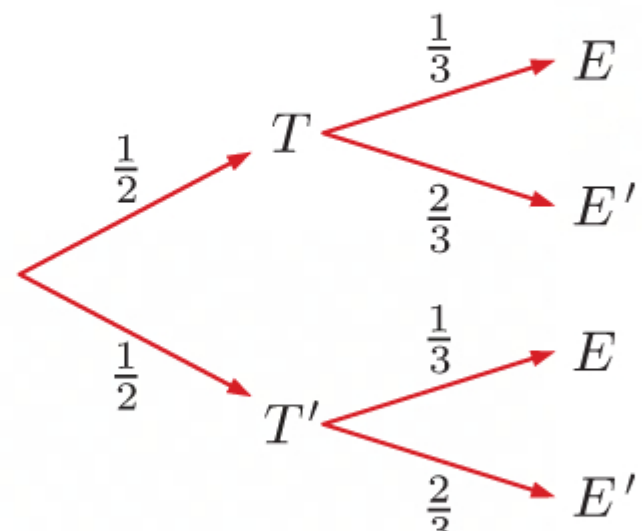
ii $P(\text{blue balls are drawn from boxes X and Z})$

$$\begin{aligned} &= P(BBB) + P(BRB) \\ &= \frac{1}{4} \times \frac{3}{5} \times \frac{2}{3} + \frac{1}{4} \times \frac{2}{5} \times \frac{2}{3} \\ &= \frac{1}{10} + \frac{1}{15} \\ &= \frac{1}{6} \end{aligned}$$

$$\begin{aligned}
 \text{iii } P(\text{at most one blue ball is drawn}) &= P(\text{no blue balls are drawn}) + P(\text{exactly one blue ball is drawn}) \\
 &= P(RRR) + [P(BRR) + P(RBR) + P(RRB)] \\
 &= \frac{3}{4} \times \frac{2}{5} \times \frac{1}{3} + \left[\frac{1}{4} \times \frac{2}{5} \times \frac{1}{3} + \frac{3}{4} \times \frac{3}{5} \times \frac{1}{3} + \frac{3}{4} \times \frac{2}{5} \times \frac{2}{3} \right] \\
 &= \frac{1}{10} + \frac{1}{30} + \frac{3}{20} + \frac{1}{5} \\
 &= \frac{29}{60}
 \end{aligned}$$

c If an extra red ball is added to box Y, the probabilities in **b i** and **b iii** will be affected.

54 a

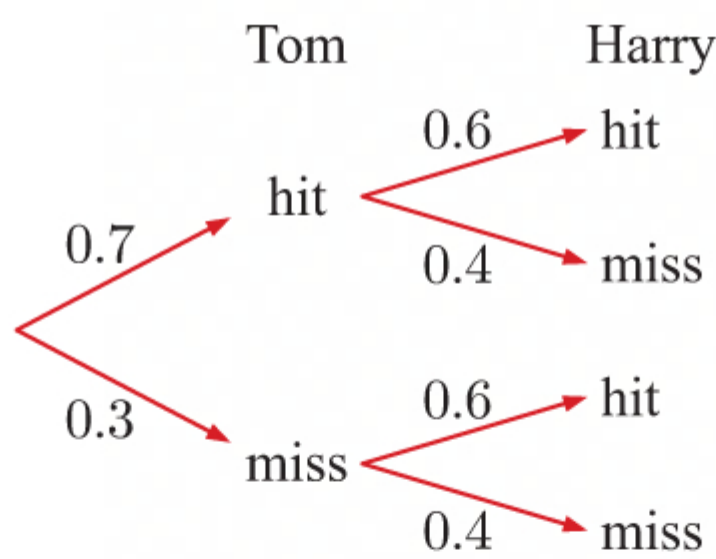
	outcome	probability
	T and E	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
	T and E'	$\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$
	T' and E	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
	T' and E'	$\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$
	total	$\frac{6}{6} = 1$

b i $P(T \cap E') = \frac{1}{3}$ {from **a**}

ii $P(T \cup E') = P(T) + P(E') - P(T \cap E')$
 $= \frac{1}{2} + \frac{2}{3} - \frac{1}{3}$ {from **a**}

$$= \frac{5}{6}$$

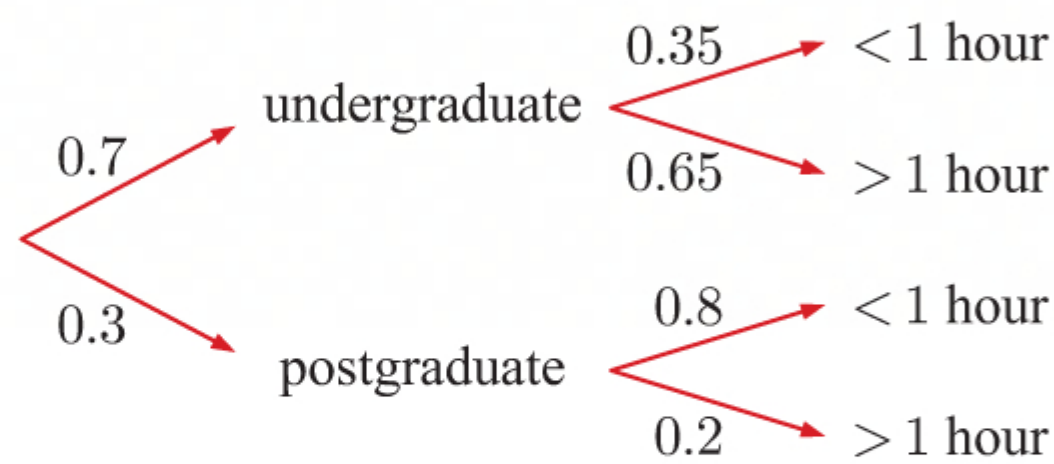
55



a $P(\text{only one of them is successful})$
 $= 0.7 \times 0.4 + 0.3 \times 0.6$
 $= 0.46$

b $P(\text{at least one is successful})$
 $= 1 - P(\text{both miss})$
 $= 1 - 0.3 \times 0.4$
 $= 1 - 0.12$
 $= 0.88$

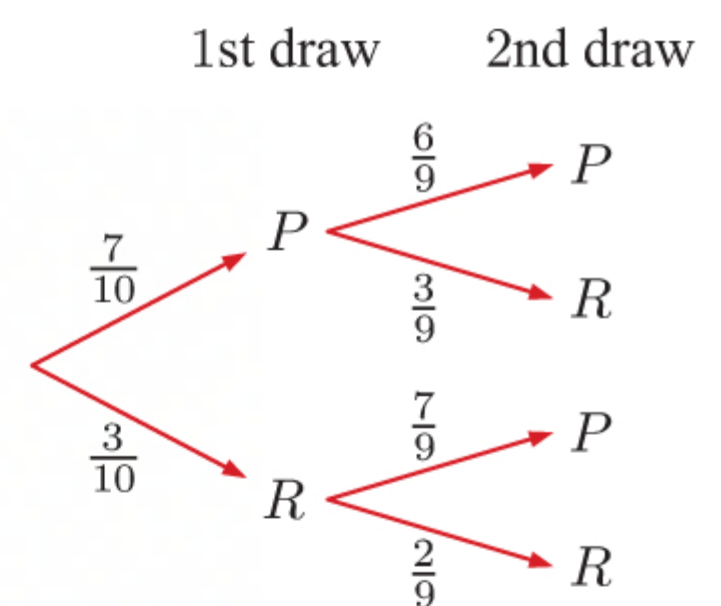
56 a



b i $P(\text{spends more than 1 hour at lunch}) = 0.7 \times 0.65 + 0.3 \times 0.2$
 $= 0.515$

ii $P(\text{is not a postgraduate student who spends less than 1 hour at lunch})$
 $= 1 - P(\text{is a postgraduate student who spends less than 1 hour at lunch})$
 $= 1 - 0.3 \times 0.8$
 $= 0.76$

57 a Let P be the event that a purple ticket is drawn, and R be the event that a red ticket is drawn.

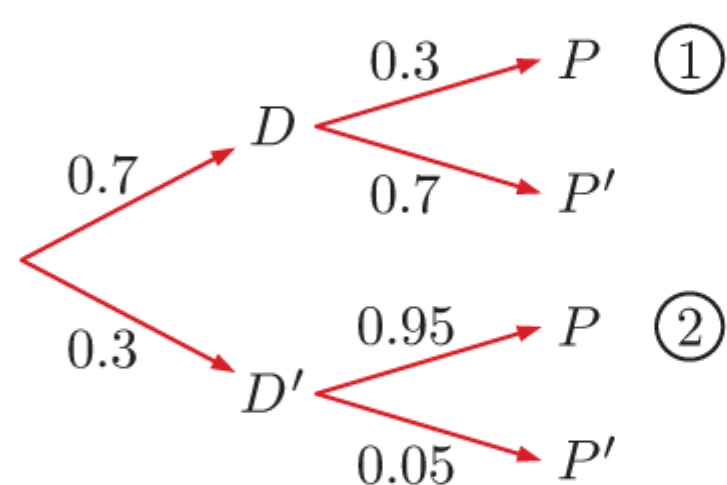


b i $P(\text{at least one red ticket})$
 $= 1 - P(\text{no red tickets})$
 $= 1 - P(P P)$
 $= 1 - \frac{7}{10} \times \frac{6}{9}$
 $= 1 - \frac{42}{90}$
 $= \frac{48}{90} = \frac{8}{15}$

ii $P(\text{one ticket of each colour})$
 $= P(P R \text{ or } R P)$
 $= \frac{7}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{7}{9}$
 $= \frac{21}{90} + \frac{21}{90}$
 $= \frac{42}{90} = \frac{7}{15}$

iii $P(\text{purple ticket second})$
 $= P(P P \text{ or } R P)$
 $= \frac{7}{10} \times \frac{6}{9} + \frac{3}{10} \times \frac{7}{9}$
 $= \frac{42}{90} + \frac{21}{90}$
 $= \frac{63}{90} = \frac{7}{10}$

- 58** Let D be the event that Donna goes shopping with Nick, and P be the event that Nick purchases a packet of potato chips.



$$\begin{aligned} \mathbf{a} \quad P(P) &= \underbrace{P(D \cap P)}_{\text{①}} + \underbrace{P(D' \cap P)}_{\text{②}} \\ &= 0.7 \times 0.3 + 0.3 \times 0.95 \\ &= 0.495 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad P(D | P) &= \frac{P(D \cap P)}{P(P)} \\ &= \frac{0.7 \times 0.3}{0.495} \quad \leftarrow \text{①} \\ &\quad \leftarrow \text{from a} \\ &\approx 0.424 \end{aligned}$$

$$\begin{aligned} \mathbf{59} \quad P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ \therefore 0.63 &= P(A) + 0.36 - P(A)P(B) \quad \{A \text{ and } B \text{ are independent}\} \\ \therefore 0.27 &= P(A) - 0.36 \times P(A) \\ \therefore 0.27 &= 0.64 \times P(A) \\ \therefore P(A) &= \frac{0.27}{0.64} \approx 0.422 \end{aligned}$$

$$\begin{aligned} \mathbf{60} \quad \mathbf{a} \quad P(2 \text{ white truffles}) &= P(\text{first is white} \cap \text{second is white}) \\ &= P(\text{first is white}) \times P(\text{second is white given first is white}) \\ &= \frac{2}{12} \times \frac{1}{11} \\ &= \frac{2}{132} \\ &= \frac{1}{66} \end{aligned}$$

$$\mathbf{b} \quad P(2 \text{ white truffles}) = \frac{1}{66} \quad \{\text{from a}\}$$

$$\begin{aligned} P(2 \text{ dark brown truffles}) &= P(\text{first is dark brown} \cap \text{second is dark brown}) \\ &= P(\text{first is dark brown}) \times P(\text{second is dark brown given first is dark brown}) \\ &= \frac{6}{12} \times \frac{5}{11} \\ &= \frac{30}{132} \\ &= \frac{5}{22} \end{aligned}$$

$$\begin{aligned} P(2 \text{ light brown truffles}) &= P(\text{first is light brown} \cap \text{second is light brown}) \\ &= P(\text{first is light brown}) \times P(\text{second is light brown given first is light brown}) \\ &= \frac{4}{12} \times \frac{3}{11} \\ &= \frac{12}{132} \\ &= \frac{1}{11} \end{aligned}$$

$$\begin{aligned} P(\text{different coloured truffles}) &= 1 - P(\text{same coloured truffles}) \\ &= 1 - \left(\frac{1}{66} + \frac{5}{22} + \frac{1}{11} \right) \\ &= 1 - \frac{1}{3} \\ &= \frac{2}{3} \end{aligned}$$

61 a

x	0	1	2	3
$P(X = x)$	k	0.2	0.5	0.1

$$\begin{aligned} \sum P(X = x) &= 1 \\ \therefore k + 0.2 + 0.5 + 0.1 &= 1 \\ \therefore k + 0.8 &= 1 \\ \therefore k &= 0.2 \end{aligned}$$

b

x	2	4	6
$P(X = x)$	$2k$	0.1	$0.6 - k$

$$\begin{aligned} \sum P(X = x) &= 1 \\ \therefore 2k + 0.1 + (0.6 - k) &= 1 \\ \therefore k + 0.7 &= 1 \\ \therefore k &= 0.3 \end{aligned}$$

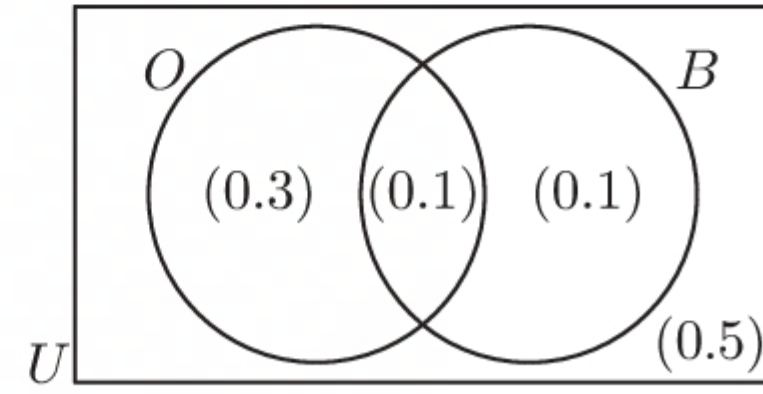
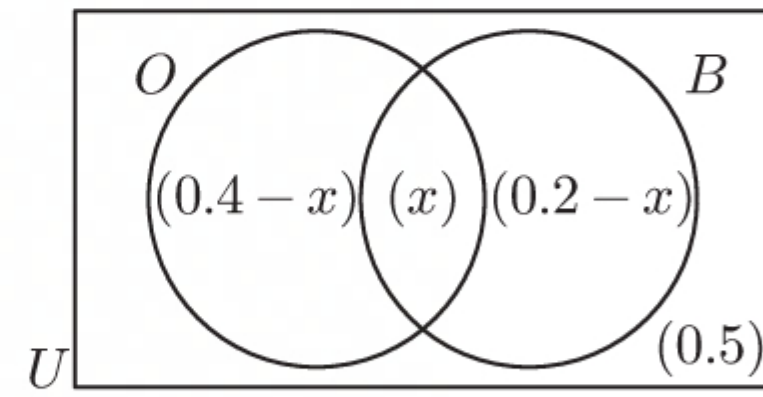
- 62 a** Let O represent a student who owns an orange highlighter, and B represent a student who owns a blue highlighter.

Let the proportion of students in $O \cap B$ be x .

\therefore the proportion in $O \cap B'$ is $0.4 - x$ and
the proportion in $O' \cap B$ is $0.2 - x$.

The proportion in $O' \cap B'$ is 0.5 .

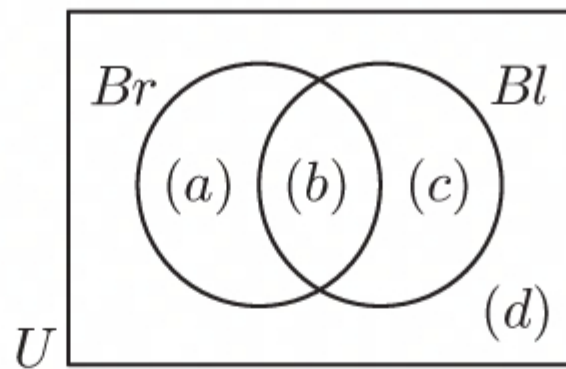
$$\begin{aligned}\therefore (0.4 - x) + x + (0.2 - x) &= 1 - 0.5 \\ \therefore 0.6 - x &= 0.5 \\ \therefore x &= 0.1\end{aligned}$$



b i
$$\begin{aligned}P(B | O) &= \frac{P(B \cap O)}{P(O)} \\ &= \frac{0.1}{0.4} \\ &= \frac{1}{4}\end{aligned}$$

ii
$$\begin{aligned}P(O | B') &= \frac{P(O \cap B')}{P(B')} \\ &= \frac{0.3}{0.8} \\ &= \frac{3}{8}\end{aligned}$$

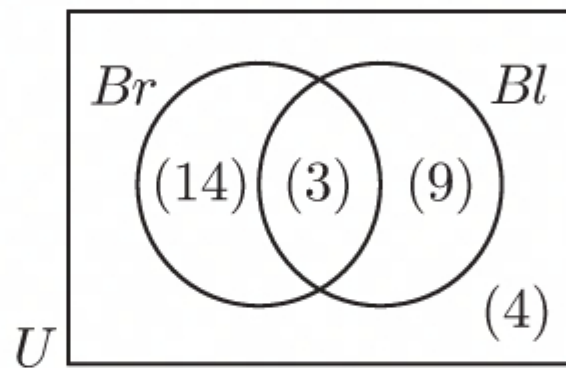
- 63 a** Let Br represent a student with brown hair, and Bl represent a student with blue eyes.



$$\begin{aligned}a + b + c + d &= 30 \\ a + b &= 17 \\ b + c &= 12 \\ d &= 4 \\ \therefore a + b + c &= 26\end{aligned}$$

Since $a + b = 17$, $17 + c = 26$
 $\therefore c = 9$

Since $b + c = 12$, $b + 9 = 12$
 $\therefore b = 3$



$$\begin{aligned}a + b &= 17 \\ \therefore a + 3 &= 17 \\ \therefore a &= 14\end{aligned}$$

b i $P(Bl \text{ but not } Br) = \frac{9}{30} = 0.3$

ii $P(Br | Bl) = \frac{3}{12} = 0.25$

- 64** $n = 180$ attempts

$$p = P(\text{Roger getting a "first serve" in}) = \frac{7}{9}$$

Roger would be expected to get $np = 180 \times \frac{7}{9} = 140$ "first serves" in.

- 65** Let H represent a head, and T represent a tail.

a
$$\begin{aligned}P(2 \text{ heads and } 1 \text{ tail}) &= P(\text{HHT, HTH, or THH}) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \\ &= \frac{3}{8}\end{aligned}$$

- b** $n = 400$ times

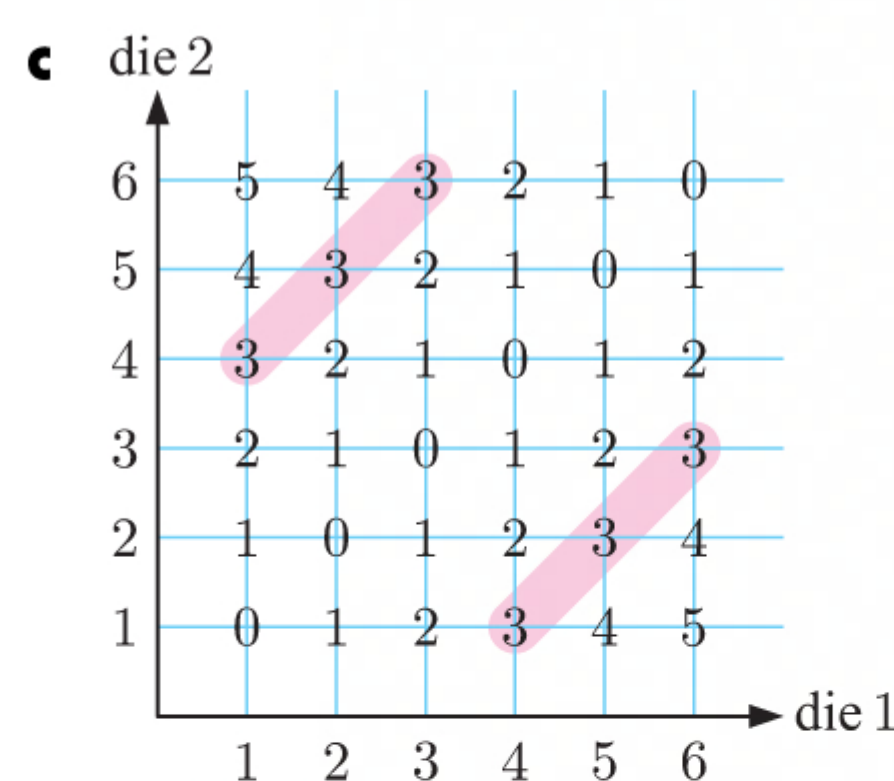
$$p = P(\text{exactly } 1 \text{ tail}) = \frac{3}{8} \quad \{\text{from a}\}$$

You would expect to see exactly one tail

$$np = 400 \times \frac{3}{8} = 150 \text{ times.}$$

- 66 a** X is the difference of a number from one die and a number from the other die. So X is a discrete random variable because X has a set of distinct possible values.

b $X = 0, 1, 2, 3, 4, 5$



$$\begin{aligned}P(X = 3) &= \frac{6}{36} \\ &= \frac{1}{6}\end{aligned}$$

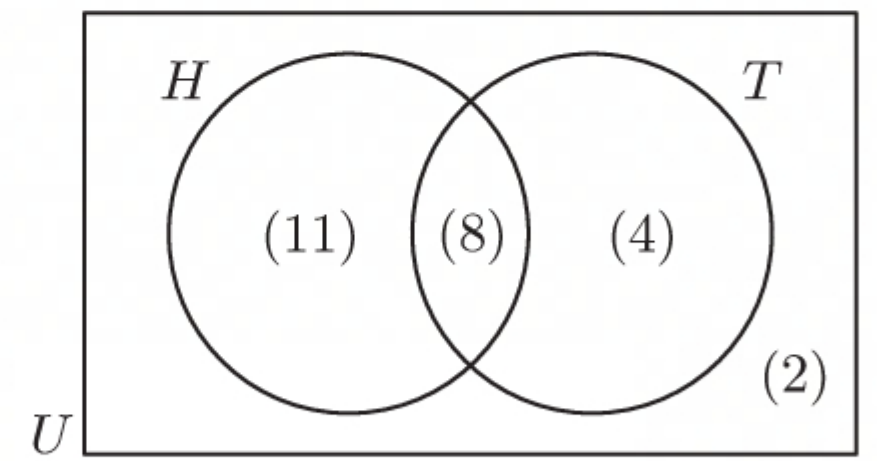
67 There are $11 + 8 + 4 + 2 = 25$ students.

$$\begin{aligned} \mathbf{a} \quad P(H) &= \frac{11+8}{25} \\ &= \frac{19}{25} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad P(T') &= \frac{11+2}{25} \\ &= \frac{13}{25} \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad P(\text{plays at least one sport}) &= P(H \cup T) \\ &= \frac{11+8+4}{25} \\ &= \frac{23}{25} \end{aligned}$$

$$\begin{aligned} \mathbf{d} \quad P(T | H) &= \frac{8}{11+8} \\ &= \frac{8}{19} \end{aligned}$$



68 a $P(x) = \frac{a}{(x-3)^2}, \quad x = 0, 1, 2$

$$\therefore P(0) = \frac{a}{9}, \quad P(1) = \frac{a}{4}, \quad P(2) = a$$

Since $P(x)$ is a probability mass function, $\sum_{i=1}^n P(x_i) = 1$

$$\therefore \frac{a}{9} + \frac{a}{4} + a = 1$$

$$\therefore \frac{49}{36}a = 1$$

$$\therefore a = \frac{36}{49}$$

b $P(X = 2) = P(2)$

$$= a$$

$$= \frac{36}{49} \quad \{\text{from a}\}$$

c Since $P(X = 2) = \frac{36}{49}$ is the greatest probability, the mode of the distribution is 2.

$$p_1 = \frac{a}{9} = \frac{\frac{36}{49}}{9} = \frac{4}{49} \approx 0.0816$$

$$p_1 + p_2 = \frac{a}{9} + \frac{a}{4} = \frac{4}{49} + \frac{\frac{36}{49}}{4} = \frac{4}{49} + \frac{9}{49} = \frac{13}{49} \approx 0.265$$

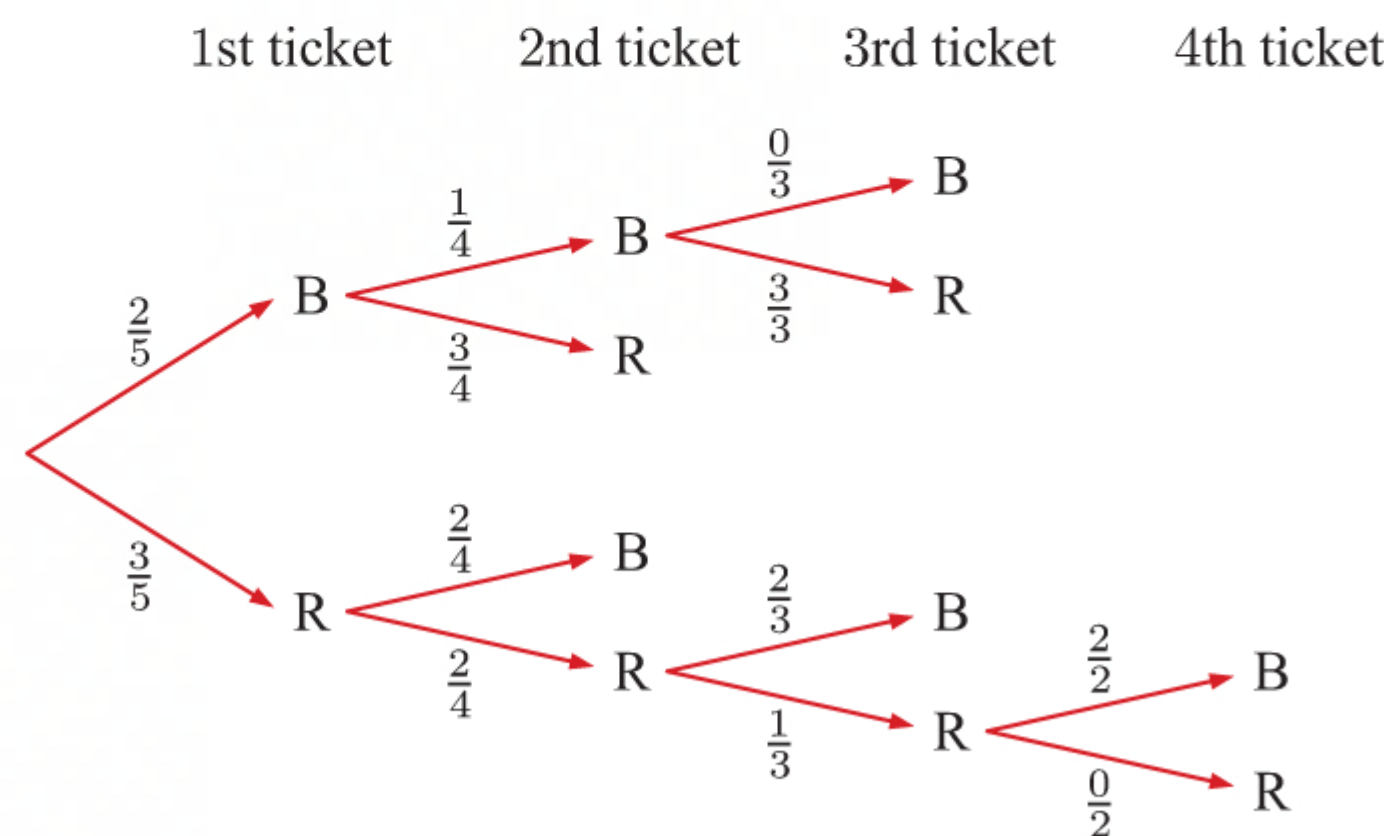
Since $p_1 + p_2 + p_3 = 1 \geq 0.5$, the median is 2.

69 a $X = 2, 3, 4$

b Let B represent a blue ticket, and R represent a red ticket.

The possible selections that can be made are:

BR BBR RRRB
RB RRB ↓
(X = 2) (X = 3) (X = 4)



$$\text{So, } P(X = 2) = \frac{2}{5} \times \frac{3}{4} + \frac{3}{5} \times \frac{2}{4} = \frac{3}{5}$$

$$P(X = 3) = \frac{2}{5} \times \frac{1}{4} \times \frac{3}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{3}{10}$$

$$P(X = 4) = \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} = \frac{1}{10}$$

\therefore the probability distribution of X is

x	2	3	4
$P(X = x)$	$\frac{3}{5}$	$\frac{3}{10}$	$\frac{1}{10}$

c It is most likely that 2 tickets are drawn, so the mode is 2.

$$\begin{aligned} \mathbf{d} \quad E(X) &= \sum_{i=1}^n x_i p_i \\ &= 2\left(\frac{3}{5}\right) + 3\left(\frac{3}{10}\right) + 4\left(\frac{1}{10}\right) \\ &= \frac{5}{2} = 2.5 \end{aligned}$$

70 $P(x) = P(X = x) = \frac{1}{24}(x + 6), \quad x = 1, 2, 3$

a $P(1) = \frac{1}{24}(1 + 6) = \frac{7}{24}$

$$P(2) = \frac{1}{24}(2 + 6) = \frac{8}{24} = \frac{1}{3}$$

$$P(3) = \frac{1}{24}(3 + 6) = \frac{9}{24} = \frac{3}{8}$$

$$\begin{aligned} \mathbf{b} \quad E(X) &= \sum_{i=1}^3 x_i P(x_i) \\ &= 1\left(\frac{7}{24}\right) + 2\left(\frac{8}{24}\right) + 3\left(\frac{9}{24}\right) \quad \{\text{from a}\} \\ &= \frac{50}{24} = 2\frac{1}{12} \end{aligned}$$

71	<i>Cups of tea</i>	0	1	2	3	4	5
	<i>Probability</i>	0.1	0.07	0.16	0.37	0.21	0.09

a Russell is most likely to drink 3 cups of tea in one day, so the mode is 3 cups.

b Expected number of cups of tea = $0 \times 0.1 + 1 \times 0.07 + 2 \times 0.16 + 3 \times 0.37 + 4 \times 0.21 + 5 \times 0.09$
 $= 2.79$ cups

\therefore on average, Russell drinks 2.79 cups of tea per day.

72 a Let X be the return from each game.

$$\begin{aligned} E(X) &= 40\left(\frac{1}{12}\right) + 20\left(\frac{3}{12}\right) + 5\left(\frac{8}{12}\right) \\ &= \frac{40 + 60 + 40}{12} \\ &= \frac{140}{12} \approx \$11.67 \end{aligned}$$

<i>Ticket colour</i>	Blue	Red	Yellow
<i>Winnings</i>	\$40	\$20	\$5
<i>Probability</i>	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{8}{12}$

b The expected return per game is \$11.67. It costs \$15 to play.

So, the expected gain $\approx \$11.67 - \$15 \approx -\$3.33$

It is not advisable to play this game many times as the player can expect to lose \$3.33 on average per game.

c Let k be the number of extra red tickets added to the bag.

<i>Ticket colour</i>	Blue	Red	Yellow
<i>Winnings</i>	\$40	\$20	\$5
<i>Probability</i>	$\frac{1}{12+k}$	$\frac{k+3}{12+k}$	$\frac{8}{12+k}$

For the game to be fair, the expected return must equal the cost of each game.

$$\therefore E(X) = 40\left(\frac{1}{12+k}\right) + 20\left(\frac{k+3}{12+k}\right) + 5\left(\frac{8}{12+k}\right) = 15 \quad \{\text{the cost of the game is \$15}\}$$

$$\therefore \frac{40}{12+k} + \frac{20k+60}{12+k} + \frac{40}{12+k} = 15$$

$$\therefore \frac{20k+140}{12+k} = 15$$

$$\therefore 20k+140 = 15(12+k)$$

$$\therefore 20k+140 = 180+15k$$

$$\therefore 5k = 40$$

$$\therefore k = 8$$

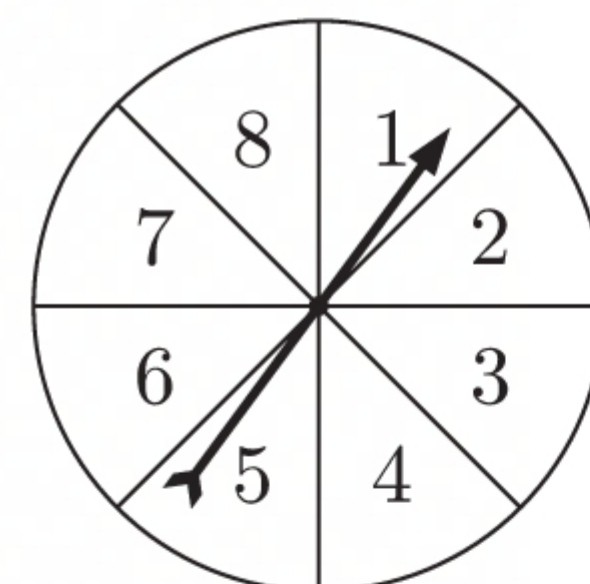
So, 8 extra red tickets should be added to the bag to make the game fair.

73 a	1	1				$n = 1$
	1	2	1			$n = 2$
	1	3	3	1		$n = 3$
	1	4	6	4	1	$n = 4$

b i Let X be the number of times we get a number greater than 3.

$$\therefore X \sim B\left(4, \frac{5}{8}\right) \text{ and } P(X = x) = \binom{4}{x} \left(\frac{5}{8}\right)^x \left(\frac{3}{8}\right)^{4-x}$$

$$\begin{aligned} \text{So, } P(X = 3) &= \binom{4}{3} \left(\frac{5}{8}\right)^3 \left(\frac{3}{8}\right)^1 \\ &= 4 \left(\frac{5}{8}\right)^3 \left(\frac{3}{8}\right) \\ &\approx 0.366 \end{aligned}$$



ii Let X be the number of times we get a number greater than 5.

$$\therefore X \sim B\left(4, \frac{3}{8}\right) \text{ and } P(X = x) = \binom{4}{x} \left(\frac{3}{8}\right)^x \left(\frac{5}{8}\right)^{4-x}$$

$$\begin{aligned} \text{So, } P(X \geq 2) &= 1 - P(X < 2) \\ &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - \binom{4}{0} \left(\frac{3}{8}\right)^0 \left(\frac{5}{8}\right)^4 - \binom{4}{1} \left(\frac{3}{8}\right)^1 \left(\frac{5}{8}\right)^3 \\ &= 1 - \left(\frac{5}{8}\right)^4 - 4 \left(\frac{3}{8}\right) \left(\frac{5}{8}\right)^3 \\ &\approx 0.481 \end{aligned}$$

iii Let X be the number of times we get a number that is spelt with 3 letters.

$$\therefore X \sim B(4, \frac{3}{8}) \text{ and } P(X = x) = \binom{4}{x} \left(\frac{3}{8}\right)^x \left(\frac{5}{8}\right)^{4-x}$$

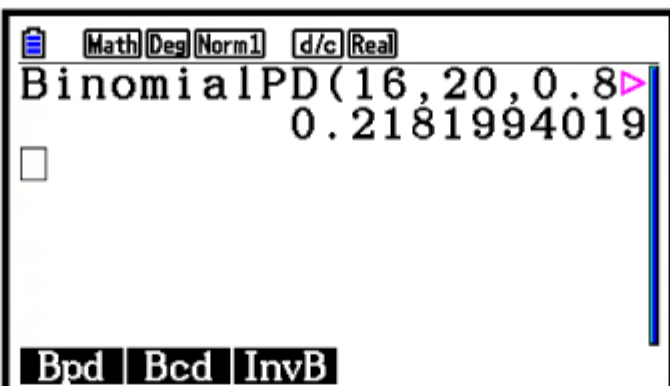
$$\begin{aligned} \text{So, } P(X \leq 1) &= P(X = 0) + P(X = 1) \\ &= \binom{4}{0} \left(\frac{3}{8}\right)^0 \left(\frac{5}{8}\right)^4 + \binom{4}{1} \left(\frac{3}{8}\right)^1 \left(\frac{5}{8}\right)^3 \\ &= \left(\frac{5}{8}\right)^4 + 4 \left(\frac{3}{8}\right) \left(\frac{5}{8}\right)^3 \\ &\approx 0.519 \end{aligned}$$

one two three four
five six seven eight

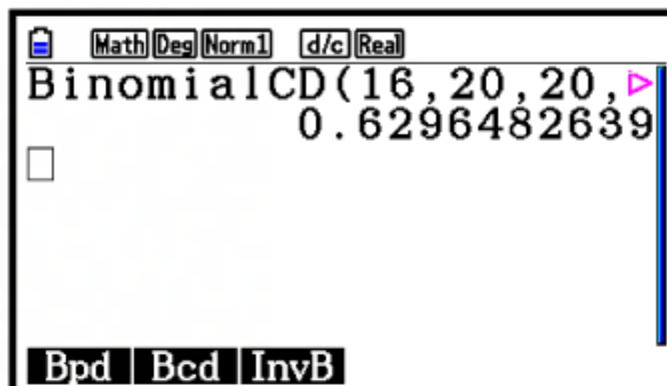
74 Let X be the number of residents who oppose the construction.

$n = 20$, so $X = 0, 1, 2, 3, \dots$, or 20 , and $p = 80\% = 0.8$

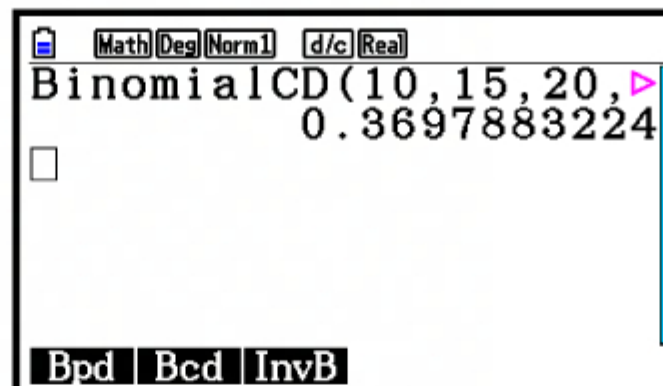
$$\therefore X \sim B(20, 0.8)$$

a 

$$P(X = 16) \approx 0.218$$

b 

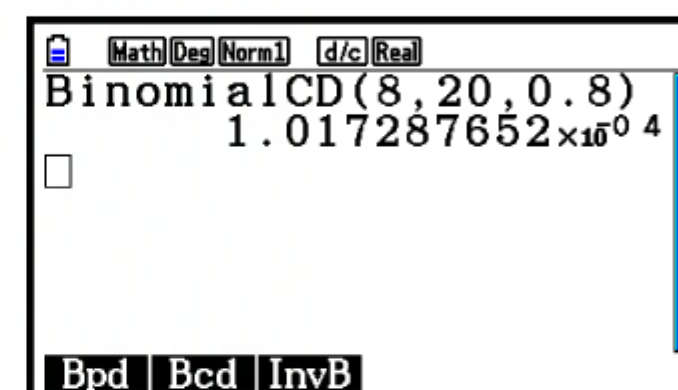
$$P(X \geq 16) \approx 0.630$$

c 

$$P(10 \leq X \leq 15) \approx 0.370$$

d If more than 8 residents support the construction, then 8 or fewer residents oppose the construction.

$$P(X \leq 8) \approx 0.000102$$



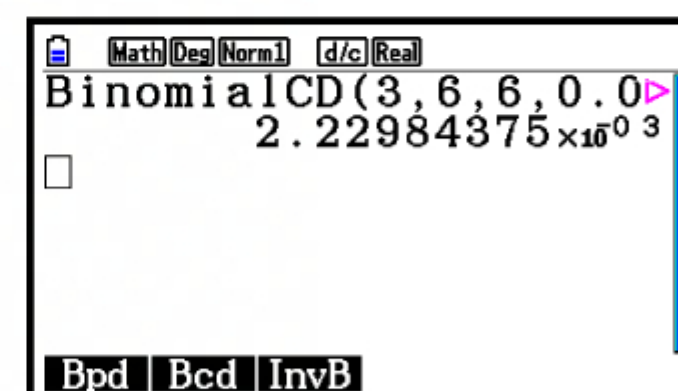
75 a Let X be the number of defective items.

$n = 6$, so $X = 0, 1, 2, 3, 4, 5$, or 6 , and $p = 5\% = 0.05$

$$\therefore X \sim B(6, 0.05)$$

$$\begin{aligned} \text{Using technology, } P(X > 2) &= P(X \geq 3) \\ &\approx 0.00223 \end{aligned}$$

\therefore the manufacturer will have to pay a refund on about $0.00223 \times 100\% \approx 0.223\%$ of boxes.



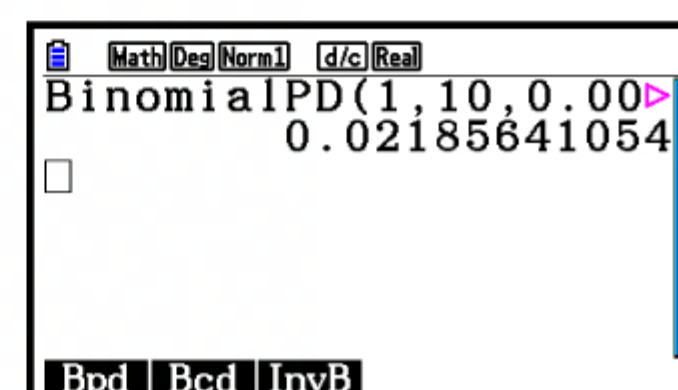
b Let Y be the number of boxes refunded.

$n = 10$, so $Y = 0, 1, 2, 3, \dots$, or 10 , and $p \approx 0.00223$ {from a}

$$\therefore Y \sim B(10, 0.00223)$$

$$\text{Using technology, } P(Y = 1) \approx 0.0219$$

So, the probability that Patrick will get a refund for exactly 1 box is about 0.0219.



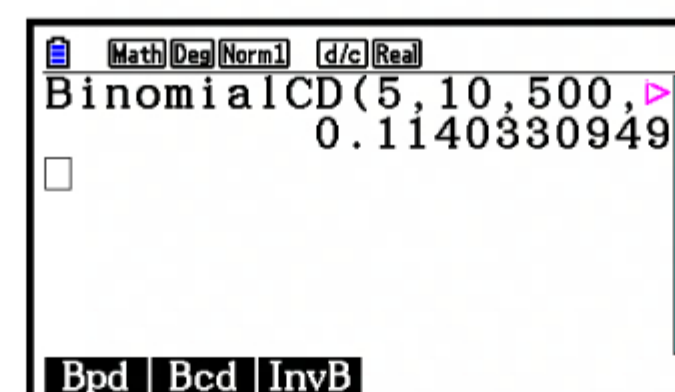
76 Let F be the event of a faulty chip.

$$\therefore P(F) = 0.03 \text{ and } P(F') = 0.97$$

If X is the number which are faulty then $X \sim B(500, 0.03)$.

1% is 5 chips and 2% is 10 chips.

$$\text{Using technology, } P(5 \leq X \leq 10) \approx 0.114$$



77

Score	1	2	3	4
Probability	$\frac{1}{12}$	k	$\frac{1}{4}$	$\frac{1}{3}$

a Since this is a probability distribution, $\sum_{i=1}^n P(x_i) = 1$

$$\begin{aligned} \therefore \frac{1}{12} + k + \frac{1}{4} + \frac{1}{3} &= 1 \\ \therefore k &= \frac{1}{3} \end{aligned}$$

b Let X be the number of 2s rolled.

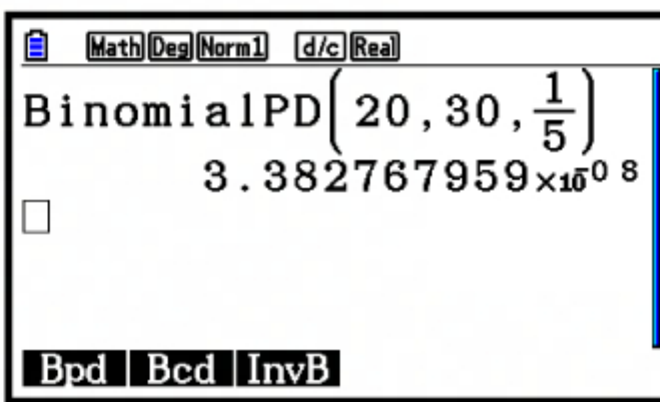
$n = 2400$, so $X = 0, 1, 2, 3, \dots$, or 2400 , and $p = \frac{1}{3}$ {from **a**}

$$\therefore X \sim B(2400, \frac{1}{3})$$

$$\begin{aligned} \text{So, } \mu &= np & \text{and } \sigma &= \sqrt{np(1-p)} \\ &= 2400 \times \frac{1}{3} & &= \sqrt{2400 \times \frac{1}{3} \times \frac{2}{3}} \\ &= 800 & &= \sqrt{\frac{1600}{3}} \\ & & &= \frac{40}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ & & &= \frac{40\sqrt{3}}{3} \end{aligned}$$

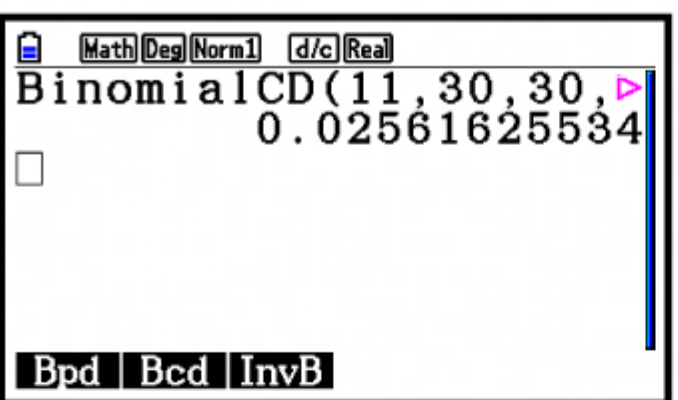
78 $Y \sim B(30, \frac{1}{5})$

$$\begin{aligned} \text{a } \mu &= np & \text{and } \sigma &= \sqrt{np(1-p)} \\ &= 30 \times \frac{1}{5} & &= \sqrt{30 \times \frac{1}{5} \times \frac{4}{5}} \\ &= 6 & &= \sqrt{\frac{24}{5}} \\ & & &\approx 2.19 \end{aligned}$$

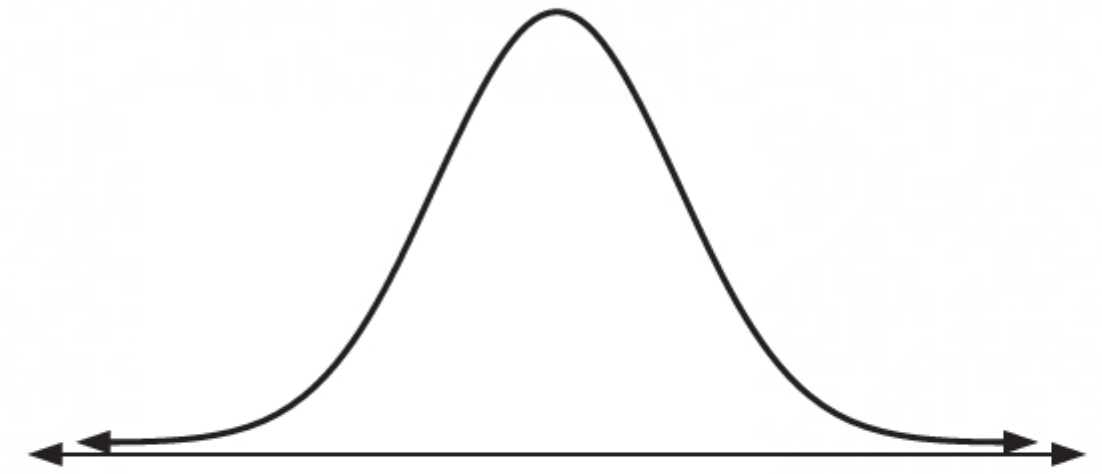
b 

$$P(Y = 20) \approx 3.38 \times 10^{-8}$$

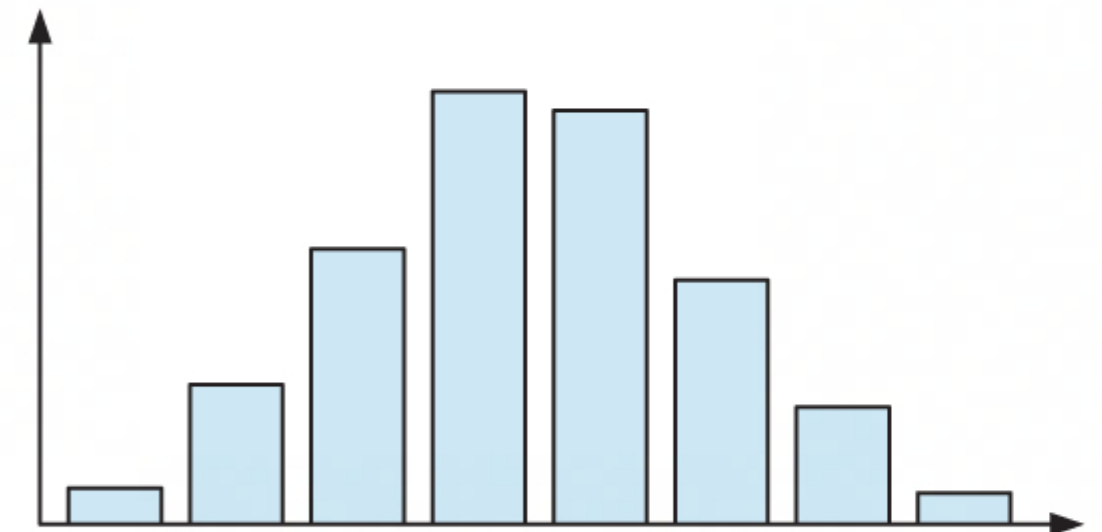
$$\begin{aligned} \text{c } P(Y \geq \mu + 2\sigma) &= P\left(Y \geq 6 + 2\sqrt{\frac{24}{5}}\right) \quad \{\text{from a}\} \\ \text{But } 6 + 2\sqrt{\frac{24}{5}} &\approx 10.38 \\ \therefore P(Y \geq \mu + 2\sigma) &= P(Y \geq 11) \\ &\approx 0.0256 \quad \{\text{using technology}\} \end{aligned}$$



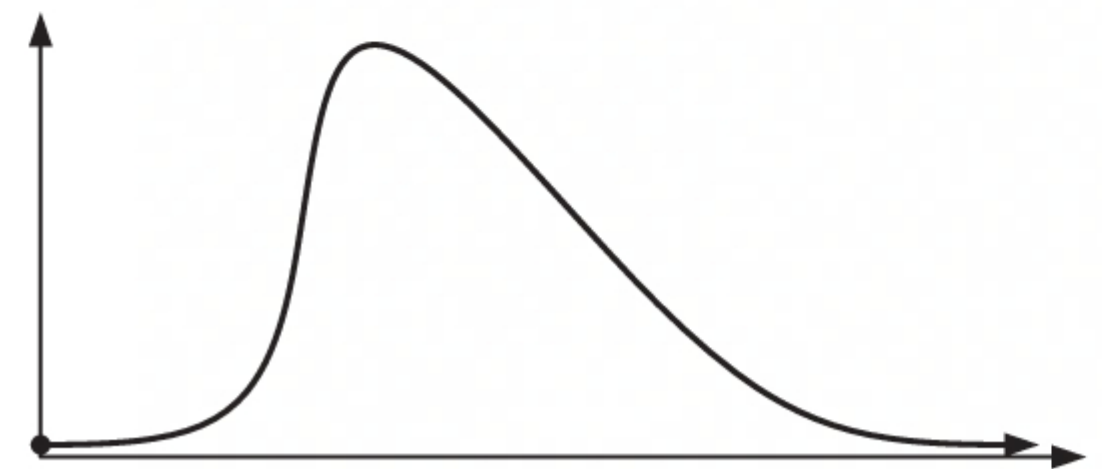
79 a The variable is likely to be normally distributed as the amount of sleep will be generally centred around the mean, but will vary due to factors such as diet, exercise, and so on.



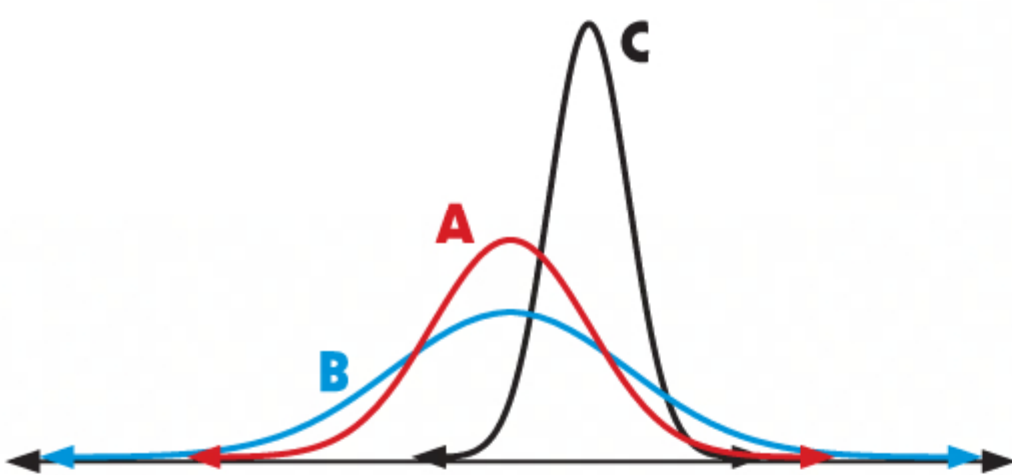
b The variable is not likely to be normally distributed as it is a discrete variable. The number of lollies may vary from bag to bag, so the distribution may appear symmetric.



c The variable is not likely to be normally distributed as it is more likely that there would be more people younger than the mean age than there are older. The distribution may be positively skewed.



80



A and **B** both have the same mean, and **C** has a greater mean.

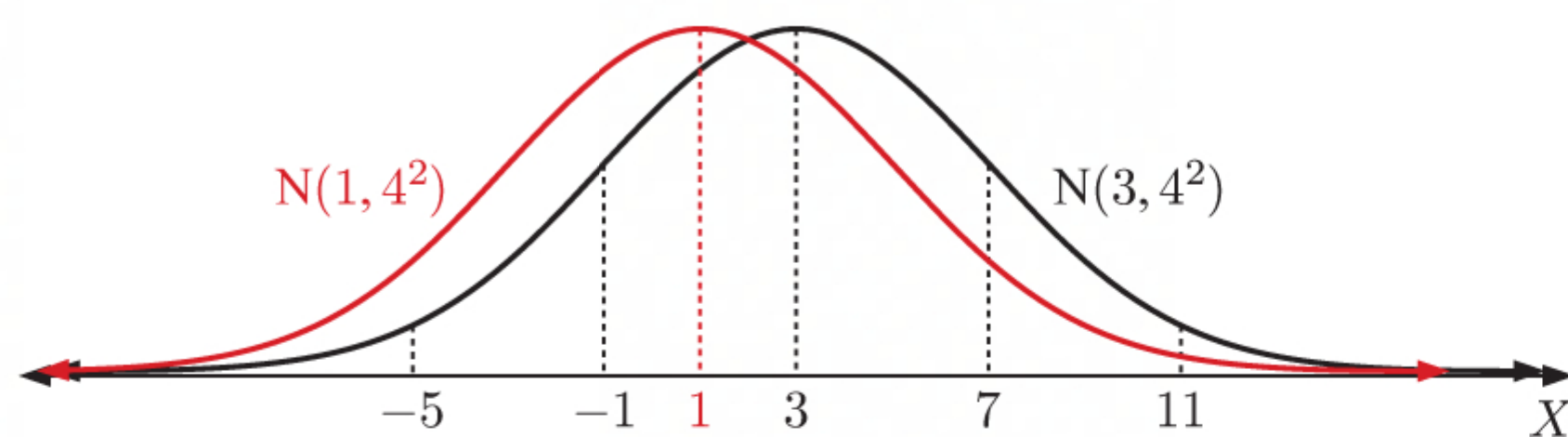
B has a greater spread, and hence a larger standard deviation than **A**.

a $\mu = 4, \sigma = 1$ corresponds to **C**

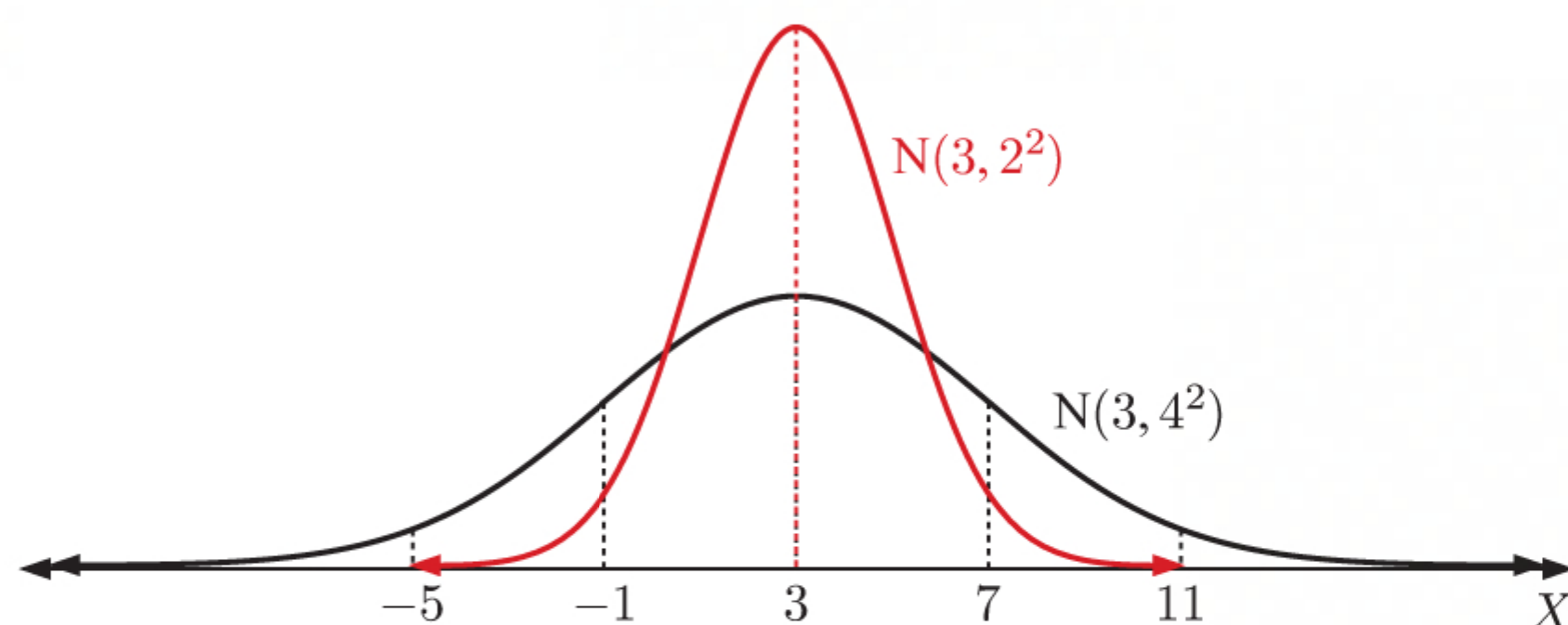
b $\mu = 2, \sigma = 2$ corresponds to **A**

c $\mu = 2, \sigma = 3$ corresponds to **B**

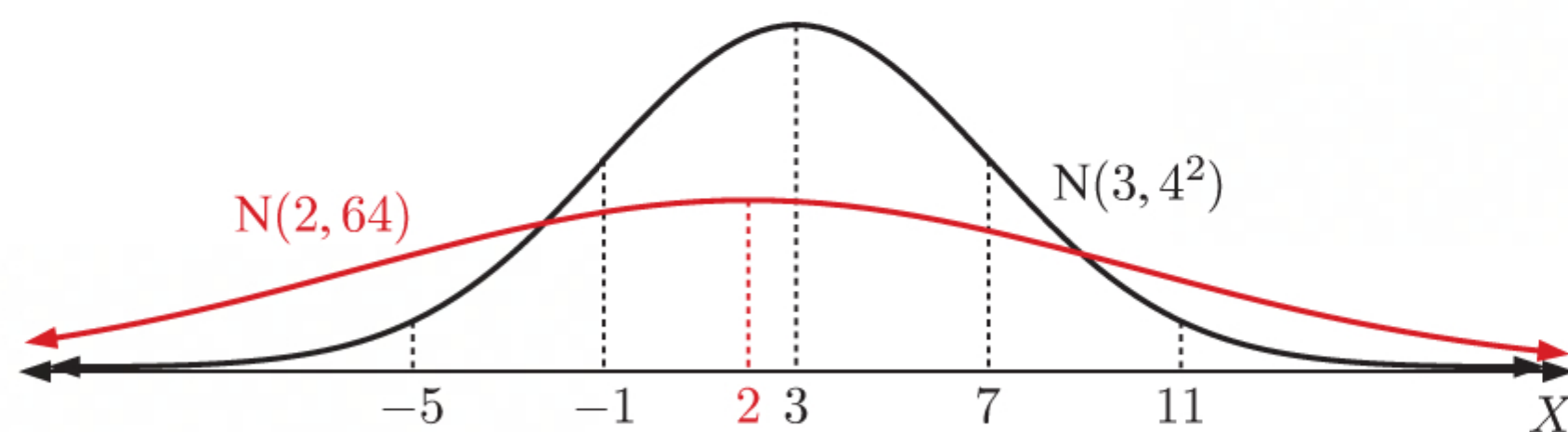
81 a



b



c



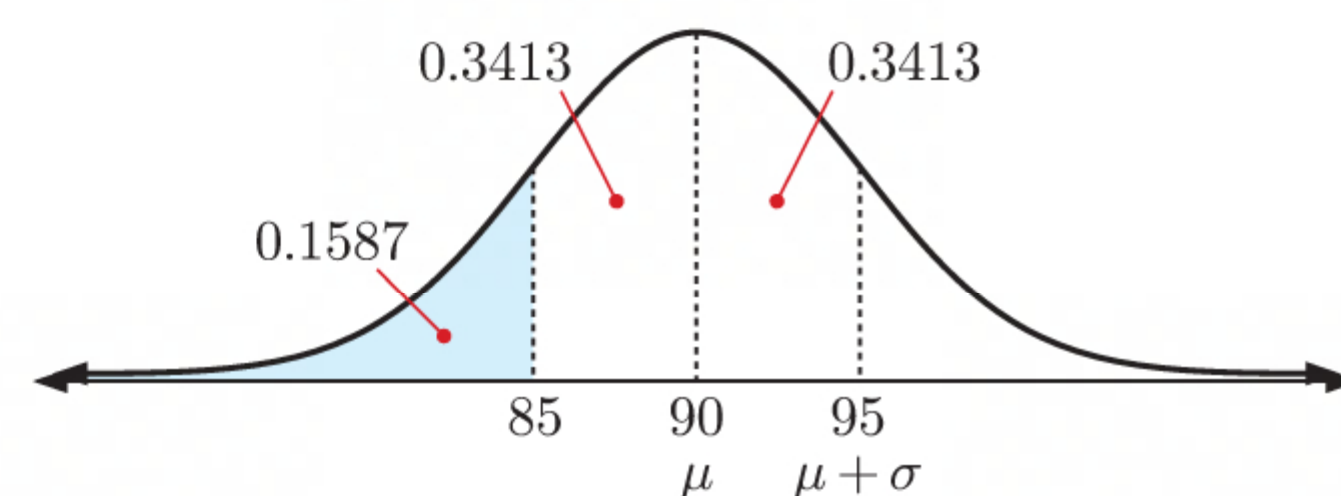
82 a Approximately 68% of the population lies between 25 and 35.

b Approximately 95% of the population lies between 20 and 40.

c Approximately 99.7% of the population lies between 15 and 45.

 83 a $P(X < 85) \approx 0.1587$

From the diagram, $P(90 < X < 95) = P(85 < X < 90)$
 $\approx 0.5 - 0.1587$
 ≈ 0.3413


 b As roughly 34.13% of scores lie between μ and $\mu + \sigma$ for the normal distribution, $\sigma \approx 5$.

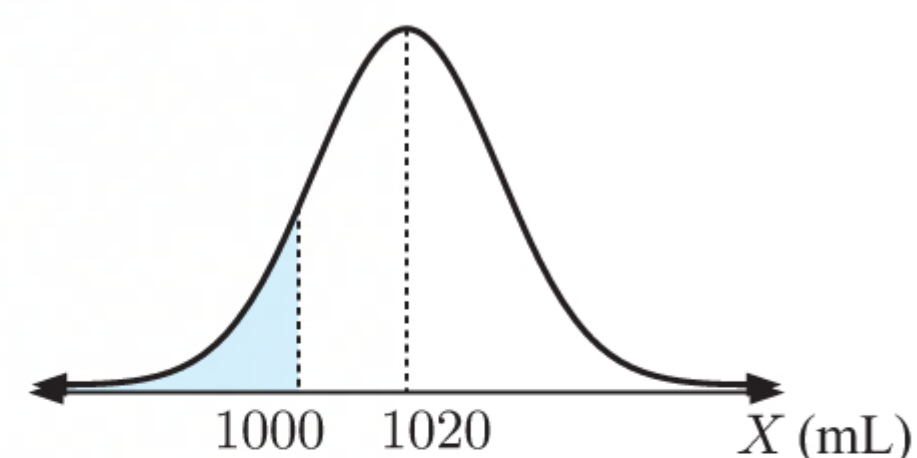
 84 Let the capacity of a randomly selected container be X mL.

 So, $X \sim N(1020, 17^2)$.

a

Normal C.D	
Data	: Variable
Lower	: -9×10^9
Upper	: 1000
σ	: 17
μ	: 1020
Save Res:	: None
	: LIST

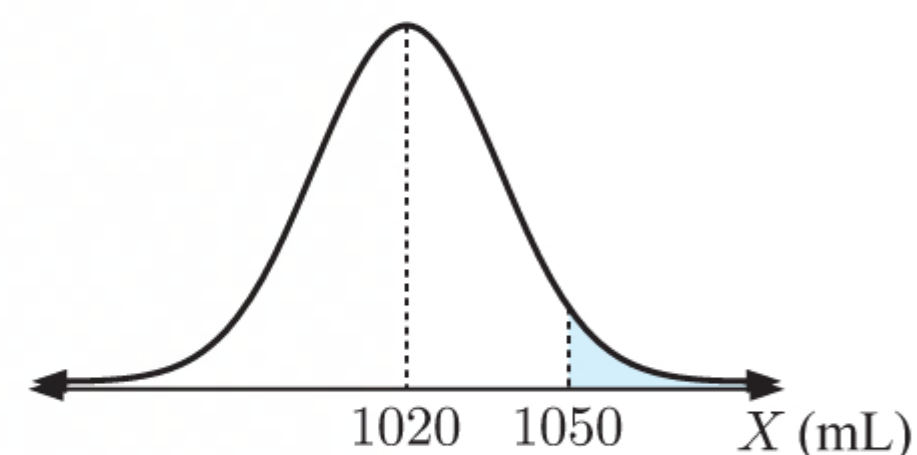
Normal C.D	
p	= 0.11970343
z: Low	= -5.294×10^8
z: Up	= -1.1764706

 $P(X \leq 1000) \approx 0.120$


b

Normal C.D	
Data	: Variable
Lower	: 1050
Upper	: 9×10^9
σ	: 17
μ	: 1020
Save Res:	: None
	: LIST

Normal C.D	
p	= 0.0388066
z: Low	= 1.76470588
z: Up	= 5.2941×10^8

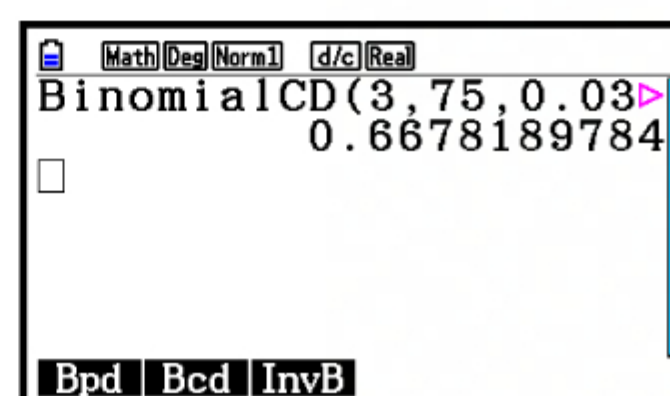
 $P(X \geq 1050) \approx 0.0388$
 \therefore about 3.88% of containers overflow.


c Let Y be the number of containers which overflow.

$n = 75$, so $Y = 0, 1, 2, 3, \dots$, or 75 and $p \approx 0.0388$ {from **b**}

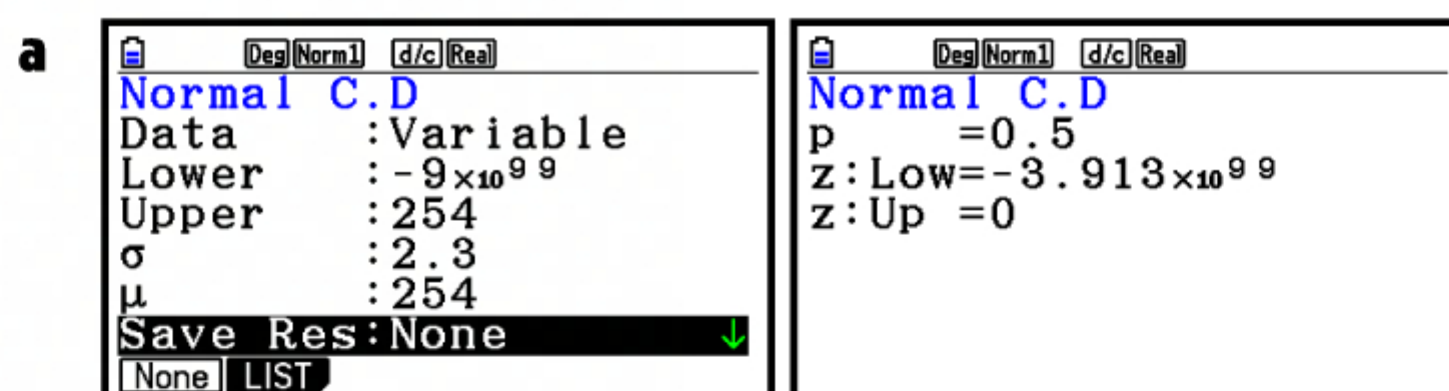
$\therefore Y \sim B(75, 0.0388)$

Using technology, $P(Y \leq 3) \approx 0.668$

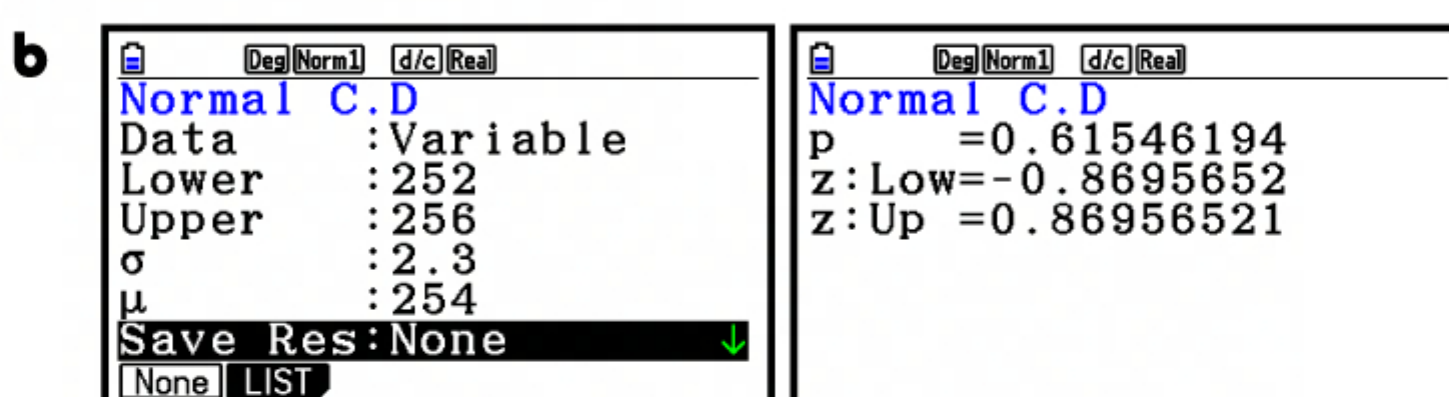
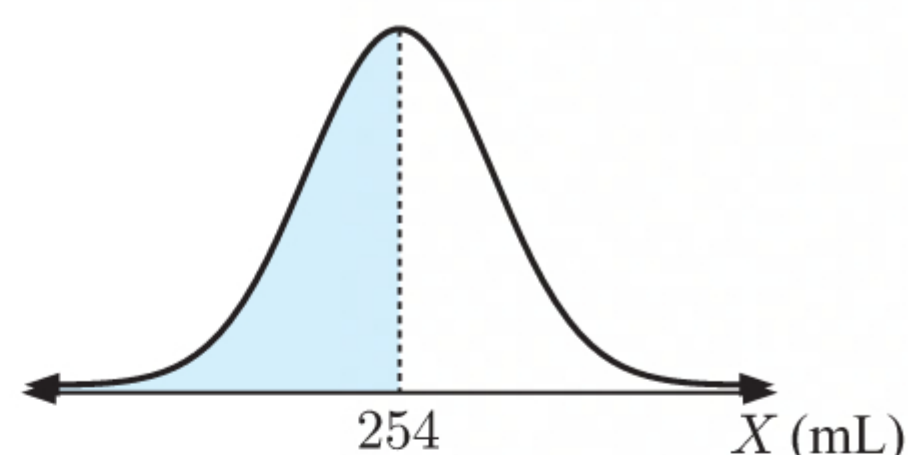


85 Let the volume of a randomly selected drink be X mL.

So, $X \sim N(254, (2.3)^2)$.

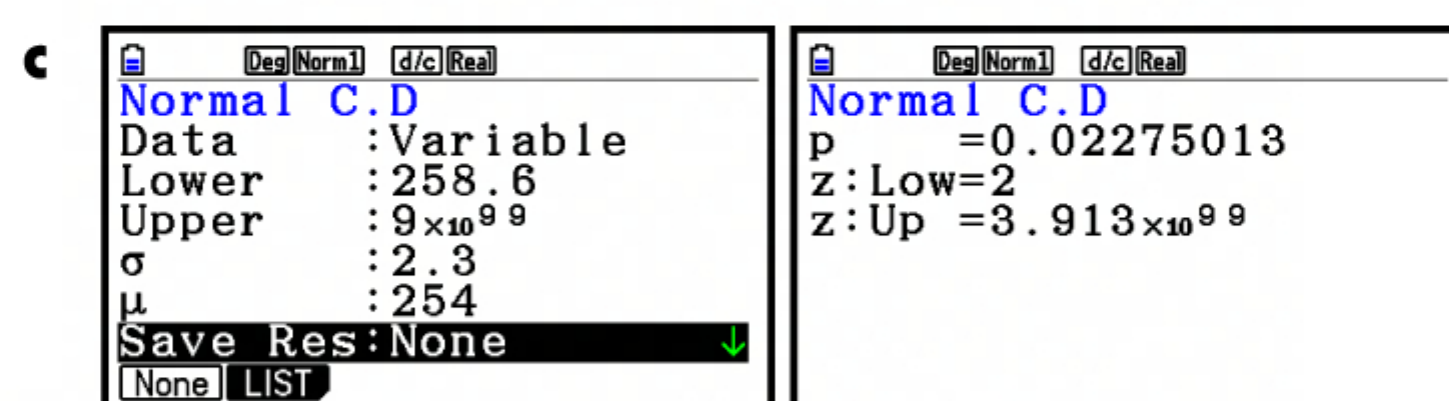
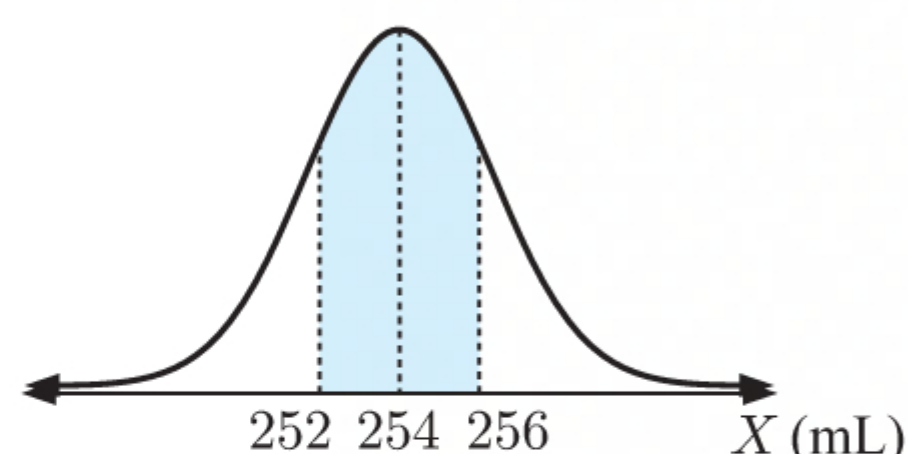


$$P(X < 254) = 0.5$$



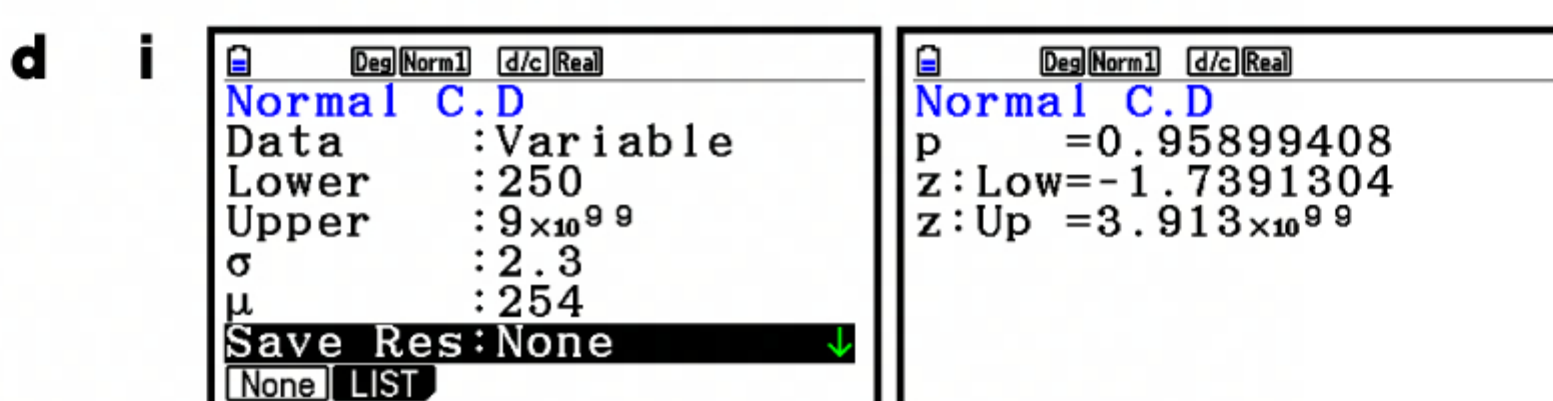
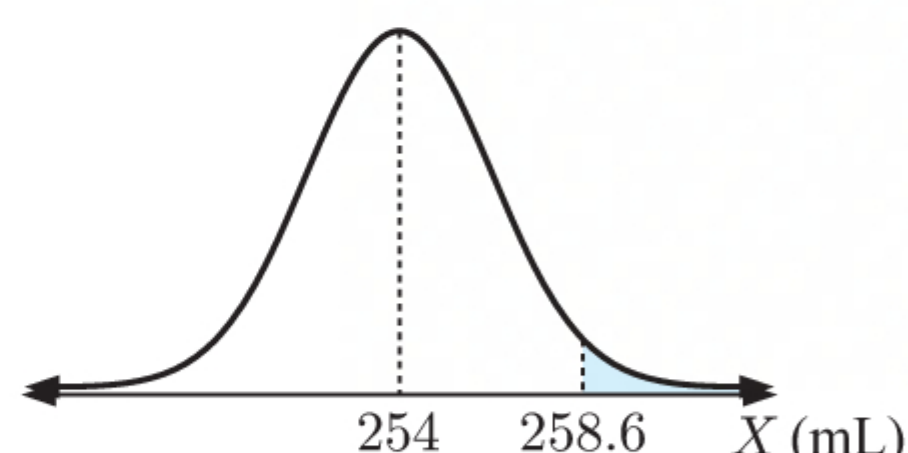
$$P(252 \leq X \leq 256) \approx 0.615$$

\therefore about 61.5% of drinks dispensed by the machine have volume between 252 mL and 256 mL.



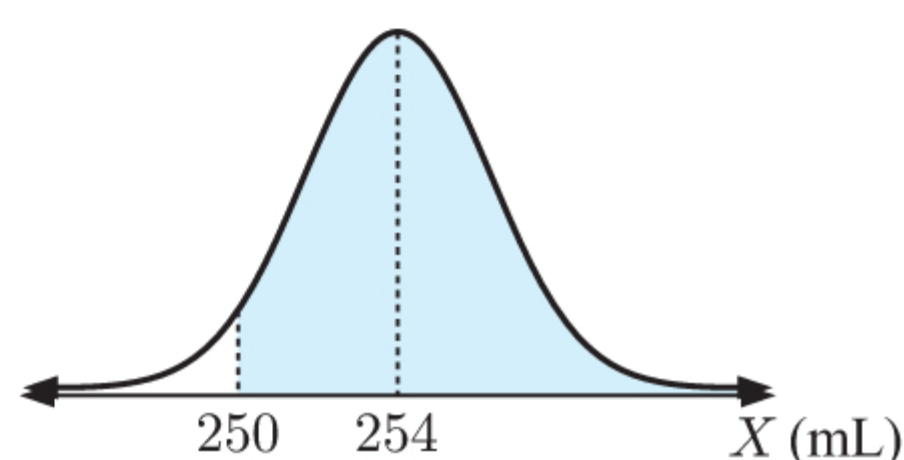
$$P(X \geq 254 + 2 \times 2.3) = P(X \geq 258.6) \approx 0.0228$$

\therefore we expect about $0.0228 \times 80 \approx 2$ drinks to have volume at least two standard deviations above the mean.

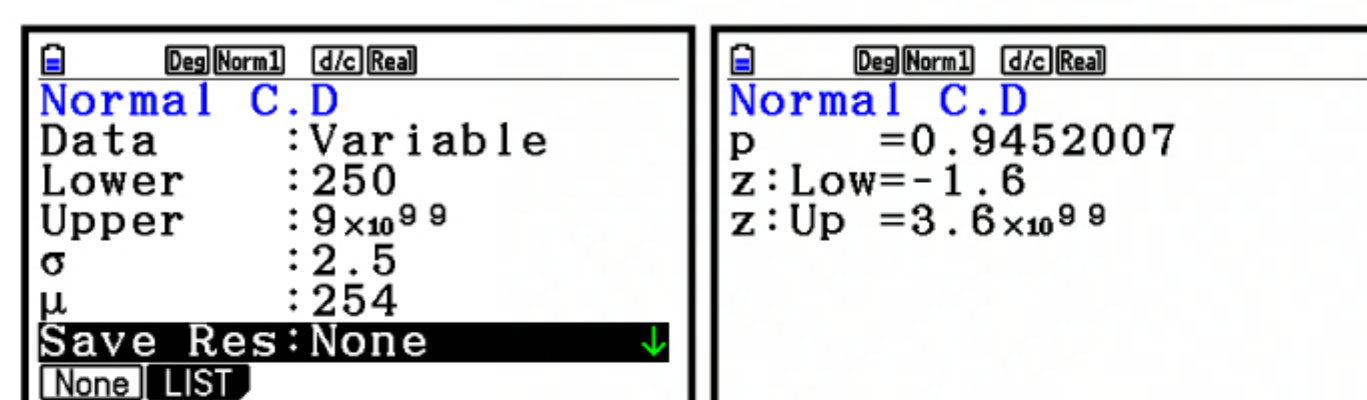


$$P(X \geq 250) \approx 0.959$$

\therefore the operator's guarantee that at least 95% of drinks will have volume at least 250 mL is valid.



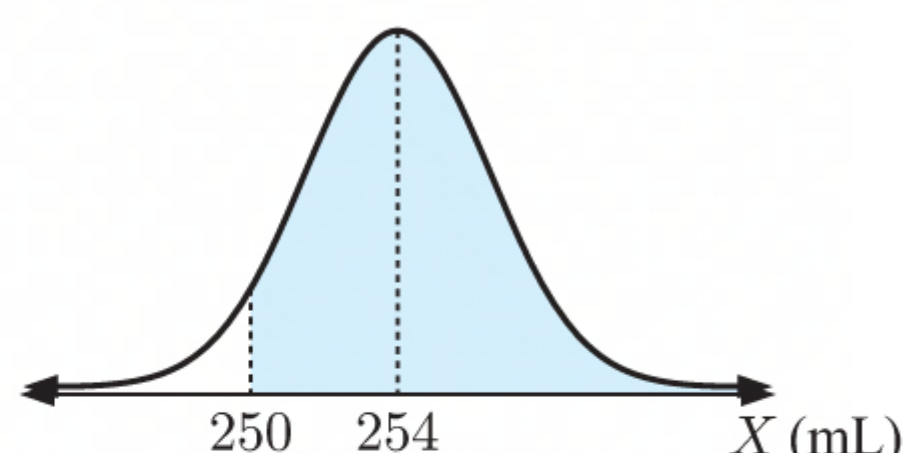
ii Suppose $X \sim N(254, (2.5)^2)$.



$$P(X \geq 250) \approx 0.945$$

\therefore about 94.5% of drinks will have volume at least 250 mL.

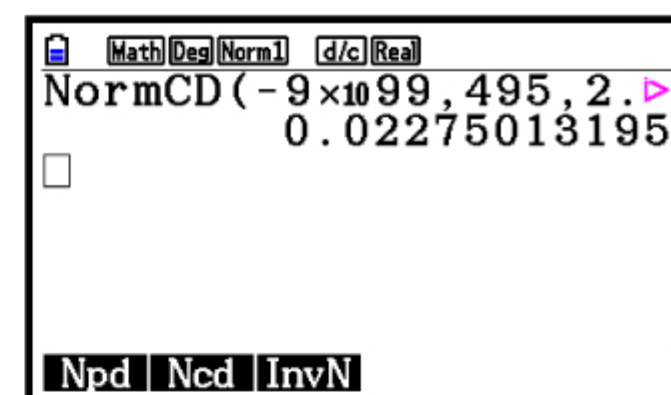
So, the operator's guarantee is no longer valid.



- 86 a** Let the volume of sauce in a randomly selected bottle be X mL.

$$X \sim N(500, (2.5)^2)$$

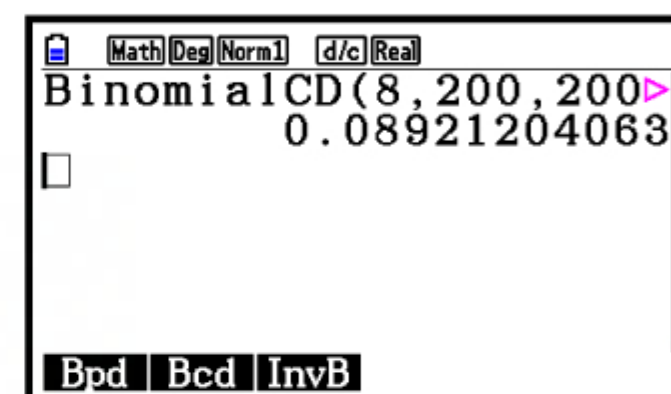
$$\therefore P(X < 495) \approx 0.0228$$



- b** Let Y be the number of bottles which require extra sauce.

$$Y \sim B(200, 0.0228) \quad \{\text{from a}\}$$

$$\therefore P(Y \geq 8) \approx 0.0892$$

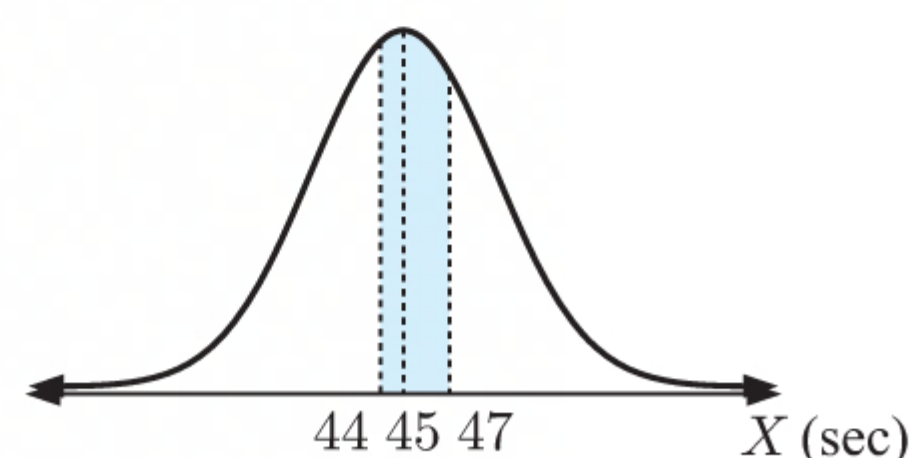
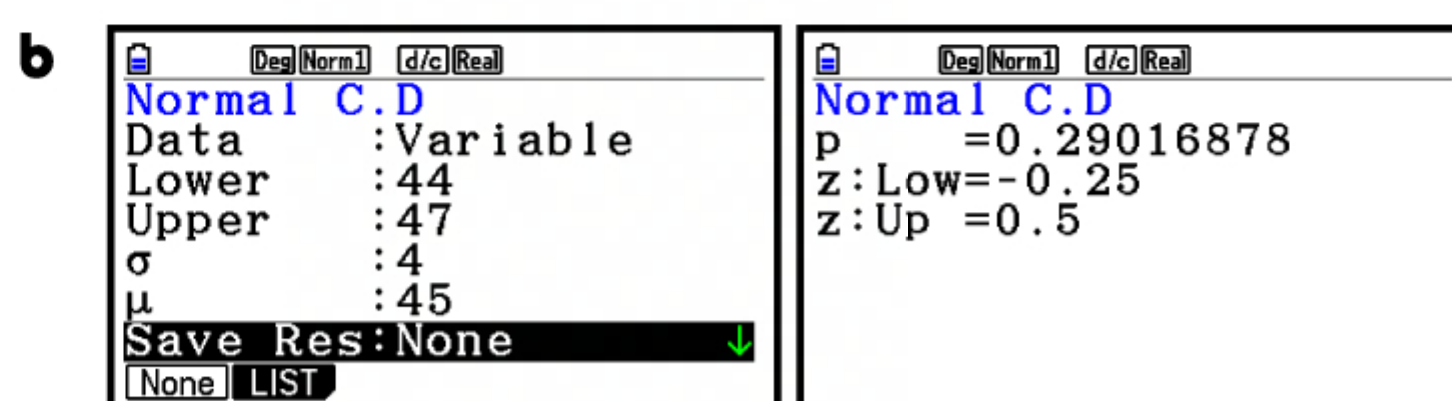
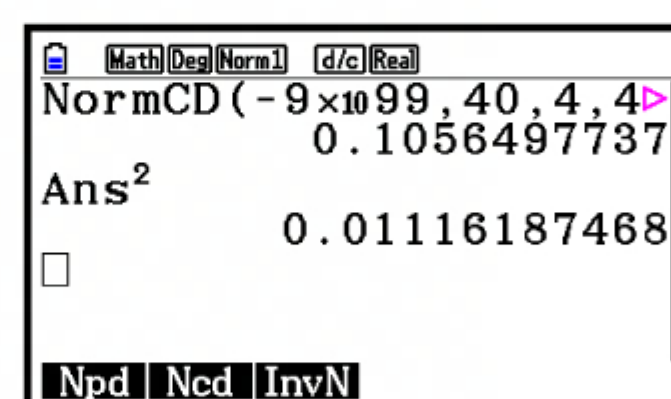


- 87 a** Let the completion time of a randomly selected run be X seconds.

$$X \sim N(45, 4^2)$$

i $P(X < 40) \approx 0.106$

ii $P(\text{two consecutive runs under 40 seconds})$
 $= [P(X < 40)]^2$
 ≈ 0.0112

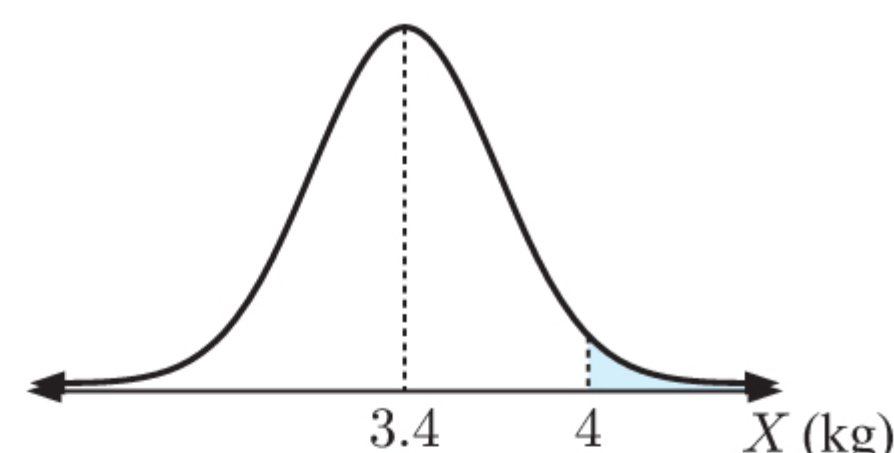
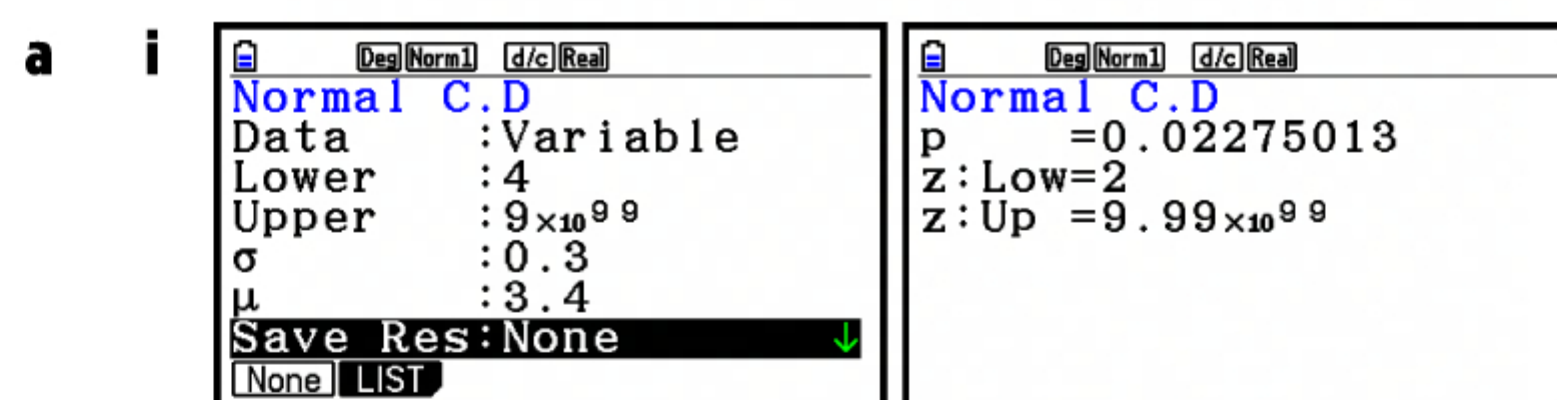


$$P(44 \leq X \leq 47) \approx 0.290$$

\therefore we expect about $0.290 \times 60 \approx 17$ runs to take between 44 seconds and 47 seconds.

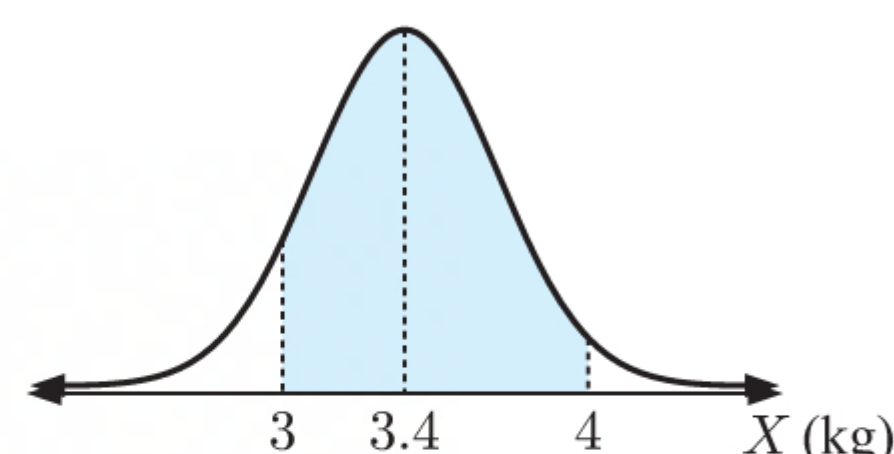
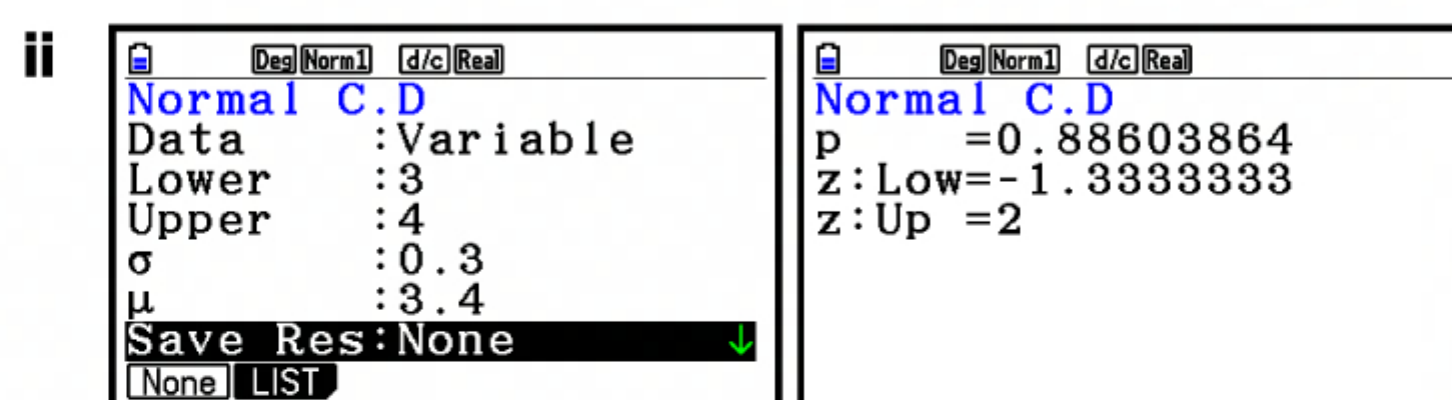
- 88** Let the birth weight of a randomly selected baby be X kg.

$$\text{So, } X \sim N(3.4, (0.3)^2).$$



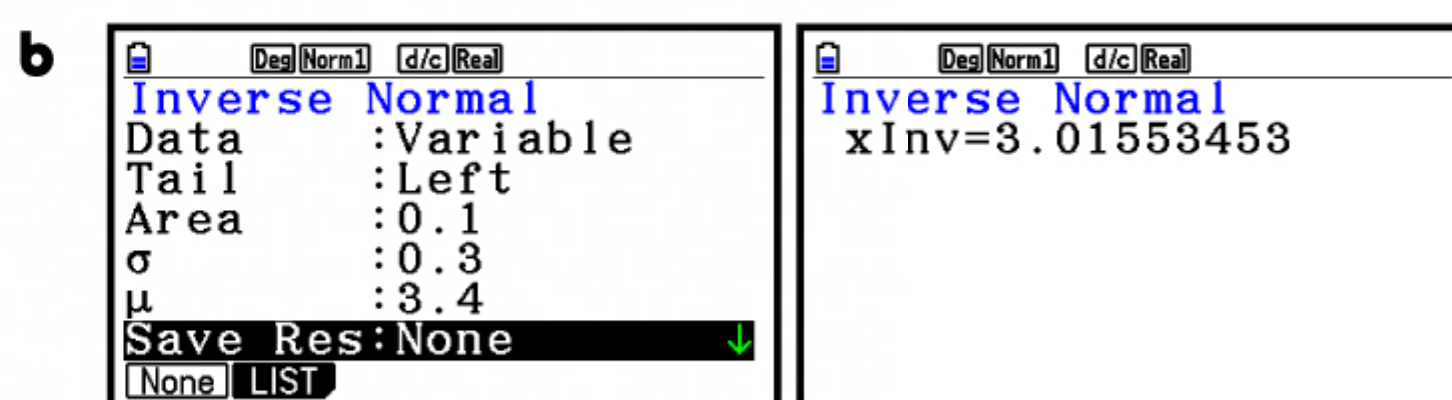
$$P(X > 4) \approx 0.0228$$

\therefore about 2.28% of babies have birth weights in excess of 4 kg.



$$P(3 \leq X \leq 4) \approx 0.886$$

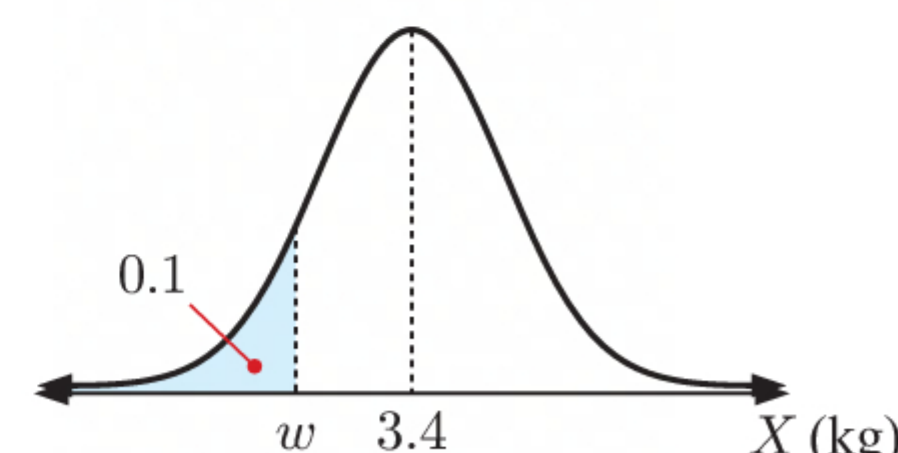
\therefore about 88.6% of babies have birth weights between 3 kg and 4 kg.



$$P(X \leq w) = 0.1$$

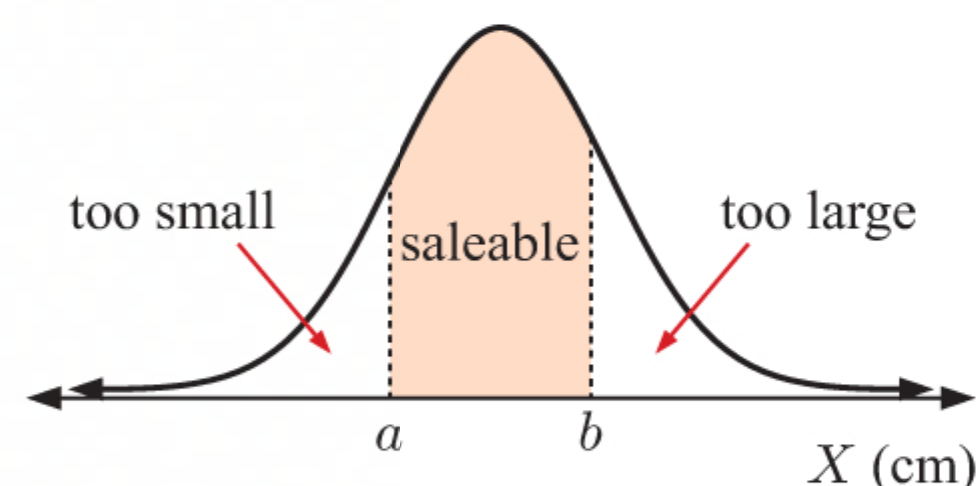
$$\therefore w \approx 3.02$$

\therefore the weight below which a baby is classified as having *low birth weight* is about 3.02 kg.



89 Let the length of a randomly selected zucchini be X cm.

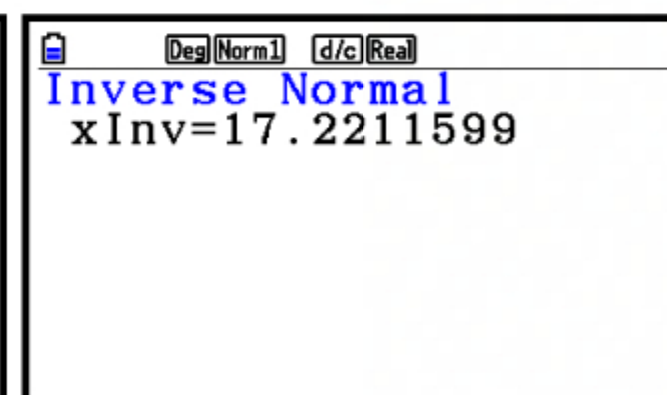
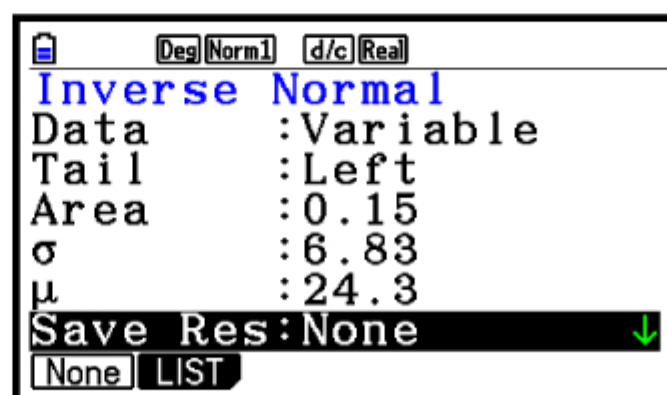
So, $X \sim N(24.3, (6.83)^2)$.



a 15% of zucchinis are too small.

$$P(X < a) = 0.15$$

$$\therefore a \approx 17.2$$

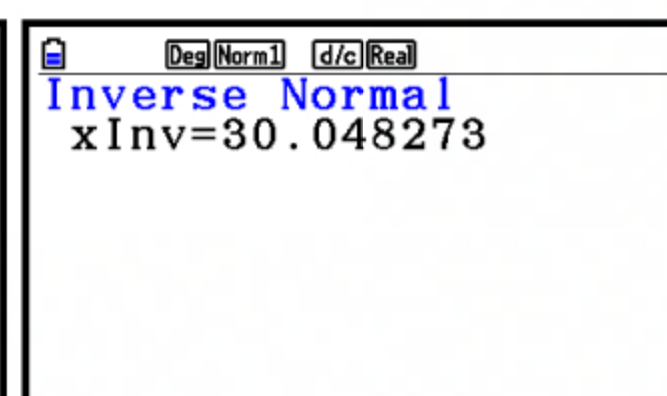
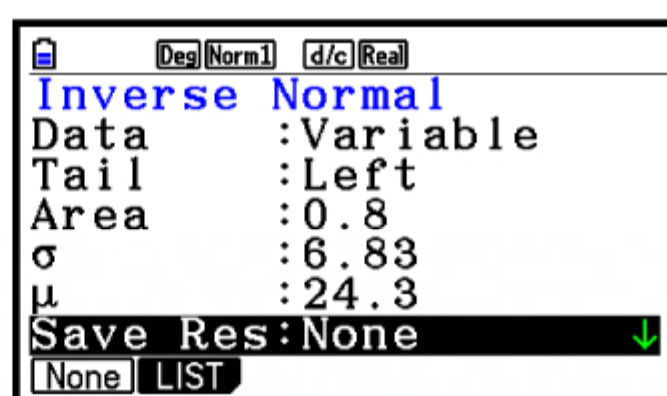


20% of zucchinis are too large.

$$P(X > b) = 0.2$$

$$\therefore P(X \leq b) = 0.8$$

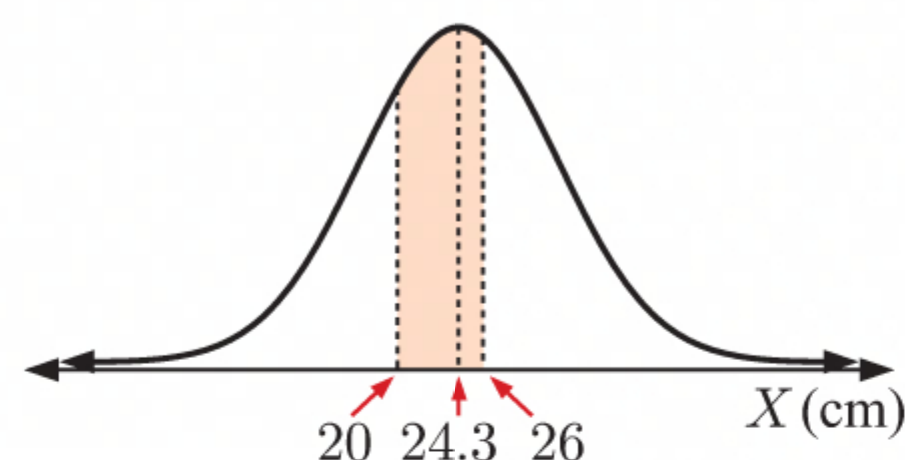
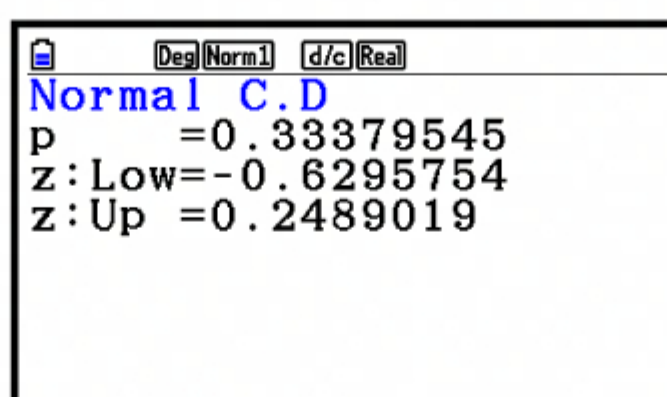
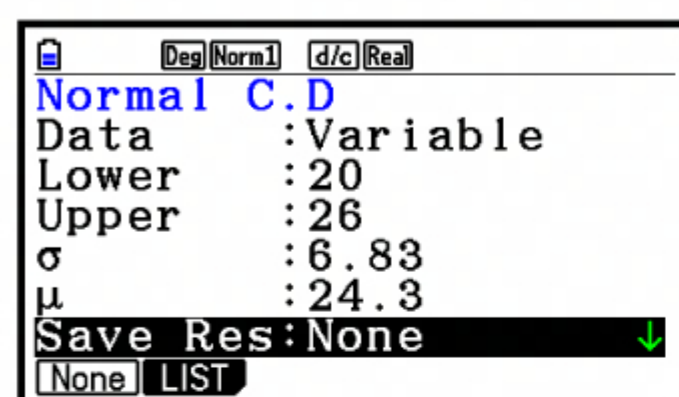
$$\therefore b \approx 30.0$$



b i $P(\text{saleable length}) = 1 - 0.2 - 0.15$

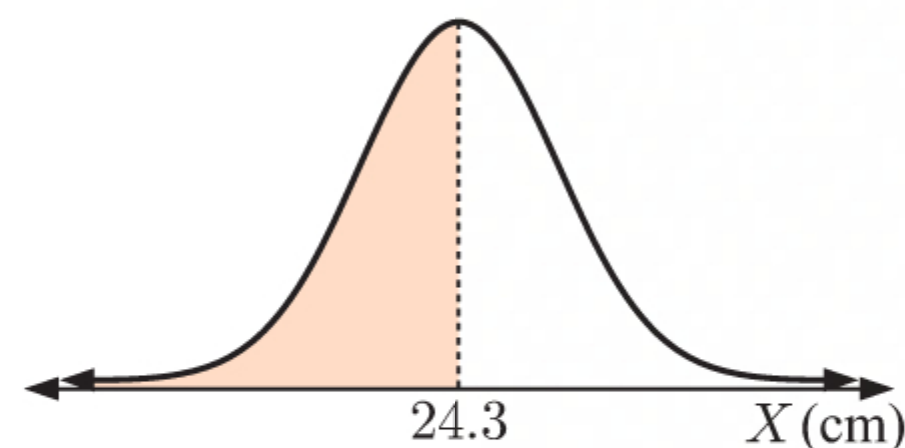
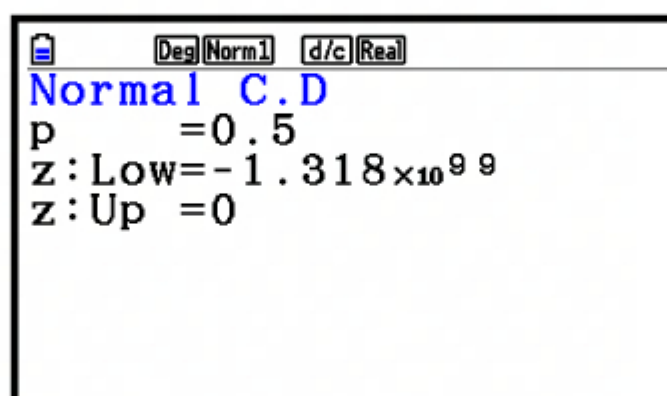
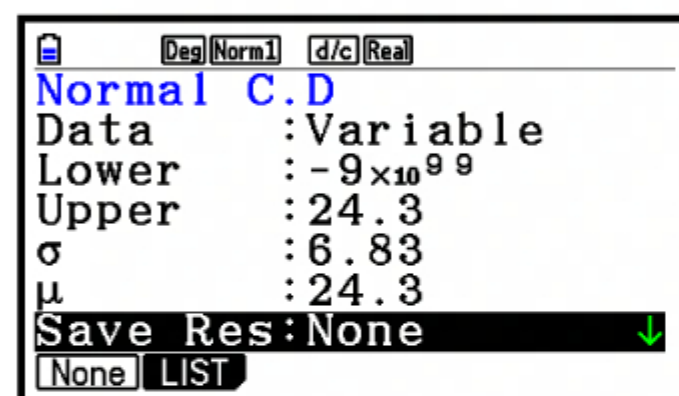
$$= 0.65$$

ii



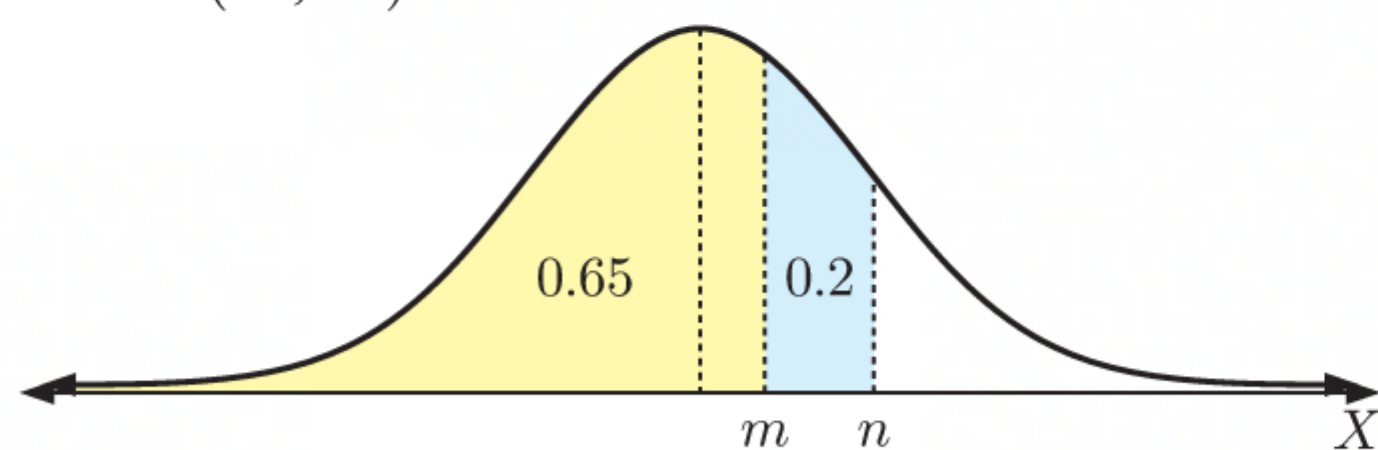
$$P(20 \leq X \leq 26) \approx 0.334$$

iii



$$P(X < 24.3) = 0.5$$

90 $X \sim N(44, 20)$



$$P(X \leq m) = 0.65$$

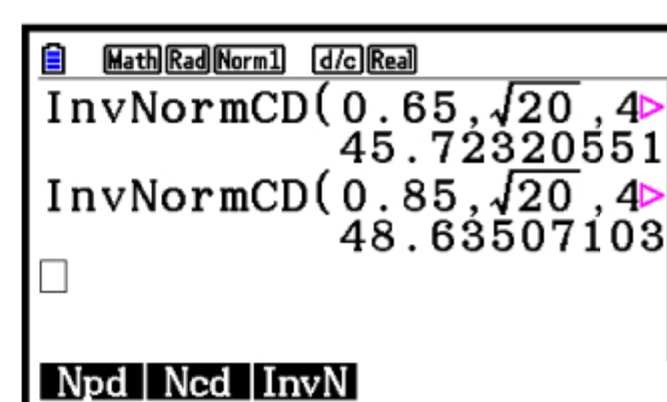
$$\therefore m \approx 45.72 \quad \{\text{using technology}\}$$

$$P(X < n) = 0.65 + 0.2 = 0.85$$

$$\therefore n \approx 48.64 \quad \{\text{using technology}\}$$

$$\therefore n - m \approx 48.64 - 45.72$$

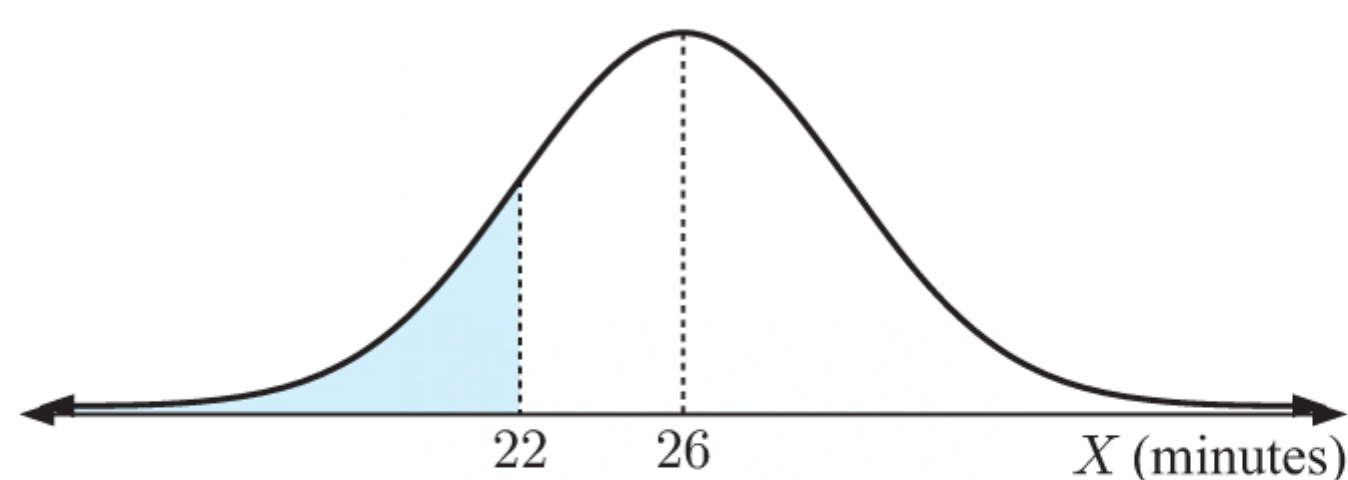
$$\approx 2.92$$



91 Let the finishing time of a randomly selected runner be X minutes.

$$\therefore X \sim N(26, 4^2)$$

a i



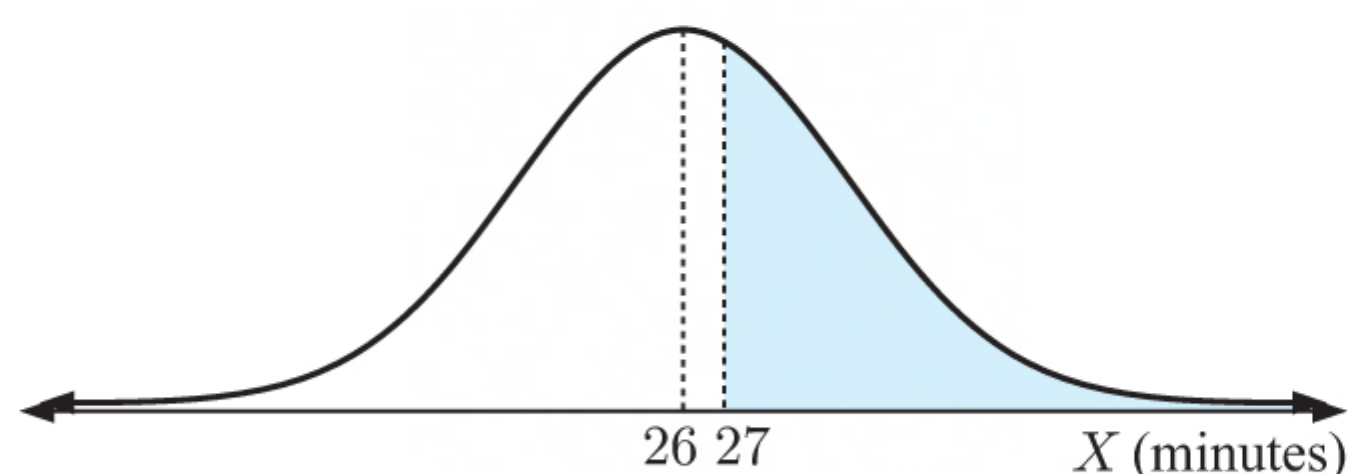
$$P(X < 22) \approx 0.159$$

\therefore we expect about $200 \times 0.159 \approx 32$ runners to have completed the course in less than 22 minutes.

Rad	Norm1	d/c	Real
Normal C.D			
Data	Variable		
Lower	:-9×10 ⁹⁹		
Upper	:22		
σ	:4		
μ	:26		
Save Res:None			
None	LIST		

Rad	Norm1	d/c	Real
Normal C.D			
p	=0.15865525		
z:Low	=-2.25×10 ⁹⁹		
z:Up	=-1		

ii



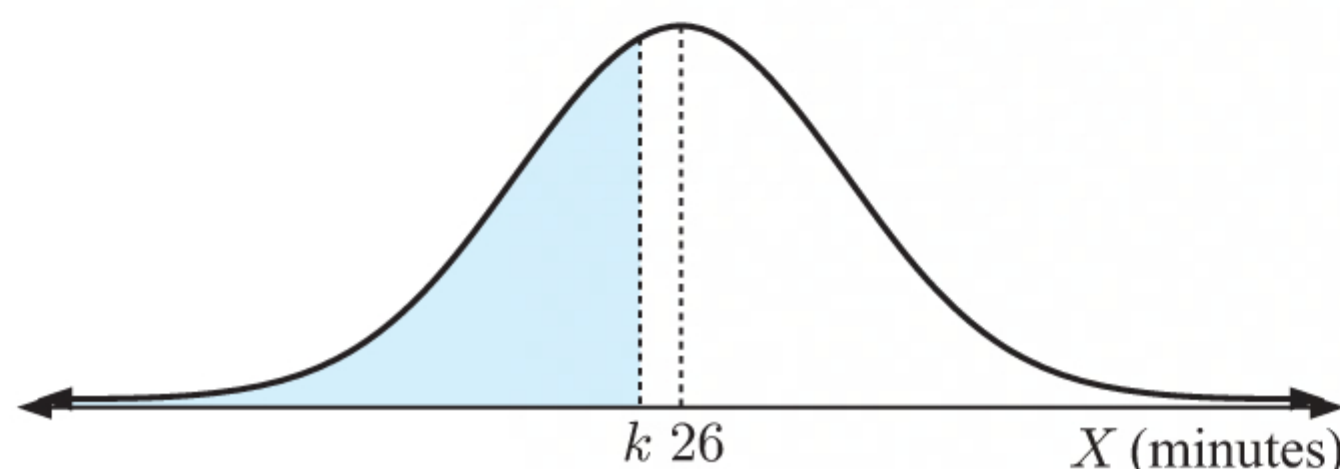
$$P(X > 27) \approx 0.401$$

\therefore we expect about $200 \times 0.401 \approx 80$ runners to have completed the course in more than 27 minutes.

Rad	Norm1	d/c	Real
Normal C.D			
Data	Variable		
Lower	:27		
Upper	:9×10 ⁹⁹		
σ	:4		
μ	:26		
Save Res:None			
None	LIST		

Rad	Norm1	d/c	Real
Normal C.D			
p	=0.40129367		
z:Low	=0.25		
z:Up	=2.25×10 ⁹⁹		

b



$$P(X < k) = 0.4$$

Using technology, $k \approx 25.0$.

The fastest 40% of runners finished quicker than about 25.0 minutes.

Rad	Norm1	d/c	Real
Inverse Normal			
Data	Variable		
Tail	:Left		
Area	:0.4		
σ	:4		
μ	:26		
Save Res:None			

Rad	Norm1	d/c	Real
Inverse Normal			
xInv	=24.9866116		

92 Let μ be the population mean time Eugene takes to walk the 5 km route.

The hypotheses that should be considered are:

$$H_0: \mu = 40 \quad \{\text{the average time is 40 minutes}\}$$

$$H_1: \mu < 40 \quad \{\text{the average time is below 40 minutes}\}$$

93 *Step 1:* Let μ be the population mean diameter of the tennis balls manufactured by the company.

The hypotheses to be considered are:

$$H_0: \mu = 6.541 \quad \{\text{the mean diameter is 6.541 cm}\}$$

$$H_1: \mu < 6.541 \quad \{\text{the mean diameter is below 6.541 cm}\}$$

Step 2: The significance level is $\alpha = 0.05$.

Step 3: $\bar{x} = 6.548$ cm, $s = 0.173$ cm, $n = 500$

$$\text{The value of the test statistic is } t = \frac{6.548 - 6.541}{\frac{0.173}{\sqrt{500}}} \approx 0.905$$

Step 4: Since $H_1: \mu < 6.541$, $p\text{-value} = P(T \leq t)$ where $T \sim t_{499}$
 $\approx P(T \leq 0.905)$
 ≈ 0.817 {using technology}

Step 5: Since $p\text{-value} > 0.05 = \alpha$, we do not have enough evidence to reject H_0 in favour of H_1 on a 5% level of significance. We therefore accept H_0 .

Step 6: Since we have accepted H_0 , there is insufficient evidence to conclude that the tennis balls produced by this company do not meet the minimum international standard diameter.

94

	Sample mean	Sample standard deviation
Before change	183	5.83
After change	184	2.35

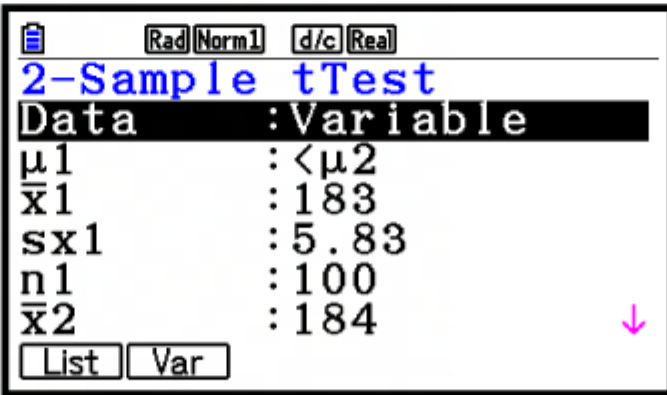
Step 1: Let μ_1 be the population mean weight before the change in fertiliser, and μ_2 be the population mean weight after the change in fertiliser.

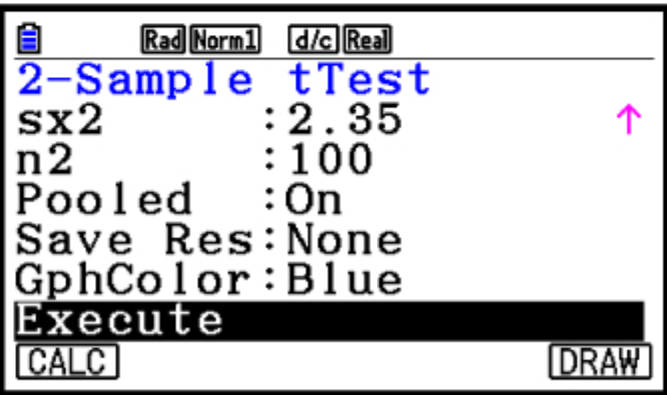
The hypotheses to be considered are:

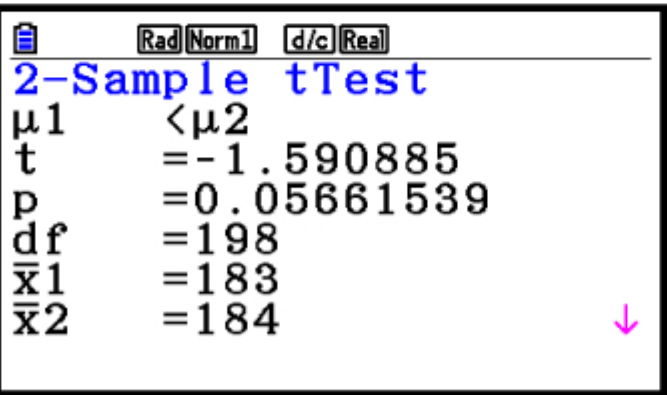
$H_0: \mu_1 = \mu_2$ {the new fertiliser is no better than the old fertiliser}
 $H_1: \mu_1 < \mu_2$ {the new fertiliser is better than the old fertiliser}

Step 2: The significance level is $\alpha = 0.05$.

Step 3:







Using technology, the value of the test statistic is $t \approx -1.59$.

Step 4: From the screenshots above, the p -value ≈ 0.0566 .

Step 5: Since the p -value $> 0.05 = \alpha$, we do not have enough evidence to reject H_0 in favour of H_1 on a 5% level of significance. We therefore accept H_0 .

Step 6: Since we have accepted H_0 , we cannot conclude that the change in fertiliser has increased the mean weight of potatoes.

95

Type of cookie	Number eaten	Expected frequency
choc-chip	789	$2143 \times 0.35 = 750.05$
oatmeal	542	$2143 \times 0.25 = 535.75$
shortbread	423	$2143 \times 0.2 = 428.6$
butter	389	$2143 \times 0.2 = 428.6$
Total	2143	

We are making a claim about population proportions, so a χ^2 goodness of fit test is appropriate.

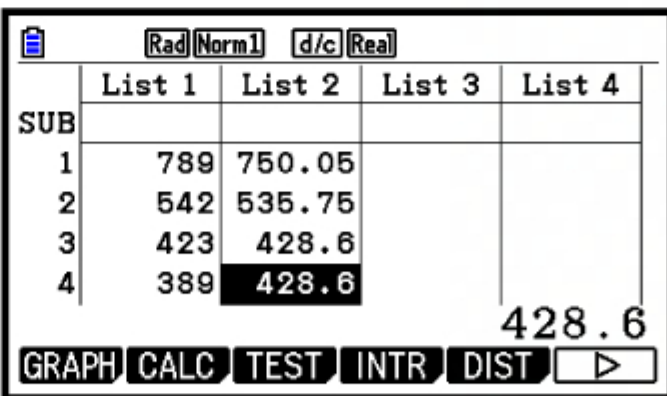
Step 1: Let p_1, p_2, p_3 , and p_4 be the population proportion of choc-chip, oatmeal, shortbread, and butter cookies respectively.

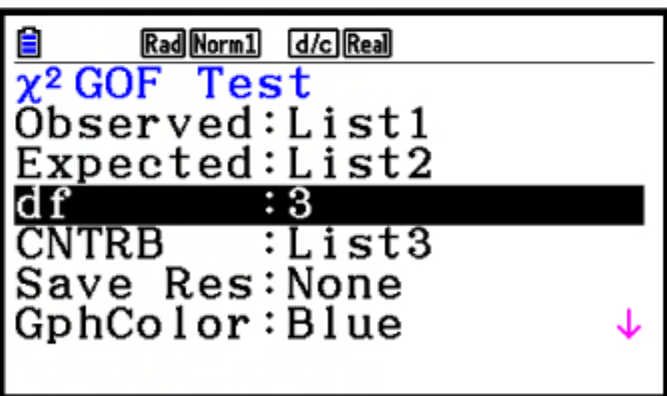
The hypotheses that should be tested are:

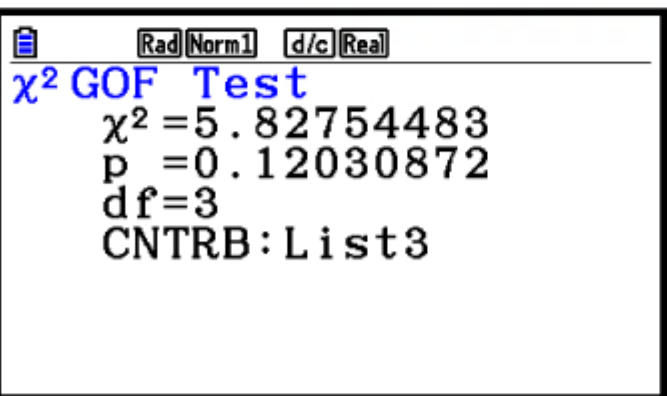
$H_0: p_1 = 0.35, p_2 = 0.25, p_3 = 0.2, p_4 = 0.2$
 $H_1: \text{at least one of } p_1 \neq 0.35, p_2 \neq 0.25, p_3 \neq 0.2, \text{ or } p_4 \neq 0.2.$

Step 2: The significance level is $\alpha = 0.01$.

Step 3: $df = 4 - 1 = 3$







Using technology, the value of the test statistic is $\chi^2_{\text{calc}} \approx 5.83$.

Step 4: From the screenshots above, the p -value ≈ 0.120 .

Step 5: Since p -value $> 0.01 = \alpha$, we do not have enough evidence to reject H_0 in favour of H_1 on a 1% level of significance. We therefore accept H_0 .

Step 6: Since we have accepted H_0 , we cannot conclude that the proportions of cookies eaten are different to the proportions of cookies that Brianna provides. Brianna should not change the proportion of cookies she provides.

96

- a H_0 : Preferred milk flavour and age are independent.
 H_1 : Preferred milk flavour and age are not independent.

b

Using technology, the value of the test statistic is $\chi^2_{\text{calc}} \approx 10.7$.

- c** Since $\chi^2_{\text{calc}} > \chi^2_{\text{crit}} = 7.81$, we have enough evidence to reject H_0 in favour of H_1 on a 5% significance level.

We conclude that *preferred milk flavour* is not independent of *age* at a 5% significance level.

97

	Variety A	Variety B	Variety C	Sum
Fertiliser	65	48	75	188
No fertiliser	40	54	58	152
Sum	105	102	133	340

- a** H_0 : The effect of fertiliser is independent of the variety of orange.
 H_1 : The effect of fertiliser is not independent of the variety of orange.

b

	Variety A	Variety B	Variety C
Fertiliser	$\frac{188 \times 105}{340} \approx 58.1$	$\frac{188 \times 102}{340} \approx 56.4$	73.5
No fertiliser	$\frac{152 \times 105}{340} \approx 46.9$	45.6	59.5

c

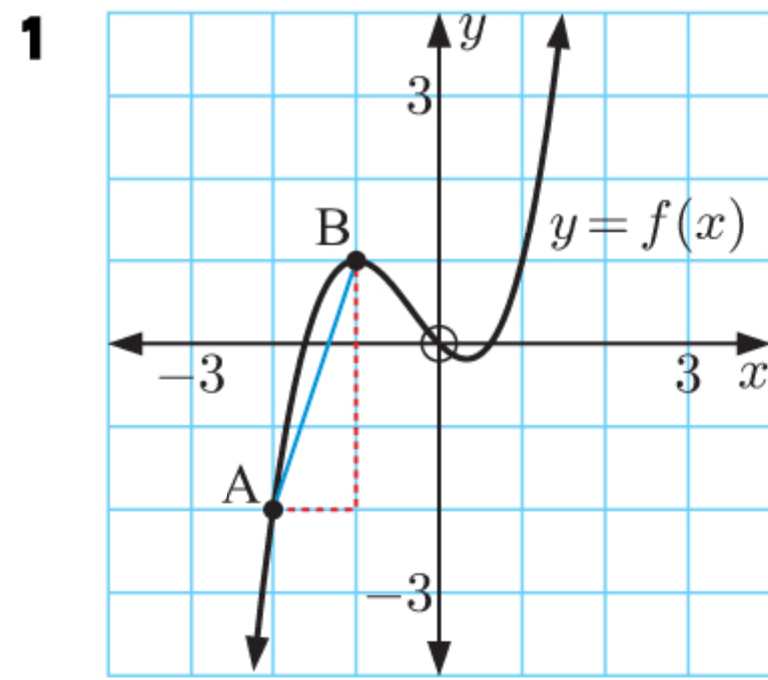
Using technology, $\chi^2_{\text{calc}} \approx 4.72$.

- d** $df = (2 - 1)(3 - 1) = 2$
e From the screenshots in **c**, p -value ≈ 0.0944 .
f The significance level is $\alpha = 0.05$.

Since p -value $> 0.05 = \alpha$, we do not have enough evidence to reject H_0 in favour of H_1 on a 5% significance level. We therefore accept H_0 .

We conclude that the *effect of the fertiliser* is independent of the *variety of orange*.

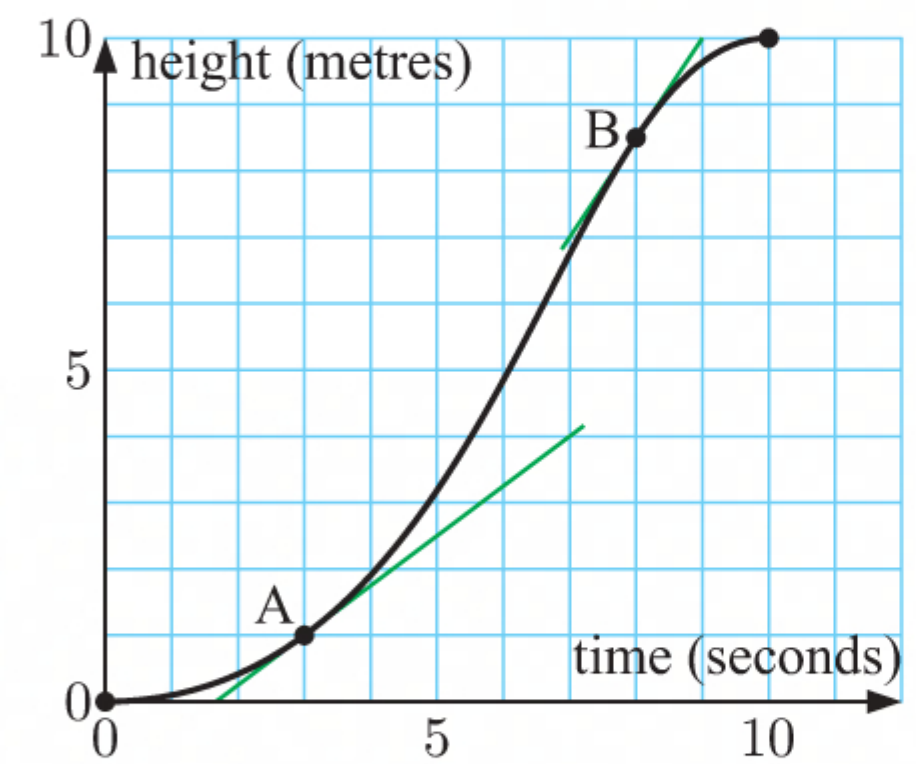
TOPIC 5 SKILL BUILDER QUESTIONS



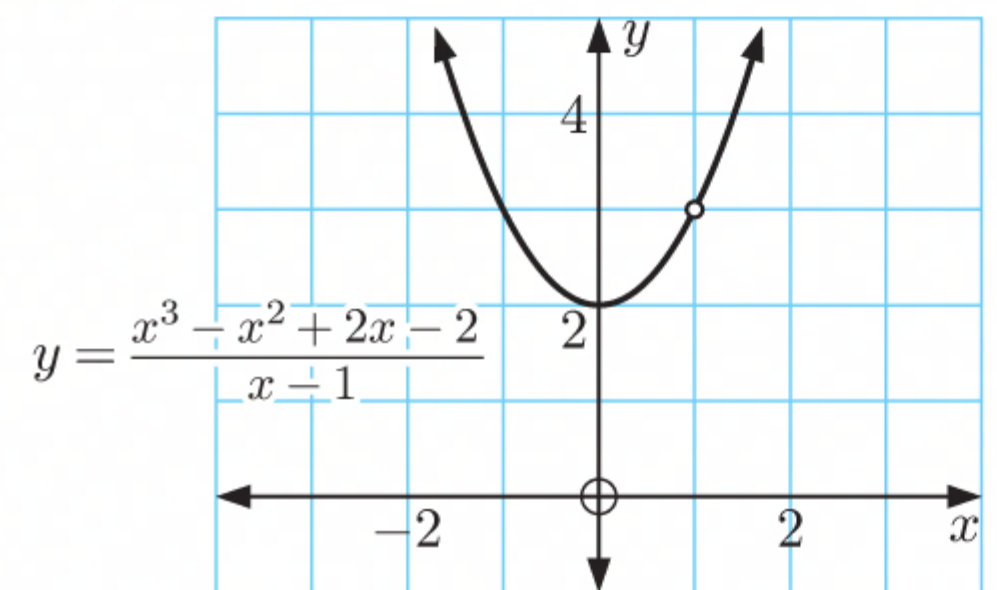
Average rate of change = gradient of chord [AB]

$$\begin{aligned}
 &= \frac{1 - (-2)}{-1 - (-2)} \\
 &= \frac{3}{1} \\
 &= 3
 \end{aligned}$$

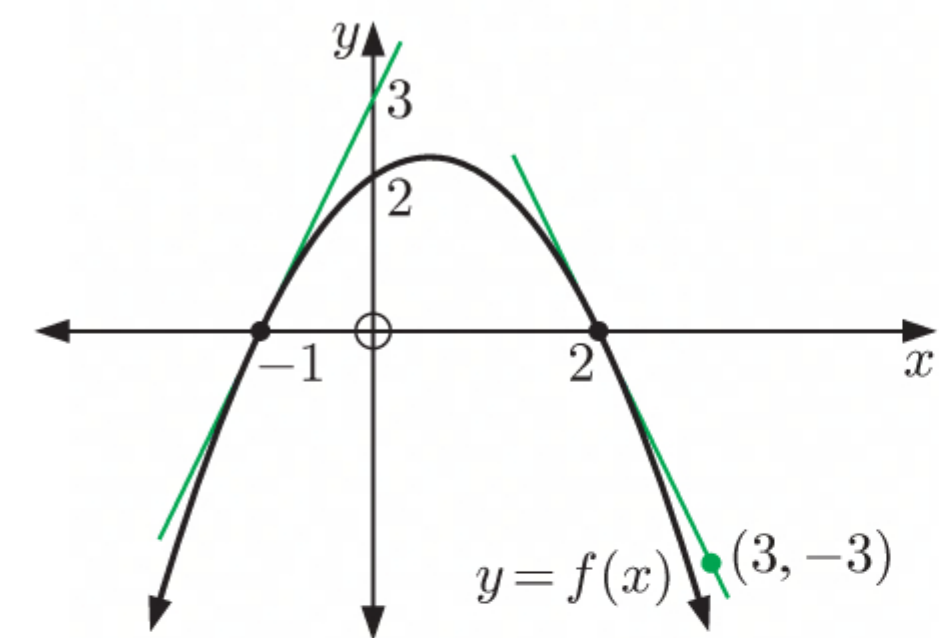
- 2 a The tangent at A has gradient $\frac{4-1}{7-3} = \frac{3}{4}$.
 \therefore the elevator's instantaneous speed after 3 seconds is 0.75 m s^{-1} .
- b The tangent at B has gradient $\frac{10-7}{9-7} = \frac{3}{2}$.
 \therefore the elevator's instantaneous speed after 8 seconds is 1.5 m s^{-1} .



- 3 a When $x = 1$, the denominator of $\frac{x^3 - x^2 + 2x - 2}{x - 1}$ is zero. So, the function is undefined at $x = 1$.
- b From the graph, we can see that $y \rightarrow 3$ as $x \rightarrow 1$ from either direction.
 So, $\lim_{x \rightarrow 1} f(x) = 3$.



- 4 a The tangent at $x = -1$ passes through $(-1, 0)$ and $(0, 3)$.
 $\therefore f'(-1) = \text{gradient of the tangent}$
 $= \frac{3-0}{0-(-1)}$
 $= \frac{3}{1}$
 $= 3$



- b The tangent at $x = 2$ passes through $(2, 0)$ and $(3, -3)$.
 $\therefore f'(2) = \text{gradient of the tangent}$
 $= \frac{-3-0}{3-2}$
 $= \frac{-3}{1}$
 $= -3$

5 a $\frac{d}{dx}(6 - 3x + 2x^2) = 0 - 3(1) + 2(2x)$
 $= -3 + 4x$

c $\frac{d}{dx}\left(\frac{1}{x} + \frac{1}{x^2}\right) = \frac{d}{dx}(x^{-1} + x^{-2})$
 $= -x^{-2} - 2x^{-3}$
 $= -\frac{1}{x^2} - \frac{2}{x^3}$

b $\frac{d}{dx}\left(\frac{1}{2}x^2 + 3x - 5\right) = \frac{1}{2}(2x) + 3(1) - 0$
 $= x + 3$

d $\frac{d}{dx}\left(\frac{2x^2 + x + 1}{x}\right) = \frac{d}{dx}(2x + 1 + x^{-1})$
 $= 2(1) + 0 - x^{-2}$
 $= 2 - \frac{1}{x^2}$

6 a $f(x) = ax^2 + bx^3$, $f'(1) = -5$, and $f'(-1) = -13$

$\therefore f'(x) = 2ax + 3bx^2$

But $f'(1) = -5$, so $2a(1) + 3b(1)^2 = -5$

$\therefore 2a + 3b = -5$

and $f'(-1) = -13$, so $2a(-1) + 3b(-1)^2 = -13$

$\therefore -2a + 3b = -13$

Solving the system of equations $\begin{cases} 2a + 3b = -5 \\ -2a + 3b = -13 \end{cases}$

simultaneously gives $a = 2$, $b = -3$.

b $f(x) = ax + \frac{b}{x^2}$, $f(1) = 8$, and $f'(1) = -7$

$= ax + bx^{-2}$

$\therefore f'(x) = a - 2bx^{-3}$

$= a - \frac{2b}{x^3}$

But $f(1) = 8$, so $a(1) + \frac{b}{1^2} = 8$

$\therefore a + b = 8$

and $f'(1) = -7$, so $a - \frac{2b}{1^3} = -7$

$\therefore a - 2b = -7$

Solving the system of equations $\begin{cases} a + b = 8 \\ a - 2b = -7 \end{cases}$

simultaneously gives $a = 3$, $b = 5$.

7 a $y = 3x - 2x^2$

$\therefore \frac{dy}{dx} = 3 - 2(2x)$
 $= 3 - 4x$

When $x = 4$, $\frac{dy}{dx} = 3 - 16 = -13$.

So, the tangent has gradient -13 .

b $y = \frac{x^2 + 4x - 1}{x^2}$
 $= 1 + \frac{4}{x} - \frac{1}{x^2}$
 $= 1 + 4x^{-1} - x^{-2}$
 $\therefore \frac{dy}{dx} = -4x^{-2} + 2x^{-3}$
 $= -\frac{4}{x^2} + \frac{2}{x^3}$

When $x = 1$, $\frac{dy}{dx} = -4 + 2 = -2$.

So, the tangent has gradient -2 .

8 $f(x) = ax^2 + bx - 7$

$\therefore f'(x) = 2ax + b$

Now at $(-1, -10)$, the tangent has gradient 1.

$\therefore f(-1) = -10$ and $f'(-1) = 1$

$\therefore a(-1)^2 + b(-1) - 7 = -10$ and $2a(-1) + b = 1$

$\therefore a - b = -3$ and $-2a + b = 1$

Solving the system of equations $\begin{cases} a - b = -3 \\ -2a + b = 1 \end{cases}$

simultaneously gives $a = 2$, $b = 5$.

9 a $y = 3x^2 + 5x + 1$

$$\therefore \frac{dy}{dx} = 6x + 5$$

$$\therefore \text{the tangent has gradient 11 when } 6x + 5 = 11$$

$$\therefore 6x = 6$$

$$\therefore x = 1$$

$$\text{When } x = 1, y = 3(1)^2 + 5(1) + 1 = 9$$

So, the tangent has gradient 11 at the point (1, 9).

b $f(x) = \frac{1}{2}x^3 - 4x - 2$

$$\therefore f'(x) = \frac{3}{2}x^2 - 4$$

$$\therefore \text{the tangent has gradient } \frac{1}{2} \text{ when } \frac{3}{2}x^2 - 4 = \frac{1}{2}$$

$$\therefore 3x^2 - 8 = 1$$

$$\therefore 3x^2 = 9$$

$$\therefore x^2 = 3$$

$$\therefore x = \pm\sqrt{3}$$

$$f(-\sqrt{3}) = \frac{1}{2}(-\sqrt{3})^3 - 4(-\sqrt{3}) - 2 \approx 2.33$$

$$f(\sqrt{3}) = \frac{1}{2}(\sqrt{3})^3 - 4(\sqrt{3}) - 2 \approx -6.33$$

So, the tangent has gradient $\frac{1}{2}$ at the points $(-\sqrt{3}, 2.33)$ and $(\sqrt{3}, -6.33)$.

10 a $y = x^2 + 2x - 5$

$$\therefore \frac{dy}{dx} = 2x + 2$$

$$\text{Now when } x = 1, y = 1^2 + 2(1) - 5 \text{ and } \frac{dy}{dx} = 2(1) + 2$$

$$= -2 \qquad \qquad \qquad = 4$$

\therefore the point of contact is (1, -2) and the gradient of the tangent is 4.

$$\therefore \text{the tangent has equation } y = 4(x - 1) - 2$$

$$\text{which is } y = 4x - 6.$$

b $y = 3 - \frac{2}{x}$

$$= 3 - 2x^{-1}$$

$$\therefore \frac{dy}{dx} = -2(-x^{-2})$$

$$= \frac{2}{x^2}$$

$$\text{Now when } x = -2, y = 3 - \frac{2}{-2} \text{ and } \frac{dy}{dx} = \frac{2}{(-2)^2}$$

$$= 4 \qquad \qquad \qquad = \frac{1}{2}$$

\therefore the point of contact is (-2, 4) and the gradient of the tangent is $\frac{1}{2}$.

$$\therefore \text{the tangent has equation } y = \frac{1}{2}(x + 2) + 4$$

$$\text{which is } y = \frac{1}{2}x + 5.$$

11 a $f(x) = -x^2 + 4x$

$$\therefore f'(x) = -2x + 4$$

b Since $f(k) = -k^2 + 4k$, the point of contact is $(k, -k^2 + 4k)$.

$$\text{Now } f'(k) = -2k + 4.$$

$$\text{The tangent has equation } y = f'(k)(x - k) + f(k)$$

$$\therefore y = (-2k + 4)(x - k) - k^2 + 4k$$

$$\therefore y = (-2k + 4)x + 2k^2 - 4k - k^2 + 4k$$

$$\therefore y = (-2k + 4)x + k^2$$

c The tangent passes through (4, 9), so

$$(-2k + 4)(4) + k^2 = 9$$

$$\therefore -8k + 16 + k^2 = 9$$

$$\therefore k^2 - 8k + 7 = 0$$

$$\therefore k = 1 \text{ or } 7 \quad \{\text{technology}\}$$

The gradient of the tangent is positive, so $-2k + 4 > 0$.

$$\text{Now if } k = 1, -2k + 4 = -2 + 4 = 2 \quad \checkmark$$

$$\text{if } k = 7, -2k + 4 = -14 + 4 = -10 \quad \times$$

So $k = 1$.

12 $y = x^3 + 2x + 1$

$$\therefore \frac{dy}{dx} = 3x^2 + 2$$

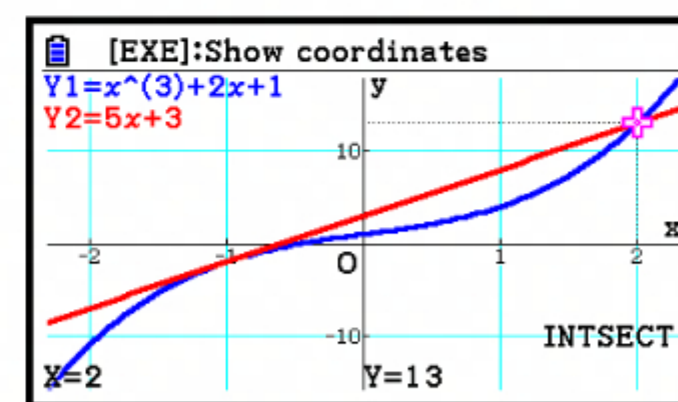
Now when $x = -1$, $y = (-1)^3 + 2(-1) + 1$ and $\frac{dy}{dx} = 3(-1)^2 + 2$

$$\begin{aligned} &= -1 - 2 + 1 & &= 3 + 2 \\ &= -2 & &= 5 \end{aligned}$$

\therefore the point of contact is $(-1, -2)$ and the gradient of the tangent is 5.

\therefore the tangent has equation $y = 5(x + 1) - 2$
which is $y = 5x + 3$.

Using technology, the tangent meets the curve again at $(2, 13)$.



13 a $y = \frac{a}{x} - x^2 + 1 = ax^{-1} - x^2 + 1$

$$\begin{aligned} \therefore \frac{dy}{dx} &= -ax^{-2} - 2x \\ &= \frac{-a}{x^2} - 2x \end{aligned}$$

Now the gradient of the tangent at $x = 2$ is -5 .

$$\therefore \frac{-a}{2^2} - 2(2) = -5$$

$$\therefore \frac{-a}{4} - 4 = -5$$

$$\therefore \frac{-a}{4} = -1$$

$$\therefore a = 4$$

14 a $f(x) = x^3 - 2x$

$$\therefore f'(x) = 3x^2 - 2$$

Now $f(1) = 1^3 - 2(1) = -1$ and

$$f'(1) = 3(1)^2 - 2 = 1$$

\therefore the point of contact is $(1, -1)$ and the gradient of the normal is -1 .

\therefore the equation of the normal is $y = -(x - 1) - 1$
which is $y = -x$.

15 a $f(x) = ax^2 + bx$

i When $x = 2$, $2 + 3y = -4$

$$\therefore 3y = -6$$

$$\therefore y = -2$$

$$\therefore f(2) = -2$$

$$\therefore a(2)^2 + b(2) = -2$$

$$\therefore 4a + 2b = -2$$

b $y = \frac{a}{x} - x^2 + 1 = \frac{4}{x} - x^2 + 1$ {from a}

When $x = 2$, $y = \frac{4}{2} - 2^2 + 1$

$$\begin{aligned} &= 2 - 4 + 1 \\ &= -1 \end{aligned}$$

So, the point of contact is $(2, -1)$.

The equation of tangent is $y = -5(x - 2) + (-1)$

$$\therefore y = -5x + 10 - 1$$

$$\therefore y = -5x + 9$$

b $f(x) = \frac{3}{x} - \frac{6}{x^2}$

$$= 3x^{-1} - 6x^{-2}$$

$$\therefore f'(x) = -3x^{-2} + 12x^{-3}$$

$$= -\frac{3}{x^2} + \frac{12}{x^3}$$

Now $f(2) = \frac{3}{2} - \frac{6}{4} = 0$ and $f'(2) = -\frac{3}{4} + \frac{12}{8} = \frac{3}{4}$

\therefore the point of contact is $(2, 0)$ and the gradient of the normal is $-\frac{4}{3}$.

\therefore the equation of the normal is $y = -\frac{4}{3}(x - 2) + 0$
which is $y = -\frac{4}{3}x + \frac{8}{3}$.

ii $f'(x) = 2ax + b$

Now the normal has equation $x + 3y = -4$

$$\therefore 3y = -4 - x$$

$$\therefore y = -\frac{4}{3} - \frac{1}{3}x$$

\therefore the normal has gradient $-\frac{1}{3}$ at $x = 2$.

\therefore the tangent has gradient 3 at $x = 2$

$$\therefore f'(2) = 3$$

$$\therefore 2a(2) + b = 3$$

$$\therefore 4a + b = 3$$

- b** Solving the system of equations $\begin{cases} 4a + 2b = -2 \\ 4a + b = 3 \end{cases}$ simultaneously gives $a = 2$, $b = -5$.

- c** From **a ii**, the normal L_1 has gradient $-\frac{1}{3}$.

\therefore the tangent L_2 has gradient $-\frac{1}{3}$.

Now from **b**, $f(x) = 2x^2 - 5x$ and $f'(x) = 4x - 5$.

\therefore the tangent has gradient $-\frac{1}{3}$ when $4x - 5 = -\frac{1}{3}$

$$\therefore 12x - 15 = -1$$

$$\therefore 12x = 14$$

$$\therefore x = \frac{14}{12} = \frac{7}{6}$$

$$f\left(\frac{7}{6}\right) = 2\left(\frac{7}{6}\right)^2 - 5\left(\frac{7}{6}\right) = -\frac{28}{9}$$

\therefore Q has coordinates $\left(\frac{7}{6}, -\frac{28}{9}\right)$.

- 16 a i** $f(x)$ is increasing for $\frac{2}{3} \leq x \leq 2$.
ii $f(x)$ is decreasing for $x \leq \frac{2}{3}$ and $x \geq 2$.

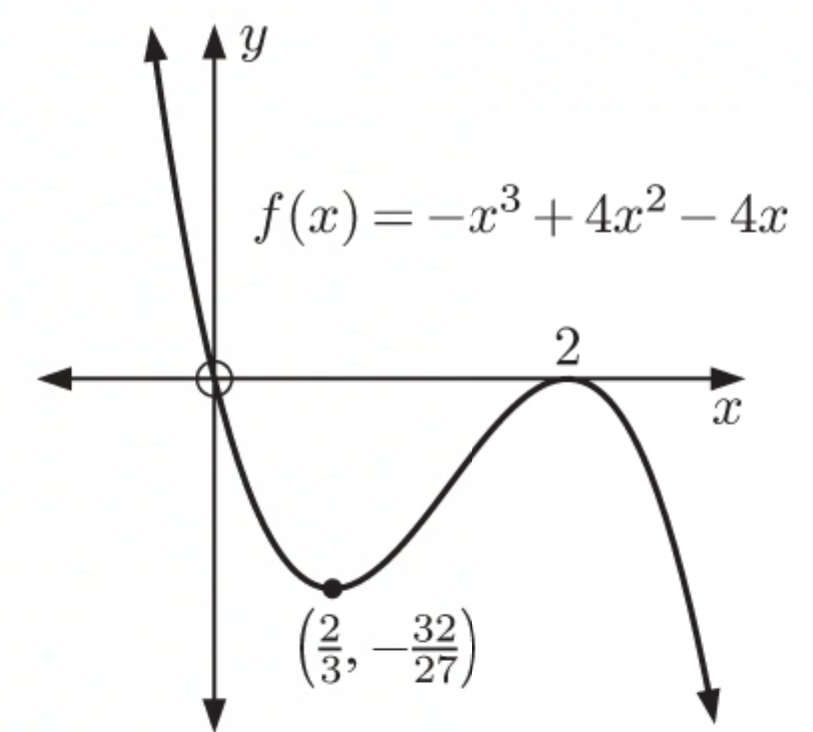
b $f(x) = -x^3 + 4x^2 - 4x$
 $\therefore f'(x) = -3x^2 + 8x - 4$

From the graph, the zeros of $f'(x)$ should be $\frac{2}{3}$ and 2.

Check: $f'\left(\frac{2}{3}\right) = -3\left(\frac{2}{3}\right)^2 + 8\left(\frac{2}{3}\right) - 4$ and $f'(2) = -3(2)^2 + 8(2) - 4$
 $= -\frac{4}{3} + \frac{16}{3} - 4 = -12 + 16 - 4 = 0$ ✓
 $= \frac{12}{3} - 4 = 0$ ✓

$f'(x)$ has sign diagram $\begin{array}{c} \leftarrow - \quad + \quad - \rightarrow \\ \frac{2}{3} \quad 2 \quad x \end{array}$

This is consistent with our observations in **a**.



- 17 a** $f(x) = 5 - 3x$
 $\therefore f'(x) = -3$
 $\therefore f'(x) < 0$ for all x .
 $\therefore f(x)$ is decreasing for all x .
- b** $f(x) = 2x^2 - 7x + 6$
 $\therefore f'(x) = 4x - 7$
 which is 0 when $x = \frac{7}{4}$
 $\therefore f'(x)$ has sign diagram $\begin{array}{c} \leftarrow - \quad + \rightarrow \\ \frac{7}{4} \quad x \end{array}$

So, $f(x)$ is increasing for $x \geq \frac{7}{4}$ and decreasing for $x \leq \frac{7}{4}$.

c $f(x) = -\frac{1}{x} = -x^{-1}$

$\therefore f'(x) = x^{-2} = \frac{1}{x^2}$ which has sign diagram $\begin{array}{c} \leftarrow + \quad + \rightarrow \\ 0 \quad x \end{array}$

So, $f(x)$ is increasing for all $x \neq 0$.

d $f(x) = 2x^3 - 9x^2 + 7x + 6$
 $\therefore f'(x) = 6x^2 - 18x + 7$

Using technology, the zeros of $f'(x)$ are ≈ 0.459 and ≈ 2.54 .

$f'(x)$ has sign diagram $\begin{array}{c} \leftarrow + \quad - \quad + \rightarrow \\ \approx 0.459 \quad \approx 2.54 \quad x \end{array}$

So, $f(x)$ is increasing for $x \leq 0.459$ and $x \geq 2.54$, and decreasing for $0.459 \leq x \leq 2.54$.

18 a $f(x) = x^3 - x^2$
 $\therefore f'(x) = 3x^2 - 2x$
 $= x(3x - 2)$

$\therefore f'(x) = 0$ when $x = 0$ or $\frac{2}{3}$.

\therefore the sign diagram for $f'(x)$ is

$$\begin{aligned} f(0) &= 0^3 - 0^2 = 0 & \text{and} & \quad f\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right)^2 \\ & & & = \frac{8}{27} - \frac{4}{9} \\ & & & = -\frac{4}{27} \end{aligned}$$

So, there is a local maximum at $(0, 0)$ and a local minimum at $\left(\frac{2}{3}, -\frac{4}{27}\right)$.

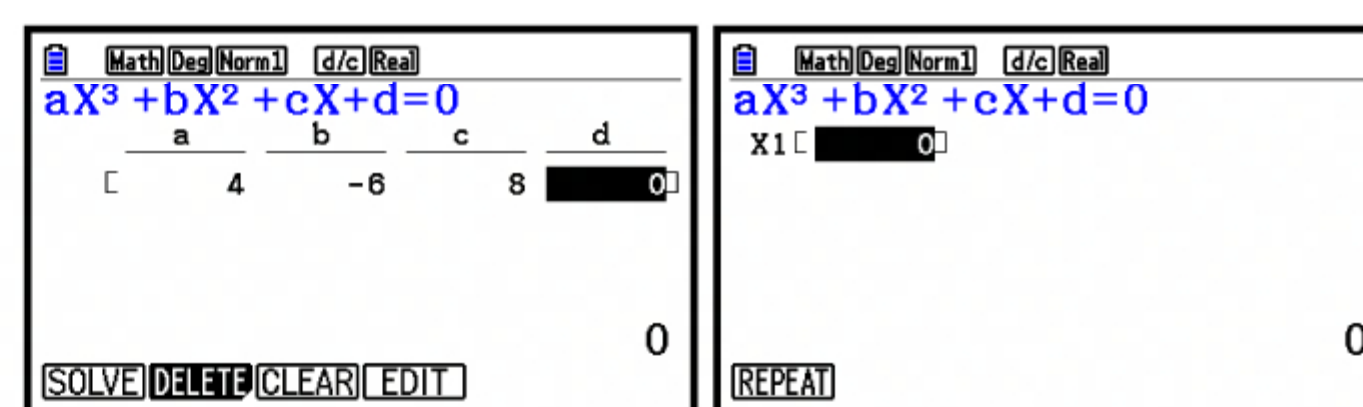
b $f(x) = x^4 - 2x^3 + 4x^2 - 8$
 $\therefore f'(x) = 4x^3 - 6x^2 + 8x$

Using technology, the only real zero of $f'(x)$ is 0.

$f'(x)$ has sign diagram

$f(0) = -8$

So, there is a local minimum at $(0, -8)$.



c $f(x) = 2x + \frac{6}{x}$
 $= 2x + 6x^{-1}$
 $\therefore f'(x) = 2 - 6x^{-2}$
 $= 2 - \frac{6}{x^2}$

which is 0 when $2 - \frac{6}{x^2} = 0$

$$\therefore \frac{6}{x^2} = 2$$

$$\therefore 2x^2 = 6$$

$$\therefore x^2 = 3$$

$$\therefore x = \pm\sqrt{3}$$

$f'(x)$ has sign diagram

$$\begin{aligned} f(-\sqrt{3}) &= -2\sqrt{3} - \frac{6}{\sqrt{3}} \approx -6.93 & \text{and} & \quad f(\sqrt{3}) = 2\sqrt{3} + \frac{6}{\sqrt{3}} \\ & & & \approx 6.93 \end{aligned}$$

So, there is a local maximum at $(-\sqrt{3}, -6.93)$ and a local minimum at $(\sqrt{3}, 6.93)$.

19 a $f(x) = 2x^3 + ax + b$
 $\therefore f'(x) = 6x^2 + a$

Now $f(x)$ has a stationary point at $(1, 1)$.

So, $f'(1) = 0$ and $f(1) = 1$

$$\therefore 6(1)^2 + a = 0 \quad \therefore 2(1)^3 - 6(1) + b = 1$$

$$\therefore a = -6 \quad \therefore 2 - 6 + b = 1$$

$$\therefore b = 5$$

b From **a**, $f(x) = 2x^3 - 6x + 5$ and $f'(x) = 6x^2 - 6$

Stationary points occur where $f'(x) = 0$

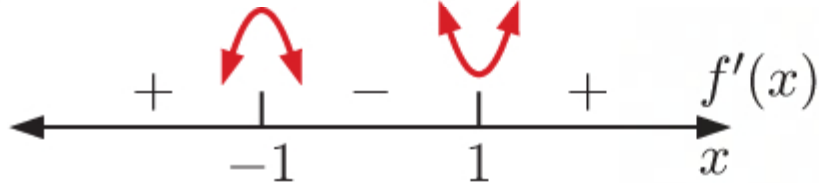
$$\therefore 6x^2 - 6 = 0$$

$$\therefore 6x^2 = 6$$

$$\therefore x^2 = 1$$

$$\therefore x = \pm 1$$

$f'(x)$ has sign diagram



$$\begin{aligned} f(-1) &= 2(-1)^3 - 6(-1) + 5 \\ &= -2 + 6 + 5 \\ &= 9 \end{aligned}$$

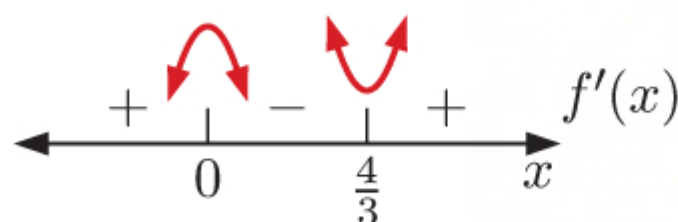
So, $(1, 1)$ is a local minimum and $(-1, 9)$ is a local maximum.

20 a $f(x) = x^3 - 2x^2$, $-1 \leq x \leq 1$

$$\begin{aligned} \therefore f'(x) &= 3x^2 - 4x \\ &= x(3x - 4) \end{aligned}$$

which is 0 when $x = 0$ or $\frac{4}{3}$.

The sign diagram of $f'(x)$ is



\therefore there is a local maximum at $x = 0$, and a local minimum at $x = \frac{4}{3}$.

Critical value (x)	$f(x)$
-1 (end point)	-3
0 (local maximum)	0
1 (end point)	-1

The greatest of these values is 0 when $x = 0$.

The least of these values is -3 when $x = -1$.

b $f(x) = x^2 - \frac{27}{x} = x^2 - 27x^{-1}$, $-6 \leq x \leq -1$

$$\therefore f'(x) = 2x + 27x^{-2}$$

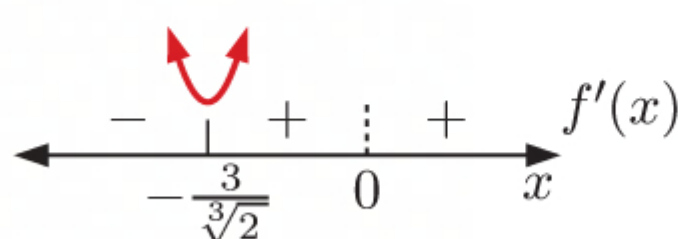
which is 0 when $2x + 27x^{-2} = 0$

$$\therefore 2x = -\frac{27}{x^2}$$

$$\therefore x^3 = -\frac{27}{2}$$

$$\therefore x = -\frac{3}{\sqrt[3]{2}}$$

The sign diagram of $f'(x)$ is



\therefore there is a local minimum at $x = -\frac{3}{\sqrt[3]{2}}$.

Critical value (x)	$f(x)$
-6 (end point)	40.5
$-\frac{3}{\sqrt[3]{2}}$ (local minimum)	≈ 17.0
-1 (end point)	28

The greatest of these values is 40.5 when $x = -6$.

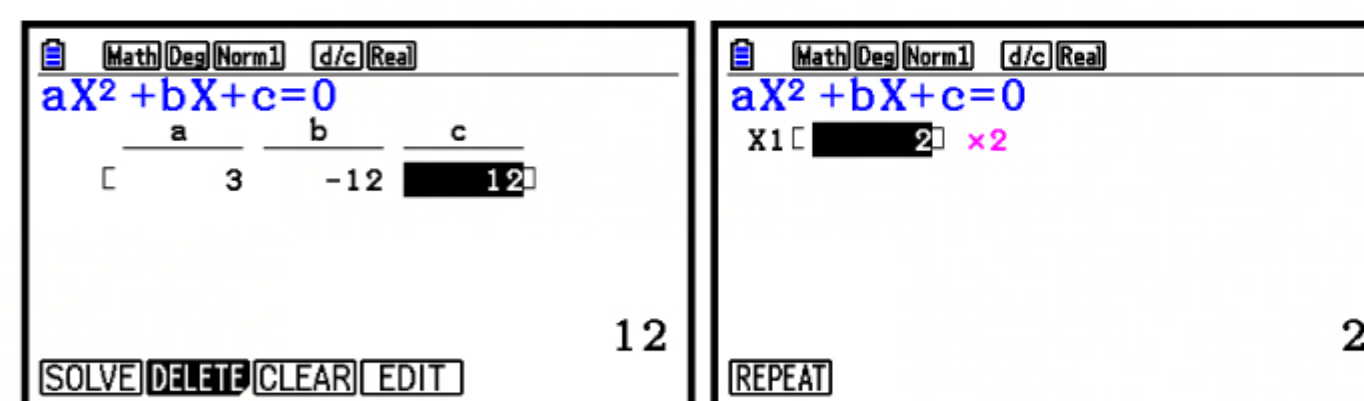
The least of these values is ≈ 17.0 when $x = -\frac{3}{\sqrt[3]{2}}$.

c $f(x) = x^3 - 6x^2 + 12x - 10, \quad 0 \leq x \leq 5$

$\therefore f'(x) = 3x^2 - 12x + 12$

which is 0 when $3x^2 - 12x + 12 = 0$

$\therefore x = 2$ {technology}



The sign diagram of $f'(x)$ is



\therefore there is a stationary inflection at $x = 2$.

Critical value (x)	$f(x)$
0 (end point)	-10
5 (end point)	25

The greatest of these values is 25 when $x = 5$.

The least of these values is -10 when $x = 0$.

21 a $C(x) = -0.2x^2 + 4x + 10$ for $0 \leq x \leq 10$

$C(5) = -0.2(5)^2 + 4(5) + 10 = \25

Hence it costs \$25 to produce 5 bracelets.

c If C is measured in dollars and x is the number of bracelets produced, then $C'(x)$ has units “dollars per bracelet”.

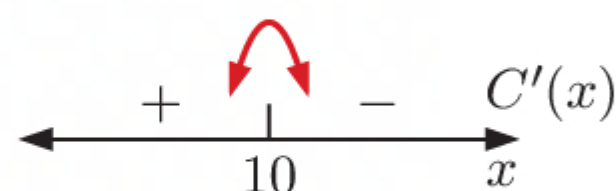
b $C'(x) = -0.2(2x) + 4$
 $= -0.4x + 4$

d $C'(5) = -0.4(5) + 4$
 $= 2$ dollars per bracelet

\therefore when 5 bracelets are produced, the cost is increasing by \$2 per bracelet.

e $C'(x) = 0$ when $-0.4x + 4 = 0$
 $\therefore -0.4x = -4$
 $\therefore x = 10$

$C'(x)$ has sign diagram:

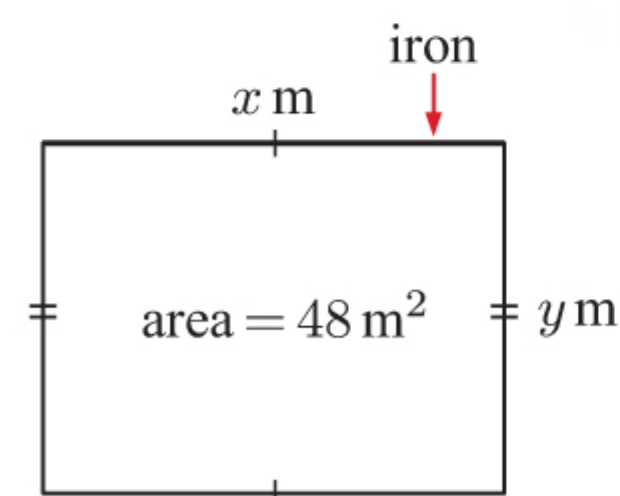


\therefore there is a maximum when $x = 10$.

$C(10) = -0.2(10)^2 + 4(10) + 10 = 30$

\therefore the maximum value of $C(x)$ on $0 \leq x \leq 10$ is \$30.

22 a



Let the length of the side adjacent to the corrugated iron fence be y m.

Now $\text{area} = xy$

$\therefore xy = 48$

$\therefore y = \frac{48}{x}$

Cost of fencing $= 18(2y + x) + 30x$

$\therefore C = 18\left(\frac{2 \times 48}{x} + x\right) + 30x$

$\therefore C = \frac{2 \times 48 \times 18}{x} + 18x + 30x$

$\therefore C = \frac{2 \times 48 \times 18}{x} + 48x$

$\therefore C = 48\left(\frac{36}{x} + x\right)$ dollars

b $\frac{dC}{dx} = 48\left(\frac{-36}{x^2} + 1\right)$

Now C is minimised when $\frac{dC}{dx} = 0$

$\therefore 48\left(\frac{-36}{x^2} + 1\right) = 0$

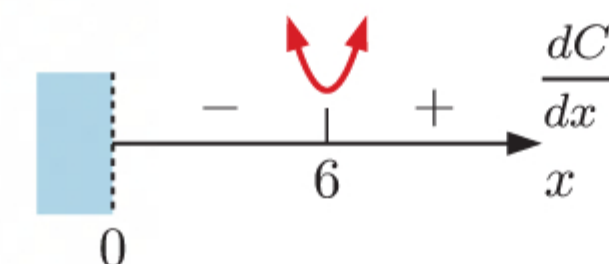
$\therefore \frac{-36}{x^2} + 1 = 0$

$\therefore \frac{36}{x^2} = 1$

$\therefore x^2 = 36$

$\therefore x = 6 \quad \{x > 0\}$

The sign diagram of $\frac{dC}{dx}$ is



$\therefore C$ is minimised when $x = 6$.

When $x = 6$, $y = \frac{48}{6} = 8$.

The dimensions that minimise the cost of fencing are 6 m \times 8 m, where one of the 6 m sides is fenced with corrugated iron.

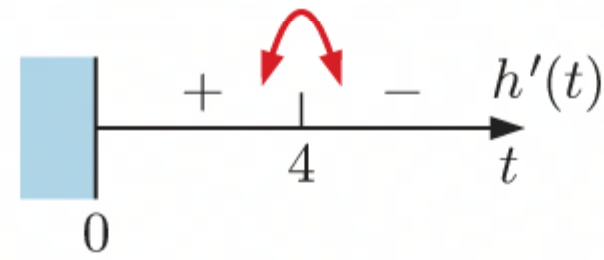
23 a $h(t) = 100 + 32t - 4t^2$

$\therefore h'(t) = 32 - 8t$

b $h'(t) = 0$ when $32 = 8t$

$\therefore t = 4$

Sign diagram for $h'(t)$:



So, the height is maximised when $t = 4$ seconds.

Now $h(4) = 100 + 32(4) - 4(4)^2 = 164$

\therefore the maximum height is 164 m.

24 a Surface area = area of triangles + area of rectangles

$= 2\left(\frac{1}{2}x^2\right) + 2(xy)$

$= x^2 + 2xy$

$\therefore x^2 + 2xy = 27$

b From **a**, $x^2 + 2xy = 27$

$\therefore 2xy = 27 - x^2$

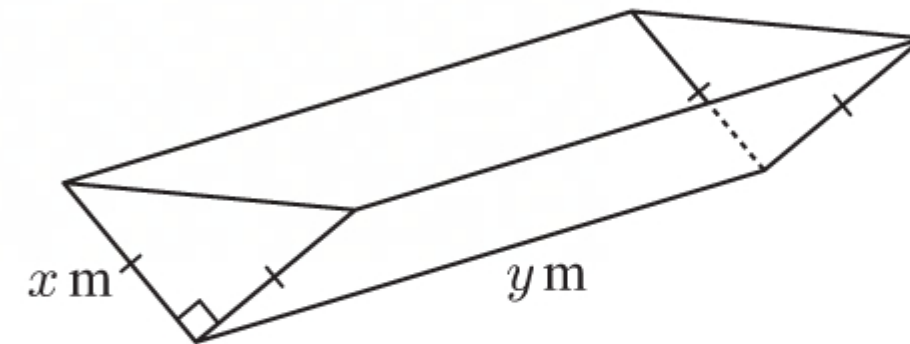
$\therefore y = \frac{27}{2x} - \frac{x}{2}$

Now volume = cross-sectional area \times depth

$\therefore V = \frac{1}{2}x^2y$

$\therefore V = \frac{1}{2}x^2\left(\frac{27}{2x} - \frac{x}{2}\right)$

$\therefore V = \frac{27x}{4} - \frac{x^3}{4}$



c $\frac{dV}{dx} = \frac{27}{4} - \frac{3}{4}x^2$

Now V is maximised when $\frac{dV}{dx} = 0$

$\therefore \frac{27}{4} - \frac{3}{4}x^2 = 0$

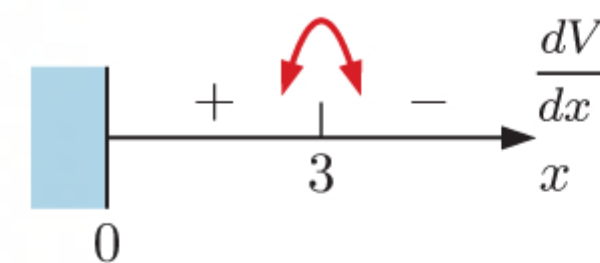
$\therefore 27 - 3x^2 = 0$

$\therefore 3x^2 = 27$

$\therefore x^2 = 9$

$\therefore x = 3 \quad \{x > 0\}$

The sign diagram of $\frac{dV}{dx}$ is



$\therefore V$ is maximised when $x = 3$.

When $x = 3$, $y = \frac{27}{2(3)} - \frac{3}{2}$

$= \frac{9}{2} - \frac{3}{2}$

$= \frac{6}{2}$

$= 3$

So, V is maximised when $x = y = 3$.

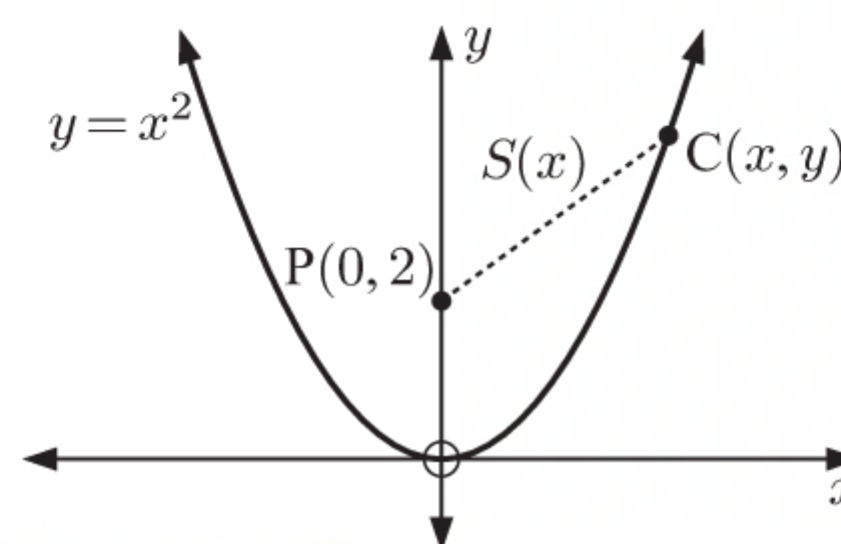
25 a C has coordinates (x, x^2) .

Now $S(x) = CP$

$= \sqrt{(x-0)^2 + (x^2-2)^2}$

$= \sqrt{x^2 + x^4 - 4x^2 + 4}$

$= \sqrt{x^4 - 3x^2 + 4}$



b $S^2 = x^4 - 3x^2 + 4$

$$\therefore \frac{d}{dx}(S^2) = 4x^3 - 6x$$

which is 0 when $4x^3 - 6x = 0$

$$\therefore 2x(2x^2 - 3) = 0$$

$$\therefore x = 0 \quad \text{or} \quad 2x^2 - 3 = 0$$

$$\therefore x^2 = \frac{3}{2}$$

$$\therefore x = \pm\sqrt{\frac{3}{2}}$$

$$\therefore x \approx -1.22 \quad \text{or} \quad 1.22$$

The sign diagram of $\frac{d}{dx}(S^2)$ is

\therefore there is a local maximum at $x = 0$, and local minima at $x \approx -1.22$ and $x \approx 1.22$.

So, S^2 is minimised when $x \approx -1.22$ or $x \approx 1.22$.

c

Critical point (x)	S^2	S
-2 (end point)	8	≈ 2.83
≈ -1.22 (local minimum)	≈ 1.75	≈ 1.32
0 (local maximum)	4	2
≈ 1.22 (local minimum)	≈ 1.75	≈ 1.32
2 (end point)	8	≈ 2.83

The greatest distance between the comet and the observer is ≈ 2.83 units when $x = \pm 2$.

The shortest distance between the comet and the observer is ≈ 1.32 units when $x \approx \pm 1.22$.

26 a

i	x_i	$f(x_i)$
0	1	13
1	1.2	13.288
2	1.4	13.384
3	1.6	13.336
4	1.8	13.192
5	2	13

$$n = 5, \quad a = 1, \quad b = 2, \quad f(x) = x^3 - 6x^2 + 11x + 7$$

$$h = \frac{b-a}{n} = \frac{1}{5}, \quad x_i = 1 + \frac{1}{5}i$$

Using the trapezoidal rule, the area

$$\approx \frac{h}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + f(x_5))$$

$$\approx 13.24 \text{ units}^2$$

b

$$\int_1^2 f(x) dx = \int_1^2 (x^3 - 6x^2 + 11x + 7) dx$$

$$= \left[\frac{1}{4}x^4 - 2x^3 + \frac{11}{2}x^2 + 7x \right]_1^2$$

$$= \left(\frac{1}{4}(2)^4 - 2(2)^3 + \frac{11}{2}(2)^2 + 7(2) \right) - \left(\frac{1}{4}(1)^4 - 2(1)^3 + \frac{11}{2}(1)^2 + 7(1) \right)$$

$$= 24 - \frac{43}{4}$$

$$= \frac{53}{4} = 13.25$$

The actual area between $y = f(x)$ and the x -axis for $1 \leq x \leq 2$ is 13.25 units^2 .

c Percentage error = $\frac{|V_A - V_E|}{V_E} \times 100\%$

$$= \frac{|13.24 - 13.25|}{13.25} \times 100\%$$

$$= \frac{0.01}{13.25} \times 100\%$$

$$\approx 0.0755\%$$

27 a $\frac{d}{dx}(x^2) = 2x$

\therefore the antiderivative of $2x$ is x^2 .

b $\frac{d}{dx}(x^3) = 3x^2$

$$\therefore \frac{d}{dx}\left(\frac{1}{9}x^3\right) = \frac{x^2}{3}$$

\therefore the antiderivative of $\frac{x^2}{3}$ is $\frac{1}{9}x^3$.

$$\begin{aligned} \mathbf{c} \quad & \frac{3}{x^2} = 3x^{-2} \\ & \frac{d}{dx}(x^{-1}) = -x^{-2} \\ \therefore & \frac{d}{dx}(-3x^{-1}) = 3x^{-2} \\ \therefore & \text{the antiderivative of } \frac{3}{x^2} \text{ is } -3x^{-1} = -\frac{3}{x}. \end{aligned}$$

$$\begin{aligned} \mathbf{28} \quad & \frac{d}{dx}\left(\frac{2}{x^2} - 3x\right) = \frac{d}{dx}(2x^{-2} - 3x) \\ & = -4x^{-3} - 3 \\ & = -\frac{4}{x^3} - 3 \\ \therefore & \frac{d}{dx}\left(-2\left(\frac{2}{x^2} - 3x\right)\right) = \frac{8}{x^3} + 6 \\ \therefore & \text{the antiderivative of } \frac{8}{x^3} + 6 \text{ is } -2\left(\frac{2}{x^2} - 3x\right) = -\frac{4}{x^2} + 6x. \\ \therefore & \int\left(\frac{8}{x^3} + 6\right) dx = -\frac{4}{x^2} + 6x + c \end{aligned}$$

$$\begin{aligned} \mathbf{29} \quad \mathbf{a} \quad & \int -3 dx = -3x + c \\ \mathbf{b} \quad & \int\left(\frac{3}{x^2} + 2x^3 - 4\right) dx \\ & = \int(3x^{-2} + 2x^3 - 4) dx \\ & = \frac{3}{-1}x^{-1} + \frac{2}{4}x^4 - 4x + c \\ & = -\frac{3}{x} + \frac{1}{2}x^4 - 4x + c \\ \mathbf{c} \quad & \int\left(\frac{1}{x} + 2x\right)^2 dx \\ & = \int\left(\left(\frac{1}{x}\right)^2 + 2\left(\frac{1}{x}\right)(2x) + (2x)^2\right) dx \\ & = \int\left(\frac{1}{x^2} + 4 + 4x^2\right) dx \\ & = \int(x^{-2} + 4 + 4x^2) dx \\ & = \frac{1}{-1}x^{-1} + 4x + \frac{4}{3}x^3 + c \\ & = -\frac{1}{x} + 4x + \frac{4}{3}x^3 + c \end{aligned}$$

$$\begin{aligned} \mathbf{30} \quad \mathbf{a} \quad & \frac{dy}{dx} = 4x \\ \therefore & y = \int 4x dx \\ & = \frac{4}{2}x^2 + c \\ & = 2x^2 + c \\ \mathbf{b} \quad & \frac{dy}{dx} = x^2 + \frac{1}{2}x + \frac{1}{3} \\ \therefore & y = \int\left(x^2 + \frac{1}{2}x + \frac{1}{3}\right) dx \\ & = \frac{1}{3}x^3 + \frac{1}{2}\left(\frac{1}{2}x^2\right) + \frac{1}{3}x + c \\ & = \frac{1}{3}x^3 + \frac{1}{4}x^2 + \frac{1}{3}x + c \\ \mathbf{c} \quad & \frac{dy}{dx} = \frac{3x^4 + 5}{x^3} \\ & = 3x + 5x^{-3} \\ \therefore & y = \int(3x + 5x^{-3}) dx \\ & = \frac{3}{2}x^2 + \frac{5}{-2}x^{-2} + c \\ & = \frac{3}{2}x^2 - \frac{5}{2x^2} + c \end{aligned}$$

$$\begin{aligned} \mathbf{31} \quad & f'(x) = 4x - 3x^2 \\ \therefore & f(x) = \int(4x - 3x^2) dx \\ & = \frac{4}{2}x^2 - x^3 + c \\ & = 2x^2 - x^3 + c \end{aligned}$$

$$\begin{aligned} \text{Now } f(3) &= -2, \text{ so } 2(3)^2 - 3^3 + c = -2 \\ \therefore 18 - 27 + c &= -2 \\ \therefore -9 + c &= -2 \\ \therefore c &= 7 \end{aligned}$$

$$\therefore f(x) = 2x^2 - x^3 + 7$$

$$\begin{aligned} \mathbf{32} \quad \mathbf{a} \quad & \int_0^1 (x^2 + x) dx \\ & = \left[\frac{1}{3}x^3 + \frac{1}{2}x^2\right]_0^1 \\ & = \left(\frac{1}{3} + \frac{1}{2}\right) - 0 \\ & = \frac{5}{6} \\ \mathbf{b} \quad & \int_{-2}^1 (x^3 - 2x^2 - 4x + 9) dx \\ & = \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 2x^2 + 9x\right]_{-2}^1 \\ & = \left(\frac{1}{4} - \frac{2}{3} - 2 + 9\right) - \left(\frac{(-2)^4}{4} - \frac{2}{3}(-2)^3 - 2(-2)^2 + 9(-2)\right) \\ & = \frac{79}{12} - \left(-\frac{50}{3}\right) \\ & = \frac{93}{4} \end{aligned}$$

$$\begin{aligned}
 \text{c } \int_3^5 \left(\frac{8}{x^2} + 3x \right) dx &= \int_3^5 (8x^{-2} + 3x) dx \\
 &= \left[-8x^{-1} + \frac{3}{2}x^2 \right]_3^5 \\
 &= \left(-\frac{8}{5} + \frac{3}{2} \times 25 \right) - \left(-\frac{8}{3} + \frac{3}{2} \times 9 \right) \\
 &= \frac{359}{10} - \frac{65}{6} \\
 &= \frac{376}{15}
 \end{aligned}$$

$$\begin{aligned}
 \text{33 a } \int_{-1}^1 3x^2 dx &= [x^3]_{-1}^1 \\
 &= 1^3 - (-1)^3 \\
 &= 1 + 1 \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 \text{b } \int_2^3 \frac{5x^3 - 2x}{x^5} dx &= \int_2^3 (5x^{-2} - 2x^{-4}) dx \\
 &= \left[-5x^{-1} + \frac{2}{3}x^{-3} \right]_2^3 \\
 &= \left(-\frac{5}{3} + \frac{2}{3} \times \frac{1}{27} \right) - \left(-\frac{5}{2} + \frac{2}{3} \times \frac{1}{8} \right) \\
 &= -\frac{133}{81} - \left(-\frac{29}{12} \right) \\
 &= \frac{251}{324}
 \end{aligned}$$

$$\begin{aligned}
 \text{c } \int_{-3}^0 (1 - 3x)^2 dx &= \int_{-3}^0 (1 - 6x + 9x^2) dx \\
 &= [x - 3x^2 + 3x^3]_{-3}^0 \\
 &= 0 - (-3 - 3(-3)^2 + 3(-3)^3) \\
 &= 0 - (-111) \\
 &= 111
 \end{aligned}$$

$$\text{34 a } \int_1^2 \sqrt{x} + x^2 dx \approx 3.55228475$$

$$\int_1^2 (\sqrt{x} + x^2) dx \approx 3.552$$

$$\text{b } \int_0^3 2^x dx \approx 10.09886529$$

$$\int_0^3 2^x dx \approx 10.10$$

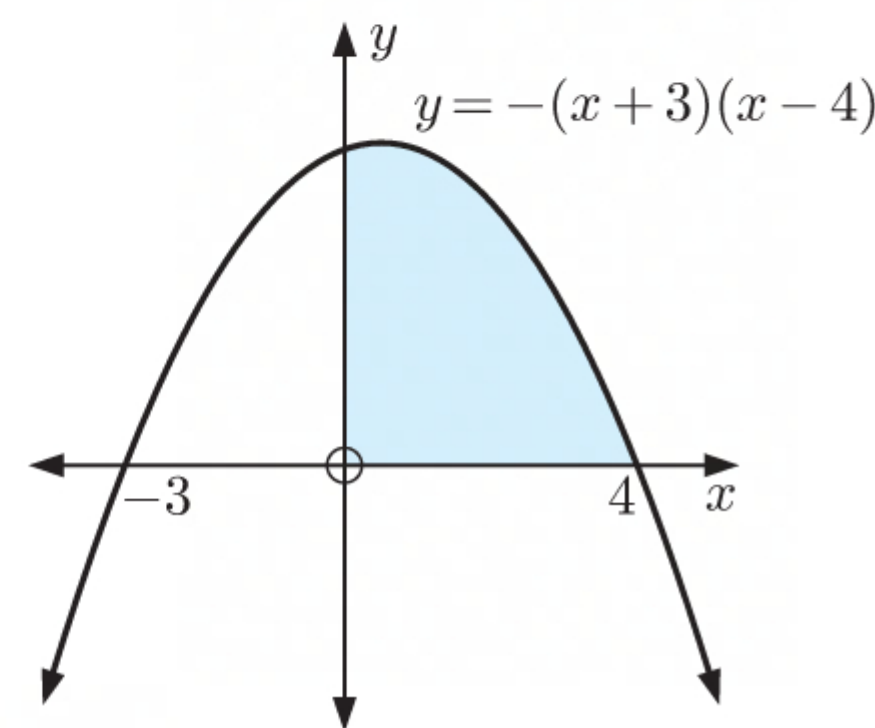
$$\text{c } \int_{-1.5}^{2.4} \frac{5x+20}{x+6} dx \approx 13.25845691$$

$$\int_{-1.5}^{2.4} \frac{5x+20}{x+6} dx \approx 13.26$$

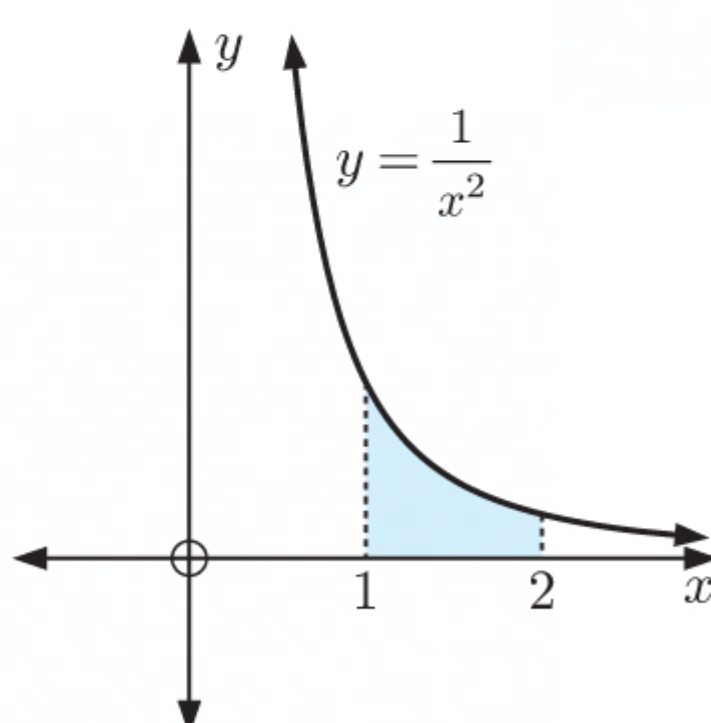
35 a The x -intercepts of $y = -(x+3)(x-4)$ are -3 and 4 .

$$\therefore \text{shaded area} = \int_0^4 -(x+3)(x-4) dx$$

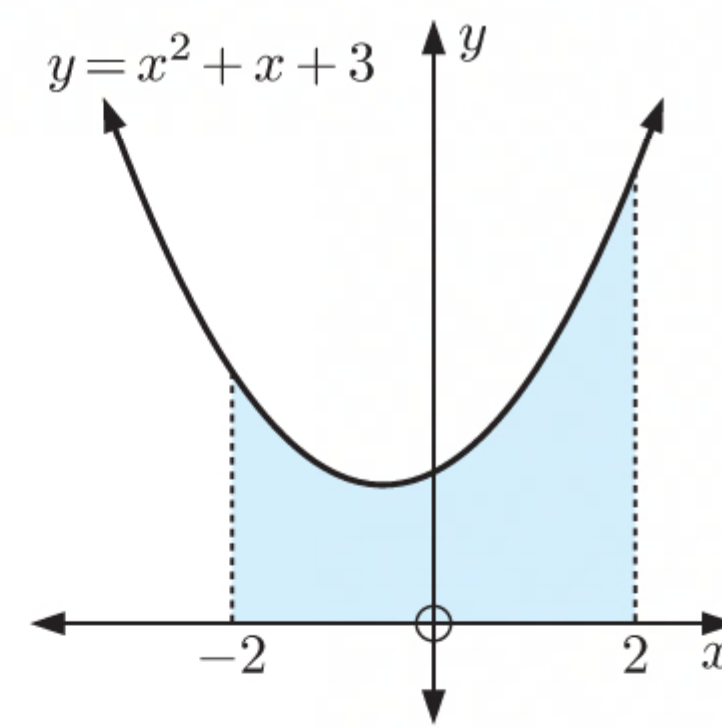
$$\begin{aligned}
 \text{b Shaded area} &= \int_0^4 -(x+3)(x-4) dx \\
 &= \int_0^4 -(x^2 - x - 12) dx \\
 &= \int_0^4 (-x^2 + x + 12) dx \\
 &= \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 12x \right]_0^4 \\
 &= \left(-\frac{4^3}{3} + \frac{4^2}{2} + 12(4) \right) - 0 \\
 &= \frac{104}{3} \\
 &= 34\frac{2}{3} \text{ units}^2
 \end{aligned}$$



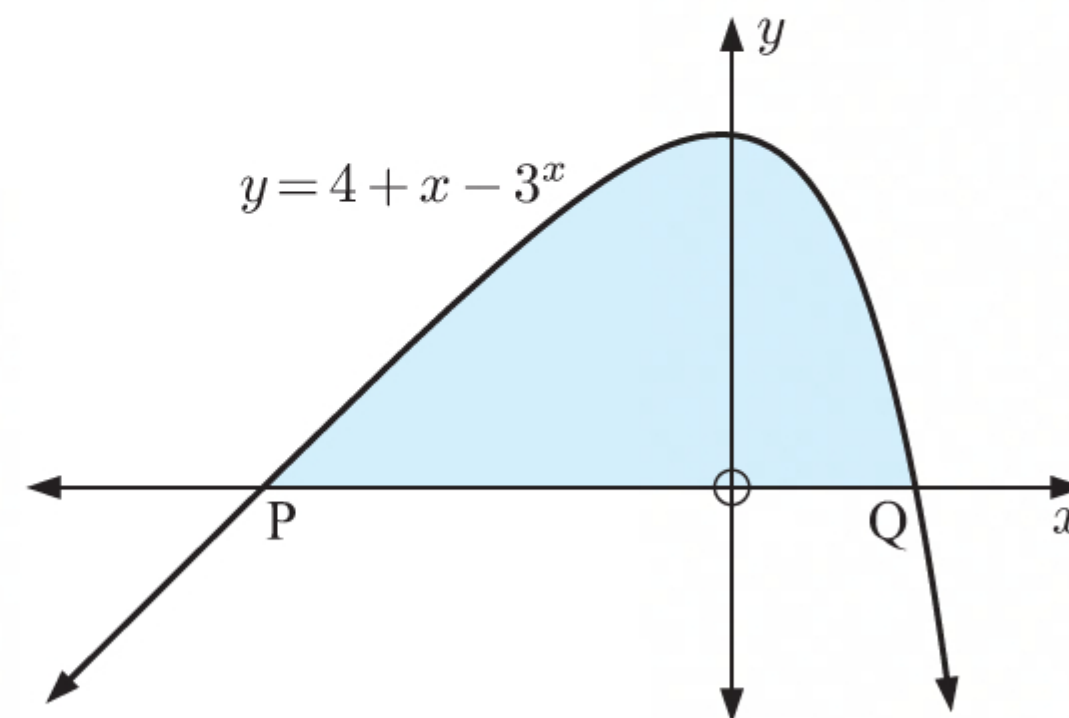
$$\begin{aligned}
 \text{36 a Area} &= \int_1^2 \frac{1}{x^2} dx \\
 &= \int_1^2 x^{-2} dx \\
 &= [-x^{-1}]_1^2 \\
 &= -\frac{1}{2} - \left(-\frac{1}{1} \right) \\
 &= \frac{1}{2} \text{ units}^2
 \end{aligned}$$



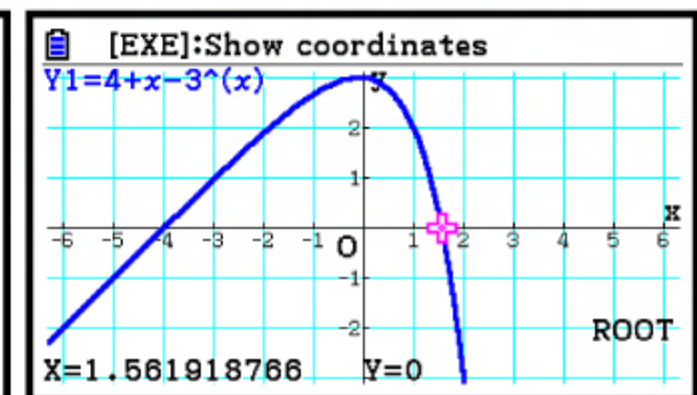
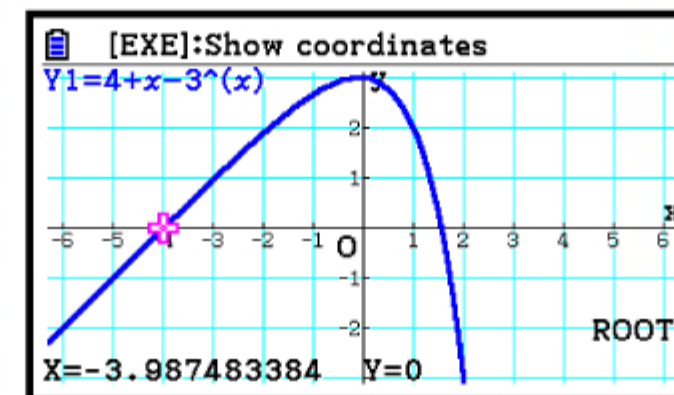
$$\begin{aligned}
 \text{b Area} &= \int_{-2}^2 (x^2 + x + 3) dx \\
 &= \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 + 3x \right]_{-2}^2 \\
 &= \left(\frac{2^3}{3} + \frac{2^2}{2} + 3(2) \right) - \left(\frac{(-2)^3}{3} + \frac{(-2)^2}{2} + 3(-2) \right) \\
 &= \frac{32}{3} - \left(-\frac{20}{3} \right) \\
 &= \frac{52}{3} \\
 &= 17\frac{1}{3} \text{ units}^2
 \end{aligned}$$



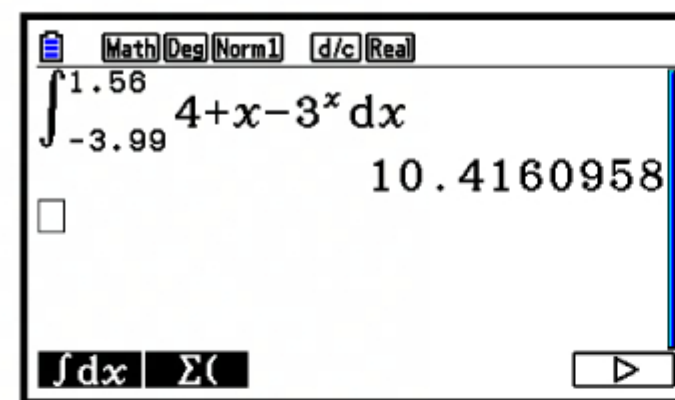
37 a P and Q are the x -intercepts of $y = 4 + x - 3^x$.



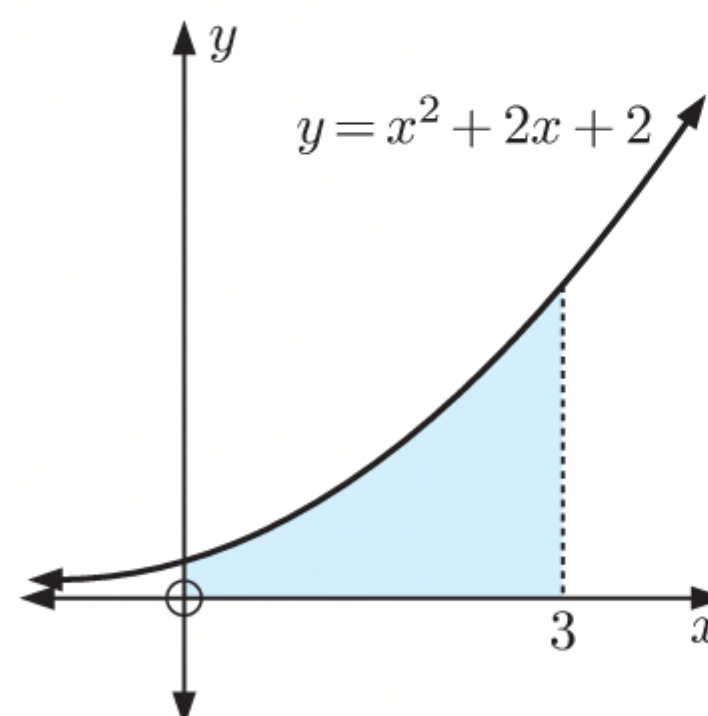
When $y = 0$, $4 + x - 3^x = 0$
 $\therefore x \approx -3.99$ or 1.56
 {technology}
 \therefore P is $(-3.99, 0)$ and Q is $(1.56, 0)$.



$$\begin{aligned}
 \text{b Area} &\approx \int_{-3.99}^{1.56} (4 + x - 3^x) dx \\
 &\approx 10.4 \text{ units}^2 \quad \text{{technology}}
 \end{aligned}$$

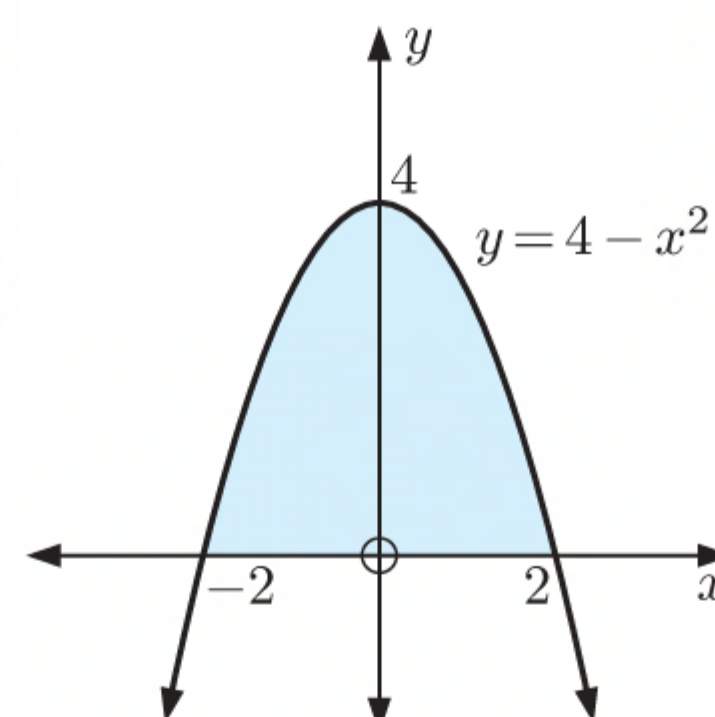


$$\begin{aligned}
 \text{38 a Area} &= \int_0^3 (x^2 + 2x + 2) dx \\
 &= \left[\frac{1}{3}x^3 + x^2 + 2x \right]_0^3 \\
 &= \left(\frac{3^3}{3} + 3^2 + 2(3) \right) - 0 \\
 &= 24 \text{ units}^2
 \end{aligned}$$



$$\begin{aligned}
 \text{b } y &= 4 - x^2 \\
 \text{When } y &= 0, \quad 4 - x^2 = 0 \\
 \therefore x^2 &= 4 \\
 \therefore x &= \pm 2 \\
 \therefore \text{ the } x\text{-intercepts are } -2 \text{ and } 2.
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{ area} &= \int_{-2}^2 (4 - x^2) dx \\
 &= \left[4x - \frac{1}{3}x^3 \right]_{-2}^2 \\
 &= \left(4(2) - \frac{2^3}{3} \right) - \left(4(-2) - \frac{(-2)^3}{3} \right) \\
 &= \frac{16}{3} - \left(-\frac{16}{3} \right) \\
 &= \frac{32}{3} \\
 &= 10\frac{2}{3} \text{ units}^2
 \end{aligned}$$



c $y = (2x + 5)^2$

When $y = 0$, $(2x + 5)^2 = 0$

$$\therefore 2x + 5 = 0$$

$$\therefore x = -\frac{5}{2}$$

\therefore the x -intercept is $-\frac{5}{2}$.

$$\therefore \text{area} = \int_{-4}^{-\frac{5}{2}} (2x + 5)^2 dx$$

$$= \int_{-4}^{-\frac{5}{2}} (4x^2 + 20x + 25) dx$$

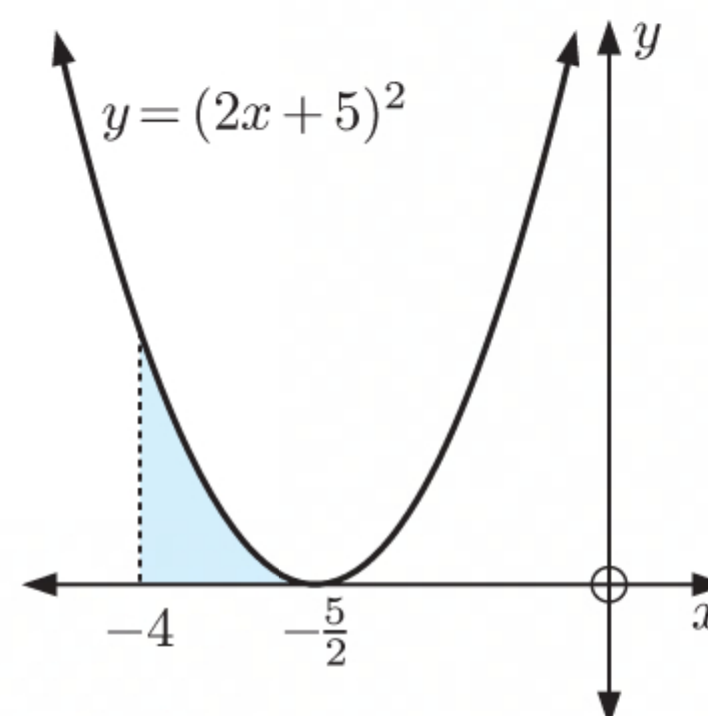
$$= \left[\frac{4}{3}x^3 + 10x^2 + 25x \right]_{-4}^{-\frac{5}{2}}$$

$$= \left(\frac{4}{3} \left(-\frac{5}{2} \right)^3 + 10 \left(-\frac{5}{2} \right)^2 + 25 \left(-\frac{5}{2} \right) \right) - \left(\frac{4}{3}(-4)^3 + 10(-4)^2 + 25(-4) \right)$$

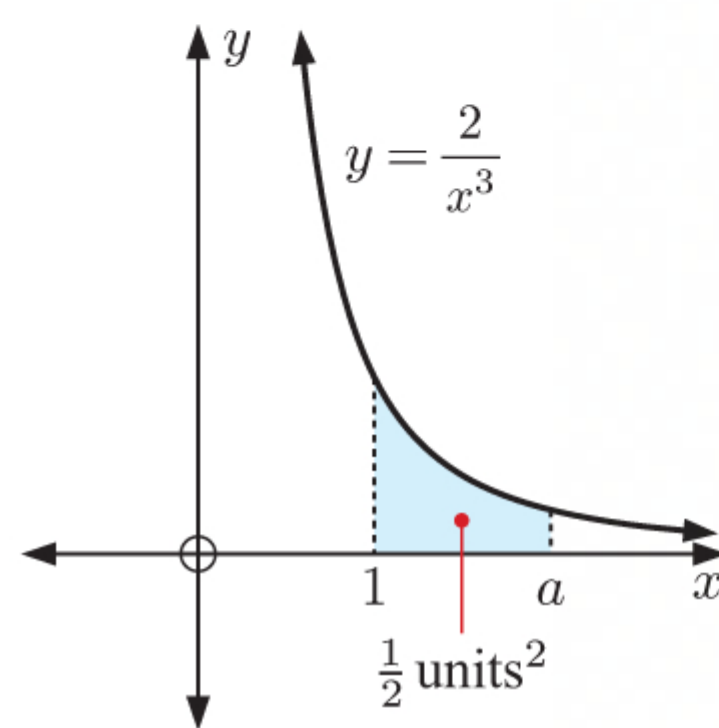
$$= -\frac{125}{6} - \left(-\frac{76}{3} \right)$$

$$= \frac{9}{2}$$

$$= 4\frac{1}{2} \text{ units}^2$$



39 a



$$\text{Area} = \frac{1}{2} \text{ units}^2$$

$$\therefore \int_1^a \frac{2}{x^3} dx = \frac{1}{2}$$

$$\therefore \int_1^a 2x^{-3} dx = \frac{1}{2}$$

$$\therefore [-x^{-2}]_1^a = \frac{1}{2}$$

$$\therefore -\frac{1}{a^2} + \frac{1}{1^2} = \frac{1}{2}$$

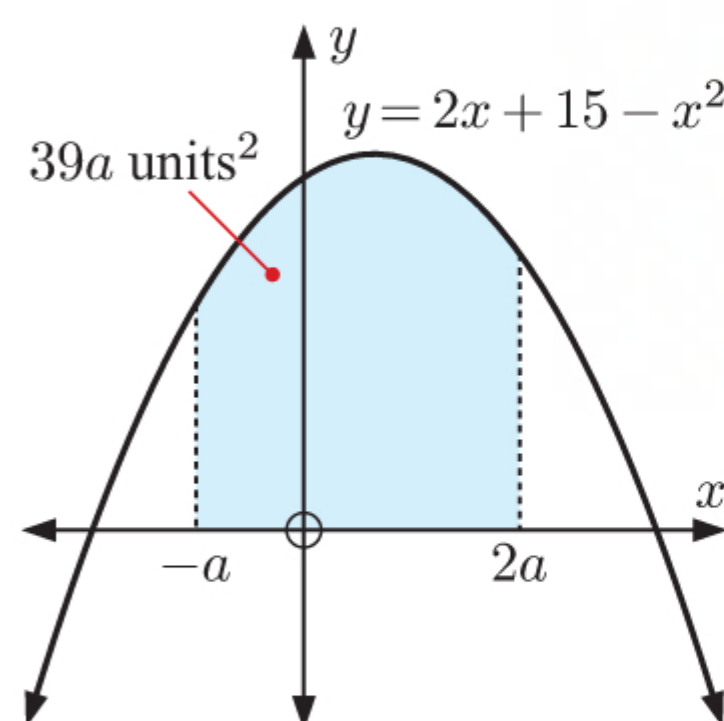
$$\therefore -\frac{1}{a^2} = -\frac{1}{2}$$

$$\therefore a^2 = 2$$

$$\therefore a = \pm\sqrt{2}$$

$$\therefore a = \sqrt{2} \quad \{a > 1\}$$

b



$$\text{Area} = 39a \text{ units}^2$$

$$\therefore \int_{-a}^{2a} (2x + 15 - x^2) dx = 39a$$

$$\therefore \left[x^2 + 15x - \frac{1}{3}x^3 \right]_{-a}^{2a} = 39a$$

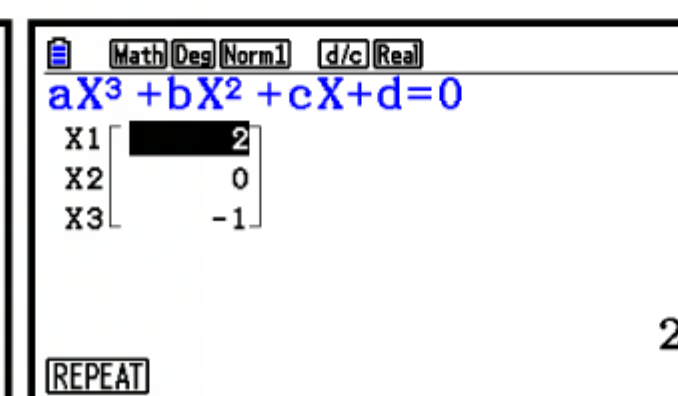
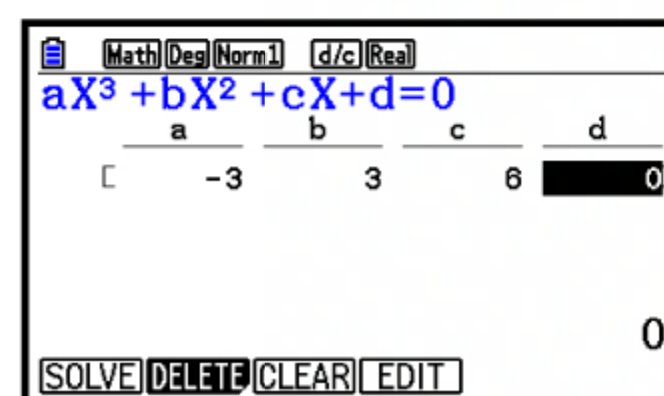
$$\therefore \left(4a^2 + 30a - \frac{8}{3}a^3 \right) - \left(a^2 - 15a + \frac{1}{3}a^3 \right) = 39a$$

$$\therefore 3a^2 + 45a - 3a^3 = 39a$$

$$\therefore 3a^2 + 6a - 3a^3 = 0$$

Using technology, $a = -1, 0$, or 2

$$\therefore a = 2 \quad \{a > 0\}$$



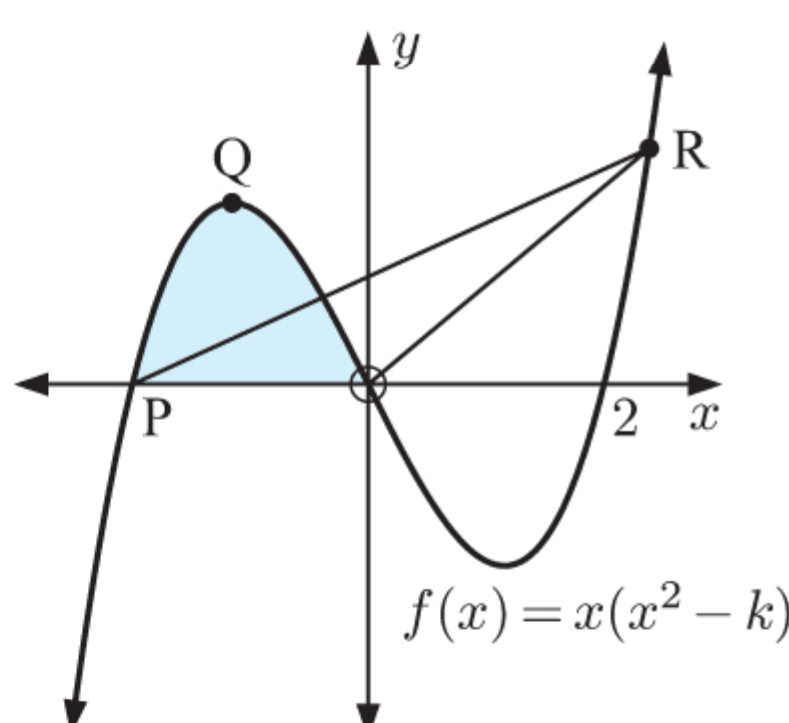
40 a 2 is an x -intercept.

$$\therefore f(2) = 0$$

$$\therefore 2(2^2 - k) = 0$$

$$\therefore 4 - k = 0$$

$$\therefore k = 4$$



b From **a**, $f(x) = x(x^2 - 4)$.

P is the other x -intercept of $y = f(x)$.

$$f(x) = 0 \text{ when } x(x^2 - 4) = 0$$

$$\therefore x = 0 \text{ or } x^2 = 4$$

$$\therefore x = \pm 2$$

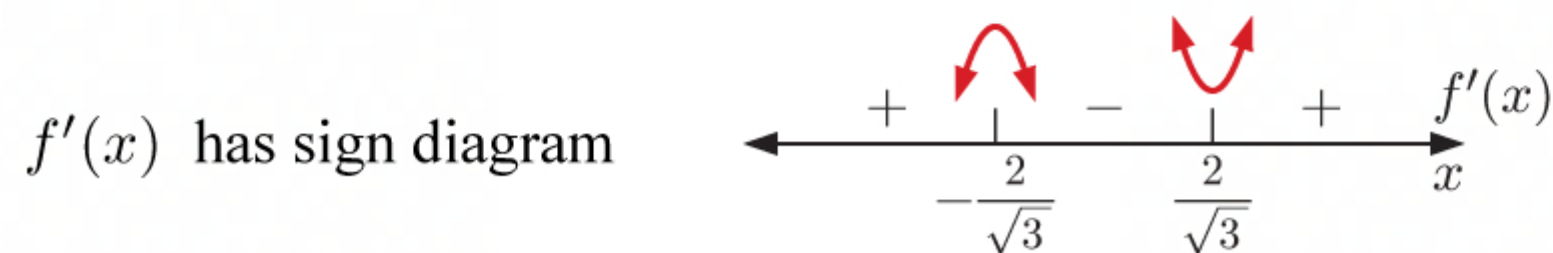
\therefore P has coordinates $(-2, 0)$.

d Stationary points occur where $f'(x) = 0$

$$\therefore 3x^2 - 4 = 0$$

$$\therefore x^2 = \frac{4}{3}$$

$$\therefore x = \pm \frac{2}{\sqrt{3}}$$

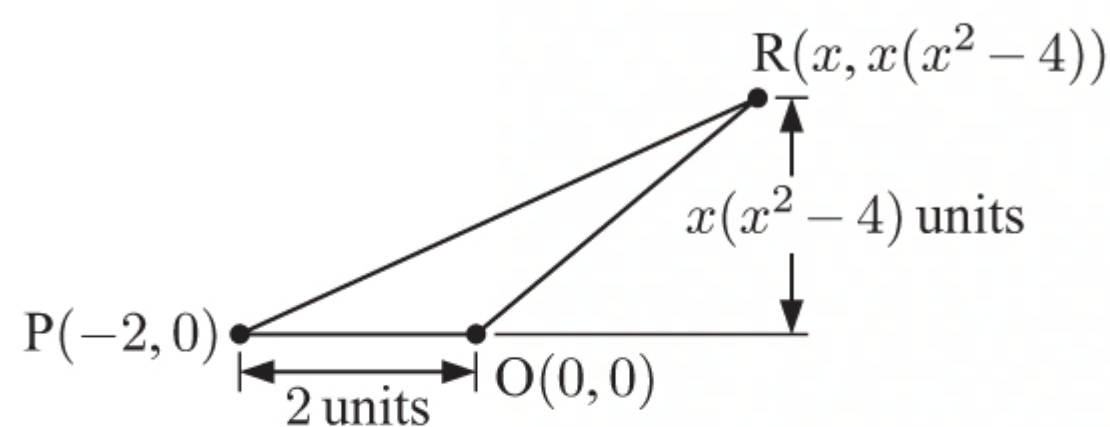


\therefore the local maximum Q has x -coordinate $-\frac{2}{\sqrt{3}}$.

$$\begin{aligned} \text{e } \int f(x) dx &= \int x(x^2 - 4) dx \\ &= \int (x^3 - 4x) dx \\ &= \frac{1}{4}x^4 - 2x^2 + c \end{aligned}$$

$$\begin{aligned} \text{f Shaded area} &= \int_{-2}^0 f(x) dx \\ &= \left[\frac{1}{4}x^4 - 2x^2 \right]_{-2}^0 \\ &= 0 - \left(\frac{(-2)^4}{4} - 2(-2)^2 \right) \\ &= 0 - (-4) \\ &= 4 \text{ units}^2 \end{aligned}$$

g R has coordinates $(x, x(x^2 - 4))$.



$$\begin{aligned} \text{Area of } \triangle POR &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 2 \times x(x^2 - 4) \\ &= x(x^2 - 4) \end{aligned}$$

Shaded area = area of $\triangle POR$

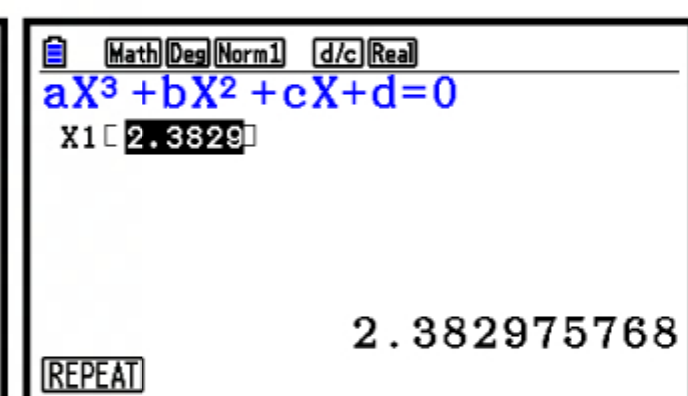
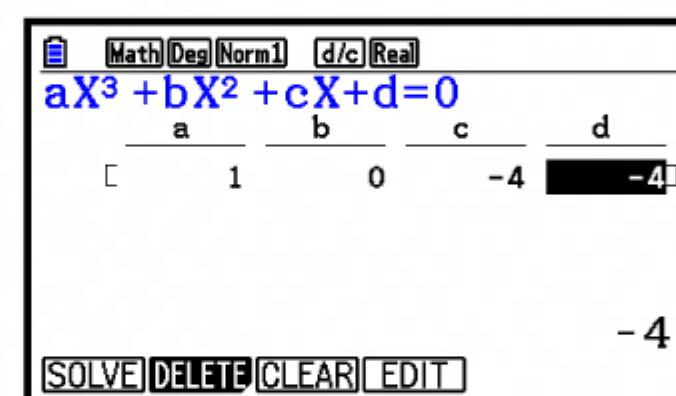
$$\therefore 4 = x(x^2 - 4)$$

$$\therefore 4 = x^3 - 4x$$

$$\therefore x^3 - 4x - 4 = 0$$

$$\therefore x \approx 2.38 \quad \{\text{technology}\}$$

\therefore R has coordinates $(\approx 2.38, 4)$.



MIXED QUESTIONS

MIXED QUESTIONS SET 1

1 $L: y = 3 - 2x$

a Substituting $x = 3$ and $y = k$ into the equation gives $k = 3 - 2(3)$

$$\therefore k = 3 - 6$$

$$\therefore k = -3$$

b Line L has gradient -2 .

c From b, the line is perpendicular to L , which has gradient -2 .

\therefore the line has gradient $\frac{1}{2}$ and passes through $P(3, -3)$. {from a}

\therefore the equation of the line is $y - (-3) = \frac{1}{2}(x - 3)$

$$\therefore y + 3 = \frac{1}{2}x - \frac{3}{2}$$

$$\therefore y = \frac{1}{2}x - \frac{9}{2}$$

2 x could be from 5.5 cm to 6.5 cm.

y could be from 3.5 cm to 4.5 cm.

z could be from 6.5 cm to 7.5 cm.

θ could be from 21.5° to 22.5° .

Now volume of prism = area of cross-section \times length

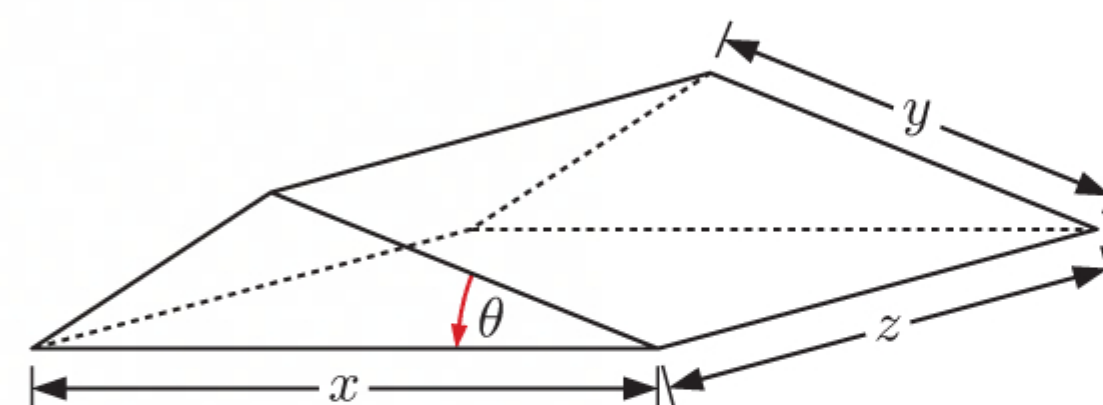
$$= \frac{1}{2}xy \sin \theta \times z$$

$$= \frac{xyz \sin \theta}{2}$$

\therefore the upper boundary of the volume is $\frac{6.5 \times 4.5 \times 7.5 \sin 22.5^\circ}{2} \approx 42.0 \text{ cm}^3$

and the lower boundary of the volume is $\frac{5.5 \times 3.5 \times 6.5 \sin 21.5^\circ}{2} \approx 22.9 \text{ cm}^3$.

The volume of the prism is between about 22.9 cm^3 and about 42.0 cm^3 .



3 a $H(t) = 80 - 5t^2 \text{ m}$

$$\therefore H(0) = 80 - 5(0)^2 = 80 \text{ m}$$

The initial height is 80 m.

b The toy hits the ground when $H(t) = 0$

$$\therefore 80 - 5t^2 = 0$$

$$\therefore 80 = 5t^2$$

$$\therefore t^2 = 16$$

$$\therefore t = \pm 4$$

But $t > 0$, so $t = 4$ seconds

So, the toy hits the ground after 4 seconds.

c $H'(t) = -10t \text{ m s}^{-1}$

$$H'(2) = -10(2) = -20 \text{ m s}^{-1}$$

After 2 seconds of flight, the toy aeroplane is travelling at 20 m s^{-1} towards the ground.

4 a Let u_0 be the original value of the car.

Since the value of the car depreciates by 10% each year, the value of the car after 3 years is

$$u_0 \times (1 - 0.1)^3 = u_0 \times (0.9)^3.$$

$$\therefore u_0 \times (0.9)^3 = 26\,244$$

$$\therefore u_0 = \frac{26\,244}{(0.9)^3}$$

$$= 36\,000$$

\therefore the original value of the car is \$36 000.

b Let u_n be the value of the car after n years.

$$\therefore u_n = u_0 \times (1 - d)^n$$

$$= 36\,000 \times (0.9)^n$$

which describes a geometric sequence with

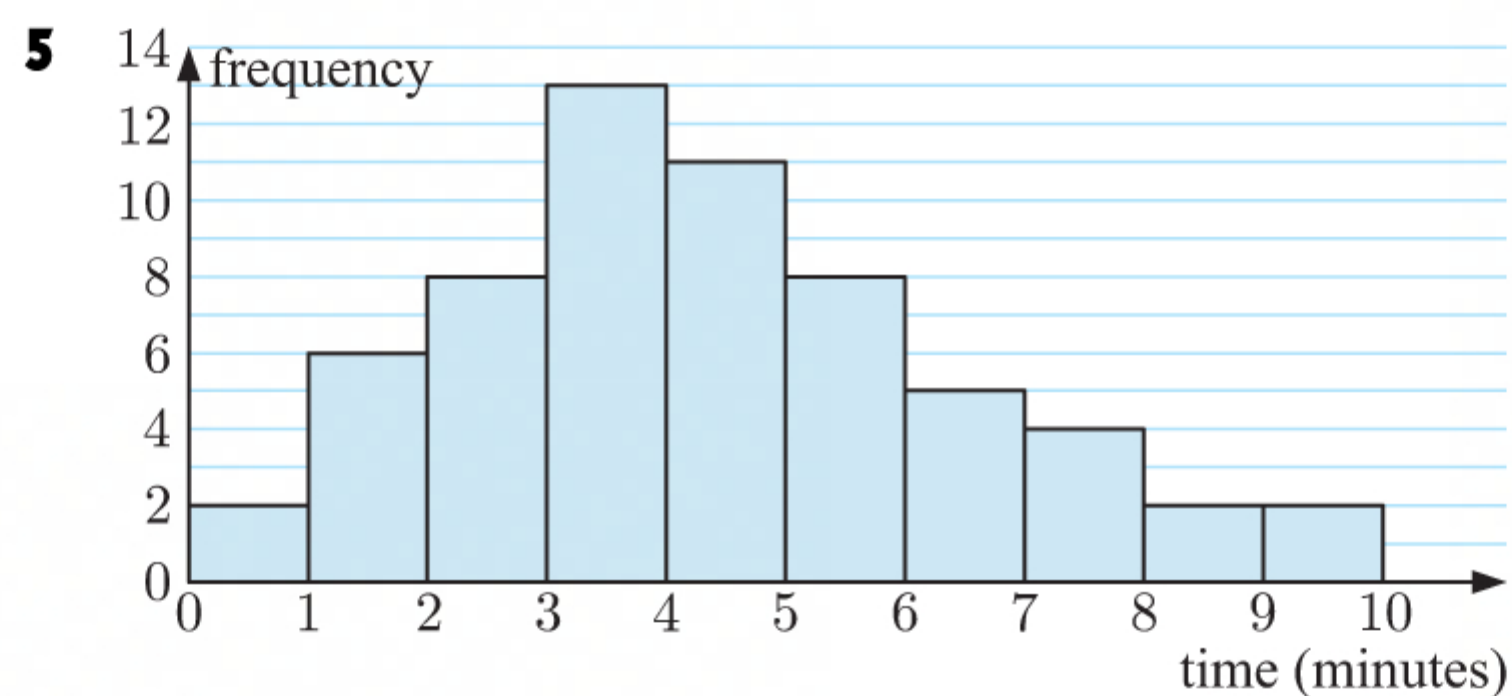
$$u_0 = 36\,000 \text{ and } r = 0.9.$$

- c** For the value of the car to fall below \$10 000, we need to find n such that $u_n < 10\,000$.

Using technology, the first term less than 10 000 is $u_{13} \approx 9150.72$.

So, in the 13th year the value of the car falls below \$10 000.

Math	Des	Norm1	d/c	Real
Y1=36000×0.9^(x)				
X	Y1			
10	12552			
11	11297			
12	10167			
13	9150.7			
	9150.716982			
FORMULA	DELETE	ROW	EDIT	GRAPH



- a** The modal class is $3 \leq t < 4$ where t is the time in minutes.

b

Duration of call (t min)	Frequency (f)	Midpoint (x)	Product (xf)
$0 \leq t < 1$	2	0.5	1
$1 \leq t < 2$	6	1.5	9
$2 \leq t < 3$	8	2.5	20
$3 \leq t < 4$	13	3.5	45.5
$4 \leq t < 5$	11	4.5	49.5
$5 \leq t < 6$	8	5.5	44
$6 \leq t < 7$	5	6.5	32.5
$7 \leq t < 8$	4	7.5	30
$8 \leq t < 9$	2	8.5	17
$9 \leq t < 10$	2	9.5	19
Total	$\sum f = 61$		$\sum xf = 267.5$

c
$$\bar{x} = \frac{\sum xf}{\sum f}$$

$$= \frac{267.5}{61}$$

$$\approx 4.39$$

\therefore the mean length of a phone call is about 4.39 minutes.

d
$$P(\geq 6 \text{ minutes}) \approx \frac{5 + 4 + 2 + 2}{61}$$

$$\approx \frac{13}{61}$$

$$\approx 0.213$$

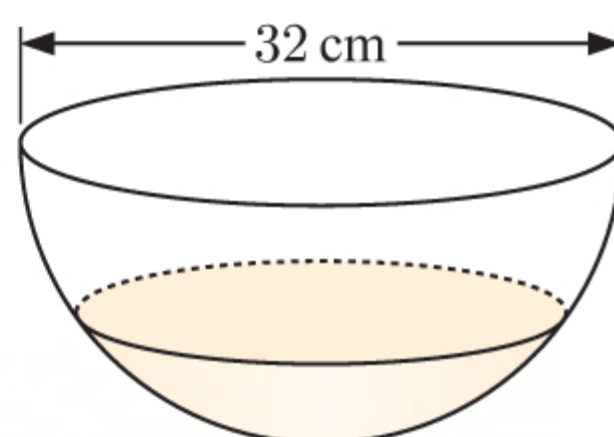
6 a
$$V = \frac{1}{2} \times \text{volume of sphere}$$

$$= \frac{1}{2} \times \frac{4}{3} \pi r^3$$

$$= \frac{2}{3} \times \pi \times \left(\frac{32}{2}\right)^3 \text{ cm}^3$$

$$= \frac{8192}{3} \pi \text{ cm}^3$$

$$\approx 8580 \text{ cm}^3$$



The capacity of the bowl is about 8580 mL or 8.58 L.

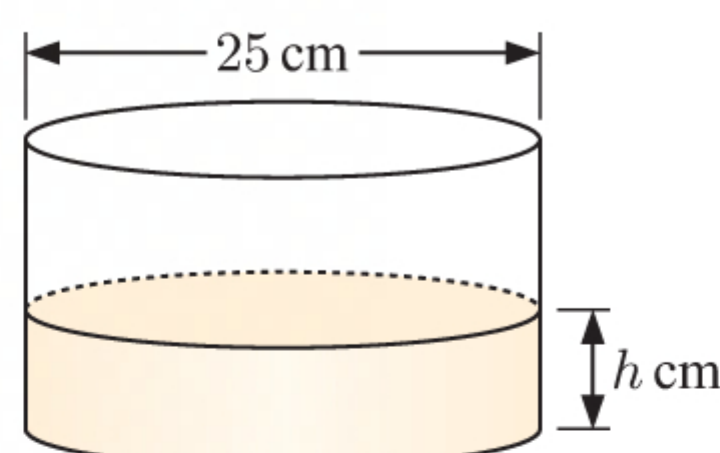
- b i** When 20% full, the bowl contains $\frac{8192}{3} \pi \times 0.2 \approx 1720 \text{ mL}$ or about 1.72 L of cake batter.

ii
$$V = \frac{8192}{3} \pi \times 0.2 \text{ cm}^3$$

$$\therefore \pi \times \left(\frac{25}{2}\right)^2 \times h = \frac{8192}{3} \pi \times 0.2$$

$$\therefore h = \frac{8192 \times 0.2}{3 \times (12.5)^2}$$

$$\approx 3.50 \text{ cm}$$



The cake batter will reach about 3.50 cm up the tin.

7 Let W denote Wollongong, P denote Picton, and C denote Canberra.

a North-west is in the direction 315° . South-west is in the direction 225° .

$$\widehat{PWN} = 360^\circ - 315^\circ = 45^\circ$$

$$\widehat{WPN} = 180^\circ - 45^\circ = 135^\circ \quad \{\text{co-interior angles}\}$$

$$\widehat{CPW} = 225^\circ - 135^\circ = 90^\circ$$

$\therefore \triangle CPW$ is right angled at P.

$$CW^2 = 36^2 + 210^2 \quad \{\text{Pythagoras}\}$$

$$\therefore CW = \sqrt{36^2 + 210^2} \quad \{\text{as } CW > 0\}$$

$$\approx 213 \text{ km}$$

So, Canberra is about 213 km from Wollongong.

b $\widehat{CPN} = 360^\circ - 225^\circ = 135^\circ$

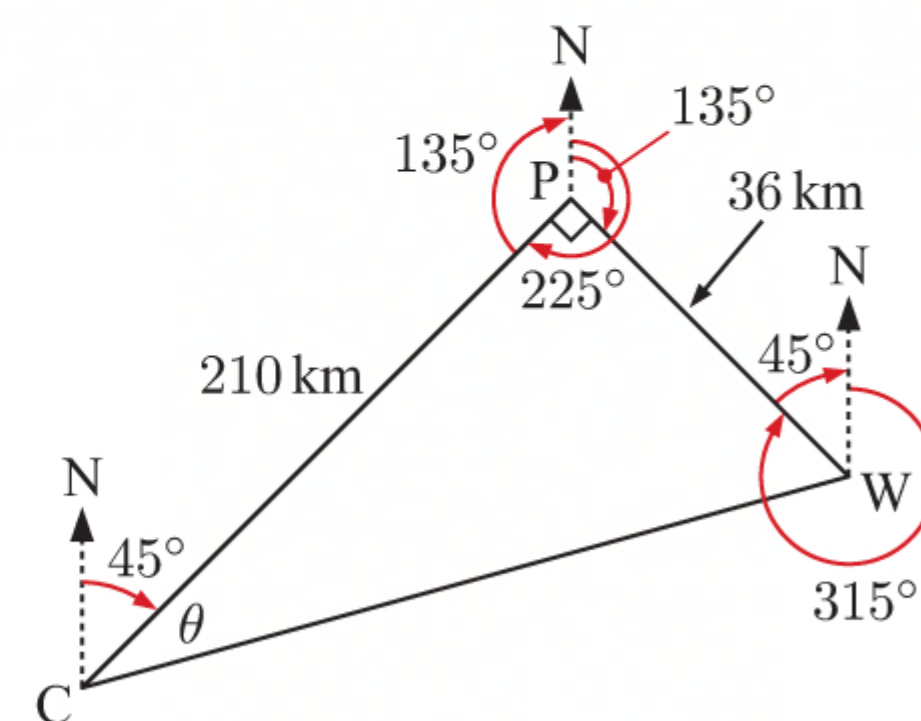
$$\widehat{NCP} = 180^\circ - 135^\circ = 45^\circ \quad \{\text{co-interior angles}\}$$

$$\tan \theta = \frac{36}{210}$$

$$\therefore \theta = \tan^{-1}\left(\frac{36}{210}\right) \approx 9.73^\circ$$

$$\therefore \text{the bearing of Wollongong from Canberra} \approx 45^\circ + 9.73^\circ$$

$$\approx 054.7^\circ$$



8 $f'(x) = \frac{a}{x^2} + bx^2$, $f(-1) = -7$, $f(1) = 7$, $f(2) = 26$

$$= ax^{-2} + bx^2$$

$$\therefore f(x) = \int (ax^{-2} + bx^2) dx$$

$$= -ax^{-1} + \frac{b}{3}x^3 + c$$

$$= -\frac{a}{x} + \frac{b}{3}x^3 + c$$

$$\text{Now } f(-1) = -7,$$

$$f(1) = 7,$$

$$\text{and } f(2) = 26$$

$$\therefore -\frac{a}{-1} + \frac{b}{3}(-1)^3 + c = -7$$

$$\therefore -\frac{a}{1} + \frac{b}{3}(1)^3 + c = 7$$

$$\therefore -\frac{a}{2} + \frac{b}{3}(2)^3 + c = 26$$

$$\therefore a - \frac{1}{3}b + c = -7$$

$$\therefore -a + \frac{1}{3}b + c = 7$$

$$\therefore -\frac{1}{2}a + \frac{8}{3}b + c = 26$$

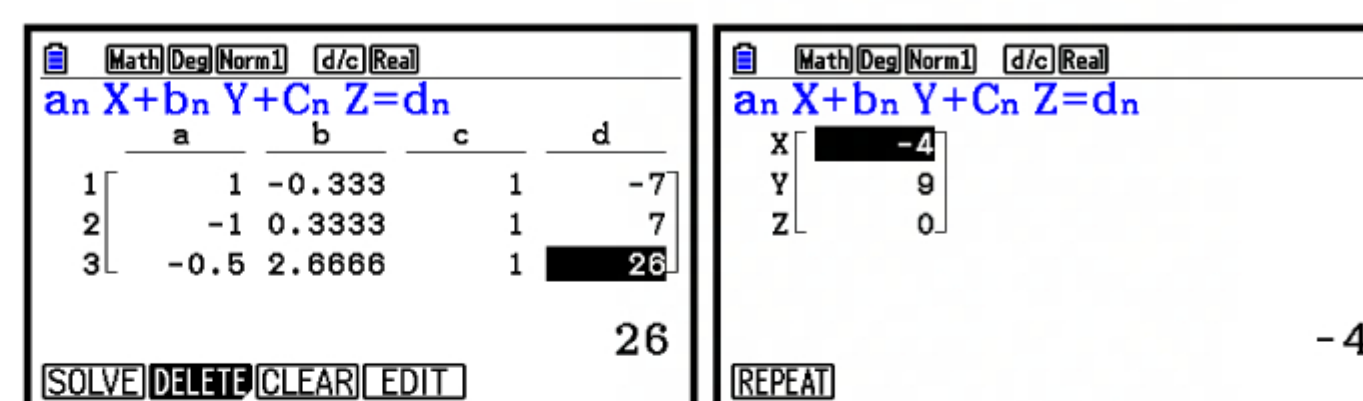
So we have the system of equations

$$\begin{cases} a - \frac{1}{3}b + c = -7 \\ -a + \frac{1}{3}b + c = 7 \\ -\frac{1}{2}a + \frac{8}{3}b + c = 26 \end{cases}$$

Solving simultaneously gives $a = -4$, $b = 9$, $c = 0$.

$$\therefore f(x) = -\frac{4}{x} + \frac{9}{3}x^3$$

$$= \frac{4}{x} + 3x^3$$



9 a Volume of jar $= \pi\left(\frac{20}{2}\right)^2 \times 30 = 3000\pi \text{ cm}^3$

$$\text{Total volume of marbles} = 0.6 \times \text{volume of jar}$$

$$= 0.6 \times 3000\pi$$

$$= 1800\pi \text{ cm}^3$$

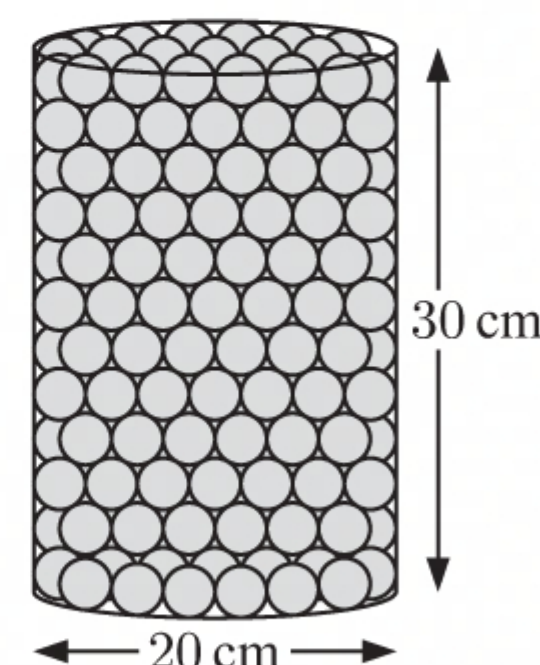
$$\text{Volume of 1 marble with radius } r \text{ cm} = \frac{4}{3}\pi r^3 \text{ cm}^3$$

$$\therefore \text{number of marbles } N = \frac{\text{total volume of marbles}}{\text{volume of 1 marble}}$$

$$= \frac{1800\pi}{\frac{4}{3}\pi r^3}$$

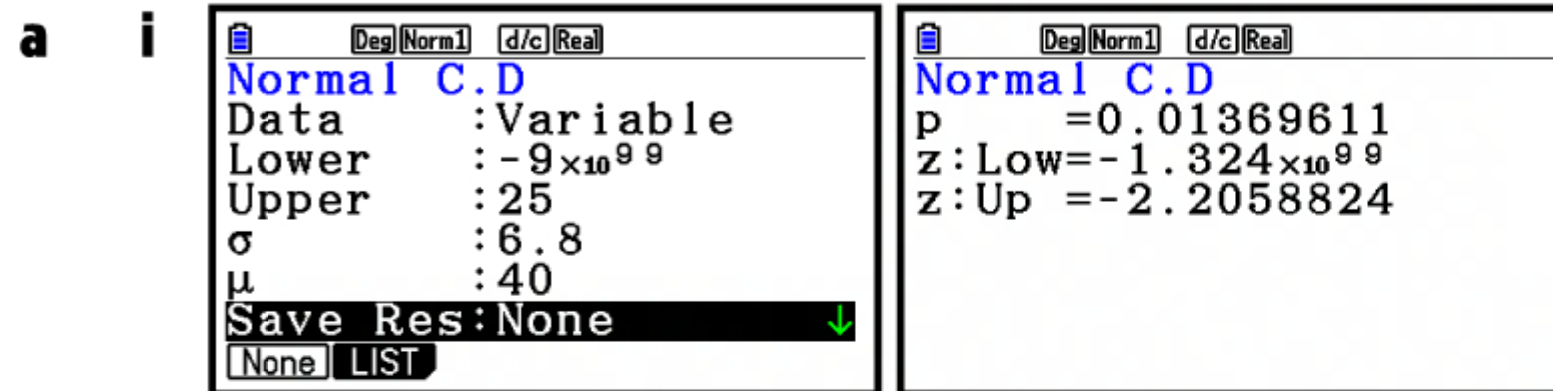
$$= \frac{1350}{r^3}$$

$$\therefore \text{the model is } N = \frac{1350}{r^3}.$$



- b** When $r = 1.5$, $N = \frac{1350}{(1.5)^3} = 400$ marbles.
- c** The estimate in **b** is an underestimate because the actual percentage of the jar's volume that the marbles occupy is greater than Julie's assumption.
- d** Percentage error $= \frac{|V_A - V_E|}{V_E} \times 100\%$
 $= \frac{|400 - 426|}{426} \times 100\%$
 $= \frac{26}{426} \times 100\%$
 $\approx 6.10\%$

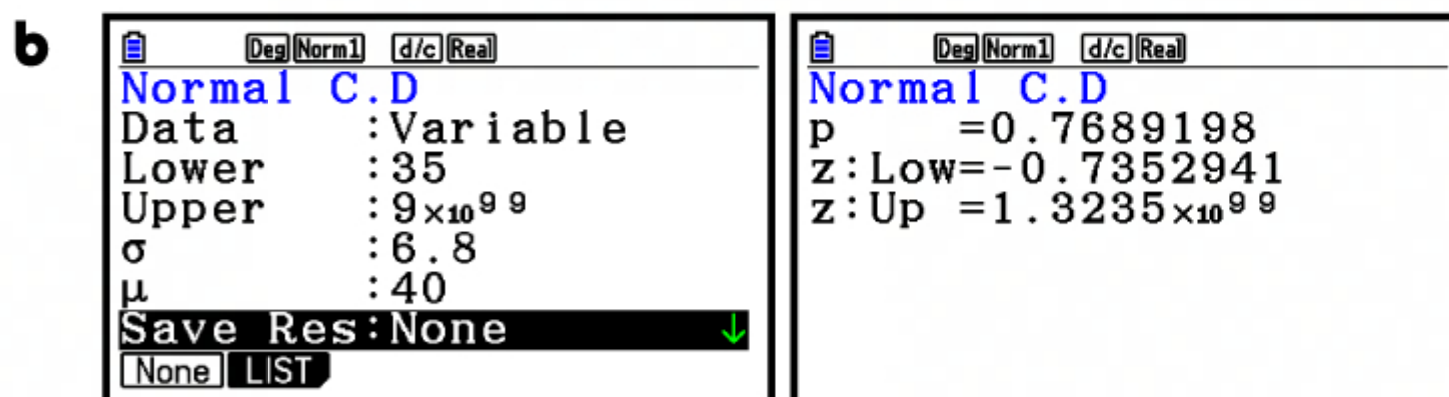
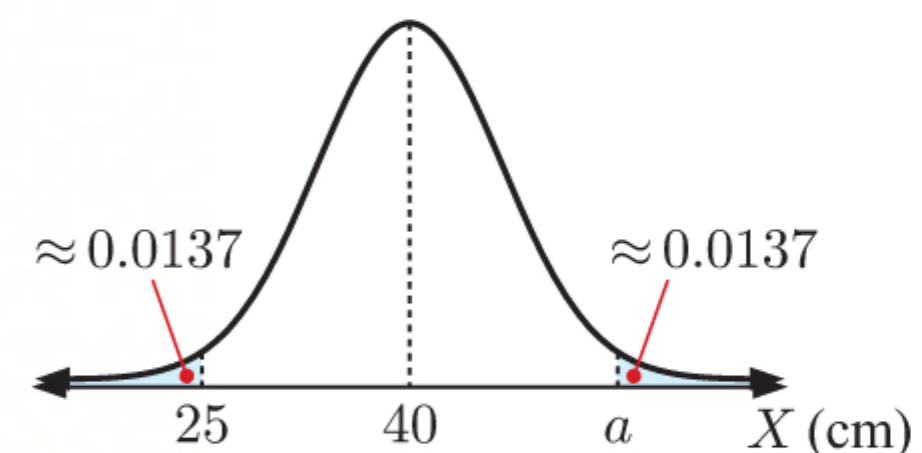
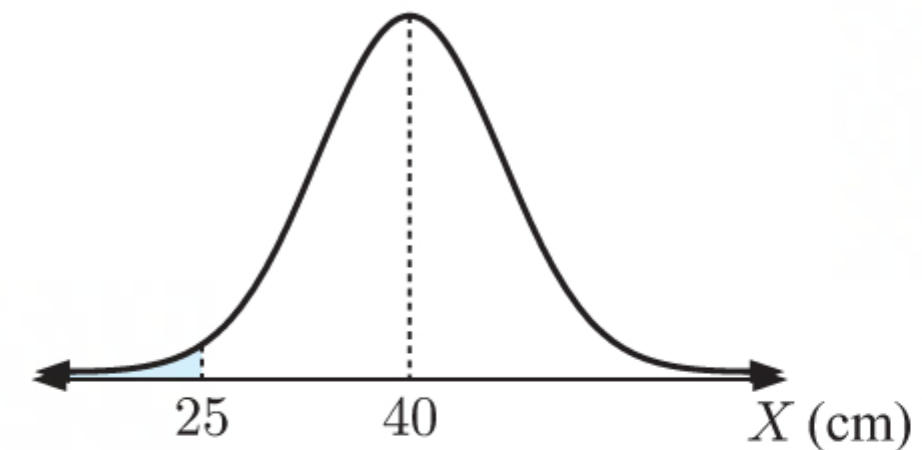
10 $X \sim N(40, (6.8)^2)$



$$P(X < 25) \approx 0.0137$$

ii 25 is 15 less than 40.

By symmetry, $P(X < 25) = P(X > 40 + 15)$
 $\therefore P(X < 25) = P(X > 55)$
 $\therefore a = 55$



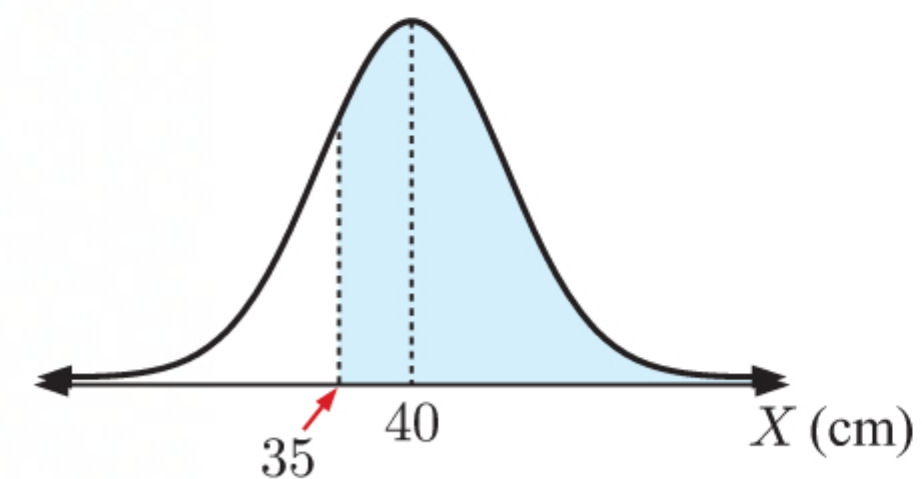
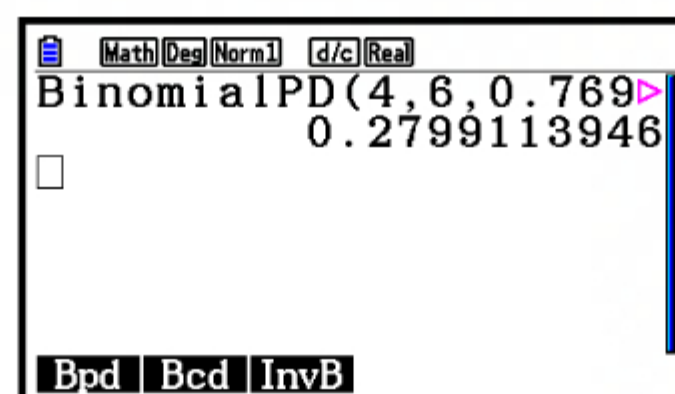
$$P(X > 35) \approx 0.769$$

Let Y be the number of maize plants more than 35 cm high.

$n = 6$, so $Y = 0, 1, 2, 3, 4, 5$, or 6 and $p \approx 0.769$.

$$Y \sim B(6, 0.769)$$

$$\text{So, } P(Y = 4) \approx 0.280$$



11

x	-2	0	3	5
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{6}$	k	$\frac{1}{12}$

- a** X can only take the values $-2, 0, 3$, or 5 .
 $\therefore X$ is a discrete random variable.

- b** Since this is a probability distribution, $P(X = -2) + P(X = 0) + P(X = 3) + P(X = 5) = 1$
 $\therefore \frac{1}{3} + \frac{1}{6} + k + \frac{1}{12} = 1$
 $\therefore k + \frac{7}{12} = 1$
 $\therefore k = \frac{5}{12}$

- c** Since $P(X = 3)$ is the greatest probability, 3 is the mode of the distribution.

$$P(X = -2) = \frac{1}{3} \approx 0.333$$

$$P(X = -2) + P(X = 0) = \frac{1}{3} + \frac{1}{6} = 0.5$$

Since $P(X = -2) + P(X = 0) \geq 0.5$, the median is 0.

$$\begin{aligned} \mathbf{d} \quad E(X) &= -2\left(\frac{1}{3}\right) + 0\left(\frac{1}{6}\right) + 3\left(\frac{5}{12}\right) + 5\left(\frac{1}{12}\right) \\ &= -\frac{2}{3} + 0 + \frac{5}{4} + \frac{5}{12} \\ &= 1 \end{aligned}$$

12

Distance (d km)	5	10	20
Signal strength (S units)	40	2.5	0.156 25

a $S \propto \frac{1}{d^4}$, so $S = \frac{k}{d^4}$ where k is a constant.

Using the first point, $40 = \frac{k}{5^4}$
 $\therefore k = 25\,000$
 $\therefore S = \frac{25\,000}{d^4}$

b When $d = 18$, $S = \frac{25\,000}{18^4} \approx 0.238$

So, the signal strength for an object 18 km away is about 0.238 units.

c If d is tripled, then

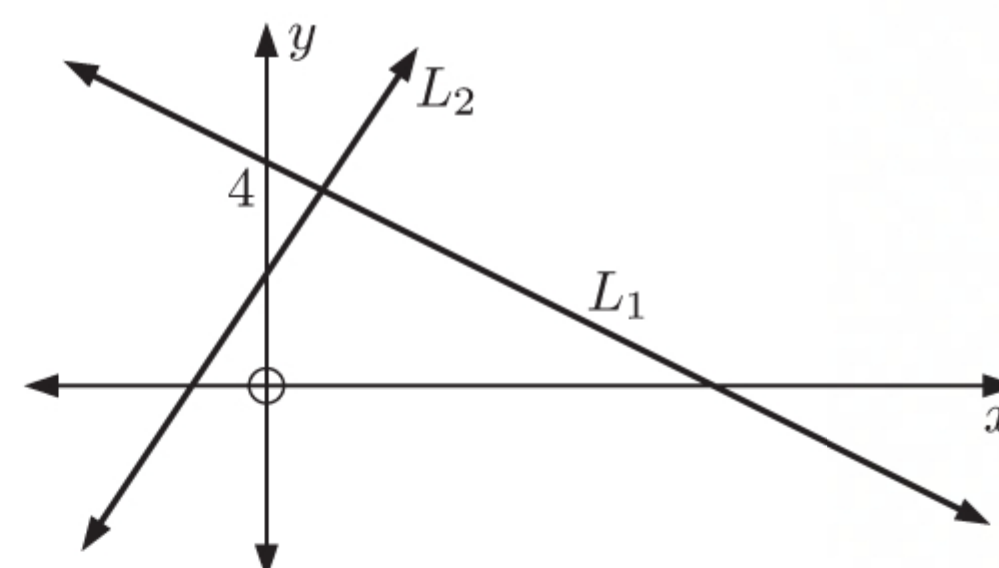
d is multiplied by 3
 $\therefore d^4$ is multiplied by $3^4 = 81$
 $\therefore \frac{1}{d^4}$ is multiplied by $\frac{1}{81}$
 $\therefore S$ is multiplied by $\frac{1}{81} \approx 0.0123$

So, the signal strength is decreased by about 98.8%.

MIXED QUESTIONS SET 2

1 a L_1 has gradient $-\frac{1}{2}$ and passes through $(0, 4)$.

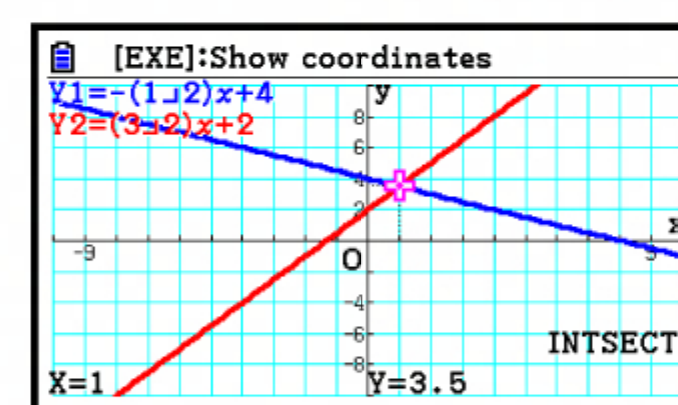
$\therefore L_1$ has equation $y - 4 = -\frac{1}{2}(x - 0)$
 $\therefore y - 4 = -\frac{1}{2}x$
 $\therefore y = -\frac{1}{2}x + 4$



b L_2 has gradient $\frac{8 - (-1)}{4 - (-2)} = \frac{9}{6} = \frac{3}{2}$, and passes through $(-2, -1)$.

$\therefore L_2$ has equation $y - (-1) = \frac{3}{2}(x - (-2))$
 $\therefore y + 1 = \frac{3}{2}(x + 2)$
 $\therefore y + 1 = \frac{3}{2}x + 3$
 $\therefore y = \frac{3}{2}x + 2$

Using technology, the point of intersection of L_1 and L_2 is $(1, \frac{7}{2})$.



2 a $\widehat{ABC} = 180^\circ - 50^\circ$ {angles in a straight line}
 $= 130^\circ$

$\therefore \widehat{ACB} = 180^\circ - (25^\circ + 130^\circ)$ {angles in a triangle}
 $= 25^\circ$

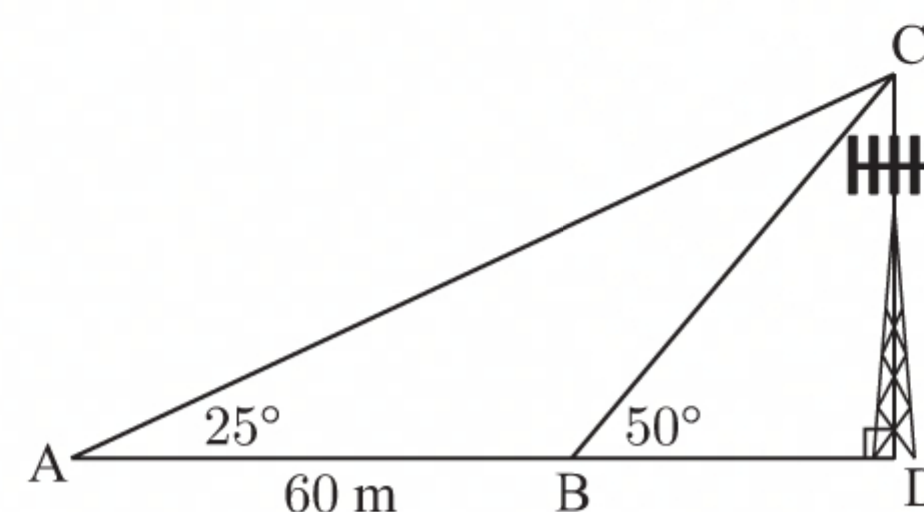
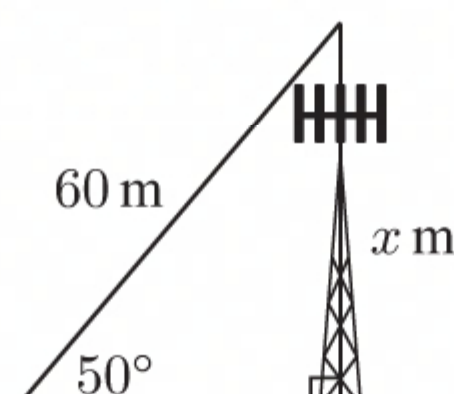
b $\triangle ACB$ is isosceles {base angles equal}

$\therefore BC = AB = 60$ m

Let the tower have height x m.

$\therefore \sin 50^\circ = \frac{x}{60}$
 $\therefore x = 60 \sin 50^\circ$
 $\therefore x \approx 46.0$

The tower is about 46.0 m high.



3 a $u_n = u_1 + (n-1)d$

Now $u_5 = 18 \quad \therefore u_1 + 4d = 18$

and $u_8 = 39 \quad \therefore u_1 + 7d = 39$

Solving the system of equations
$$\begin{cases} u_1 + 4d = 18 \\ u_1 + 7d = 39 \end{cases}$$

simultaneously gives $u_1 = -10$ and $d = 7$.

Math Des Norm1 d/c Real
an X+bn Y=Cn
a b c
1 1 4 18
2 1 7 39
SOLVE DELETE CLEAR EDIT 39

Math Des Norm1 d/c Real
an X+bn Y=Cn
X [-10]
Y [7]
REPEAT -10

b $u_{12} = u_1 + 11d$
 $= -10 + 11(7) \quad \{\text{using a}\}$
 $= -10 + 77$
 $= 67$

c $S_{10} = \frac{10}{2}(2u_1 + 9d)$
 $= 5(2(-10) + 9(7))$
 $= 5(-20 + 63)$
 $= 5(43)$
 $= 215$

4 a The tangent to the curve $y = ax^3 - bx^2$ at the point where $x = 3$ is $y = x - 6$.

\therefore the tangent has gradient 1, and the point of contact is $(3, 3 - 6)$ which is $(3, -3)$.

Now $f(x) = ax^3 - bx^2$

$\therefore f'(x) = 3ax^2 - 2bx$

So, $f'(3) = 1$ and $f(3) = -3$

$\therefore 3a(3)^2 - 2b(3) = 1 \quad \therefore a(3)^3 - b(3)^2 = -3$

$\therefore 27a - 6b = 1 \quad \therefore 27a - 9b = -3$

Solving the system of equations
$$\begin{cases} 27a - 6b = 1 \\ 27a - 9b = -3 \end{cases}$$

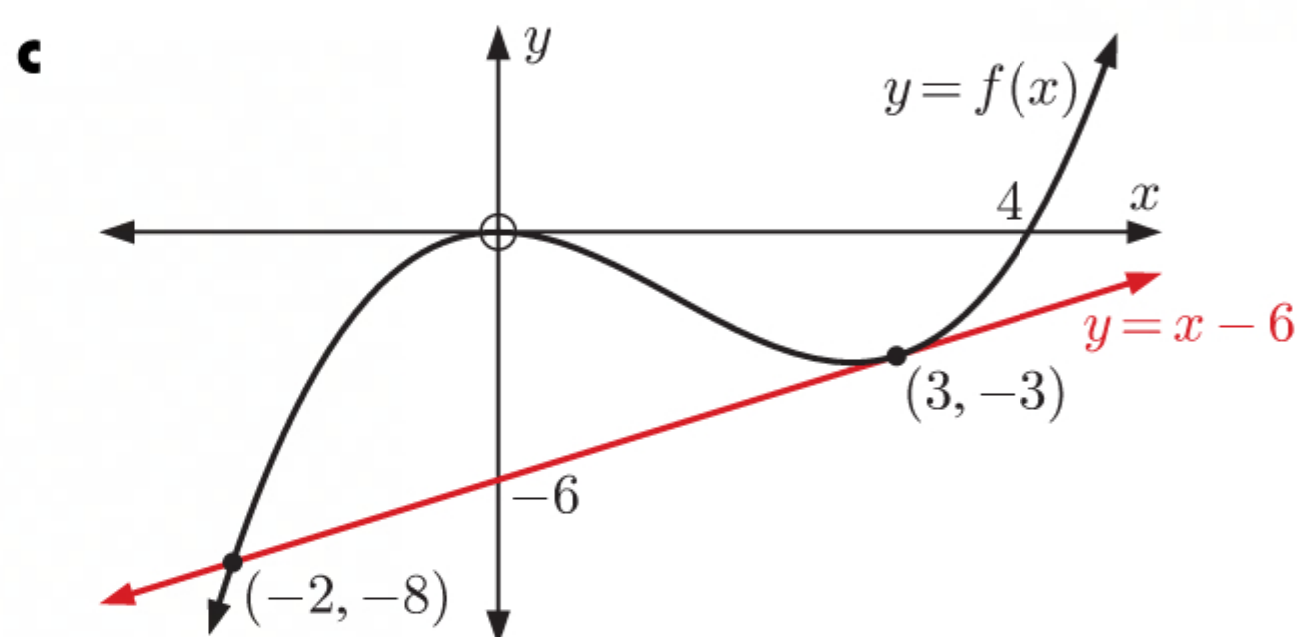
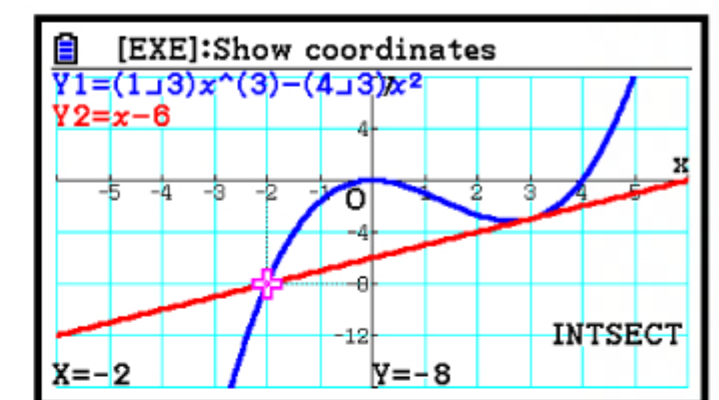
simultaneously gives $a = \frac{1}{3}$ and $b = \frac{4}{3}$.

Math Des Norm1 d/c Real
an X+bn Y=Cn
a b c
1 27 -6 1
2 27 -9 -3
SOLVE DELETE CLEAR EDIT -3

Math Des Norm1 d/c Real
an X+bn Y=Cn
X [0.3333]
Y [1.3333]
REPEAT 1/3

b From **a**, $f(x) = \frac{1}{3}x^3 - \frac{4}{3}x^2$.

Using technology, the tangent $y = x - 6$ meets the curve $y = f(x)$ again at $(-2, -8)$.



5 a i $(2, 2)$ lies in cell B, so it is closest to site B.

ii $(-5, -3)$ lies in cell A, so it is closest to site A.

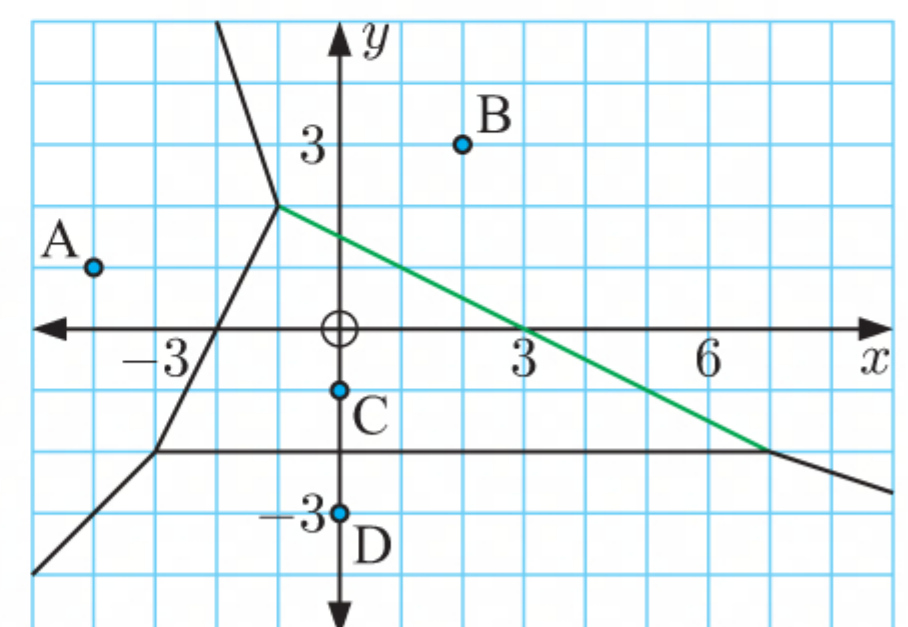
iii $(6, -4)$ lies in cell D, so it is closest to site D.

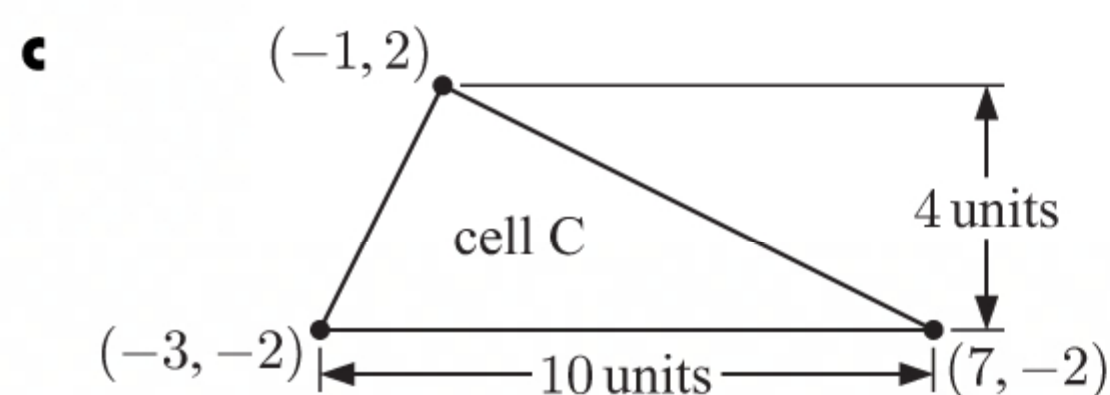
b The green edge passes through $(-1, 2)$ and $(7, -2)$.

The gradient $= \frac{-2 - 2}{7 - (-1)} = \frac{-4}{8} = \frac{-1}{2}$.

\therefore its equation is $x + 2y = (-1) + 2(2)$

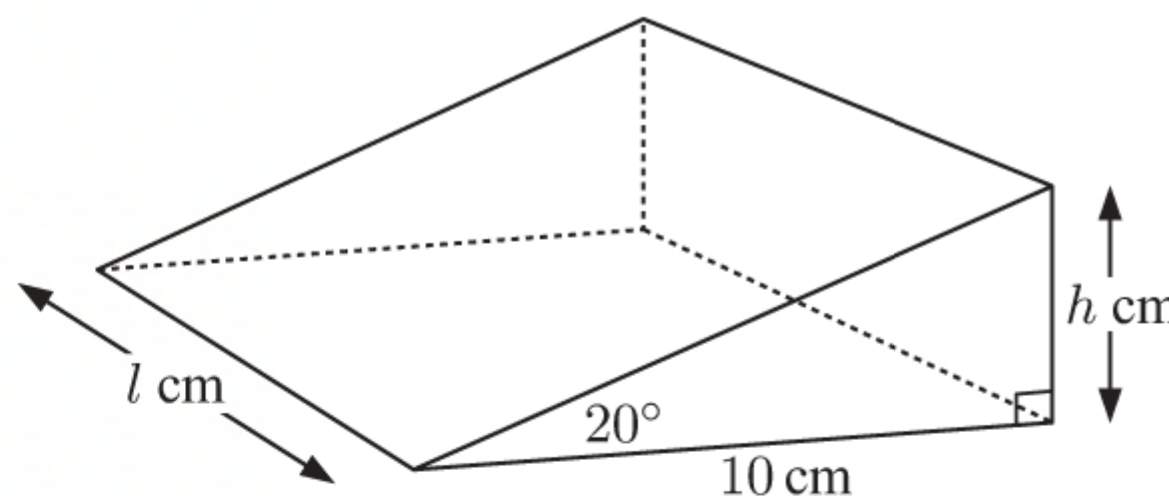
or $x + 2y - 3 = 0$





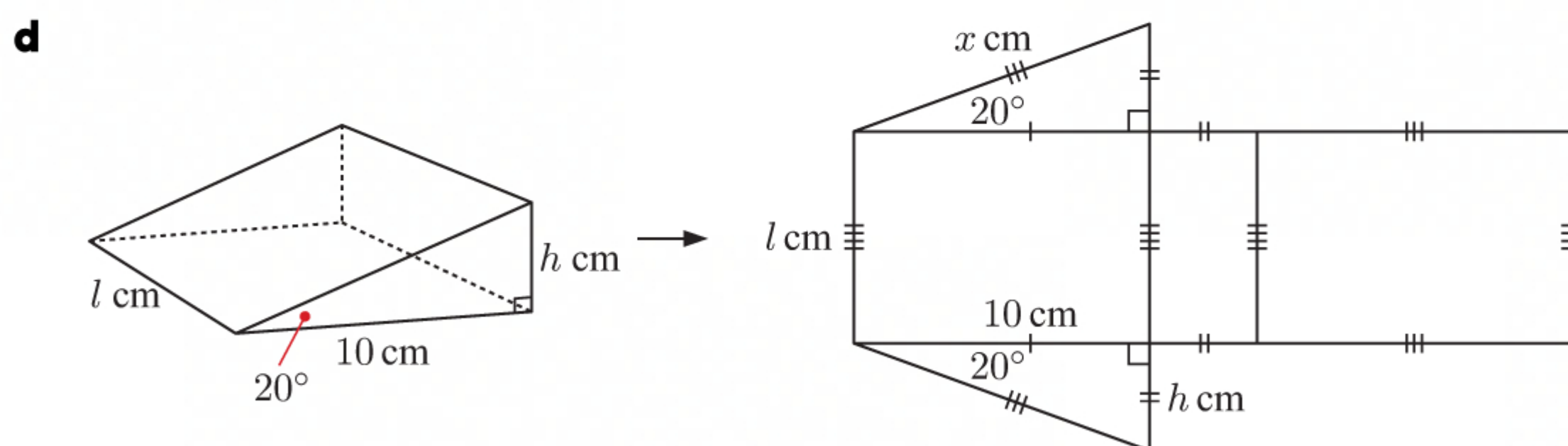
$$\begin{aligned}\text{Area of cell C} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 10 \times 4 \\ &= 20 \text{ units}^2\end{aligned}$$

6 a $\tan 20^\circ = \frac{h}{10}$
 $\therefore h = 10 \tan 20^\circ$
 ≈ 3.640



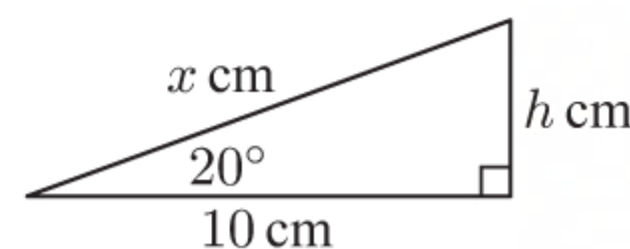
b Area of triangular end $= \frac{1}{2} \times \text{base} \times \text{height}$
 $= \frac{1}{2} \times 10 \times h$
 $= 5 \times 10 \tan 20^\circ$ {from **a**}
 $= 50 \tan 20^\circ \text{ cm}^2$
 $\approx 18.2 \text{ cm}^2$

c Volume of door-stop $= 60 \text{ cm}^3$
 $\therefore \text{area of triangular end} \times \text{length} = 60$
 $\therefore 50 \tan 20^\circ \times l = 60$ {from **b**}
 $\therefore l = \frac{60}{50 \tan 20^\circ}$
 $\therefore l = \frac{6}{5 \tan 20^\circ}$
 $\therefore l \approx 3.30$



Let the hypotenuse of the triangular end be x cm.

$$\begin{aligned}\cos 20^\circ &= \frac{10}{x} \\ \therefore x &= 10 \cos 20^\circ\end{aligned}$$



$$\begin{aligned}\text{Surface area} &= (10 \times l) + (h \times l) + (x \times l) + 2 \times \left(\frac{1}{2} \times 10 \times h\right) \\ &= (10 + h + x) \times l + 10h \\ &= (10 + 10 \tan 20^\circ + 10 \cos 20^\circ) \times \frac{6}{5 \tan 20^\circ} + 10 \times 10 \tan 20^\circ \quad \{\text{from **a** and **c**}\} \\ &\approx 112 \text{ cm}^2\end{aligned}$$

- 7 a**
- The survey is likely to under-represent full-time weekday workers.
 - The survey was taken at a suburban shopping centre, so the people surveyed are likely to prefer suburban shopping. Therefore the sample is likely to be biased toward suburban shopping.

b The conclusion is unreasonable since the survey is likely to contain a coverage error, as in **a**, and so the results may not accurately represent the opinions of the whole population.

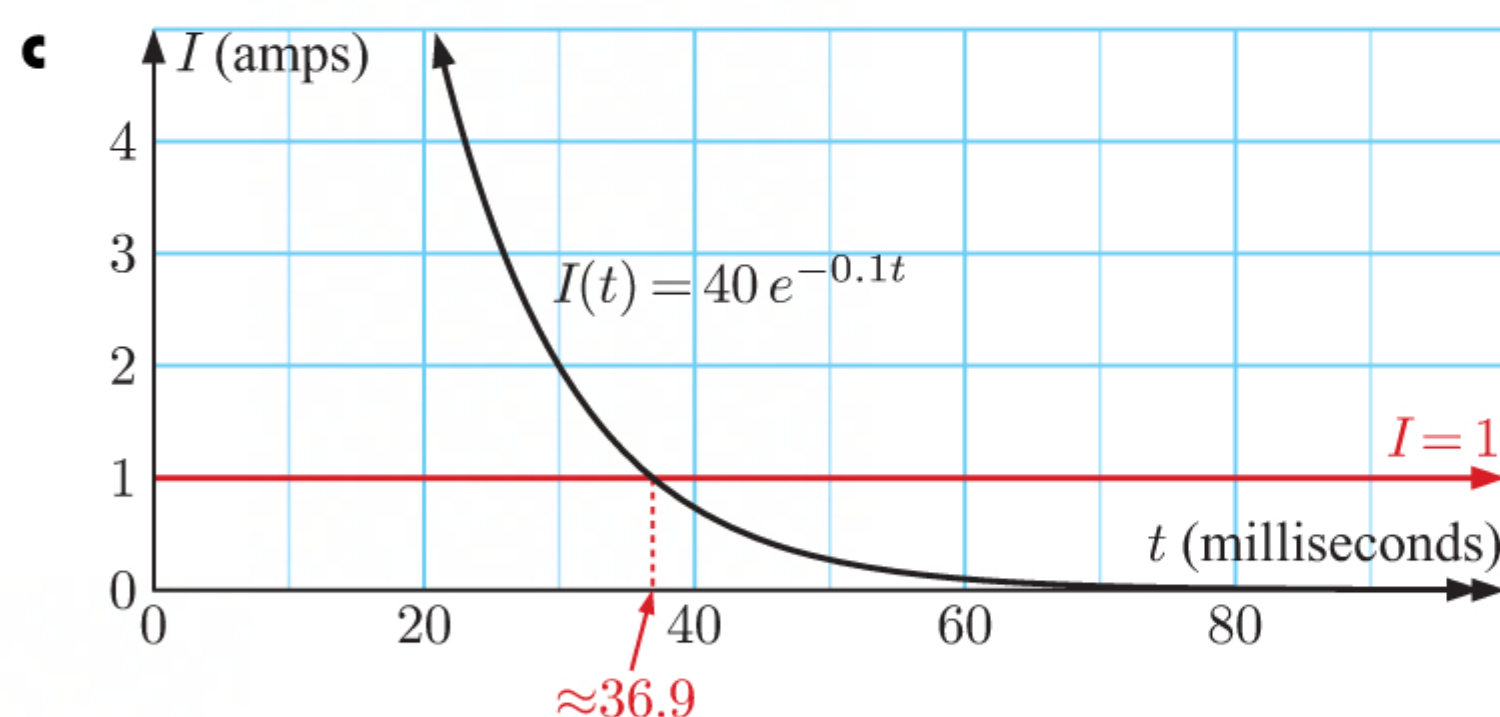
8 $I(t) = 40e^{-0.1t}$ amps

a $I(0) = 40e^0$
 $= 40$

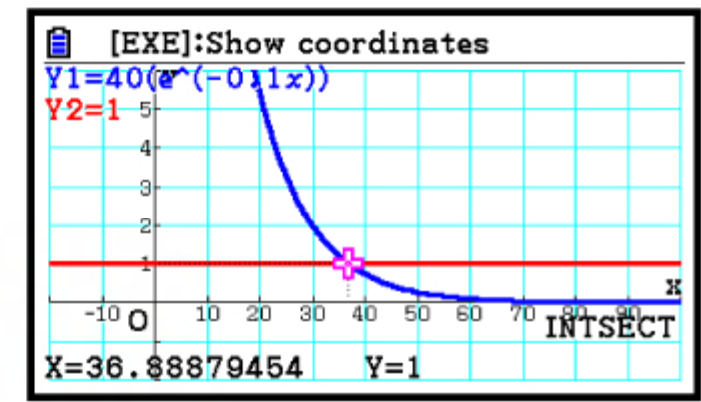
\therefore there was 40 amps of current flowing through the circuit initially.

b $I(100) = 40e^{-0.1 \times 100}$
 ≈ 0.00182

\therefore after 100 milliseconds, there was about 0.00182 amps flowing through the circuit.



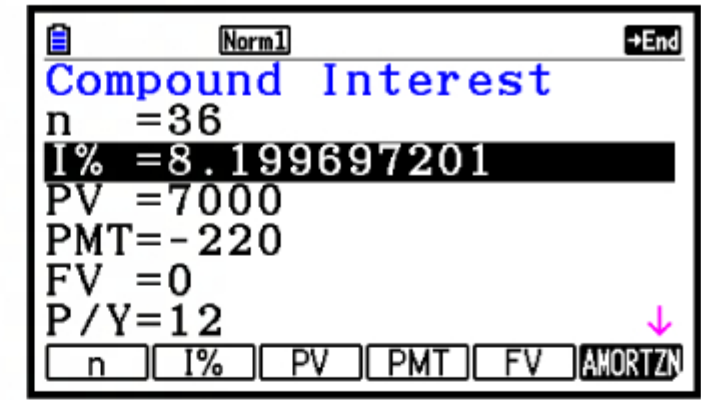
- d** Using technology, it took about 36.9 milliseconds for the current to fall to 1 amp.



- 9 a** $N = 3 \times 12 = 36$, $PV = 7000$, $PMT = -220$, $FV = 0$, $P/Y = 12$, $C/Y = 12$

$$\therefore I\% \approx 8.20$$

$$\therefore r \approx 8.20$$

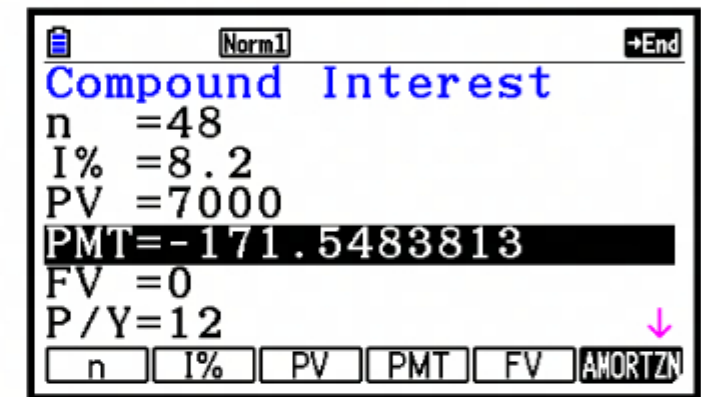


- b** Total interest = total repayment – starting principal
 $= £220 \times 36 - £7000$
 $= £920$

- c i** $N = 4 \times 12 = 48$, $I\% = 8.20$, $PV = 7000$, $FV = 0$, $P/Y = 12$, $C/Y = 12$

$$\therefore PMT \approx 171.55$$

Soraya's monthly repayment is £171.55.



- ii** Total interest = total repayment – starting principal
 $= £171.55 \times 48 - £7000$
 $= £1234.40$
 \therefore extra interest = $£1234.40 - £920$
 $= £314.40$

- 10** $N(t) = at^3 + bt^2 + ct + d$

- a** $N(0) = 2000$ {2000 people at 12 pm}
 $N'(6) = 200$ {increasing at 200 people per hour at 6 pm}
 $N'(8) = -500$ {decreasing at 500 people per hour at 8 pm}
 $N(10) = 0$ {closed at 10 pm}

- b** $N(0) = 2000 \quad \therefore a(0)^3 + b(0)^2 + c(0) + d = 2000$
 $\therefore d = 2000$

$$N(10) = 0 \quad \therefore a(10)^3 + b(10)^2 + c(10) + 2000 = 0$$

$$\therefore 1000a + 100b + 10c = -2000$$

$$\therefore 100a + 10b + c = -200$$

$$N'(t) = 3at^2 + 2bt + c$$

$$N'(6) = 200 \quad \therefore 3a(6)^2 + 2b(6) + c = 200$$

$$\therefore 108a + 12b + c = 200$$

$$N'(8) = -500 \quad \therefore 3a(8)^2 + 2b(8) + c = -500$$

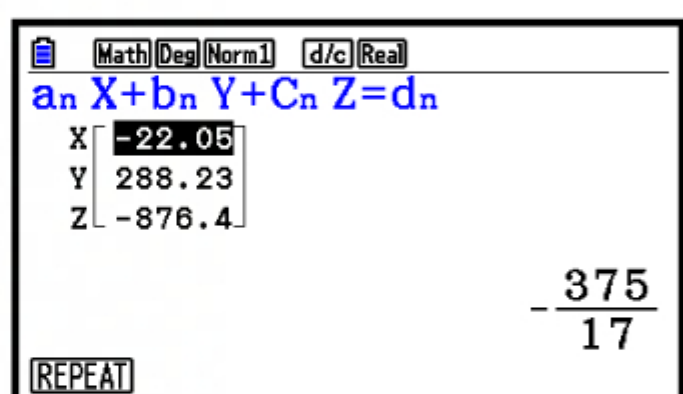
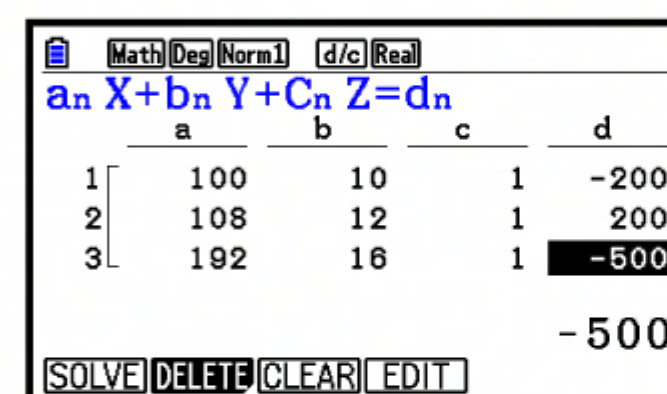
$$\therefore 192a + 16b + c = -500$$

So we have the system of equations

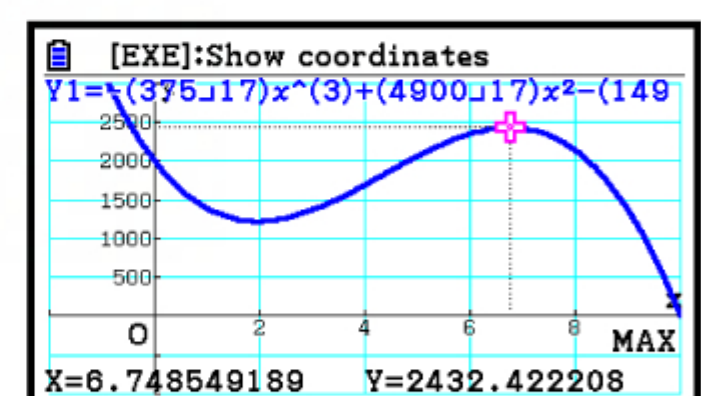
$$\begin{cases} 100a + 10b + c = -200 \\ 108a + 12b + c = 200 \\ 192a + 16b + c = -500 \end{cases}$$

Using technology to solve this system simultaneously gives $a = -\frac{375}{17}$, $b = \frac{4900}{17}$, and $c = -\frac{14900}{17}$.

$$\therefore N(t) = -\frac{375}{17}t^3 + \frac{4900}{17}t^2 - \frac{14900}{17}t + 2000$$



- c** Using technology, $N(t)$ has a maximum of about 2432 when $t \approx 6.75$.
 \therefore the maximum number of people at the festival was about 2432 at about 6:45 pm.



- 11** We extend the table to include totals for each row and column.

	Defective	Not defective	Total
Corn	37	581	618
Pineapple	24	617	641
Total	61	1198	1259

- a** There were 1259 tins included in the sample.
- b** **i** 1198 of the 1259 tins were not defective.
 $\therefore P(\text{is not defective}) \approx \frac{1198}{1259} \approx 0.952$
- ii** 24 of the 1259 tins were defective tins of pineapple.
 $\therefore P(\text{is a defective tin of pineapple})$
 $\approx \frac{24}{1259} \approx 0.0191$
- iii** 37 of the 618 tins of corn were defective.
 $\therefore P(\text{is defective, given it is a tin of corn}) \approx \frac{37}{618} \approx 0.0599$
- 12 a** Let $p_1, p_2, p_3, p_4, p_5, p_6$, and p_7 be the probabilities of a break-in on Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, and Sunday respectively.
 $H_0: p_1 = \frac{1}{7}, p_2 = \frac{1}{7}, \dots, p_7 = \frac{1}{7}$
 $H_1: \text{at least one of } p_1 \neq \frac{1}{7}, p_2 \neq \frac{1}{7}, \dots, \text{ or } p_7 \neq \frac{1}{7}$
- b** Expected number of break-ins per day $= 140 \times \frac{1}{7}$
 $= 20$
- c** $df = 7 - 1 = 6$

Sub	List 1	List 2	List 3	List 4
1	15	20		
2	11	20		
3	17	20		
4	18	20		

15

GRAPH CALC TEST INTR DIST

χ^2 GOF Test
 Observed: List1
 Expected: List2
 df: 6
 CNTRB: List3
 Save Res: None
 GphColor: Blue

χ^2 GOF Test
 $\chi^2 = 12.9$
 $p = 0.04465172$
 $df = 6$
 CNTRB: List3

Using technology, $\chi^2_{\text{calc}} = 12.9$.

- d** Since $\chi^2_{\text{calc}} > 12.59 = \chi^2_{\text{crit}}$, we have enough evidence to reject H_0 in favour of H_1 on a 5% level of significance.
 \therefore break-ins are *not* equally likely to occur on each day of the week.

MIXED QUESTIONS SET 3

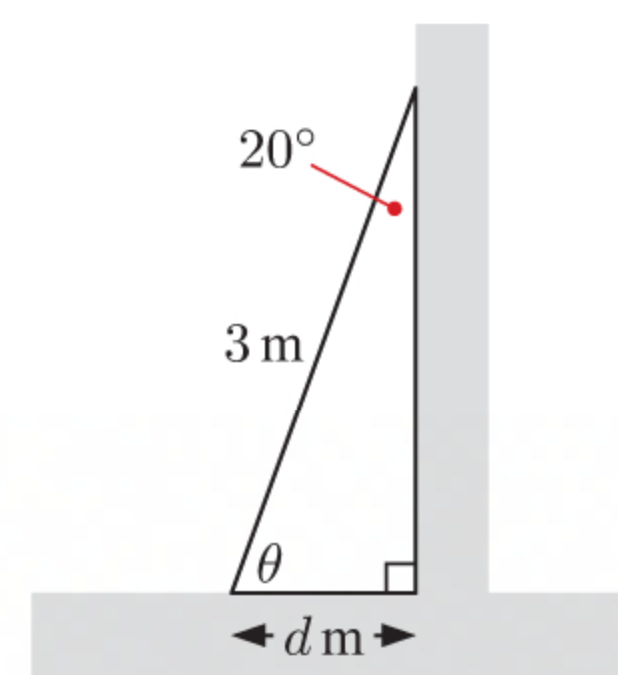
- 1 a** Let the distance be d m.

$$\sin 35^\circ = \frac{\text{OPP}}{\text{HYP}} = \frac{d}{3}$$

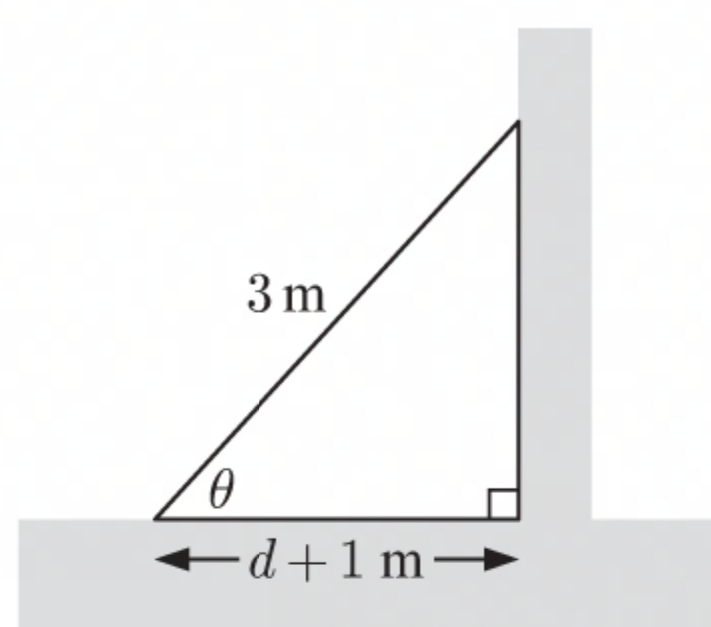
$$\therefore d = 3 \sin 20^\circ$$

$$\therefore d \approx 1.03$$

The foot of the ladder is about 1.03 m from the base of the wall.



b



$$\begin{aligned} \cos \theta &= \frac{\text{ADJ}}{\text{HYP}} \\ &= \frac{d+1}{3} \\ &= \frac{3 \sin 20^\circ + 1}{3} \quad \{\text{from a}\} \\ \therefore \theta &= \cos^{-1} \left(\frac{3 \sin 20^\circ + 1}{3} \right) \\ &\approx 47.5^\circ \end{aligned}$$

- 2 a** Amount of fluoride = concentration \times volume
 $= (3 \times 10^{-4}) \times (5.6 \times 10^8)$
 $= 1.68 \times 10^5 \text{ g}$

b Volume = $\frac{\text{amount of fluoride}}{\text{concentration}}$
 $= \frac{4.13 \times 10^7}{3 \times 10^{-4}}$
 $\approx 1.38 \times 10^{11} \text{ litres}$

3 a The y -intercept = 9, so $c = 9$

b The axis of symmetry is $x = -\frac{b}{2a}$

$$\therefore -\frac{b}{2a} = 1$$

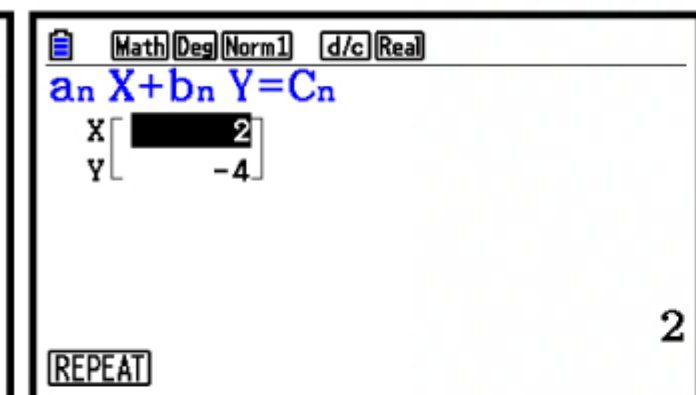
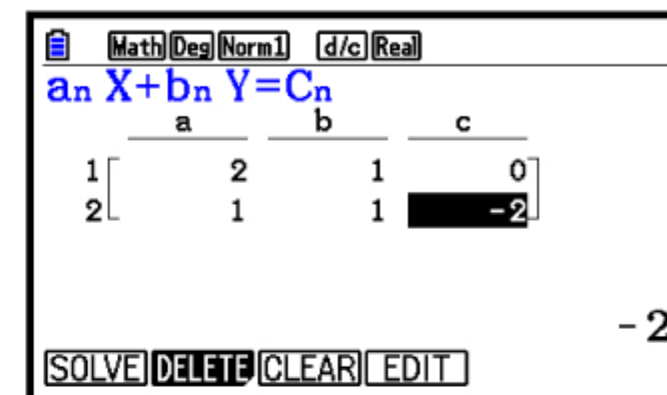
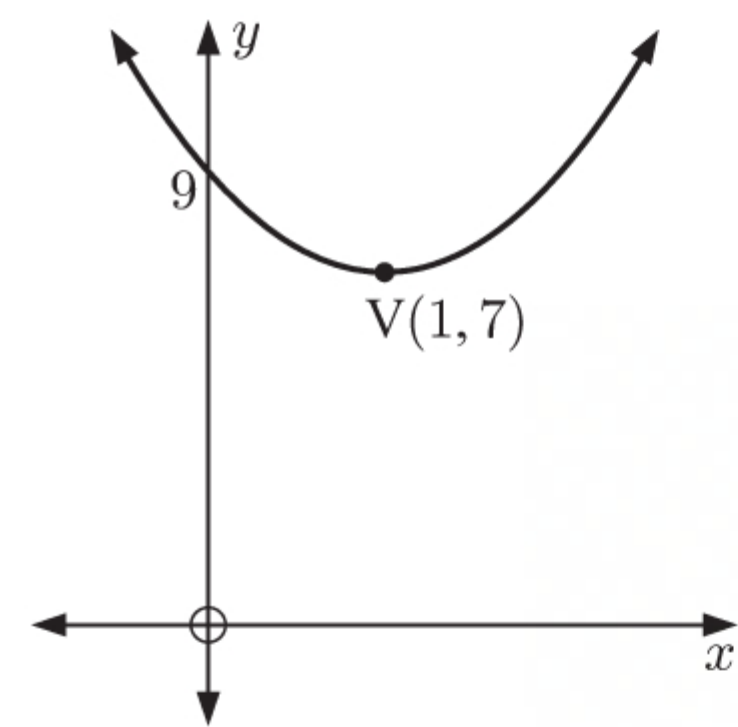
$$\therefore -b = 2a$$

$$\therefore 2a + b = 0 \quad \dots (1)$$

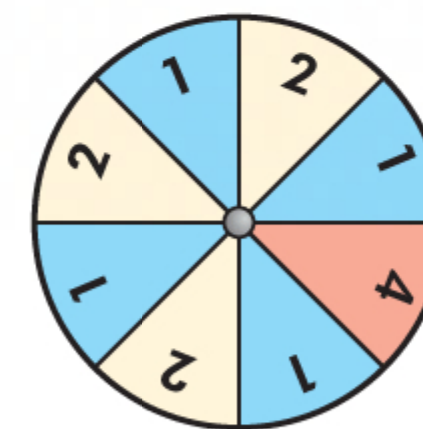
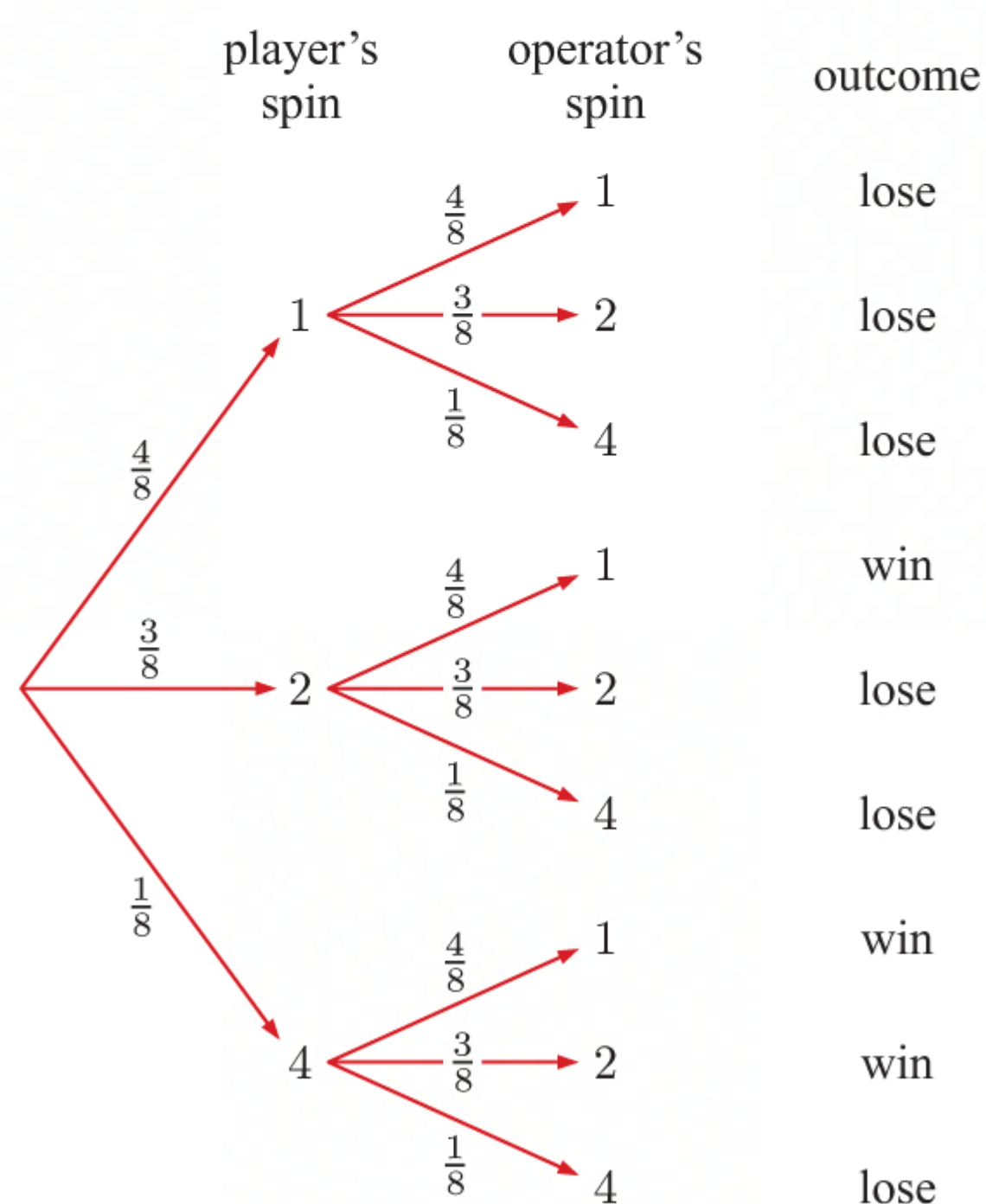
c The point $(1, 7)$ lies on the graph, so $a(1)^2 + b(1) + 9 = 7$

$$\therefore a + b = -2 \quad \dots (2)$$

d Solving (1) and (2) simultaneously gives $a = 2$ and $b = -4$.



4 We first construct a tree diagram of the possible outcomes.



The player wins if their spin is higher than the operator.

$$\begin{aligned} \therefore P(\text{win}) &= \left(\frac{3}{8} \times \frac{4}{8}\right) + \left(\frac{1}{8} \times \frac{4}{8}\right) + \left(\frac{1}{8} \times \frac{3}{8}\right) \\ &= \frac{12}{64} + \frac{4}{64} + \frac{3}{64} \\ &= \frac{19}{64} \end{aligned}$$

Outcome	Win	Lose
Winnings	\$a	\$0
Probability	$\frac{19}{64}$	$\frac{41}{64}$

Let X denote the return from one game.

$$\begin{aligned} E(X) &= \left(a \times \frac{19}{64}\right) + \left(0 \times \frac{41}{64}\right) \\ &= \frac{19a}{64} \text{ dollars} \end{aligned}$$

It costs $\$k$ to play a game, so the expected gain = $\frac{19a}{64} - k$ dollars.

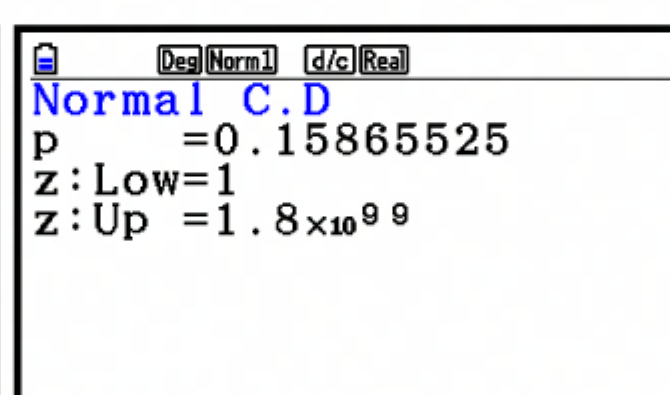
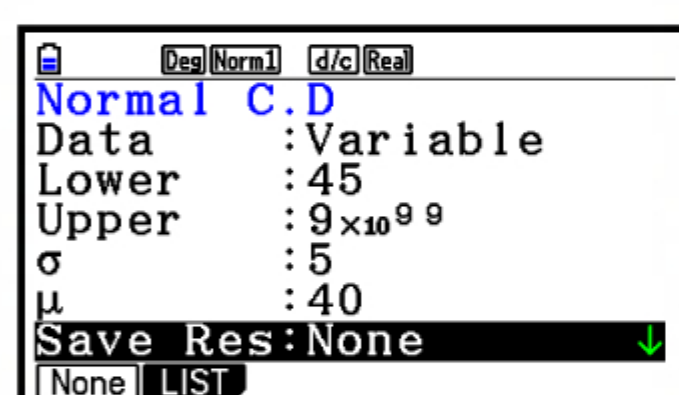
The game is fair when the expected gain is 0.

$$\therefore \frac{19a}{64} - k = 0 \quad \text{or} \quad 19a = 64k$$

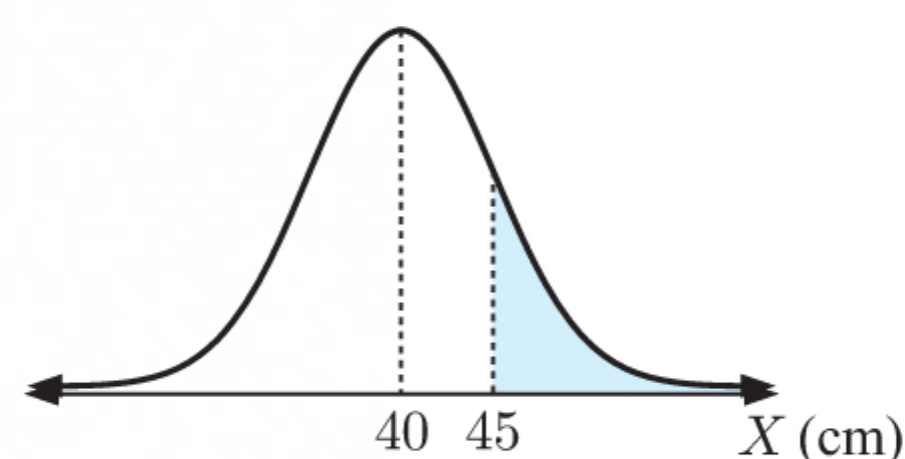
5 Let the length of a randomly selected adult fish be X cm.

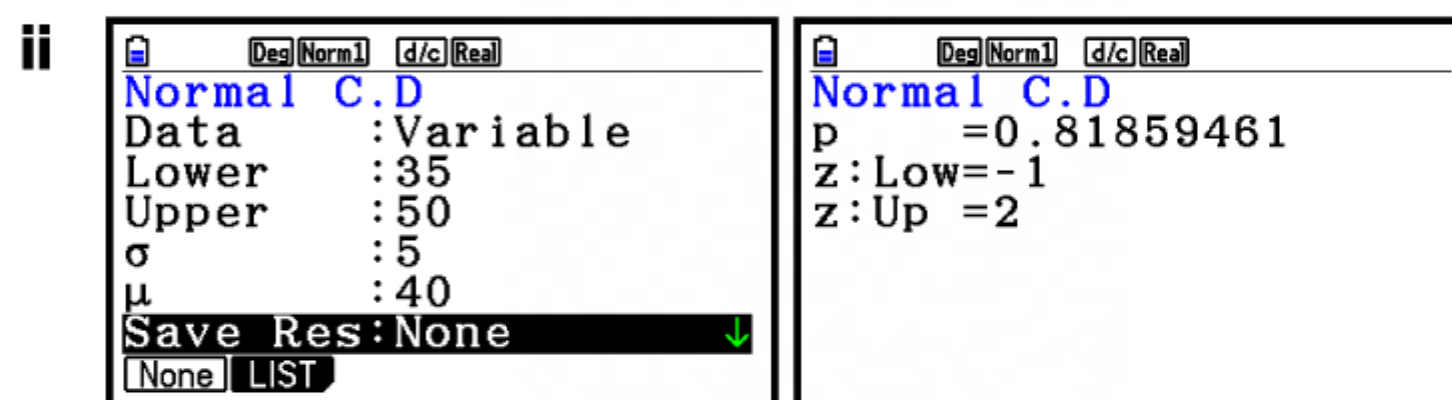
So, $X \sim N(40, 5^2)$.

a i

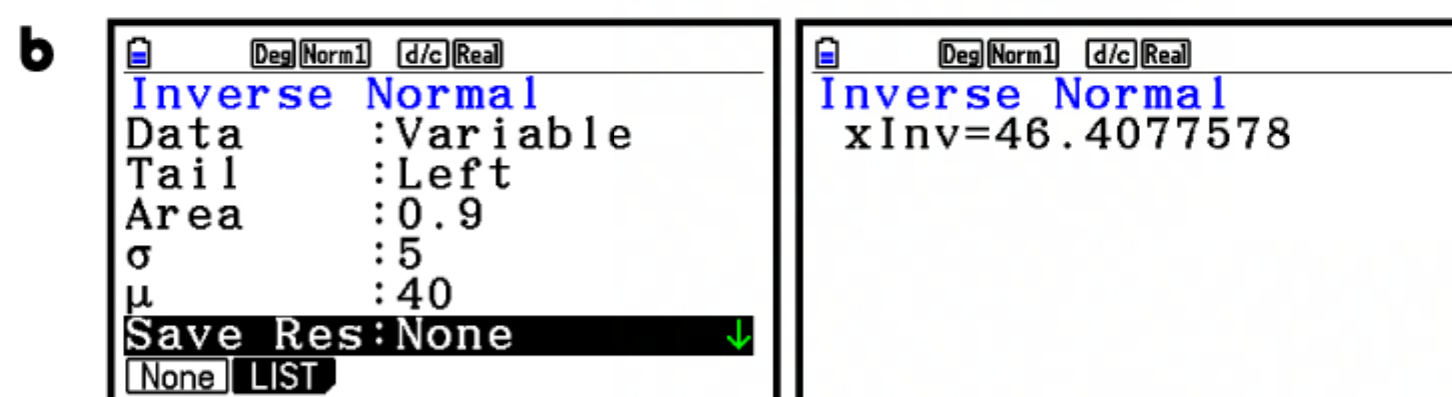
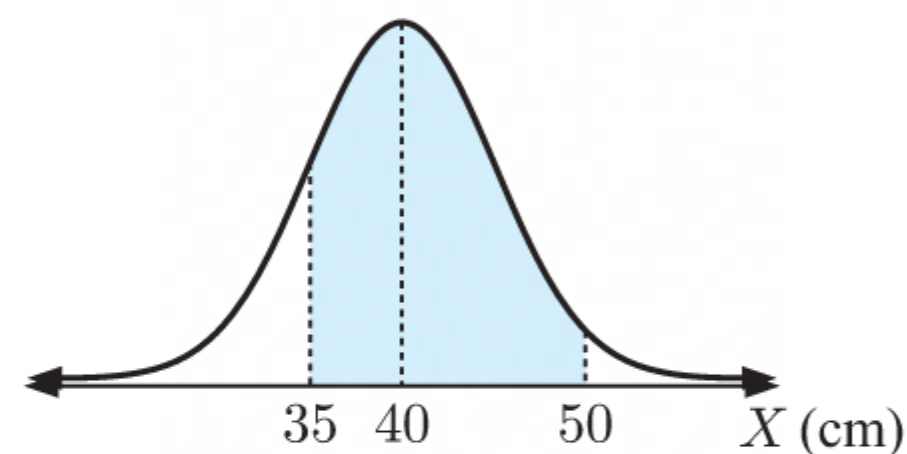


$$P(X > 45) \approx 0.159$$





$$P(35 \leq X \leq 50) \approx 0.819$$

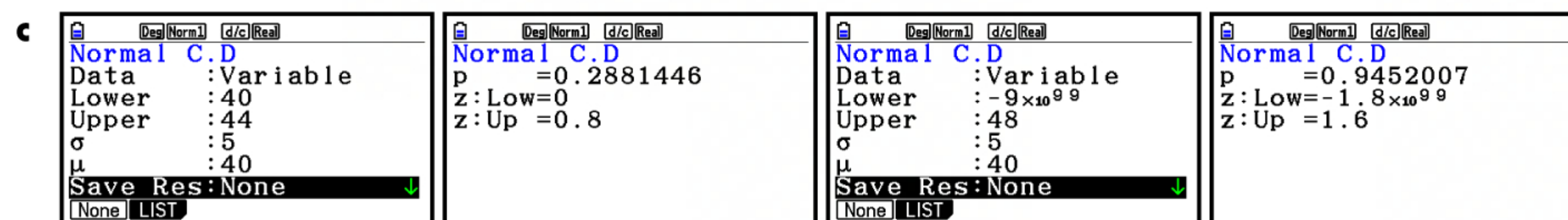
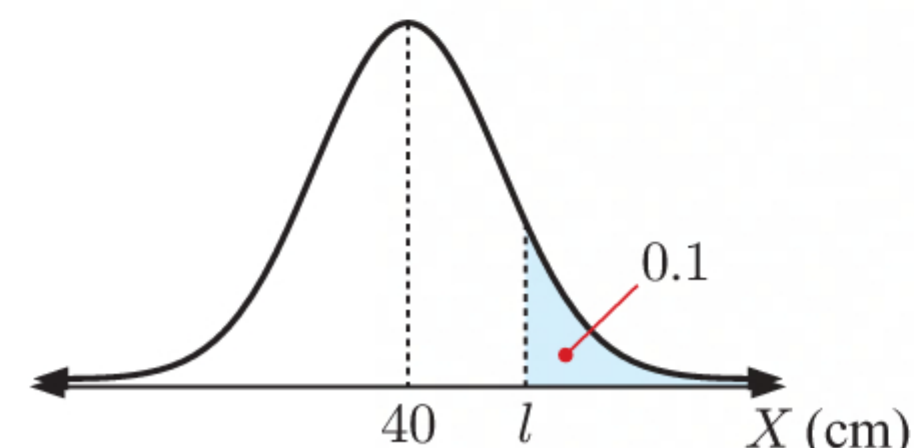


$$P(X > l) = 0.1$$

$$\therefore P(X \leq l) = 0.9$$

$$\therefore l = 46.4$$

\therefore the minimum length of the longest 10% of fish is about 46.4 cm.



$$\begin{aligned}
 P(40 \leq X \leq 44 \mid X < 48) &= \frac{P((40 \leq X \leq 44) \cap (X < 48))}{P(X < 48)} \\
 &= \frac{P(40 \leq X \leq 44)}{P(X < 48)} \\
 &\approx \frac{0.288}{0.945} \\
 &\approx 0.305
 \end{aligned}$$

6 a $f(x) = -x^2 + bx + 5$

$$\therefore f'(x) = -2x + b$$

L has gradient 5

$$\therefore f'(-1) = 5$$

$$\therefore -2(-1) + b = 5$$

$$\therefore 2 + b = 5$$

$$\therefore b = 3$$

b The tangent at P is perpendicular to L .

\therefore the tangent has gradient $-\frac{1}{5}$.

$$\text{Now } f'(x) = -\frac{1}{5} \text{ when } -2x + 3 = -\frac{1}{5}$$

$$\therefore -2x = -\frac{16}{5}$$

$$\therefore x = \frac{8}{5}$$

$$f\left(\frac{8}{5}\right) = -\left(\frac{8}{5}\right)^2 + 3\left(\frac{8}{5}\right) + 5$$

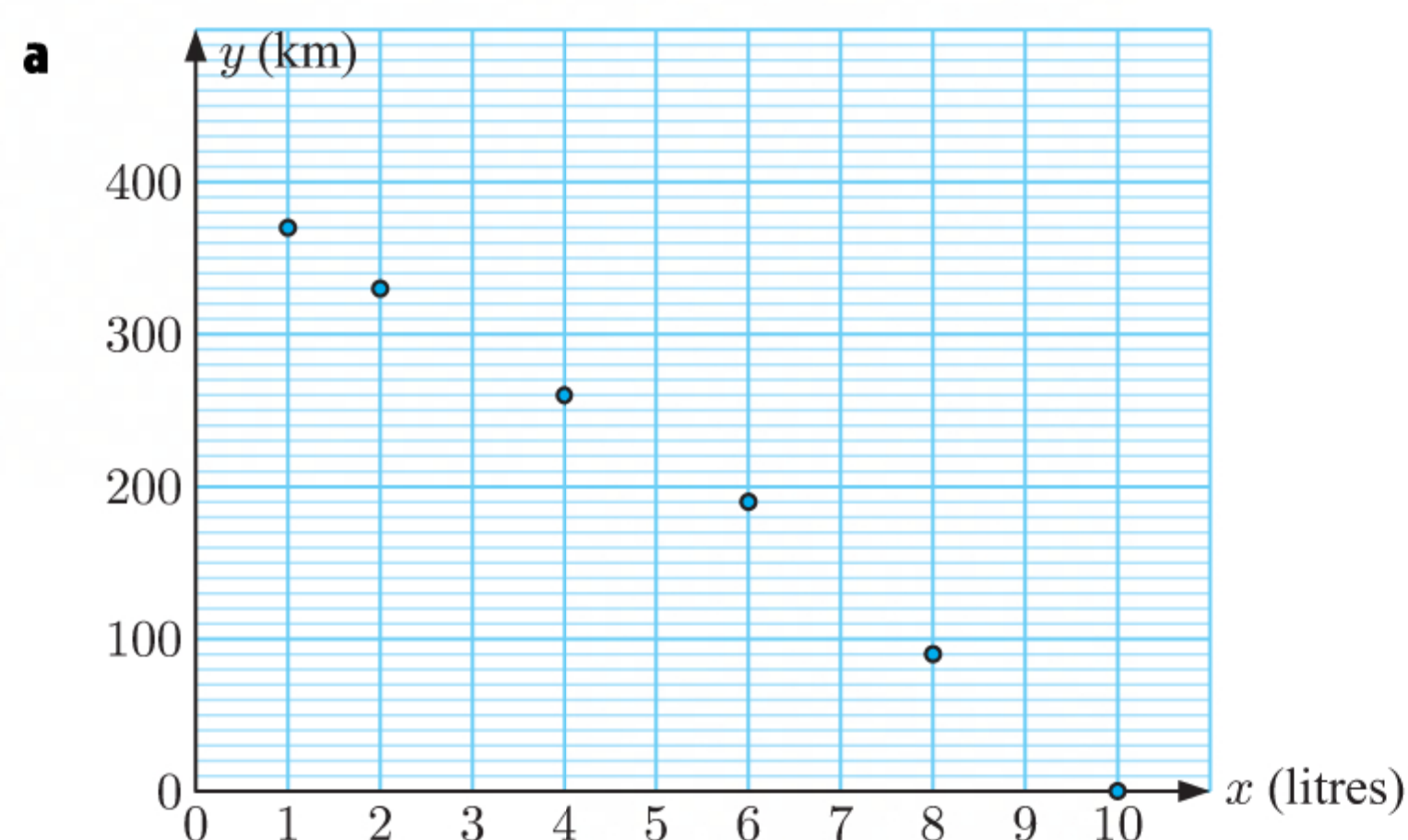
$$= -\frac{64}{25} + \frac{24}{5} + 5$$

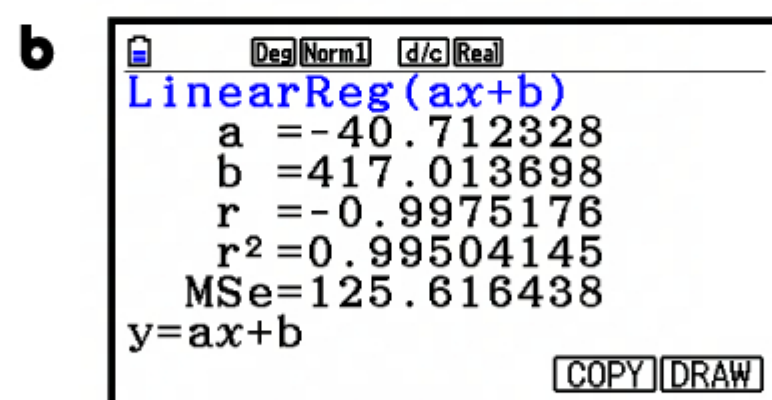
$$= \frac{181}{25}$$

\therefore P has coordinates $\left(\frac{8}{5}, \frac{181}{25}\right)$.

7

Remaining fuel (x litres)	10	8	6	4	2	1
Distance (y km)	0	90	190	260	330	370





Using technology, the regression line is $y \approx -40.7x + 417$.

c The y -intercept of the regression line ≈ 417 . This indicates that the motorbike can travel about 417 km on a full tank of petrol.

d i When $y = 220$, $220 \approx -40.7x + 417$

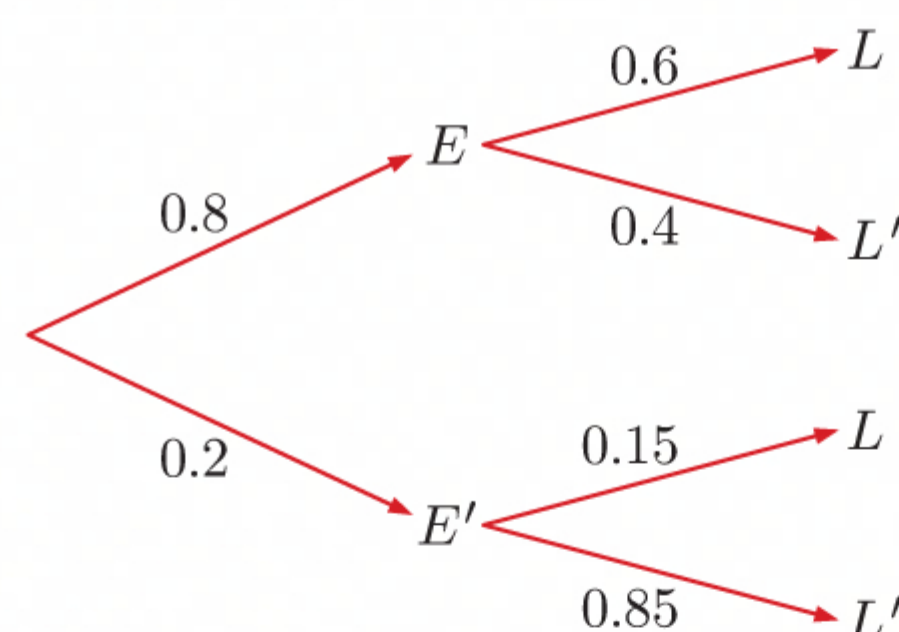
$$\therefore -197 \approx -40.7x$$

$$\therefore x \approx 4.84$$

\therefore there is about 4.84 litres of fuel left in the tank after the motorbike has travelled 220 km.

ii Average distance travelled per litre $\approx \frac{220}{10 - 4.84} \approx 42.6$ km per litre.

8 a Let E be the event that Mark wakes up early, and L be the event that Mark packs his lunch.



$$\begin{aligned} \mathbf{b} \quad P(L) &= P(E \cap L) + P(E' \cap L) \\ &= 0.8 \times 0.6 + 0.2 \times 0.15 \\ &= 0.51 \end{aligned}$$

9 a In $\triangle ABC$, by the cosine rule:

$$BC^2 = 65^2 + 104^2 - 2 \times 65 \times 104 \times \cos 60^\circ$$

$$\therefore BC = \sqrt{65^2 + 104^2 - 2 \times 65 \times 104 \times \cos 60^\circ} \quad \{\text{as } BC > 0\}$$

$$\therefore BC = 91 \text{ m}$$

$$\begin{aligned} \mathbf{b} \quad \text{Area of } \triangle ABC &= \frac{1}{2} \times AB \times AC \times \sin \widehat{BAC} \\ &= \frac{1}{2} \times 65 \times 104 \times \sin 60^\circ \\ &= 65 \times 52 \times \frac{\sqrt{3}}{2} \\ &= 1690\sqrt{3} \text{ m}^2 \\ &\approx 2930 \text{ m}^2 \end{aligned}$$

\therefore the total area of the field is about 2930 m².

$$\begin{aligned} \mathbf{c} \quad \text{Area of } A_1 &= \frac{1}{2} \times AB \times AD \times \sin \widehat{BAD} \\ &= \frac{1}{2} \times 65 \times x \times \sin 30^\circ \\ &= \frac{65x}{2} \times \frac{1}{2} \\ &= \frac{65x}{4} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of } A_2 &= \frac{1}{2} \times AC \times AD \times \sin \widehat{CAD} \\ &= \frac{1}{2} \times 104 \times x \times \sin 30^\circ \\ &= 52x \times \frac{1}{2} \\ &= 26x \text{ m}^2 \end{aligned}$$

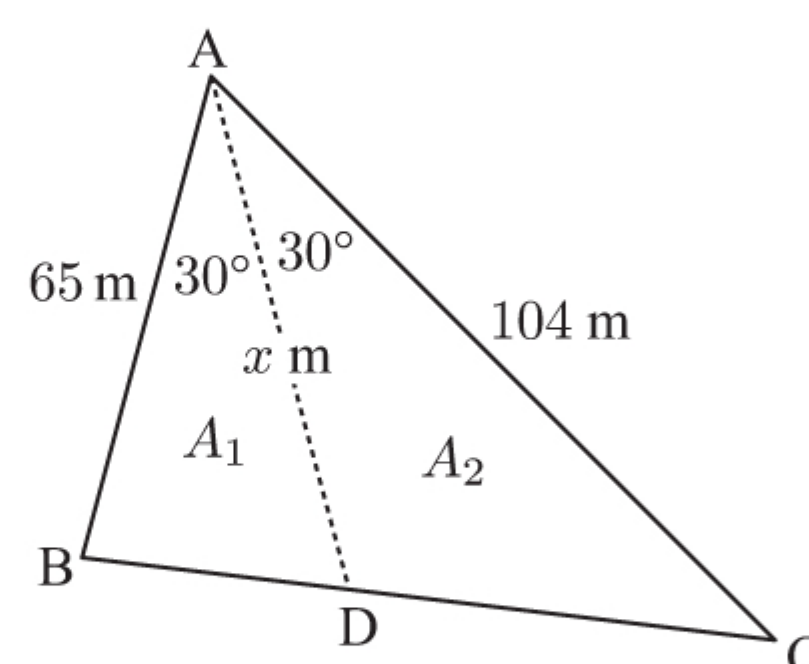
Now, the total area of the field = $A_1 + A_2$

$$\therefore 1690\sqrt{3} = \frac{65x}{4} + 26x \quad \{\text{from a}\}$$

$$\therefore 1690\sqrt{3} = x\left(\frac{65}{4} + 26\right)$$

$$\therefore x = \frac{1690\sqrt{3}}{\frac{65}{4} + 26}$$

$$\therefore x \approx 69.3$$



10 a $D'(t) = 0.8t - 8$

$$\therefore D(t) = \int (0.8t - 8) dt$$

$$= 0.4t^2 - 8t + c$$

Now $D(0) = 42 \quad \therefore 0.4(0)^2 - 8(0) + c = 42$

$$\therefore c = 42$$

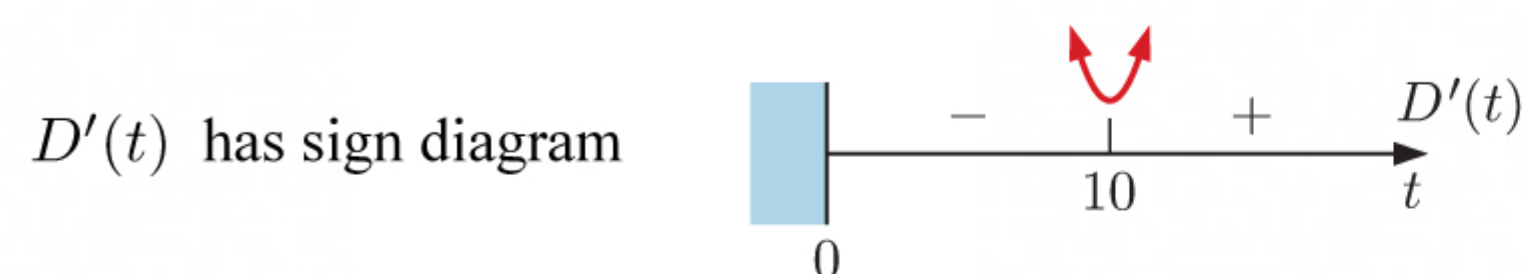
$$\therefore D(t) = 0.4t^2 - 8t + 42$$

c The minimum distance occurs when $D'(t) = 0$

$$\therefore 0.8t - 8 = 0$$

$$\therefore 0.8t = 8$$

$$\therefore t = 10$$



$$\therefore D \text{ is minimised when } t = 10.$$

$$D(10) = 0.4(10)^2 - 8(10) + 42$$

$$= 40 - 80 + 42$$

$$= 2$$

\therefore the minimum distance between the motorcyclists is 2 m which occurs 10 seconds after they start riding.

11 a $N = 7 \times 12 = 84$, $I\% = 9.25$, $PV = 55\,000$, $FV = 0$, $P/Y = 12$, $C/Y = 12$

$$\therefore PMT \approx -891.90$$

The monthly repayment is £891.90

b $N = 2\frac{1}{2} \times 12 = 30$, $I\% = 9.25$, $PV = 55\,000$, $PMT = -891.90$, $P/Y = 12$, $C/Y = 12$

$$\therefore FV \approx 39\,273.68$$

The outstanding debt is £39 273.68

c Depreciated value after 7 years $= £55\,000 \times (0.85)^7$

$$= £17\,631.74$$

$$\text{Total repayments} = 84 \times £891.90$$

$$= £74\,919.60$$

$$\therefore \text{cost of car to Melinda} = £74\,919.60 - £17\,631.74$$

$$= £57\,287.86$$

12 a Using (1), $10a + b = 30$

Using (2), $20a + b = 110$

b Solving the system of equations $\begin{cases} 10a + b = 30 \\ 20a + b = 110 \end{cases}$

simultaneously gives $a = 8$ and $b = -50$.

c From **b**, the model is $T = 8r - 50$.

For small values of r , $T \approx -50$, which does not make any physical sense.

So, this model is not appropriate for small values of r .

b $D(2) = 0.4(2)^2 - 8(2) + 42$

$$= 1.6 - 16 + 42$$

$$= 27.6$$

\therefore after 2 seconds, the distance between the motorcyclists is 27.6 m.

Norm1	
Compound Interest	
n	=84
I%	=9.25
PV	=55000
PMT	=-891.8933025
FV	=0
P/Y	=12
n	I% PV PMT FV AMORTIZ

Norm1	
Compound Interest	
n	=30
I%	=9.25
PV	=55000
PMT	=-891.9
FV	=-39273.68067
P/Y	=12
n	I% PV PMT FV AMORTIZ

Math Deg Norm1 d/c Real	
a _n X + b _n Y = C _n	
	a b c
1	10 1 30
2	20 1 110
110	
SOLVE DELETE CLEAR EDIT	

Math Deg Norm1 d/c Real	
a _n X + b _n Y = C _n	
X	8
Y	-50
8	
REPEAT	

d For Globe Park, $r = 30$.

$$\begin{aligned}\therefore T &= 8(30) - 50 \\ &= 240 - 50 \\ &= 190\end{aligned}$$

\therefore it will take 190 minutes to maintain Globe Park.

f i $T = \text{time to mow interior} + \text{time to trim perimeter}$

Now $\text{time to mow interior} \propto \text{area of circle} = \pi r^2$

$\therefore \text{time to mow interior} = k_1 \pi r^2$, where k_1 is a constant

and $\text{time to trim perimeter} \propto \text{circumference of circle} = 2\pi r$

$\therefore \text{time to trim perimeter} = 2k_2 \pi r$, where k_2 is a constant

$$\therefore T = k_1 \pi r^2 + 2k_2 \pi r$$

This model has the form $T = pr^2 + qr$ where p and q are constants.

ii When $r = 10$, $T = 30$

$$\therefore 30 = p(10)^2 + q(10)$$

$$\therefore 30 = 100p + 10q$$

$$\therefore 10p + q = 3$$

When $r = 20$, $T = 110$

$$\therefore 110 = p(20)^2 + q(20)$$

$$\therefore 110 = 400p + 20q$$

$$\therefore 40p + 2q = 11$$

So we have the system of equations
$$\begin{cases} 10p + q = 3 \\ 40p + 2q = 11 \end{cases}$$

Using technology to solve the system simultaneously gives $p = \frac{1}{4}$ and $q = \frac{1}{2}$.

iii From **ii**, $T = \frac{1}{4}r^2 + \frac{1}{2}r$

For Globe Park, $r = 30$

$$\begin{aligned}\therefore T &= \frac{1}{4}(30)^2 + \frac{1}{2}(30) \\ &= 240\end{aligned}$$

This model is better at predicting the time taken to maintain Globe Park as the predicted value is closer to the actual time than our estimate in **d**.

MIXED QUESTIONS SET 4

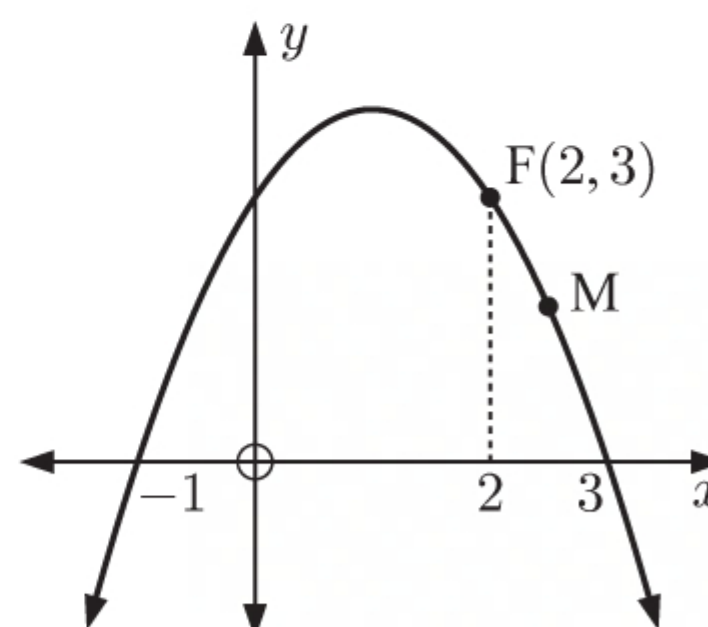
1 The distance Eliza walked could be from 6.05 km to 6.15 km.

The time it took Eliza to walk this distance could be from 81.5 minutes to 82.5 minutes.

$$\begin{aligned}\therefore \text{lower bound of Eliza's average speed} &= \frac{6.05}{\frac{82.5}{60}} \\ &= 4.4 \text{ km h}^{-1}\end{aligned}$$

2 $y = 3 + 2x - x^2$

$$\begin{aligned}\text{a } y\text{-coordinate of M} &= 3 + 2(2 + h) - (2 + h)^2 \\ &= 3 + 4 + 2h - (4 + 4h + h^2) \\ &= 3 - 2h - h^2\end{aligned}$$



$$\begin{aligned} \text{b Gradient of [FM]} &= \frac{3 - 2h - h^2 - 3}{2 + h - 2} \\ &= \frac{-2h - h^2}{h} \end{aligned}$$

$$\begin{aligned} \text{c Gradient of tangent} &= \lim_{h \rightarrow 0} \frac{-2h - h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{-\cancel{h}(2 + h)}{\cancel{h}} \\ &= \lim_{h \rightarrow 0} -(2 + h) \quad \{h \neq 0\} \\ &= -2 \end{aligned}$$

3 Neighbourhood A:

275	281	320	265	305	258	310	430	285
290	297	345	195	230	269	300	258	273

Neighbourhood B:

325	300	412	370	297	505	340	333	290
428	305	520	360	410	275	320	431	410

a The sale price of a house can be counted, so it is a discrete variable.

b Neighbourhood A:

1-Variable	
n	=18
minX	=195
Q1	=265
Med	=283
Q3	=305
maxX	=430

minimum = \$195 000
 Q_1 = \$265 000
 median = \$283 000
 Q_3 = \$305 000
 maximum = \$430 000

Neighbourhood B:

1-Variable	
n	=18
minX	=275
Q1	=305
Med	=350
Q3	=412
maxX	=520

minimum = \$275 000
 Q_1 = \$305 000
 median = \$350 000
 Q_3 = \$412 000
 maximum = \$520 000

c For Neighbourhood A, $IQR = 305\,000 - 265\,000 = 40\,000$

Test for outliers:	upper boundary	and	lower boundary
	= upper quartile + $1.5 \times IQR$		= lower quartile - $1.5 \times IQR$
	= $305\,000 + 1.5 \times 40\,000$		= $265\,000 - 1.5 \times 40\,000$
	= 365 000		= 205 000

\$430 000 is above the upper boundary, so it is an outlier.

\$195 000 is below the lower boundary, so it is an outlier.

For Neighbourhood B, $IQR = 412\,000 - 305\,000 = 107\,000$

Test for outliers:	upper boundary	and	lower boundary
	= upper quartile + $1.5 \times IQR$		= lower quartile - $1.5 \times IQR$
	= $412\,000 + 1.5 \times 107\,000$		= $305\,000 - 1.5 \times 107\,000$
	= 572 500		= 144 500

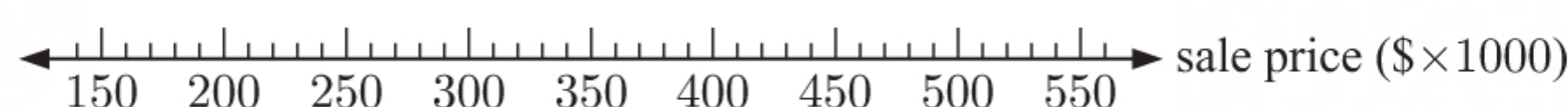
\therefore there are no outliers.



Neighbourhood A



Neighbourhood B



d Both sets of data are positively skewed. The sale price of houses in Neighbourhood B are generally higher than those in Neighbourhood A. With the outliers removed, there is more variation in the sale price of houses in Neighbourhood B compared to Neighbourhood A.

4 a

b (bags)	30	35	40	45	50
P (rupiah)	38 000	36 000	34 000	32 000	30 000

We see that for every increase of 5 bags of rice, the price decreases by 2000 rupiah.

$\therefore P(b)$ is a linear function with gradient $\frac{-2000}{5} = -400$

$\therefore P(b) = -400b + c$ for some constant c .

Now $P(50) = 30\,000$

$\therefore -400 \times 50 + c = 30\,000$

$\therefore c = 50\,000$

$\therefore P(b) = -400b + 50\,000$

b $P(60) = -400 \times 60 + 50\,000 = 26\,000$

\therefore the total cost $= 60 \times 26\,000 = 1\,560\,000$ rupiah

c $P(150) = -400 \times 150 + 50\,000$
 $= -10\,000$

This model should not be used to predict the cost of 150 bags of rice because it would yield a negative value.

5 a $y = f(x) = a(x - p)(x - q)$ has x -intercepts -1 and 5 .

$\therefore p = 5$ and $q = -1$ $\{p > q\}$

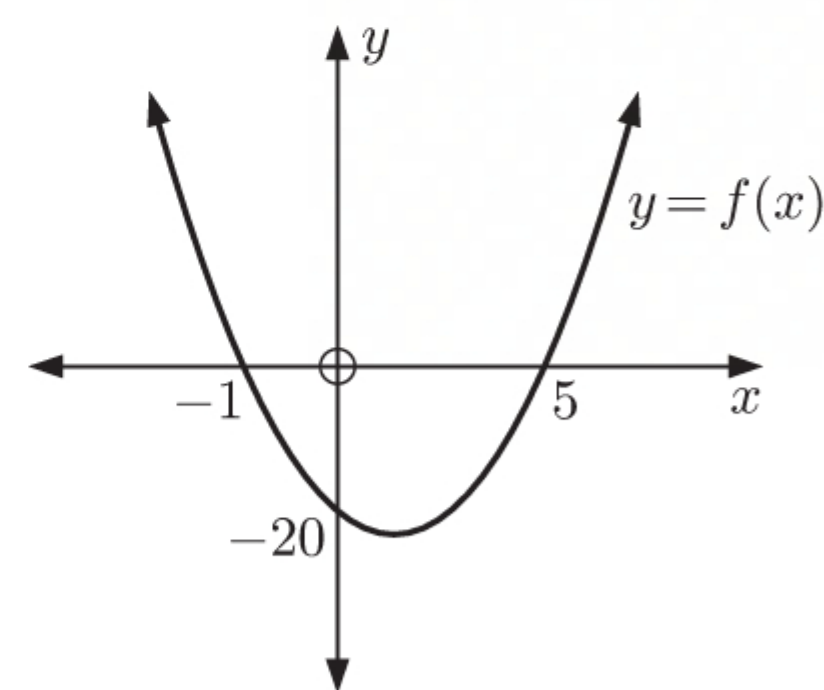
b From **a**, $f(x) = a(x - 5)(x + 1)$.

From the graph, $f(0) = -20$

$\therefore -20 = a(-5)(1)$

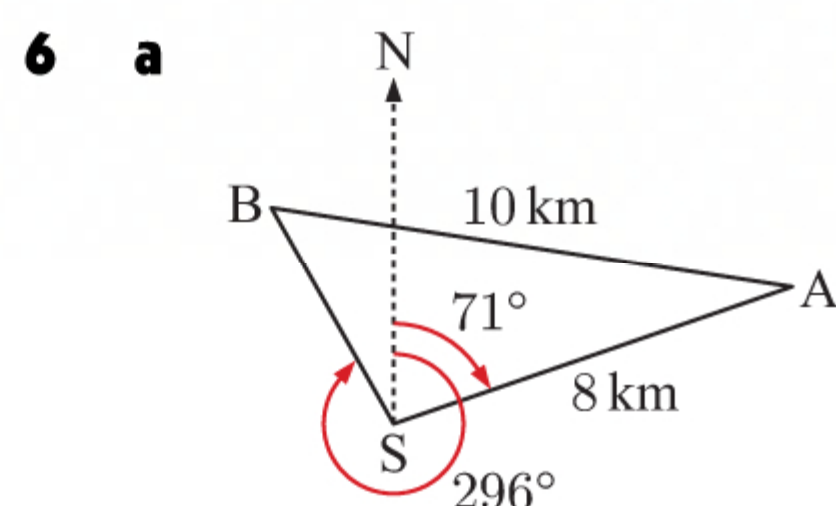
$\therefore -20 = -5a$

$\therefore a = 4$



c The axis of symmetry is midway between the x -intercepts.

\therefore the axis of symmetry is $x = \frac{-1 + 5}{2} = 2$.



b $N_1\hat{S}B = 360^\circ - 296^\circ$ {angles at a point}
 $= 64^\circ$

$\therefore B\hat{S}A = 64^\circ + 71^\circ = 135^\circ$

$\therefore \frac{\sin \alpha}{8} = \frac{\sin 135^\circ}{10}$ {sine rule}

$\therefore \sin \alpha = \frac{8 \sin 135^\circ}{10}$

$\therefore \alpha = \sin^{-1}\left(\frac{8 \sin 135^\circ}{10}\right) \approx 34.4^\circ$

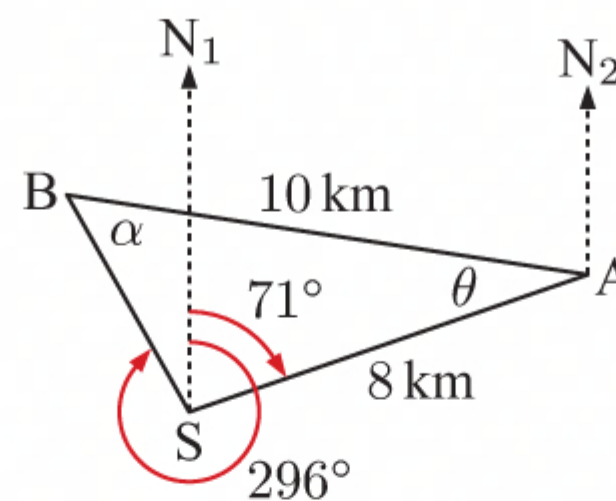
Now $\theta = 180^\circ - 135^\circ - \alpha$ {angles in a triangle}

$\approx 180^\circ - 135^\circ - 34.4^\circ$

$\approx 10.6^\circ$

$N_2\hat{A}S = 180^\circ - 71^\circ$ {co-interior angles}
 $= 109^\circ$

\therefore bearing of B from A $\approx 360^\circ - 109^\circ + 10.6^\circ \approx 262^\circ$



c By the cosine rule:

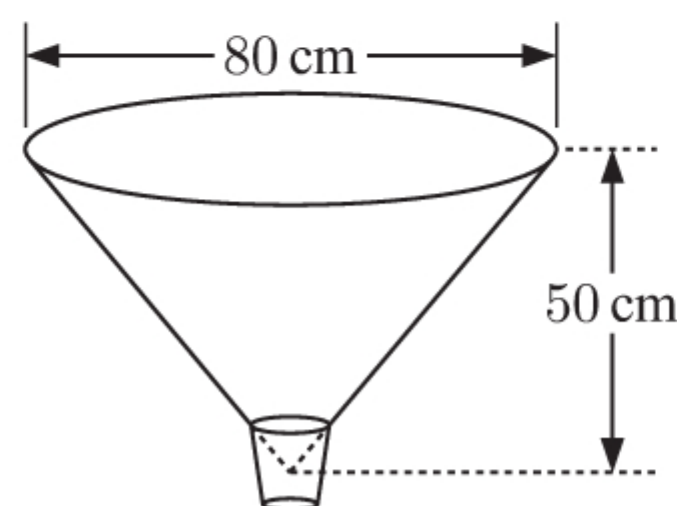
$$\begin{aligned} BS^2 &= 10^2 + 8^2 - 2(10)(8) \cos \theta \\ \therefore BS &= \sqrt{10^2 + 8^2 - 2(10)(8) \cos \theta} \\ \therefore BS &\approx \sqrt{10^2 + 8^2 - 2(10)(8) \cos 10.6^\circ} \quad \{\text{from b}\} \\ \therefore BS &\approx 2.59 \text{ km} \approx 2590 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Now time} &= \frac{\text{distance}}{\text{speed}} \\ &\approx \frac{2590}{7} \\ &\approx 370 \text{ seconds} \end{aligned}$$

It will take about 370 seconds or 6 minutes 10 seconds for train B to reach the train station.

7 a $V \approx$ volume of cone

$$\begin{aligned} &\approx \frac{1}{3} \pi r^2 h \\ &\approx \frac{1}{3} \times \pi \times \left(\frac{80}{2}\right)^2 \times 50 \text{ cm}^3 \\ &\approx \frac{80\,000}{3} \pi \text{ cm}^3 \\ &\approx 83\,800 \text{ cm}^3 \end{aligned}$$

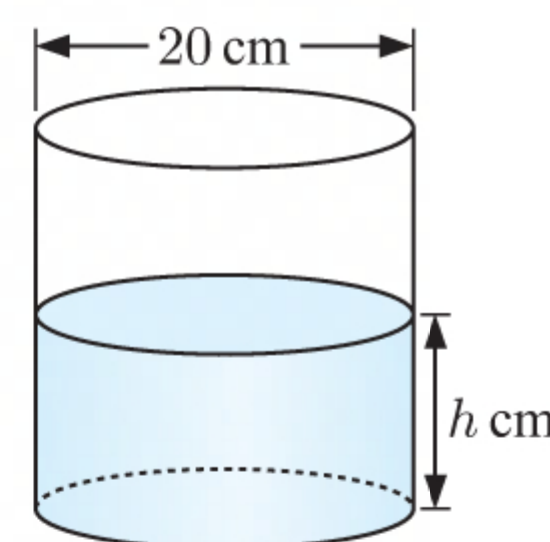


The capacity of the funnel is about 83 800 mL or 8.38×10^4 mL.

b When half full, the funnel contains about $\frac{80\,000}{3} \pi \times 0.5 \approx \frac{40\,000}{3} \pi$ mL of liquid.

$$\begin{aligned} V &\approx \frac{40\,000}{3} \pi \text{ cm}^3 \\ \therefore \pi \times \left(\frac{20}{2}\right)^2 \times h &\approx \frac{40\,000}{3} \pi \\ \therefore h &\approx \frac{40\,000}{3 \times 10^2} \\ \therefore h &\approx 133 \text{ cm} \end{aligned}$$

The liquid will reach about 133 cm up the tube.



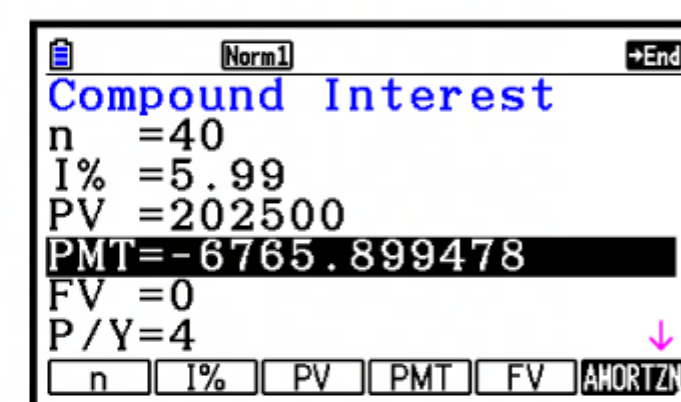
8 a A deposit of $\text{€}225\,000 \times 10\% = \text{€}22\,500$ is required.

\therefore it is necessary to borrow $\text{€}225\,000 - \text{€}22\,500 = \text{€}202\,500$

$$N = 10 \times 4 = 40, \quad I\% = 5.99, \quad PV = 202\,500, \quad FV = 0, \quad P/Y = 4, \quad C/Y = 4$$

$$\therefore PMT \approx -6765.90$$

The quarterly repayments are €6765.90

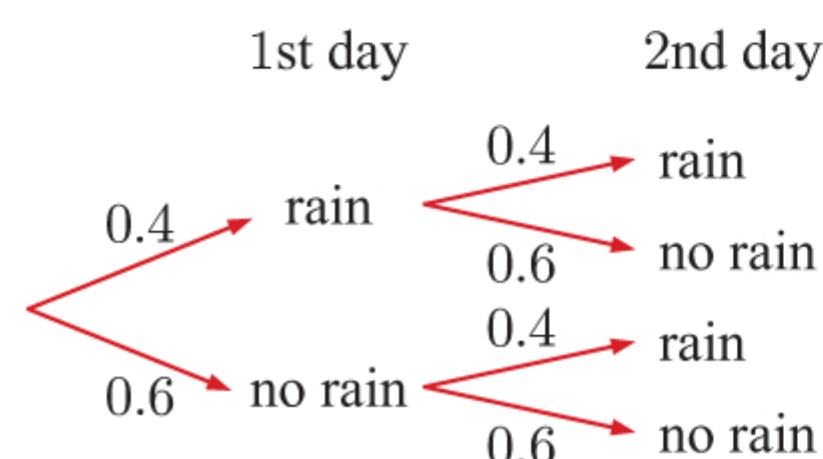


b Total interest = total repayment – starting principal

$$\begin{aligned} &= \text{€}6765.90 \times 10 \times 4 - \text{€}202\,500 \\ &= \text{€}68\,136 \end{aligned}$$

c The indexed value = $\text{€}225\,000 \times (1.035)^{10}$
 $= \text{€}317\,384.72$

9 a



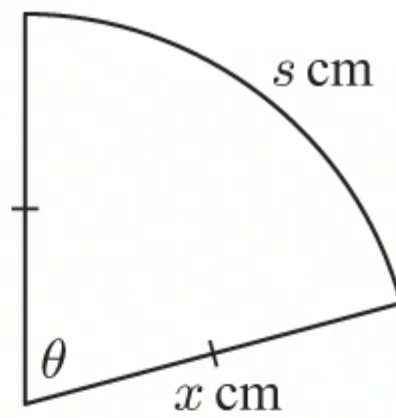
b i $P(\text{rain on both days}) = P(\text{rain} \cap \text{rain})$
 $= 0.4 \times 0.4$
 $= 0.16$

ii $P(\text{no rain on one day}) = P(\text{rain} \cap \text{no rain}) + P(\text{no rain} \cap \text{rain})$
 $= 0.4 \times 0.6 + 0.6 \times 0.4$
 $= 0.48$

$$\begin{aligned}
 \text{c} \quad P(\text{no rain on both days}) &= P(\text{no rain} \cap \text{no rain}) \\
 &= 0.6 \times 0.6 \\
 &= 0.36 \\
 \therefore P(\text{rain on at least one day}) &= 1 - 0.36 \\
 &= 0.64
 \end{aligned}$$

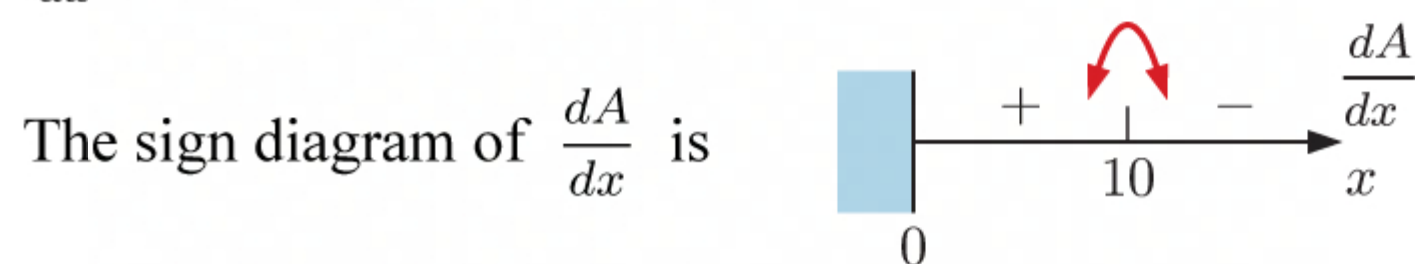
$$\begin{aligned}
 \text{So, } P(\text{rain on 2nd day} \mid \text{rain on at least one day}) &= \frac{P(\text{rain on 2nd day} \cap \text{rain on at least one day})}{P(\text{rain on at least one day})} \\
 &= \frac{P(\text{rain on 2nd day})}{P(\text{rain on at least one day})} \\
 &= \frac{0.4}{0.64} \\
 &= 0.625
 \end{aligned}$$

$$\begin{aligned}
 \text{10 a} \quad \text{Perimeter} &= 2x + s \\
 &= 2x + \frac{\theta}{360} \times 2\pi x \\
 \therefore 2x + \frac{\theta}{360} \times 2\pi x &= 40 \\
 \therefore x + \frac{\theta}{360} \times \pi x &= 20 \\
 \therefore \frac{\theta}{360} \times \pi x &= 20 - x \\
 \therefore \theta &= \frac{360}{\pi x}(20 - x)
 \end{aligned}$$



$$\begin{aligned}
 \text{b Area } A &= \frac{\theta}{360} \times \pi x^2 \\
 &= \frac{\frac{360}{\pi x}(20 - x)}{360} \times \pi x^2 \quad \{\text{using a}\} \\
 &= (20 - x)x \\
 &= 20x - x^2 \text{ cm}^2
 \end{aligned}$$

$$\text{c } \frac{dA}{dx} = 20 - 2x \text{ which is 0 when } x = 10.$$



$$\text{When } x = 10, \theta = \frac{360}{\pi(10)}(20 - 10) = \frac{360}{\pi} \approx 115^\circ.$$

$\therefore A$ is a maximum when $x = 10$ and $\theta \approx 115^\circ$.

$$\text{11 } T(t) = A \times B^{-t} + 3$$

a i The initial internal temperature of the refrigerator was 27°C .

$$\begin{aligned}
 \text{So, } T(0) &= 27 \\
 \therefore 27 &= A + 3 \\
 \therefore A &= 24
 \end{aligned}$$

ii After 3 hours, the internal temperature was 6°C .

$$\begin{aligned}
 \text{So, } T(3) &= 6 \\
 \therefore 6 &= 24 \times B^{-3} + 3 \quad \{\text{using i}\} \\
 \therefore 24 \times B^{-3} &= 3 \\
 \therefore B^{-3} &= \frac{1}{8} \\
 \therefore B^3 &= 8 \\
 \therefore B &= 2
 \end{aligned}$$

$$\begin{aligned}
 \text{b } T(t) &= 24 \times 2^{-t} + 3 \quad \{\text{using a}\} \\
 \therefore T(5) &= 24 \times 2^{-5} + 3 \\
 &= 3.75
 \end{aligned}$$

\therefore the internal temperature is 3.75°C after 5 hours.

c As $t \rightarrow \infty$, $2^{-t} \rightarrow 0$

$$\therefore T(t) \rightarrow 24 \times 0 + 3 = 3$$

\therefore the minimum temperature that the refrigerator could be expected to reach is 3°C .

- 12 a** Let μ_1 be the population mean battery life of *Mega* speakers, and let μ_2 be the population mean battery life of *Micro* speakers.

The hypotheses to be considered are:

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 > \mu_2$$

b

	Des (Norm1)	d/c (Real)
1	22.4	20.8
2	23.5	21.2
3	24.1	22.1
4	22.3	20.7
		22.4

	Des (Norm1)	d/c (Real)
2-Sample tTest		
Data	: List	
μ_1	> μ_2	
t	= 2.4270426	
p	= 0.01239673	
df	= 20	
\bar{x}_1	= 22.7	
\bar{x}_2	= 21.76	

Using technology, the test statistic $t \approx 2.43$ and p -value ≈ 0.0124 .

- c** Since p -value < 0.1 , we have enough evidence to reject H_0 on a 10% level of significance.
 \therefore we conclude that the company's claim is valid.

MIXED QUESTIONS SET 5

1 a $f(1) = 1^2 - 2(1) = -1$

$$g(1) = \frac{1}{\sqrt{1}} = 1$$

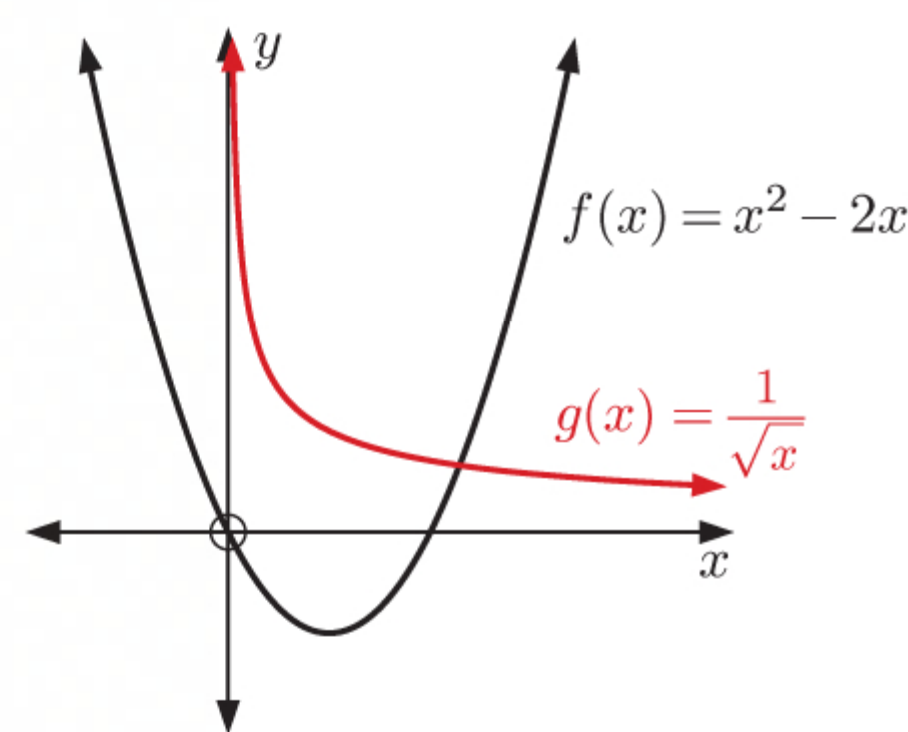
- b** g is one-to-one, so it is invertible.

f is not one-to-one, so it is not invertible.

c $g^{-1}(x) = 4$

$$\therefore x = g(4)$$

$$\therefore x = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

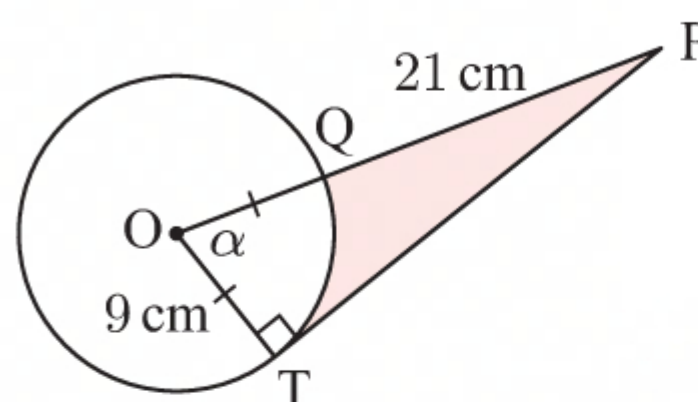


2 a $\widehat{OTP} = 90^\circ$ {radius-tangent}

$\therefore \triangle OPT$ is right angled at T .

$$\therefore \cos \alpha = \frac{9}{30}$$

$$\therefore \alpha = \cos^{-1}\left(\frac{9}{30}\right) \approx 72.5^\circ$$



b Area of $\triangle OPT = \frac{1}{2} \times 9 \times 30 \times \sin \alpha$
 $= 135 \sin \alpha \text{ cm}^2$

$$\begin{aligned} \text{Area of sector OQT} &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{\alpha}{360} \times \pi \times 9^2 \\ &= \frac{9\alpha\pi}{40} \text{ cm}^2 \end{aligned}$$

So, shaded area = area of $\triangle OPT$ - area of sector OQT

$$\begin{aligned} &= 135 \sin \alpha - \frac{9\alpha\pi}{40} \\ &= 135 \sin\left(\cos^{-1}\left(\frac{9}{30}\right)\right) - \frac{9\pi}{40} \cos^{-1}\left(\frac{9}{30}\right) \quad \{\text{using a}\} \\ &\approx 77.5 \text{ cm}^2 \end{aligned}$$

- 3 a** Francesca adds \$0.50 in the first week, \$1 the next, \$1.50 the next, adding an additional \$0.50 each subsequent week.
 \therefore in the n th week, Francesca adds $0.50n$ dollars to her money box.

Now the last week before her 11th birthday is the 51st week.

\therefore in the last week before her 11th birthday, Francesca added $\$0.50 \times 51 = \25.50 to her money box.

- b** Let $P(n)$ dollars be the amount Pierre had added to his money box after n weeks, and $F(n)$ dollars be the amount Francesca had added to her money box after n weeks.

Pierre adds \$10 each week, so after n weeks he has added $10n$ dollars.

$$\text{So, } P(n) = 10n$$

$$\therefore P(8) = 10 \times 8 = 80$$

After 8 weeks Pierre had added \$80 to his money box.

From **a**, Francesca adds $0.50n$ dollars in the n th week, so after n weeks she has added $0.50 + 1 + 1.50 + \dots + 0.50n$ dollars.

Now $0.50 + 1 + 1.50 + \dots + 0.50n$ is an arithmetic series with $u_1 = 0.5$ and $d = 0.5$.

$$\begin{aligned}\therefore 0.50 + 1 + 1.50 + \dots + 0.50n &= \frac{n}{2}(2u_1 + (n-1)d) \\ &= \frac{n}{2}(2 \times 0.5 + (n-1) \times 0.5) \\ &= \frac{n}{2}(1 + 0.5n - 0.5) \\ &= \frac{n}{2}(0.5 + 0.5n) \\ &= 0.25n + 0.25n^2\end{aligned}$$

So, $F(n) = 0.25n + 0.25n^2$

$$\therefore F(8) = 0.25 \times 8 + 0.25 \times 8^2 = 18$$

After 8 weeks, Francesca added \$18 to her money box.

c There are 52 weeks in 1 year.

$$\text{Now } P(52) = 10 \times 52 = 520$$

$$\text{and } F(52) = 0.25 \times 52 + 0.25 \times 52^2 = 689$$

\therefore after 1 year, Pierre had $\$520 + \$100 = \$620$ in his money box, and Francesca had $\$689 + \$100 = \$789$ in her money box.

So, Francesca had more money in her money box after 1 year.

4 a B is $(6, 6, 0)$ and C is $(0, 6, 0)$.

$$\begin{aligned}\text{b Volume} &= \frac{1}{3} \times \text{area of base} \times \text{height} \\ &= \frac{1}{3} \times 6 \times 6 \times 7 \\ &= 84 \text{ units}^3\end{aligned}$$

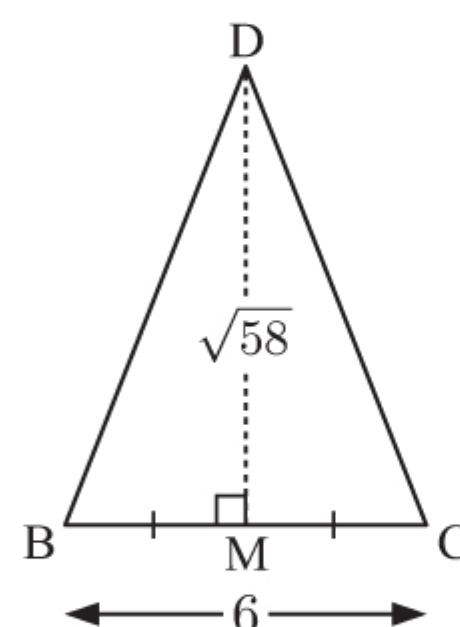
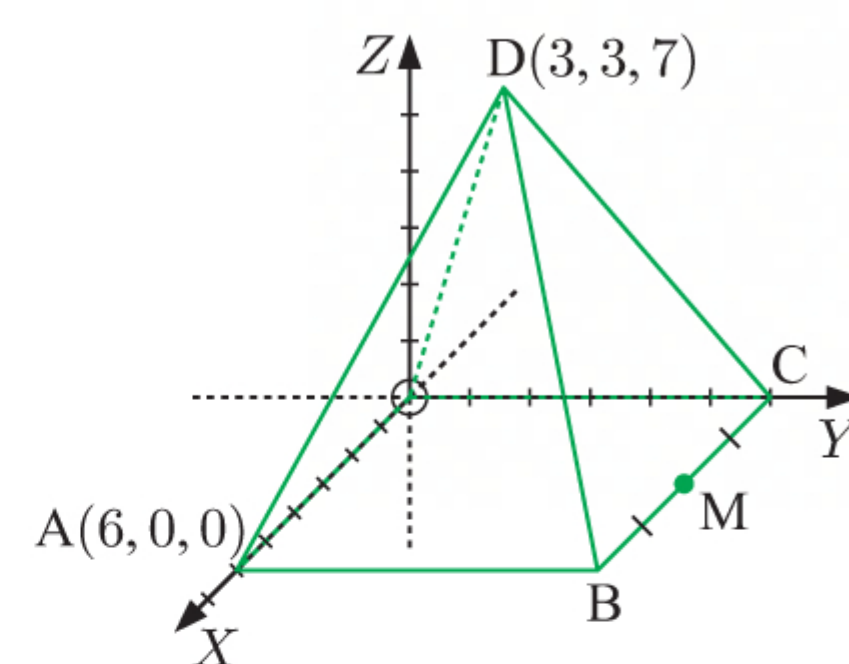
c The midpoint M of [BC] is $\left(\frac{6+0}{2}, \frac{6+6}{2}, \frac{0+0}{2}\right)$ which is $(3, 6, 0)$.

$$\begin{aligned}\text{d MD} &= \sqrt{(3-3)^2 + (3-6)^2 + (7-0)^2} \\ &= \sqrt{0^2 + (-3)^2 + 7^2} \\ &= \sqrt{0 + 9 + 49} \\ &= \sqrt{58} \text{ units}\end{aligned}$$

$$\begin{aligned}\text{Area of triangle BCD} &= \frac{1}{2} \times 6 \times \sqrt{58} \\ &= 3\sqrt{58} \text{ units}^2\end{aligned}$$

Surface area of pyramid = area of base + area of 4 triangular faces

$$\begin{aligned}&= 6 \times 6 + 4 \times \text{area of } \triangle BCD \\ &= 36 + 4 \times 3\sqrt{58} \\ &= 36 + 12\sqrt{58} \text{ units}^2 \\ &\approx 127 \text{ units}^2\end{aligned}$$



5 a $y = x^3 - 5$

$$\text{Now } \frac{dy}{dx} = 3x^2, \text{ so at } x = 1,$$

$$\frac{dy}{dx} = 3(1)^2 = 3$$

The tangent at $(1, -4)$ has equation $y = 3(x - 1) - 4$

$$\therefore y = 3x - 7$$

b The tangent cuts the x -axis where $y = 0$

$$\therefore 3x - 7 = 0$$

$$\therefore x = \frac{7}{3}$$

6	<i>Lake</i>	A	B	C	D	E	F	G	H
	Surface area (x hectares)	25	10	35	16	19	27	14	16
	Population (y)	5620	840	6125	1280	1805	3645	980	1110

a	
----------	--

So, $r_p \approx 0.923$.

b	<i>Lake</i>	A	B	C	D	E	F	G	H
	rank of x	6	1	8	3.5	5	7	2	3.5
	rank of y	7	1	8	4	5	6	2	3

c	
----------	--

So, $r_s \approx 0.970$.

d There is a very strong, positive relationship between x and y .

e $1180 > 1110$ and $1180 < 1805$, so the rank of lake D's y -value does not change.
 \therefore the value of r_s is not affected.

7 $f'(x) = (x^2 + 2)^2 = x^4 + 4x^2 + 4$, $f(1) = \frac{8}{15}$

$$\therefore f(x) = \int (x^4 + 4x^2 + 4) dx$$

$$= \frac{1}{5}x^5 + \frac{4}{3}x^3 + 4x + c$$

Now $f(1) = \frac{8}{15}$, $\therefore \frac{1}{5}(1)^5 + \frac{4}{3}(1)^3 + 4(1) + c = \frac{8}{15}$

$$\therefore \frac{1}{5} + \frac{4}{3} + 4 + c = \frac{8}{15}$$

$$\therefore \frac{83}{15} + c = \frac{8}{15}$$

$$\therefore c = -5$$

$$\therefore f(x) = \frac{1}{5}x^5 + \frac{4}{3}x^3 + 4x - 5$$

8 a Let X be the number of committee members who attend a randomly selected meeting.

$$\therefore X \sim B(15, 0.7)$$

Now $P(X \geq 10) \approx 0.722$ {using technology}

\therefore approximately 72.2% of meetings will go ahead.

--

b Suppose there are n committee members, and let Y be the number of committee members who attend a randomly selected meeting.

$$\therefore Y \sim B(n, 0.7)$$

We require $P(Y \geq 10) \geq 0.9$

If $n = 16$, $P(Y \geq 10) \approx 0.825$ ✗

If $n = 17$, $P(Y \geq 10) \approx 0.895$ ✗

If $n = 18$, $P(Y \geq 10) \approx 0.940$ ✓

So, 18 committee members are required to ensure that at least 90% of the meetings go ahead.

9 a Sites C(6, -4) and E(-4, 2) are currently in the same cell, so the missing edge must be the perpendicular bisector of [CE].

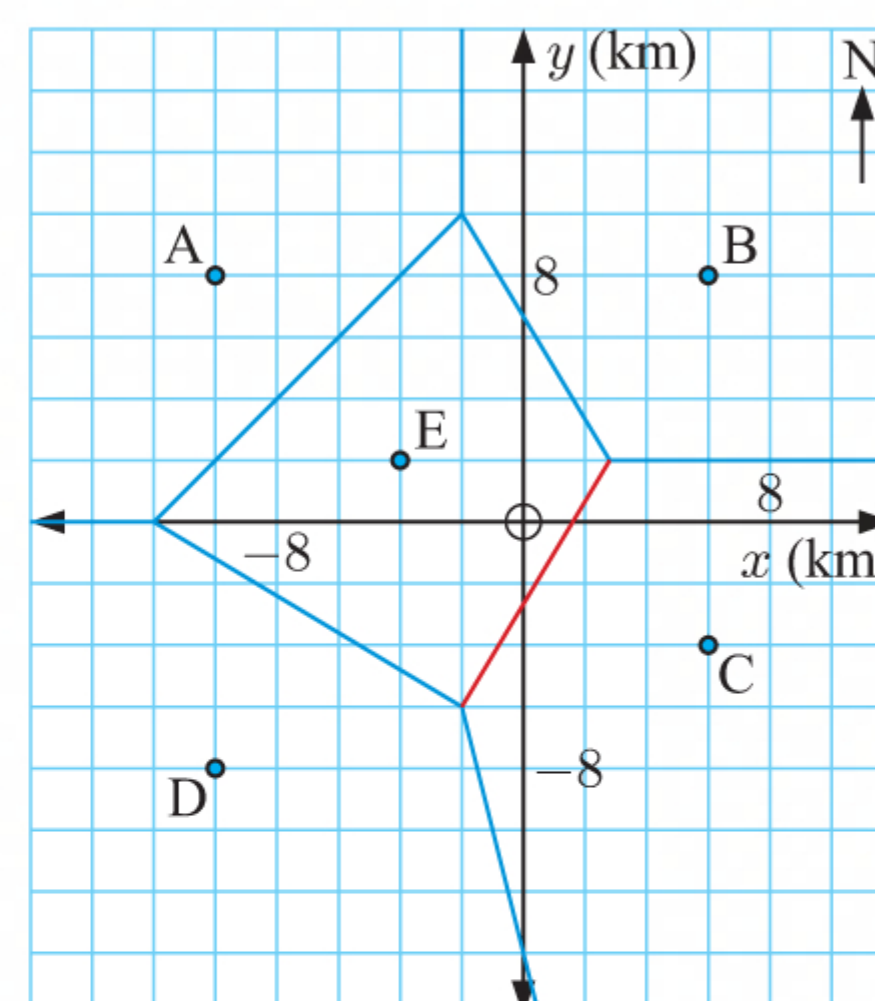
The midpoint of [CE] is $\left(\frac{6-4}{2}, \frac{-4+2}{2}\right)$ or $(1, -1)$.

The gradient of [CE] is $\frac{2-(-4)}{-4-6} = \frac{6}{-10} = -\frac{3}{5}$.

So, the perpendicular bisector of [CE] has gradient $\frac{5}{3}$.

\therefore its equation is $5x - 3y = 5(1) - 3(-1)$

$$\text{or } 5x - 3y - 8 = 0$$



b Cell D represents all the points that are closer to petrol station D than any other petrol station.

c i Riley's location is the y -intercept of the edge between cell C and E.

When $x = 0$, $5(0) - 3y - 8 = 0$ {using **a**}

$$\therefore -3y = 8$$

$$\therefore y = -\frac{8}{3}$$

$$\therefore \text{Riley's location is } R(0, -\frac{8}{3}).$$

ii Riley is closest to petrol stations C and E.

Distance Riley has to drive

$$= RC$$

$$= \sqrt{(6-0)^2 + (-4 - (-\frac{8}{3}))^2}$$

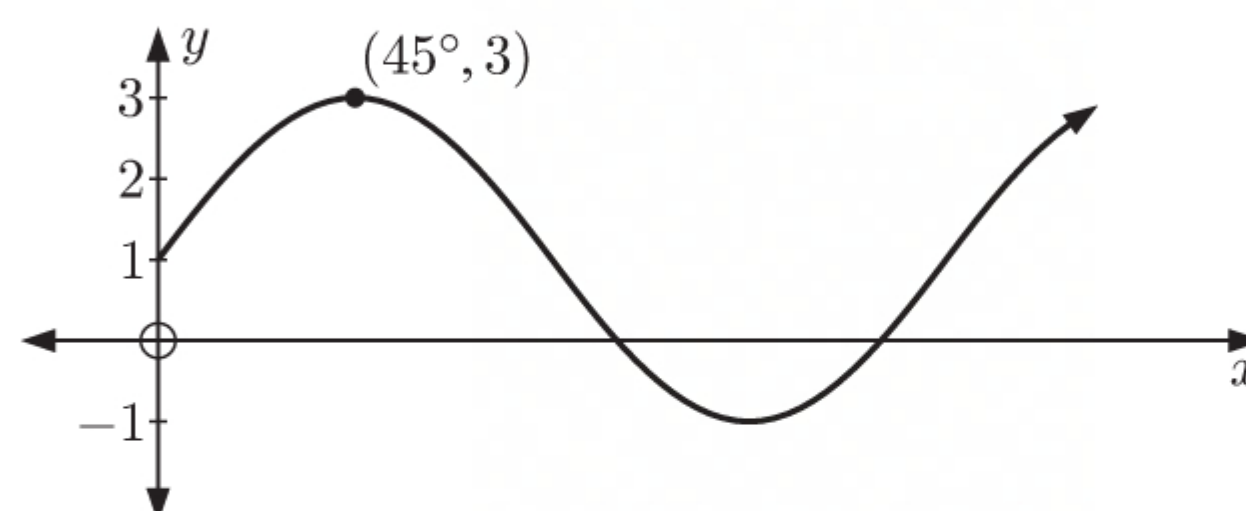
$$= \sqrt{6^2 + (-\frac{4}{3})^2}$$

$$= \sqrt{36 + \frac{16}{9}}$$

$$= \sqrt{\frac{340}{9}}$$

$$\approx 6.15 \text{ km} > 6 \text{ km}$$

\therefore Riley will not be able to drive to a petrol station before his car runs out of petrol.



10 The sine function has the form $y = a \sin bx + d$, where a , b , and d are constants.

When $x = 0^\circ$, $y = 1$

$$\therefore a \sin 0^\circ + d = 1$$

$$\therefore 0 + d = 1$$

$$\therefore d = 1$$

When $x = 45^\circ$, $y = 3$

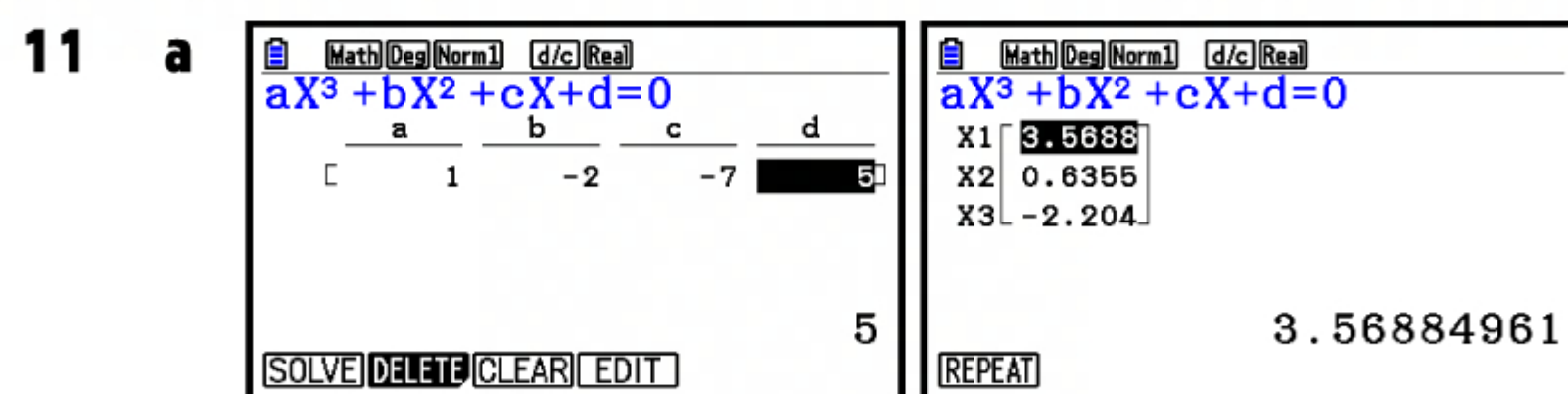
$$\therefore a \sin 90^\circ + 1 = 3$$

$$\therefore a = 2$$

The period of the graph is 180° . $\therefore \frac{360^\circ}{b} = 180^\circ$

$$\therefore b = 2$$

\therefore the sine function has equation $y = 2 \sin 2x + 1$.



Using technology, the solutions to $x^3 - 2x^2 - 7x + 5 = 0$ are $x \approx -2.20$, $x \approx 0.636$, and $x \approx 3.57$.

b When $x = 0$, $y = 5$.

\therefore the y -intercept is 5.

\therefore the height of the triangle is 5 units.

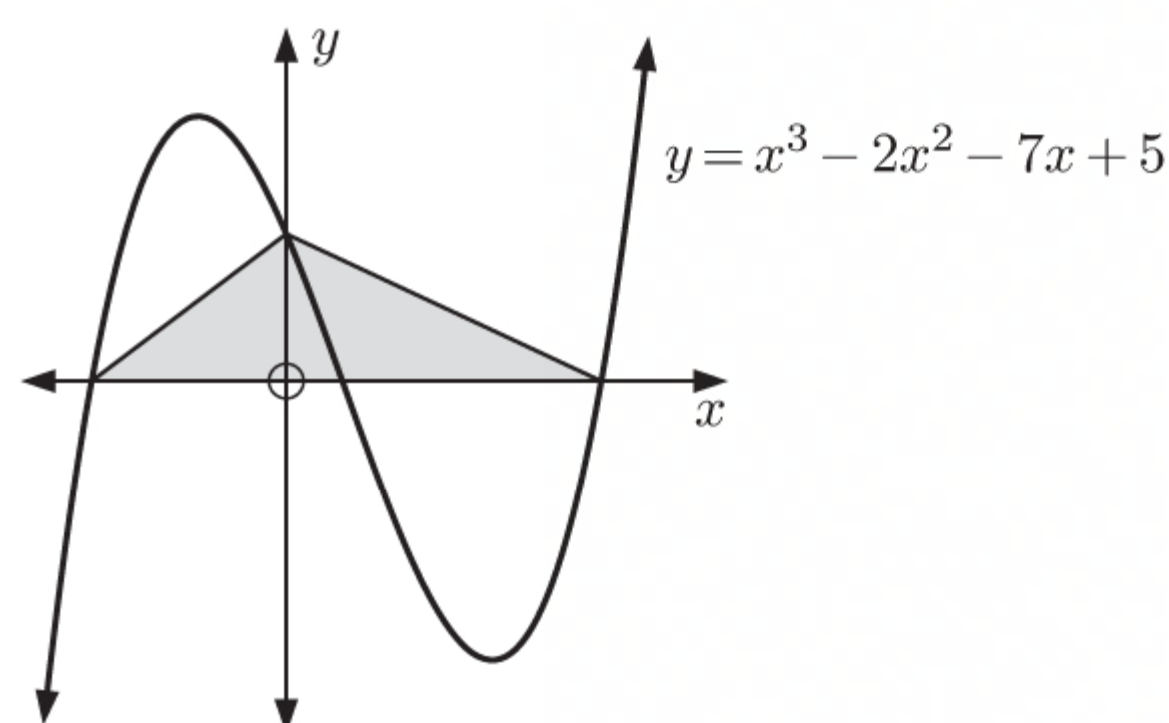
Base of triangle $\approx 3.57 - (-2.20)$ {using **a**}

$$\approx 5.77 \text{ units}$$

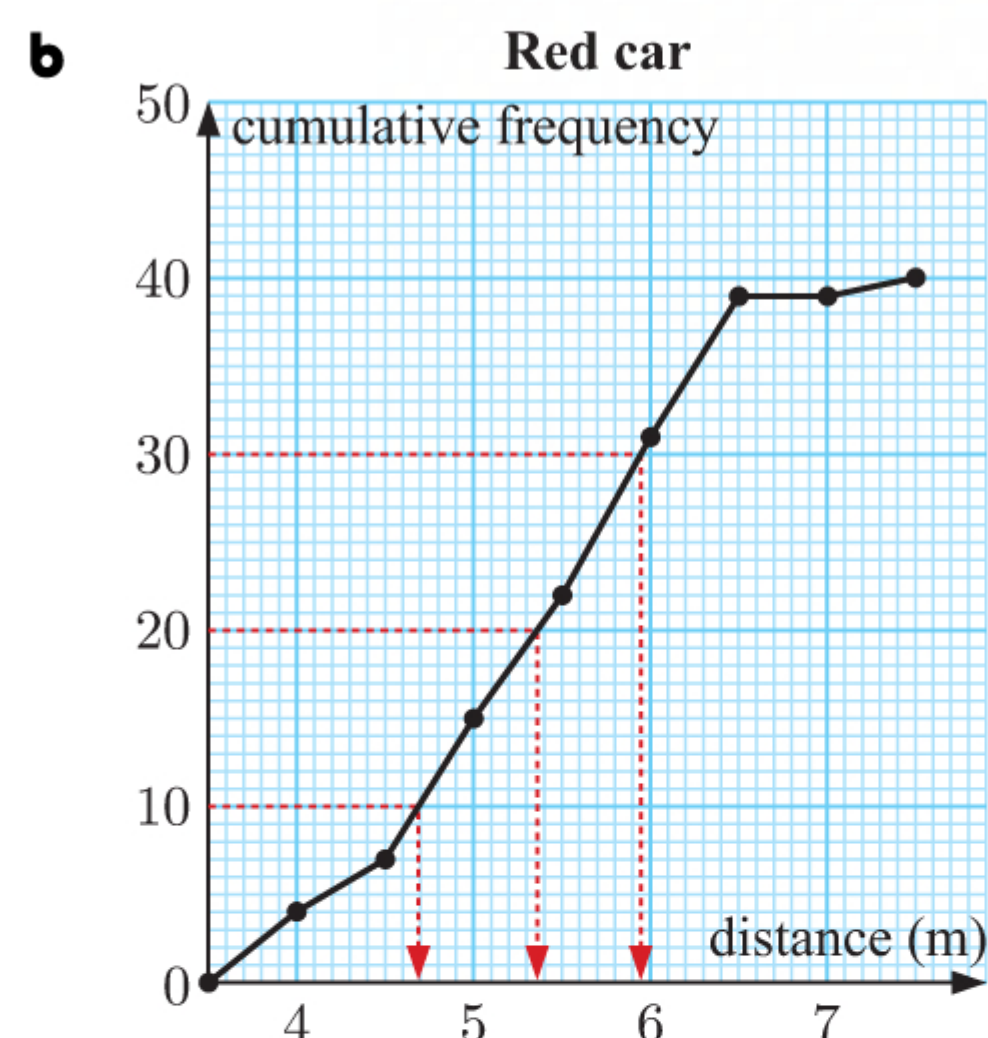
$$\therefore \text{area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\approx \frac{1}{2} \times 5.77 \times 5$$

$$\approx 14.4 \text{ units}^2$$



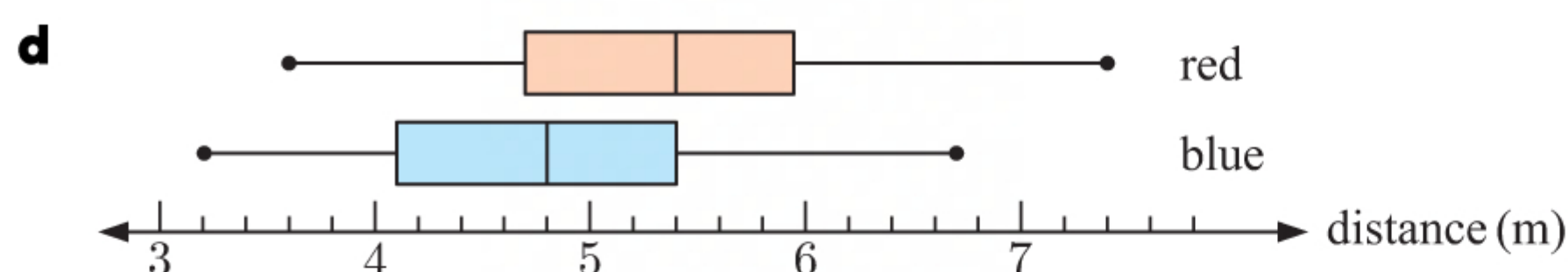
Distance (m)	Frequency	Cumulative frequency
$3.5 \leq d < 4$	4	4
$4 \leq d < 4.5$	3	7
$4.5 \leq d < 5$	8	15
$5 \leq d < 5.5$	7	22
$5.5 \leq d < 6$	9	31
$6 \leq d < 6.5$	8	39
$6.5 \leq d < 7$	0	39
$7 \leq d < 7.5$	1	40



c i Median ≈ 5.4

ii $Q_1 \approx 4.7$

iii $Q_3 \approx 5.9$



- e** All values of the five-number summary (min, Q_1 , median, Q_3 , and max) for the red car are higher than those for the blue car. This evidence is strongly against the view that the cars were made by the same machine.

MIXED QUESTIONS SET 6

1 $f(x) = 3 - 4^{-x}$

a $f(2) = 3 - 4^{-2} = 3 - \frac{1}{16}$
 $= 2\frac{15}{16}$

$\therefore p = 2\frac{15}{16}$

$f(-2) = 3 - 4^2 = -13$

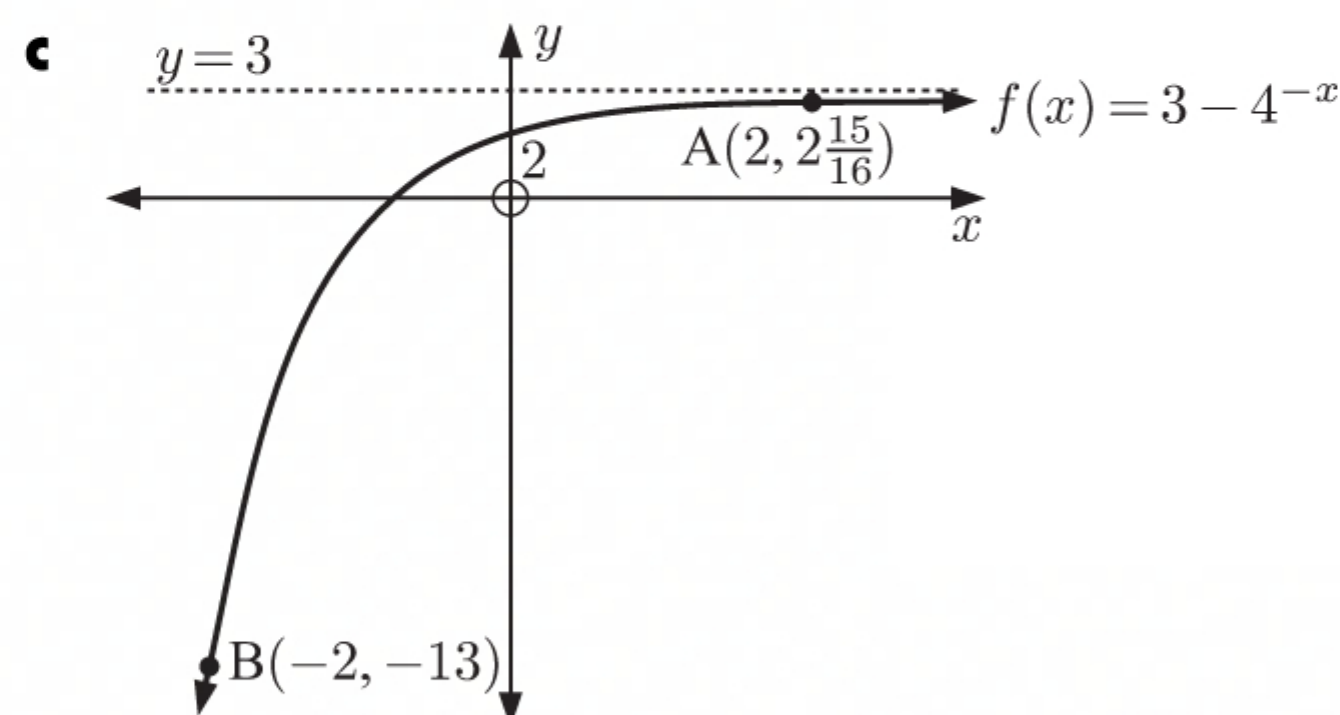
$\therefore q = -13$

b i $f(0) = 3 - 4^0 = 2 \therefore$ the y -intercept is 2.

ii As $x \rightarrow \infty$, $4^{-x} \rightarrow 0$ and so $y \rightarrow 3$

$\therefore y = 3$ is the horizontal asymptote.

d The range is $\{y \mid y < 3\}$.

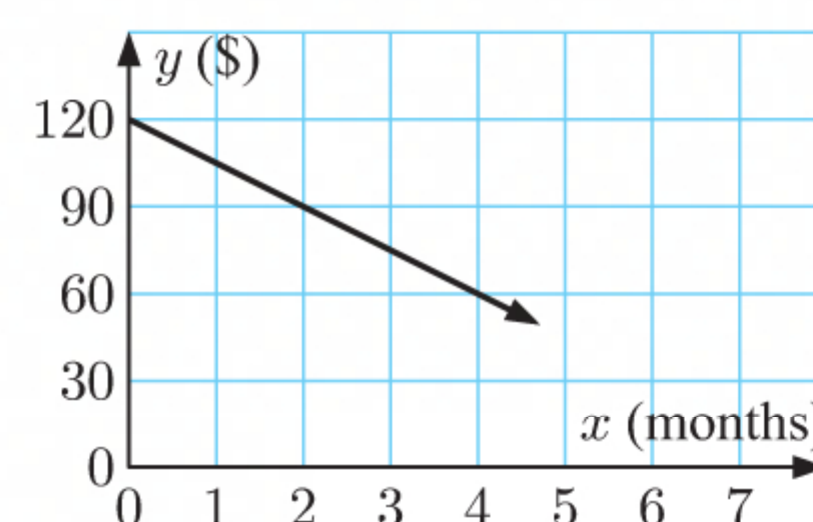


- 2 a** The line passes through $(0, 120)$ and $(2, 90)$, so the gradient is

$$\frac{90 - 120}{2 - 0} = \frac{-30}{2} = -15.$$

This means that the amount of money left in the subscription account decreases by \$15 each month.

The y -intercept is 120. This means that the initial balance was \$120.



- b** The gradient is -15 and the y -intercept is 120, so the equation of the line is $y = -15x + 120$.

- c** The account runs out of money when $y = 0$

$$\therefore -15x + 120 = 0$$

$$\therefore 15x = 120$$

$$\therefore x = 8$$

The account will run out of money after 8 months.

3 a $y = \frac{a}{x} + 3 = ax^{-1} + 3$

$$\therefore \frac{dy}{dx} = -ax^{-2} = -\frac{a}{x^2}$$

The normal to $y = \frac{a}{x} + 3$ at the point where $x = 2$ has gradient 2.

\therefore the tangent to $y = \frac{a}{x} + 3$ at the point where $x = 2$ has gradient $-\frac{1}{2}$.

So, when $x = 2$, $\frac{dy}{dx} = -\frac{1}{2}$

$$\therefore -\frac{a}{2^2} = -\frac{1}{2}$$

$$\therefore -\frac{a}{4} = -\frac{1}{2}$$

$$\therefore a = 2$$

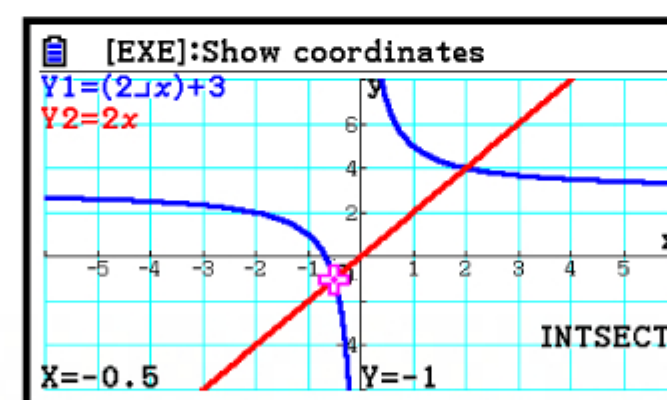
b $y = \frac{2}{x} + 3$

When $x = 2$, $y = \frac{2}{2} + 3 = 4$

So, the point of contact is $(2, 4)$.

$$\begin{aligned} \therefore \text{the equation of the normal is } y &= 2(x - 2) + 4 \\ &= 2x - 4 + 4 \\ &= 2x \end{aligned}$$

We use technology to find where the normal meets the curve again:



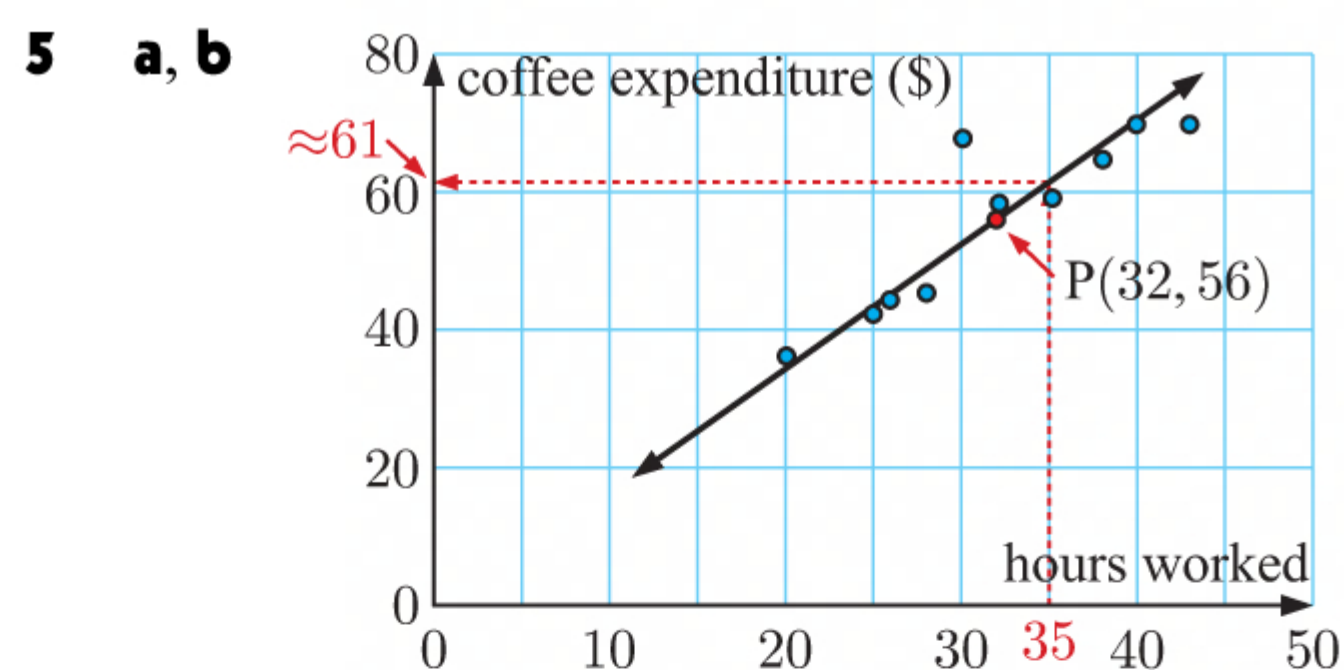
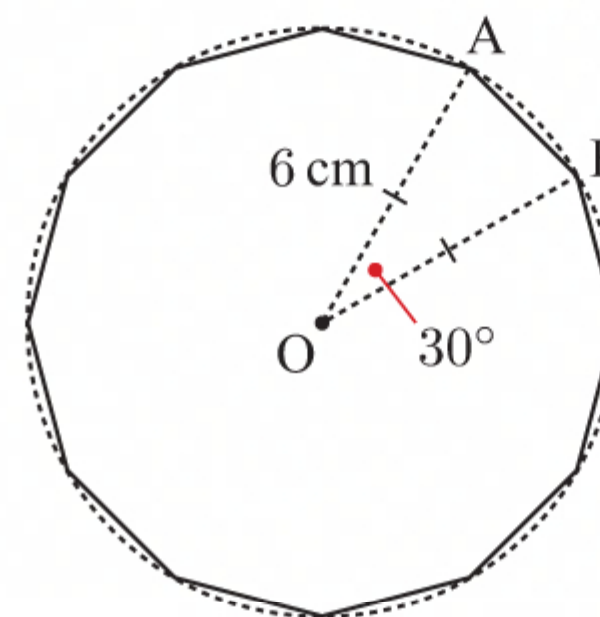
The normal meets the curve again at $(-0.5, -1)$.

- 4 a There are twelve equal angles at the centre of the dodecagon.

$$\therefore \widehat{AOB} = \frac{360^\circ}{12} = 30^\circ$$

b Area of $\triangle AOB = \frac{1}{2} \times 6 \times 6 \times \sin 30^\circ$
 $= 9 \text{ cm}^2$

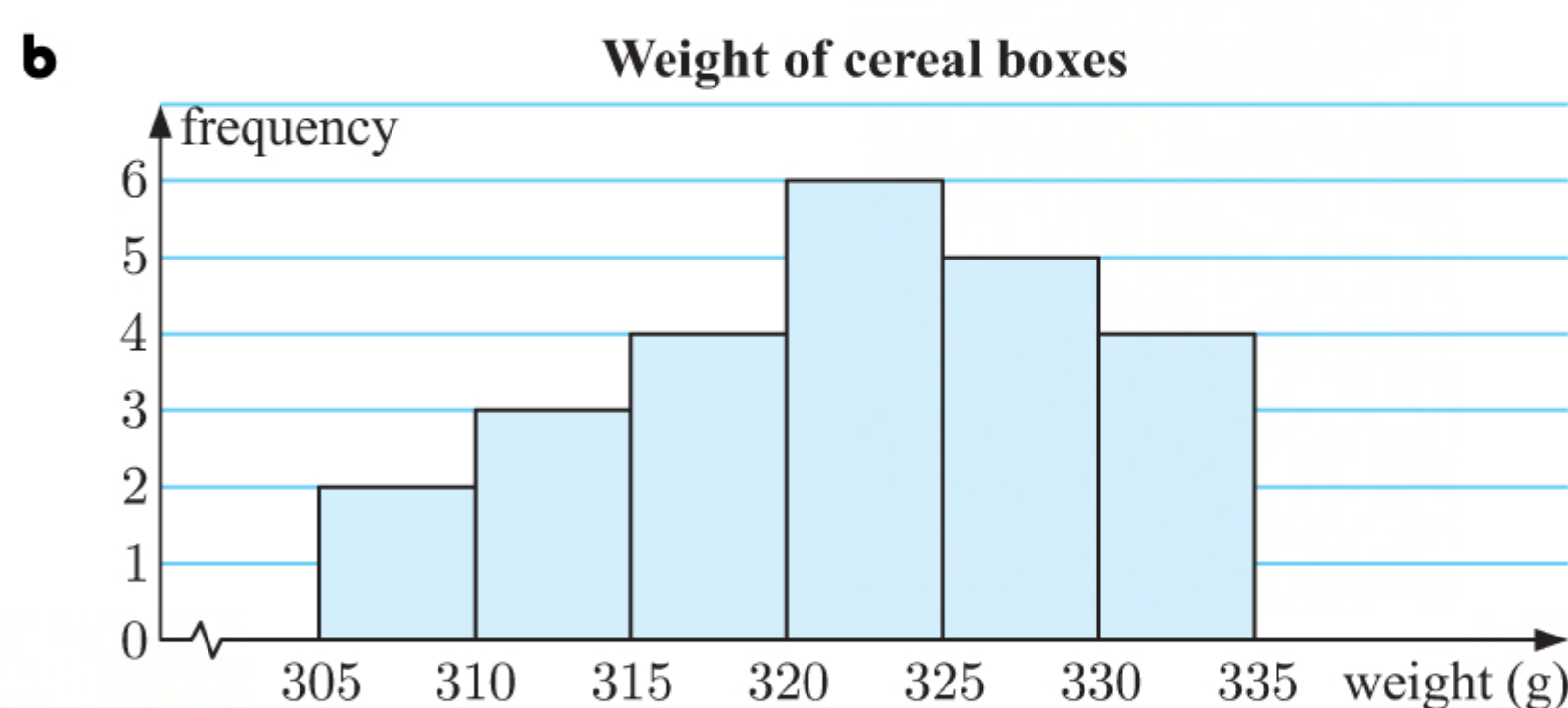
c Area of dodecagon $= 12 \times 9$
 $= 108 \text{ cm}^2$



- c From the graph, if James works a 35 hour week, he spends about \$61.
 d There is a strong positive linear relationship between the length of time James works and the amount he spends on coffee. Since the prediction in c was an interpolation on strongly correlated data, it is a reliable estimate.

6 a

Weight (w g)	Frequency
$305 \leq w < 310$	2
$310 \leq w < 315$	3
$315 \leq w < 320$	4
$320 \leq w < 325$	6
$325 \leq w < 330$	5
$330 \leq w < 335$	4



- c The data is slightly negatively skewed.
 d The modal class is the interval $320 \leq w < 325$ because it has the highest frequency.

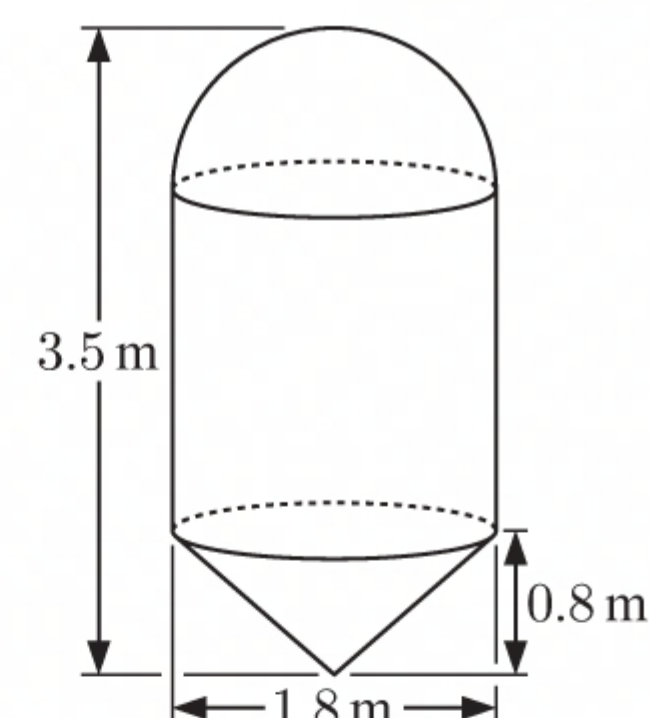
e Mean $= \frac{312 + 320 + \dots + 324}{24}$
 $= \frac{7696}{24}$
 ≈ 320.67

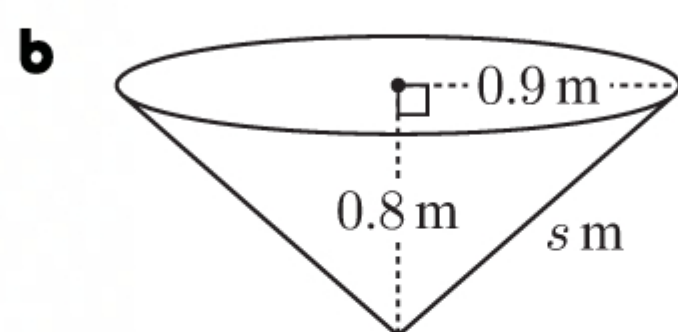
The mean of the sample is reasonably close to the average weight that the manufacturer claims.

- 7 a Height of cylinder = total height of silo – height of cone – height of hemisphere

$$= 3.5 - 0.8 - \frac{1.8}{2} \text{ m}$$

$$= 1.8 \text{ m}$$





Let the slant height of the cone be s m.

$$s^2 = (0.8)^2 + (0.9)^2$$

$$\therefore s = \sqrt{(0.8)^2 + (0.9)^2} \quad \{s > 0\}$$

$$\approx 1.204 \text{ m}$$

$$\begin{aligned} \text{Surface area of cone} &= \pi r s \\ &\approx \pi \times 0.9 \times 1.204 \\ &\approx 3.40 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Surface area of cylinder} &= 2\pi r h \\ &= 2 \times \pi \times 0.9 \times 1.8 \\ &\approx 10.2 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Surface area of hemisphere} &= 2\pi r^2 \\ &= 2 \times \pi \times (0.9)^2 \\ &\approx 5.09 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{total surface area} &\approx 3.40 + 10.2 + 5.09 \text{ m}^2 \\ &\approx 18.7 \text{ m}^2 \end{aligned}$$

So, about 18.7 m^2 of sheet metal was used to make the silo.

8 $P(x) \approx \frac{x}{\ln x}$

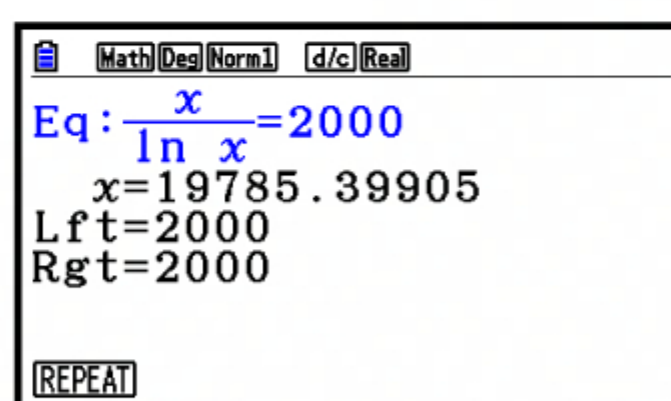
a $P(1\,000\,000) \approx \frac{1\,000\,000}{\ln 1\,000\,000}$

$$\approx 72\,400$$

We estimate there are about 72 400 prime numbers less than 1 000 000.

c $P(x) \approx 2000$ when $\frac{x}{\ln x} \approx 2000$.

Using technology, $x \approx 19\,785$.



So, there are about 2000 prime numbers less than or equal to 19 785.

9 The distance travelled d is directly proportional to the square of the time taken t .

$$\therefore d = kt^2 \text{ where } k \text{ is a constant.}$$

When $d = 19.6$ m, $t = 2$ s, so $19.6 = k(2)^2$

$$\therefore k = \frac{19.6}{4} = 4.9$$

So, $d = 4.9t^2$.

a When $t = 3$ s, $d = 4.9(3)^2$

$$= 44.1 \text{ m}$$

c Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times 0.9^2 \times 0.8$$

$$\approx 0.679 \text{ m}^3$$

$$\begin{aligned} \text{Volume of cylinder} &= \pi r^2 h \\ &= \pi \times 0.9^2 \times 1.8 \\ &\approx 4.58 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume of hemisphere} &= \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right) \\ &= \frac{1}{2} \times \frac{4}{3} \times \pi \times 0.9^3 \\ &\approx 1.53 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \therefore \text{total volume} &\approx 0.679 + 4.58 + 1.53 \text{ m}^3 \\ &\approx 6.79 \text{ m}^3 \end{aligned}$$

Now $6.79 \text{ m}^3 \equiv 6.79 \text{ kL}$

So, the capacity of the silo is about 6.79 kL.

b Percentage error $= \frac{|V_A - V_E|}{V_E} \times 100\%$

$$\approx \frac{|72\,400 - 78\,498|}{78\,498} \times 100\%$$

$$\approx 7.8\%$$

b When $d = 100$ m, $100 = 4.9t^2$

$$\therefore t^2 = \frac{100}{4.9}$$

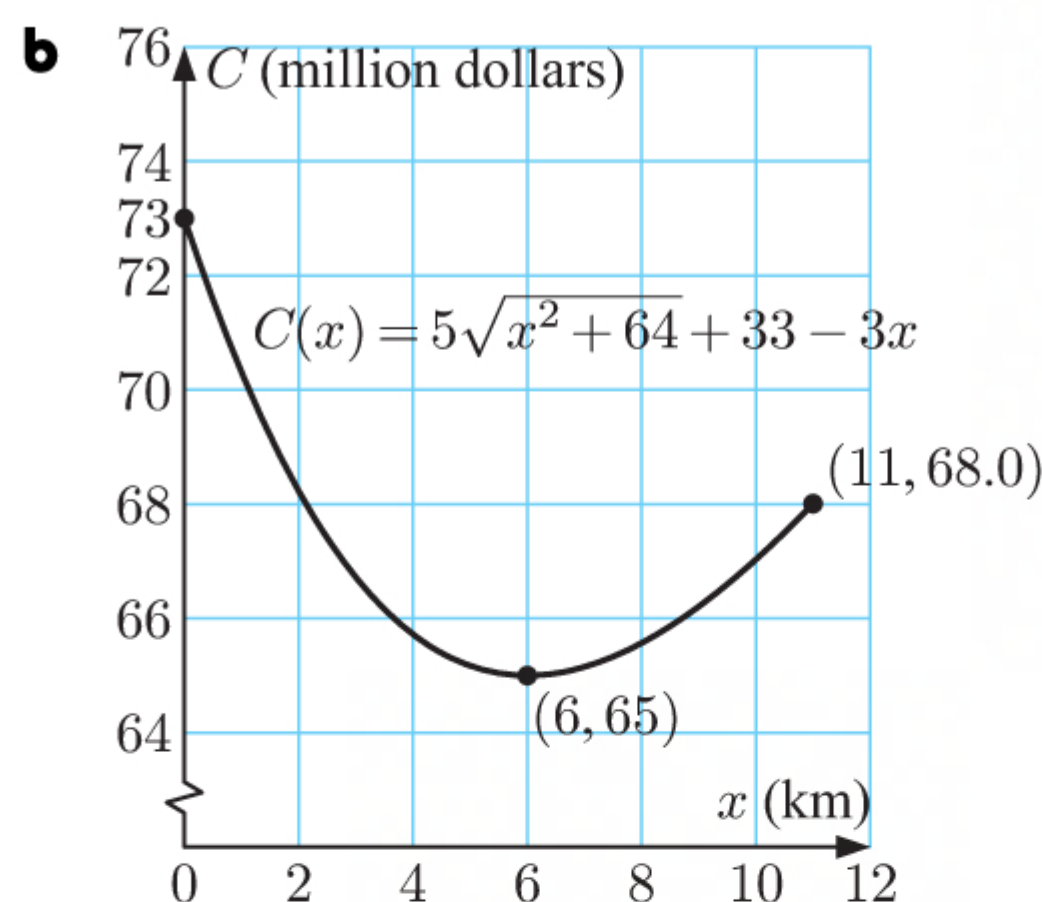
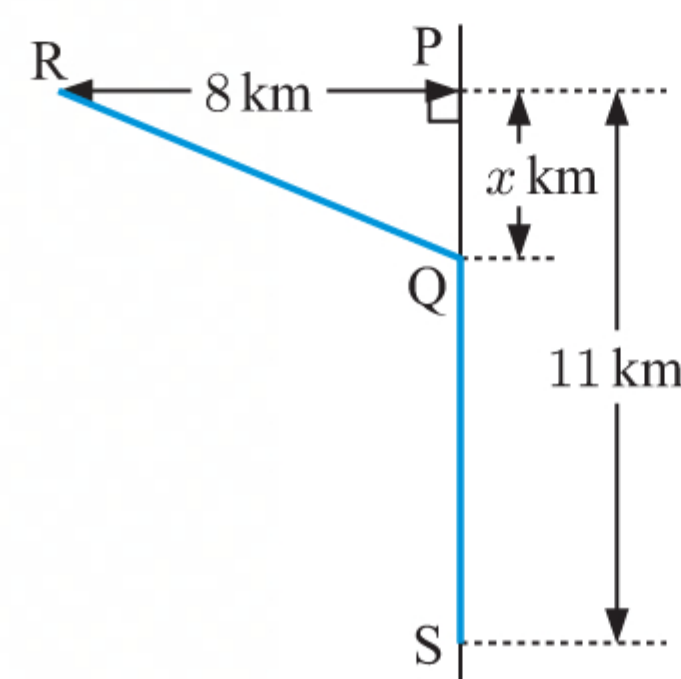
$$\therefore t = \sqrt{\frac{100}{4.9}} \quad \{\text{as } t \geq 0\}$$

$$\therefore t \approx 4.52 \text{ s}$$

10 a $QR^2 = x^2 + 8^2$ {Pythagoras}
 $\therefore QR = \sqrt{x^2 + 64}$ {as $QR > 0$ }
 Also $QS = PS - PQ$
 $= 11 - x$

So, the length of pipeline under the sea is $\sqrt{x^2 + 64}$ km,
 and the length of pipeline overland is $(11 - x)$ km.

\therefore the cost $C(x) = 5\sqrt{x^2 + 64} + 3(11 - x)$ million dollars
 $= 5\sqrt{x^2 + 64} + 33 - 3x$ million dollars.



c From the graph in **b**, there is a local minimum at $(6, 65)$.

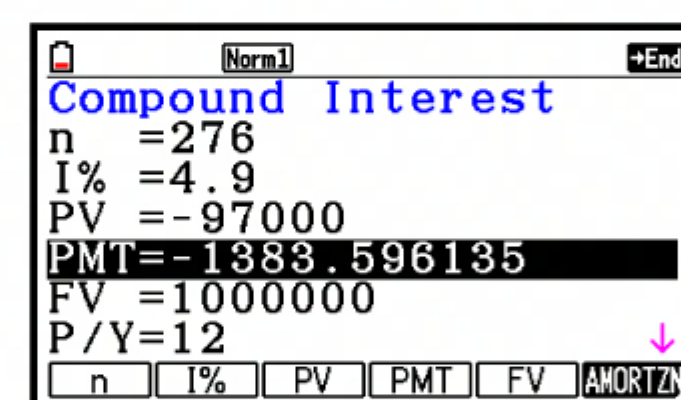
\therefore the minimum cost of the pipeline is 65 million dollars, when Q is 6 km from P.

11 a Stig has $55 - 32 = 23$ years before he retires.

$N = 23 \times 12 = 276$, $I\% = 4.9$, $PV = -97\,000$, $FV = 1\,000\,000$,
 $P/Y = 12$, $C/Y = 4$

$\therefore PMT \approx -1383.60$

Stig should contribute \$1383.60 to his fund each month.

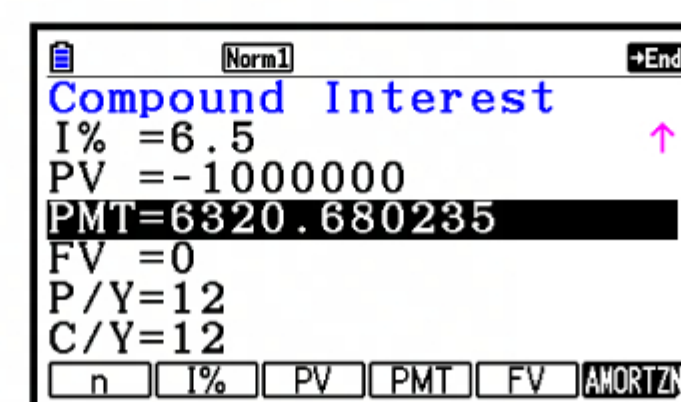


b Interest generated $\approx \$1\,000\,000 - \$97\,000 - \$1383.60 \times 276$
 $\approx \$521\,126.40$

c $N = 30 \times 12 = 360$, $I\% = 6.5$, $PV = -1\,000\,000$, $FV = 0$, $P/Y = 12$,
 $C/Y = 12$

$\therefore PMT \approx 6320.68$

Stig can afford to withdraw \$6320.68 each month.



d To index Stig's standard of living for inflation, we increase it by 3.7% each year for 23 years.

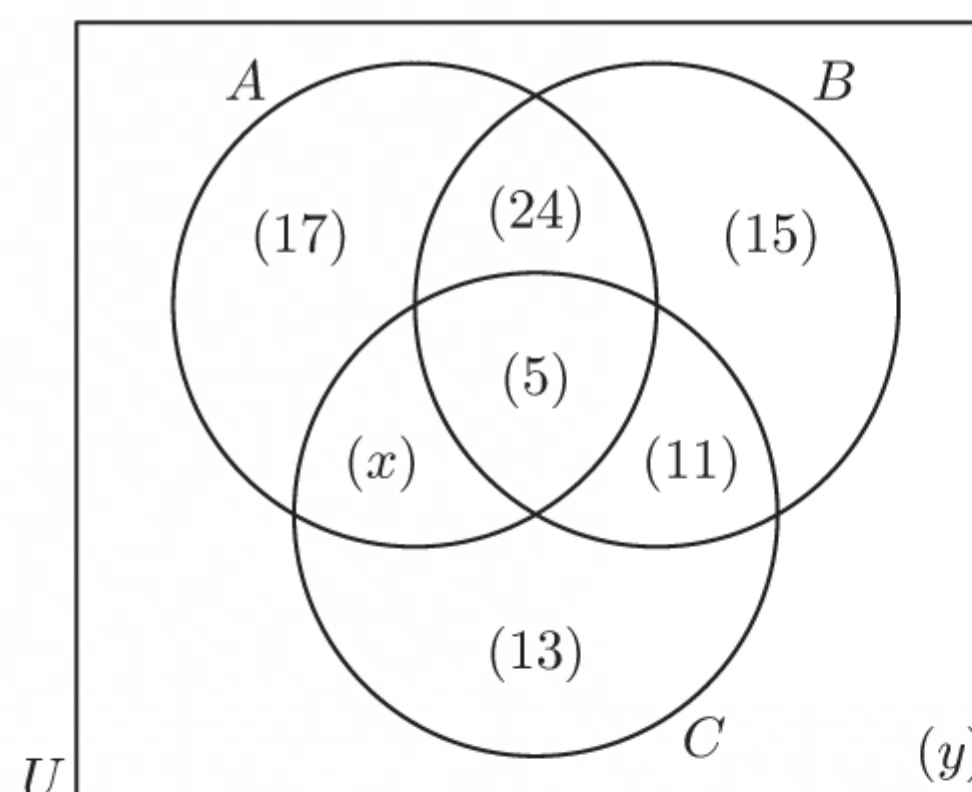
\therefore indexed value $= \$2700 \times (1.037)^{23}$
 $= \$6226.95$

This is less than our answer in **c**, so Stig can maintain his standard of living at the time of his retirement.

12 a $n(A) = 48$
 $\therefore 17 + 24 + 5 + x = 48$
 $\therefore 46 + x = 48$
 $\therefore x = 2$

Now $n(U) = 100$

$\therefore 17 + 24 + 5 + 2 + 15 + 11 + 13 + y = 100$
 $\therefore 87 + y = 100$
 $\therefore y = 13$



b $n(A) = 48$, $n(B) = 24 + 5 + 15 + 11 = 55$, $n(C) = 2 + 5 + 11 + 13 = 31$

\therefore course B was the most popular.

$$\begin{aligned} \text{c i } P(\text{liked all the courses}) &= \frac{5}{100} \\ &= \frac{1}{20} \end{aligned}$$

$$\begin{aligned} \text{iii } P(\text{liked exactly 2 courses} \mid C) &= \frac{n(\text{liked } C \text{ and exactly one other course})}{n(C)} \\ &= \frac{2 + 11}{2 + 5 + 11 + 13} \\ &= \frac{13}{31} \end{aligned}$$

$$\begin{aligned} \text{ii } P(B \cap C') &= \frac{24 + 15}{100} \\ &= \frac{39}{100} \end{aligned}$$

$$\begin{aligned} \text{iv } P(\text{liked none} \mid B') &= \frac{n(\text{liked none})}{n(B')} \\ &= \frac{13}{17 + 2 + 13 + 13} \\ &= \frac{13}{45} \end{aligned}$$

MIXED QUESTIONS SET 7

- 1 a For £1000 in sales, the salesperson makes $\text{£}1000 \times 0.05 = \text{£}50$ commission.

$$\therefore k = 50$$

- b For sales above £1000, the gradient of the line is $\frac{90 - k}{1500 - 1000} = \frac{90 - 50}{1500 - 1000} = 0.08$.

\therefore the salesperson makes 8% commission on sales above £1000.

- c For £1800 in sales, the salesperson makes $\text{£}50 + \text{£}800 \times 0.08 = \text{£}114$ commission.

- d Let $\text{£}x$ be the value in sales needed to earn £150 commission.

$$\therefore 50 + (x - 1000) \times 0.08 = 150$$

$$\therefore (x - 1000) \times 0.08 = 100$$

$$\therefore x - 1000 = 1250$$

$$\therefore x = 2250$$

\therefore £2250 in sales are needed to earn £150 commission.

- 2 a There are $n = 5 \times 4 = 20$ time periods.

Each time period the investment increases by $i = \frac{4.4\%}{4} = 1.1\%$.

$$\begin{aligned} \therefore \text{the amount after 5 years is } u_{20} &= u_0 \times (1 + i)^{20} \\ &= 2000 \times (1.011)^{20} \quad \{1.1\% = 0.011\} \\ &\approx 2489.16 \end{aligned}$$

The final value of the investment is \$2489.16.

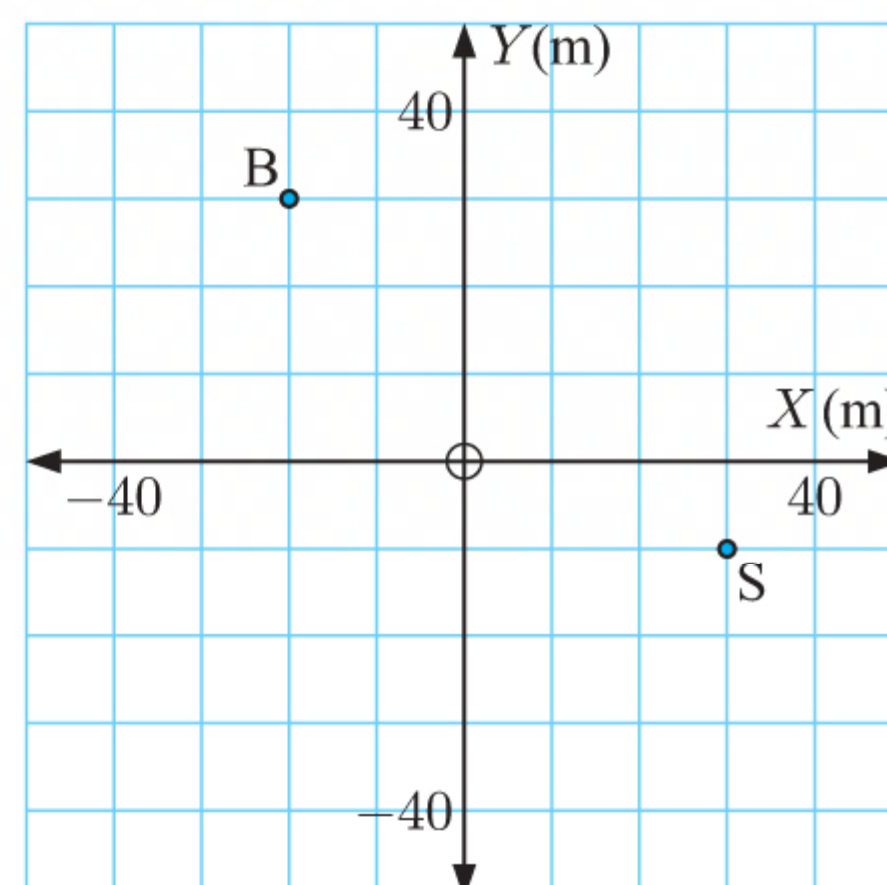
- b Interest = \$2489.16 - \$2000
= \$489.16

$$\begin{aligned} \text{c real value} \times (1.025)^5 &= \$2489.16 \\ \therefore \text{real value} &= \frac{\$2489.16}{(1.025)^5} \\ &= \$2200.05 \end{aligned}$$

- 3 a i The anchor has coordinates $A(-20, 30, -50)$.
ii The shipwreck has coordinates $S(30, -10, -40)$.

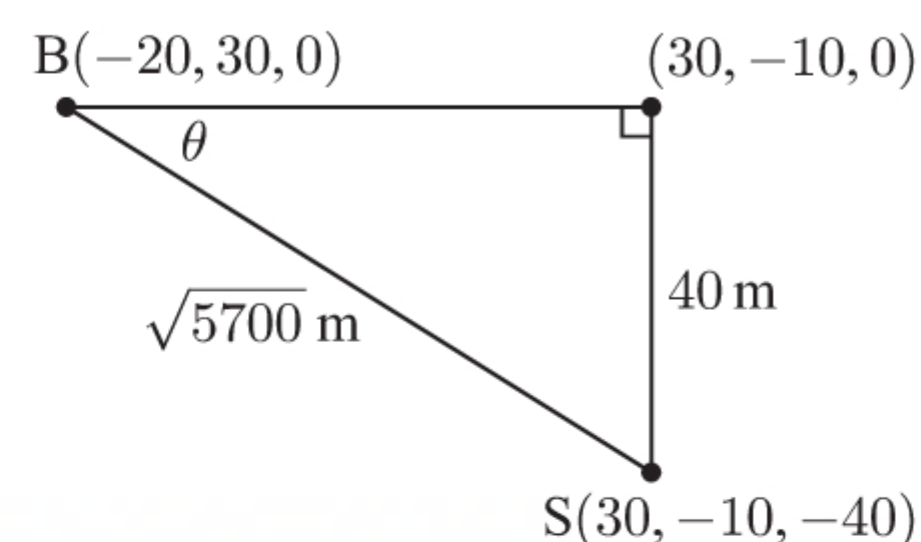
$$\begin{aligned} \text{b } BS &= \sqrt{(30 - (-20))^2 + (-10 - 30)^2 + (-40 - 0)^2} \\ &= \sqrt{50^2 + (-40)^2 + (-40)^2} \\ &= \sqrt{5700} \\ &\approx 75.5 \text{ m} \end{aligned}$$

\therefore the diver has to swim about 75.5 m.

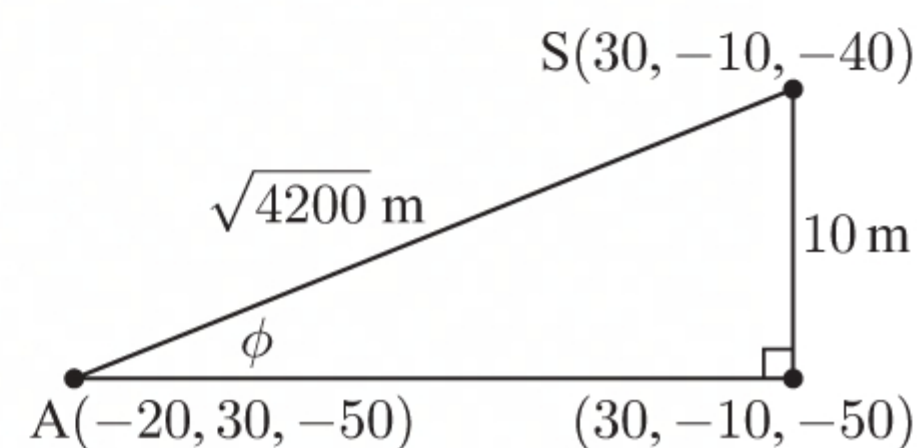


$$\begin{aligned} \text{c i } \sin \theta &= \frac{40}{\sqrt{5700}} \\ \therefore \theta &= \sin^{-1}\left(\frac{40}{\sqrt{5700}}\right) \\ \therefore \theta &\approx 32.0^\circ \end{aligned}$$

The angle of depression from the boat to the shipwreck is about 32.0° .



$$\begin{aligned}
 \text{ii } AS &= \sqrt{(-20 - 30)^2 + (30 - -10)^2 + (-50 - -40)^2} \\
 &= \sqrt{(-50)^2 + 40^2 + (-10)^2} \\
 &= \sqrt{4200} \text{ m} \\
 \therefore \sin \phi &= \frac{10}{\sqrt{4200}} \\
 \therefore \phi &= \sin^{-1}\left(\frac{10}{\sqrt{4200}}\right) \\
 \therefore \phi &\approx 8.88^\circ
 \end{aligned}$$



The angle of elevation from the anchor to the shipwreck is about 8.88° .

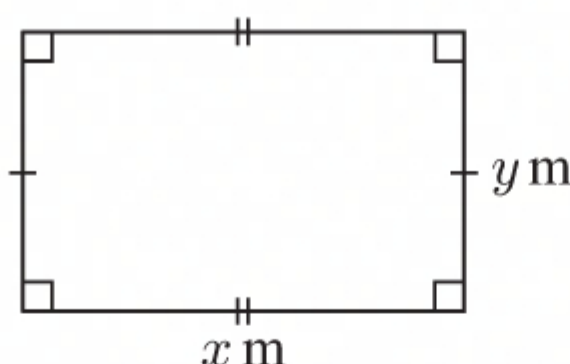
- 4 a Let the other side of the field be y m.

$$2x + 2y = 160$$

$$\therefore 2y = 160 - 2x$$

$$\therefore y = 80 - x$$

$$\therefore \text{area } A = x(80 - x) \text{ m}^2$$



b $A = 80x - x^2$

$$\therefore \frac{dA}{dx} = 80 - 2x$$

$$\begin{aligned}
 \text{Now } \frac{dA}{dx} = 0 \text{ when } 80 - 2x &= 0 \\
 \therefore 2x &= 80 \\
 \therefore x &= 40
 \end{aligned}$$

The sign diagram of $\frac{dA}{dx}$ is:

$\therefore A$ is maximised when $x = 40$, and $y = 80 - 40 = 40$.

\therefore the maximum area occurs when the field is a $40 \text{ m} \times 40 \text{ m}$ square.

c i $A = 1200$

$$\therefore x(80 - x) = 1200$$

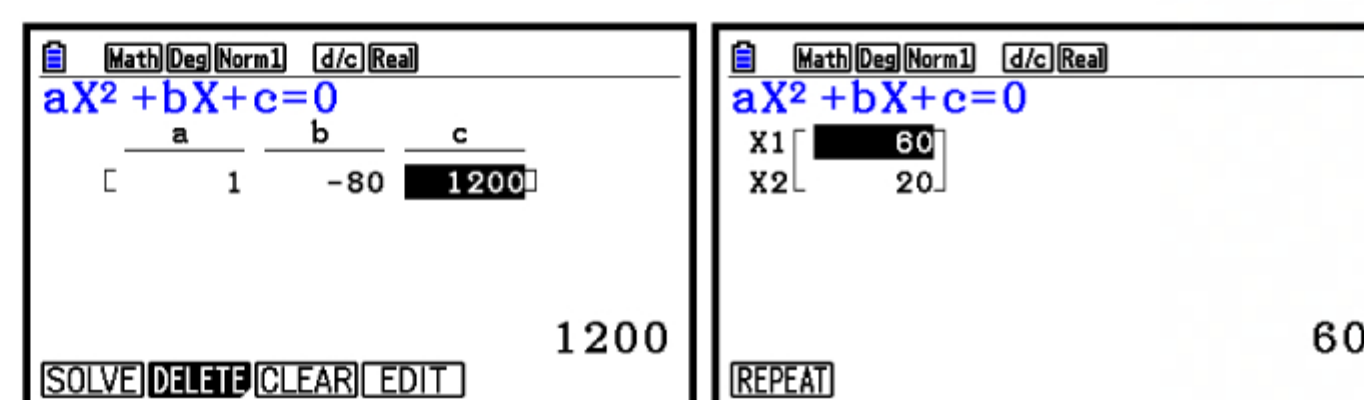
$$\therefore x^2 - 80x + 1200 = 0$$

Using technology, $x = 60$ or 20

When $x = 60$, $y = 80 - 60 = 20$.

When $x = 20$, $y = 80 - 20 = 60$.

\therefore the field is $60 \text{ m} \times 20 \text{ m}$.



- ii The maximum area is 1600 m^2 .

We lose $1600 - 1200 = 400 \text{ m}^2$ of productive land

$$\begin{aligned}
 \therefore \text{the lost production} &= 400 \times 6.5 \\
 &= 2600 \text{ kg}
 \end{aligned}$$

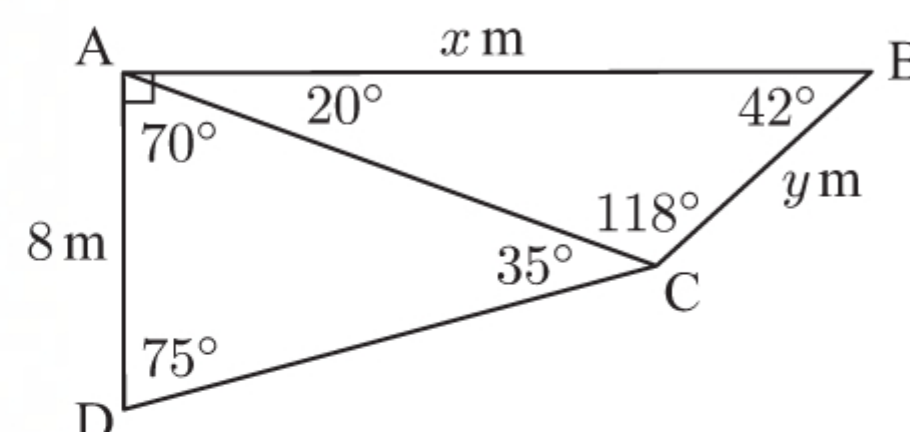
- 5 In triangle ACD, $\widehat{CAD} = 180^\circ - 75^\circ - 35^\circ = 70^\circ$ {angles in a triangle}

$$\begin{aligned}
 \text{Using the sine rule, } \frac{AC}{\sin 75^\circ} &= \frac{8}{\sin 35^\circ} \\
 \therefore AC &= \frac{8 \sin 75^\circ}{\sin 35^\circ} \\
 &\approx 13.47 \text{ m}
 \end{aligned}$$

$$\text{Now } \widehat{BAC} = 90^\circ - 70^\circ = 20^\circ$$

$$\begin{aligned}
 \text{In triangle ABC, } \widehat{ACB} &= 180^\circ - 42^\circ - 20^\circ \text{ {angles in a triangle}} \\
 &= 118^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{Now, using the sine rule in triangle ABC, } \frac{x}{\sin 118^\circ} &= \frac{AC}{\sin 42^\circ} & \text{and } \frac{y}{\sin 20^\circ} &= \frac{AC}{\sin 42^\circ} \\
 \therefore x &\approx \frac{13.47 \sin 118^\circ}{\sin 42^\circ} & \therefore y &\approx \frac{13.47 \sin 20^\circ}{\sin 42^\circ} \\
 &\approx 17.8 & &\approx 6.89
 \end{aligned}$$



6	Height (h cm)	Mid-interval value (x)	Frequency (f)	xf
	$80 \leq h < 90$	85	8	680
	$90 \leq h < 100$	95	12	1140
	$100 \leq h < 110$	105	17	1785
	$110 \leq h < 120$	115	30	3450
	$120 \leq h < 130$	125	13	1625
	Total		$\sum f = 80$	$\sum xf = 8680$

$$\begin{aligned} \mathbf{a} \quad \bar{x} &= \frac{\sum xf}{\sum f} \\ &= \frac{8680}{80} \\ &= 108.5 \end{aligned}$$

b Using technology, $s \approx 12.1$ cm.

We estimate the mean height to be about 108.5 cm.

$$\mathbf{7} \quad P(B) = 0.3 \quad \text{and} \quad P(A \cup B) = 0.55$$

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ {addition law of probability}

$\therefore P(A \cup B) = P(A) + P(B)$ { A and B are mutually exclusive}

$$\therefore 0.55 = P(A) + 0.3$$

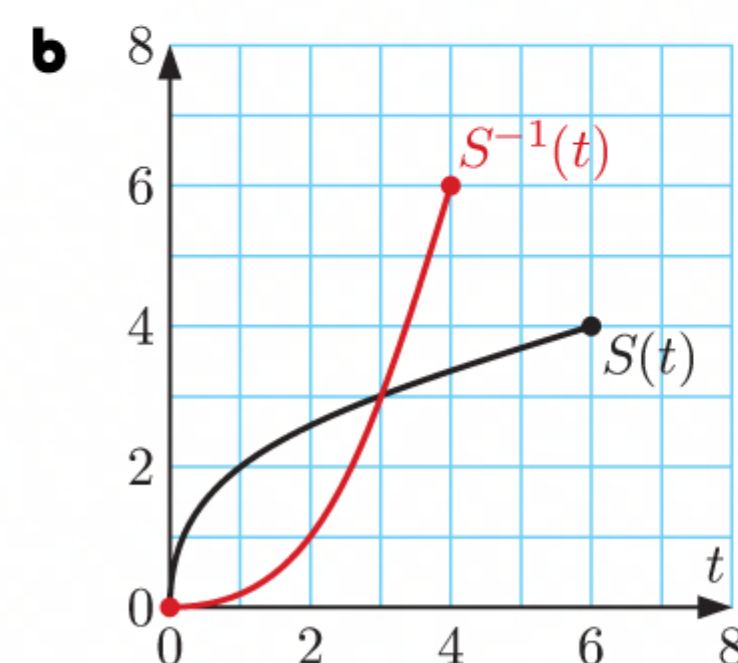
$$\therefore P(A) = 0.25$$

$$\mathbf{8} \quad \mathbf{a} \quad \text{The domain is } \{t \mid 0 \leq t \leq 6\}.$$

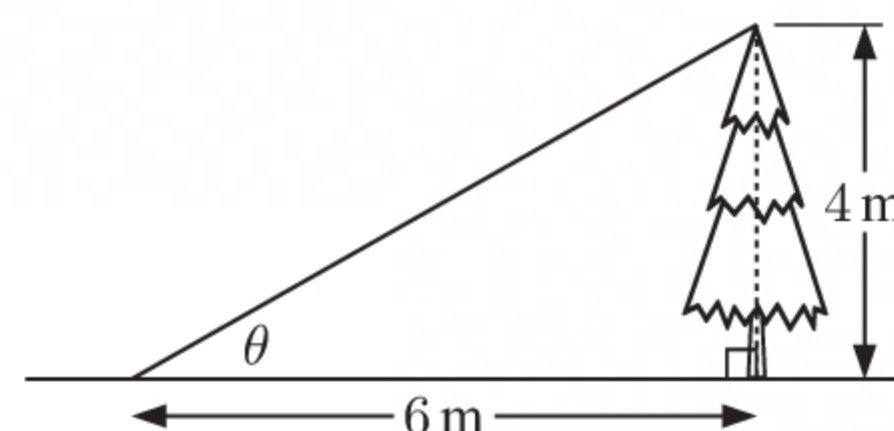
The range is $\{S \mid 0 \leq S \leq 4\}$.

$$\mathbf{c} \quad \text{From the graph in } \mathbf{b}, S^{-1}(2) = 1.$$

This means that it takes 1 second for the cyclist to reach a speed of 2 m s^{-1} .



$$\begin{aligned} \mathbf{9} \quad \mathbf{a} \quad \sin \theta &\approx \frac{4}{6} \\ \therefore \theta &\approx \sin^{-1}\left(\frac{4}{6}\right) \\ &\approx 41.8^\circ \end{aligned}$$



$$\mathbf{b} \quad \text{The height of the tree could be from } 3\frac{1}{2} \text{ m to } 4\frac{1}{2} \text{ m.}$$

The distance to the tree could be from $5\frac{1}{2}$ m to $6\frac{1}{2}$ m.

As θ increases from 0° to 90° , $\sin \theta$ increases.

$$\therefore \text{the lower boundary for } \theta \text{ is } \sin^{-1}\left(\frac{3\frac{1}{2}}{6\frac{1}{2}}\right) \approx 32.6^\circ$$

$$\text{and the upper boundary for } \theta \text{ is } \sin^{-1}\left(\frac{4\frac{1}{2}}{5\frac{1}{2}}\right) \approx 54.9^\circ.$$

$$\mathbf{c} \quad \text{Percentage error} = \frac{|\theta_A - \theta_E|}{\theta_E} \times 100\%$$

$$\begin{aligned} \text{If the exact angle } \theta_E &\approx 32.6^\circ, \text{ the percentage error} \approx \frac{|41.8^\circ - 32.6^\circ|}{32.6^\circ} \times 100\% \\ &\approx 28.2\% \end{aligned}$$

$$\begin{aligned} \text{If the exact angle } \theta_E &\approx 54.9^\circ, \text{ the percentage error} \approx \frac{|41.8^\circ - 54.9^\circ|}{54.9^\circ} \times 100\% \\ &\approx 23.9\% \end{aligned}$$

\therefore the maximum percentage error in our estimate in **a** is about 28.2%.

10 $y = (2x + 7)(2 - x)$

a $y = 0$ when $(2x + 7)(2 - x) = 0$

$$\therefore 2x + 7 = 0 \quad \text{or} \quad 2 - x = 0$$

$$\therefore 2x = -7 \quad \text{or} \quad x = 2$$

$$\therefore x = -\frac{7}{2} \quad \text{or} \quad x = 2$$

$$\therefore \text{shaded area} = \int_{-\frac{7}{2}}^0 (2x + 7)(2 - x) dx \text{ units}^2$$

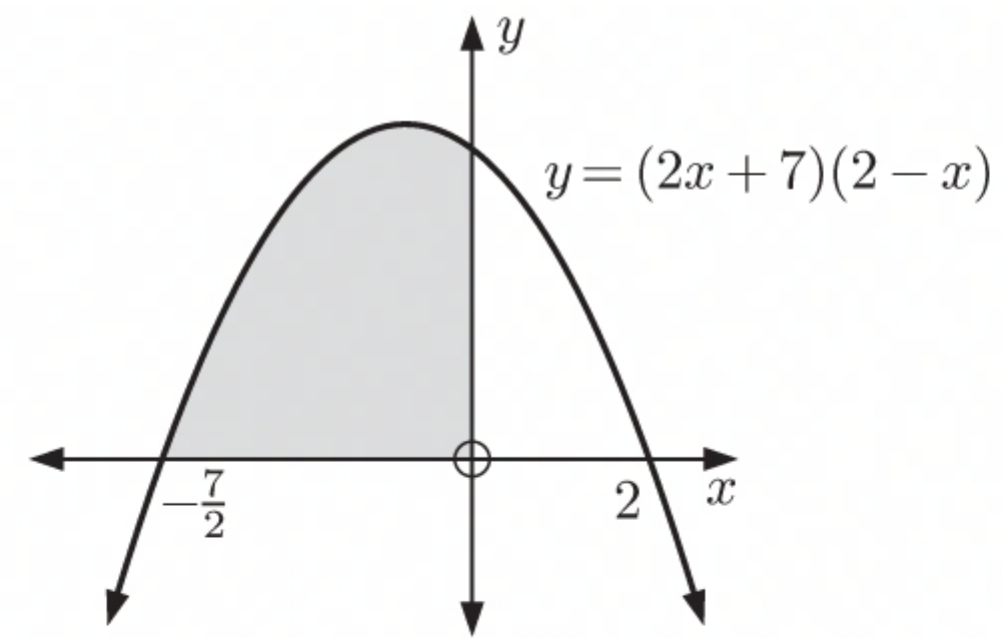
b Shaded area $= \int_{-\frac{7}{2}}^0 (4x - 2x^2 + 14 - 7x) dx$

$$= \int_{-\frac{7}{2}}^0 (-2x^2 - 3x + 14) dx$$

$$= \left[-\frac{2}{3}x^3 - \frac{3}{2}x^2 + 14x \right]_{-\frac{7}{2}}^0$$

$$= 0 - \left(-\frac{2}{3}\left(-\frac{7}{2}\right)^3 - \frac{3}{2}\left(-\frac{7}{2}\right)^2 + 14\left(-\frac{7}{2}\right) \right)$$

$$\approx 38.8 \text{ units}^2$$



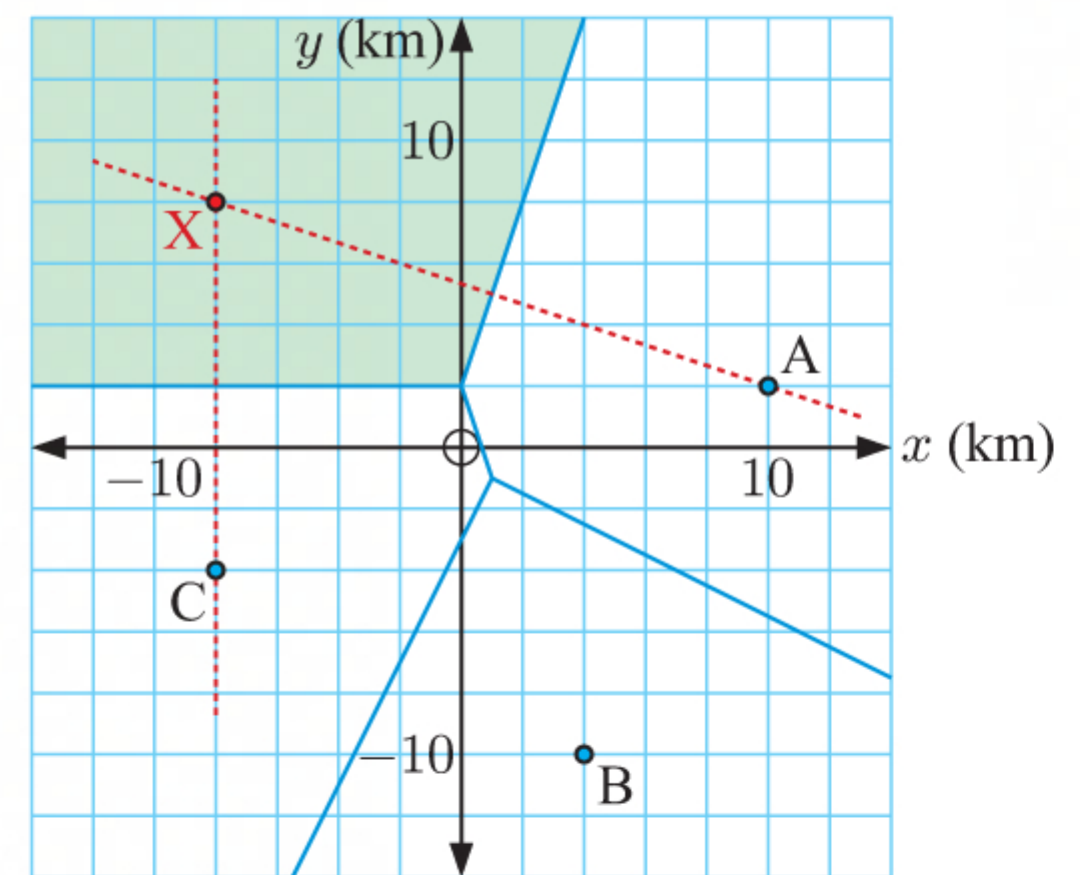
- 11 a, b** The missing hospital X must lie in the shaded cell, as this cell currently has no hospital.

The perpendicular bisector of [AX] has gradient 3, so [AX] has gradient $-\frac{1}{3}$.

The perpendicular bisector of [CX] is horizontal, so [CX] is vertical.

We draw (AX) and (CX) through A and C respectively. Their intersection is point X.

We observe that X has coordinates $(-8, 8)$.



- c i** $(-1, 2)$ lies on the edge adjacent to cells C and X, so it is equally closest to hospitals C and X.

- ii** $(4, -1)$ lies in cell A, so it is closest to hospital A.

- d i** A has coordinates $(10, 2)$, and the patient's house has coordinates $(2, 8)$.

$$\therefore \text{the distance from hospital A to the patient's house} = \sqrt{(2 - 10)^2 + (8 - 2)^2}$$

$$= \sqrt{(-8)^2 + 6^2}$$

$$= \sqrt{64 + 36}$$

$$= 10 \text{ km}$$

- ii** The patient's house lies on the perpendicular bisector of [AX].

\therefore the patient's house is equidistant from hospital A and hospital X.

- iii** Total distance travelled $= 2 \times 10 \text{ km} = 20 \text{ km}$ {from **i**}

$$\text{Now speed} = \frac{\text{distance}}{\text{time}}$$

$$\therefore \text{time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{20 \text{ km}}{40 \text{ km h}^{-1}}$$

$$= \frac{1}{2} \text{ h}$$

$$= 30 \text{ min}$$

- 12 a** $P(\text{win}) = P(\text{1st ball red} \cap \text{2nd ball red} \cap \text{3rd ball red})$

$$= \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}$$

$$= \frac{1}{6}$$

b If X is the number of wins when the game is played 60 times, then $X \sim B(60, \frac{1}{6})$.

$$\begin{aligned} \text{i} \quad \mu &= np & \sigma &= \sqrt{np(1-p)} \\ &= 60(\frac{1}{6}) & &= \sqrt{60(\frac{1}{6})(\frac{5}{6})} \\ &= 10 & &= \sqrt{\frac{25}{3}} \\ & & &\approx 2.89 \end{aligned}$$

$$\begin{aligned} \text{ii} \quad P(X = \mu) &= P(X = 10) \\ &= \binom{60}{10} (\frac{1}{6})^{10} (\frac{5}{6})^{50} \\ &\approx 0.137 \end{aligned}$$

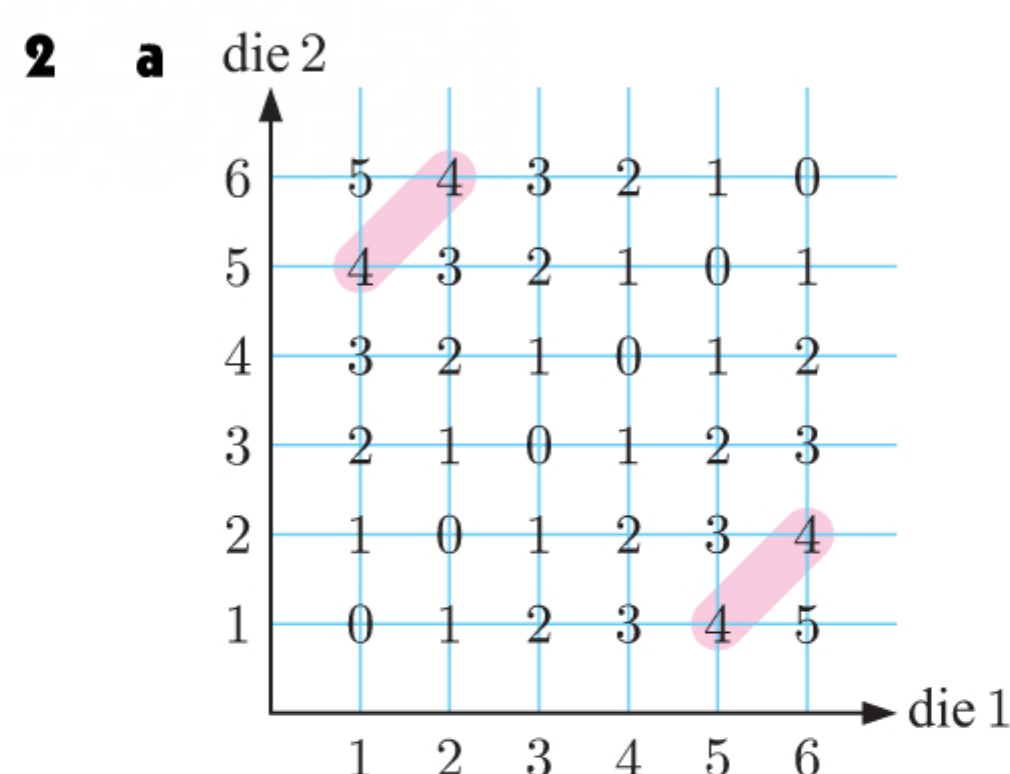
$$\begin{aligned} \text{iii} \quad P(\mu - \sigma \leq X \leq \mu + \sigma) &= P\left(10 - \sqrt{\frac{25}{3}} \leq X \leq 10 + \sqrt{\frac{25}{3}}\right) \\ &= P(7.11 \leq X \leq 12.9) \\ &= P(8 \leq X \leq 12) \\ &\approx 0.614 \quad \{\text{using technology}\} \end{aligned}$$

MIXED QUESTIONS SET 8

1 a $r = \frac{0.25}{0.125} = 2$

b Using $u_n = u_1 \times r^{n-1}$,
 $u_{20} = 0.125 \times 2^{19}$
 $= 65\,536$

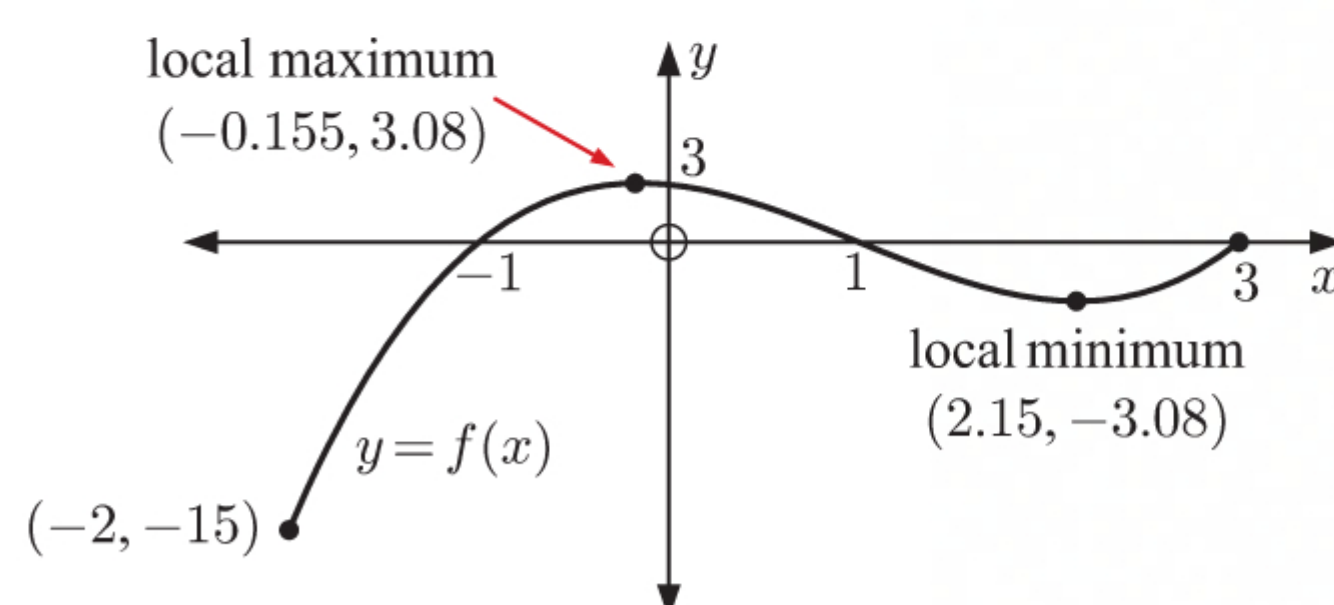
c Using $S_n = \frac{u_1(r^n - 1)}{r - 1}$,
 $S_{10} = \frac{0.125(2^{10} - 1)}{2 - 1}$
 $= 127.875$



b There are 4 outcomes where the difference is 4.

As all outcomes are equally possible, the probability of the difference being 4 is $\frac{4}{36} = \frac{1}{9}$.

3 a $f(x) = x^3 - 3x^2 - x + 3, \quad -2 \leq x \leq 3$

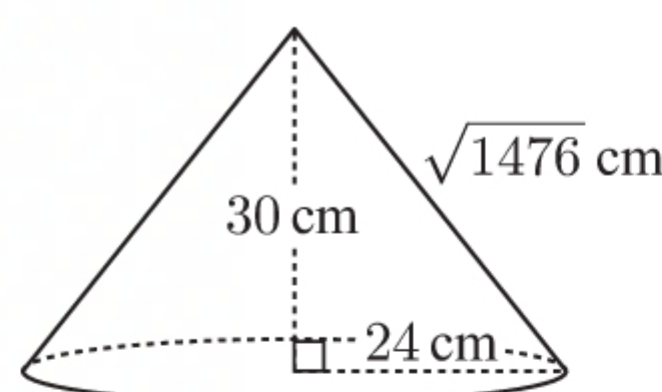


b The range of f is $\{y \mid -15 \leq y \leq 3.08\}$.

4 a

$$\begin{aligned} s^2 &= 24^2 + 30^2 \quad \{\text{Pythagoras}\} \\ \therefore s^2 &= 1476 \\ \therefore s &= \sqrt{1476} \quad \{s > 0\} \\ \therefore s &\approx 38.4 \text{ cm} \end{aligned}$$

b Surface area $= \pi r s + \pi r^2$
 $= \pi(24)\sqrt{1476} + \pi(24)^2$
 $\approx 4710 \text{ cm}^2$
 $\approx 4.71 \times 10^3 \text{ cm}^2$



5 a

Number of weeds	Frequency
0 - 4	9
5 - 9	15
10 - 14	10
15 - 19	p
20 - 24	5
25 - 29	2
Total	50

Total number of sample spots = 50

$$\therefore 9 + 15 + 10 + p + 5 + 2 = 50$$

$$\therefore p + 41 = 50$$

$$\therefore p = 9$$

b

Number of weeds	Midpoint (x)	Frequency (f)	xf
0 - 4	2	9	18
5 - 9	7	15	105
10 - 14	12	10	120
15 - 19	17	9	153
20 - 24	22	5	110
25 - 29	27	2	54
Total		$\sum f = 50$	$\sum xf = 560$

$$\bar{x} = \frac{\sum xf}{\sum f}$$

$$= \frac{560}{50}$$

$$= 11.2$$

We estimate the mean number of weeds per spot to be 11.2.

c Percentage fewer than 10 weeds = $\frac{9+15}{50} \times 100\%$

$$= \frac{24}{50} \times 100\%$$

$$= 48\%$$

6 $f(x) = \frac{6}{x^3} - \frac{2}{x} = 6x^{-3} - 2x^{-1}$

a $f'(x) = -18x^{-4} + 2x^{-2}$

$$= -\frac{18}{x^4} + \frac{2}{x^2}$$

$$= \frac{-18 + 2x^2}{x^4}$$

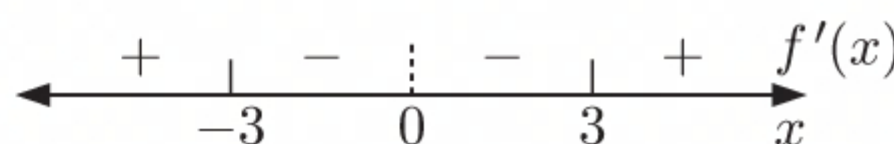
$$= \frac{2(x^2 - 9)}{x^4}$$

Now $(x+3)(x-3) = x^2 - 3x + 3x - 9$

$$= x^2 - 9$$

$$\therefore f'(x) = \frac{2(x+3)(x-3)}{x^4}$$

b The sign diagram for $f'(x)$ is:



So, $y = f(x)$ is increasing when $x \leq -3$ and $x \geq 3$, and decreasing when $-3 \leq x < 0$ and $0 < x \leq 3$.

7 a $N = 3 \times 12 = 36$, $I\% = 8.5$, $PV = 7000$, $FV = 0$, $P/Y = 12$, $C/Y = 12$

$$\therefore PMT \approx -220.98$$

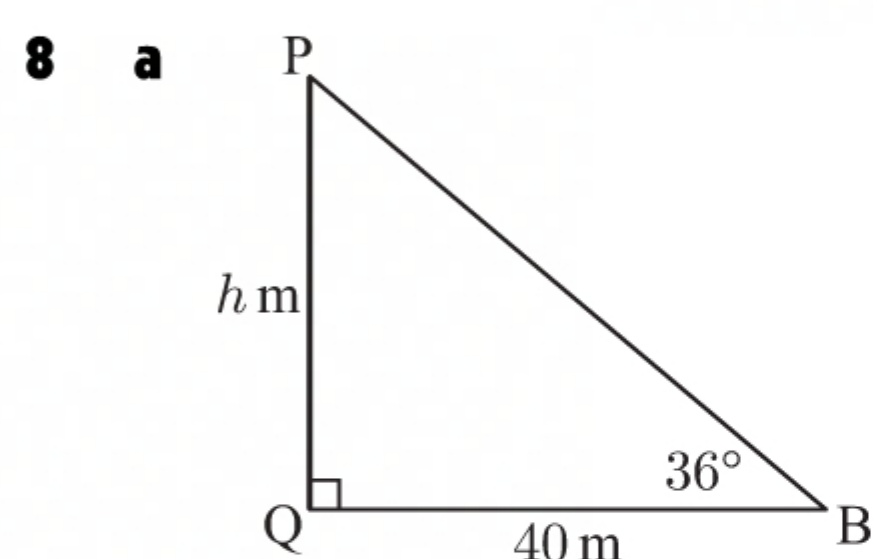
The repayments are £220.98 per month.

Compound Interest	
n	=36
I%	=8.5
PV	=7000
PMT	=-220.972762
FV	=0
P/Y	=12
C/Y	=12

b Interest = total repayment – amount borrowed

$$= £220.98 \times 36 - £7000$$

$$= £955.28$$

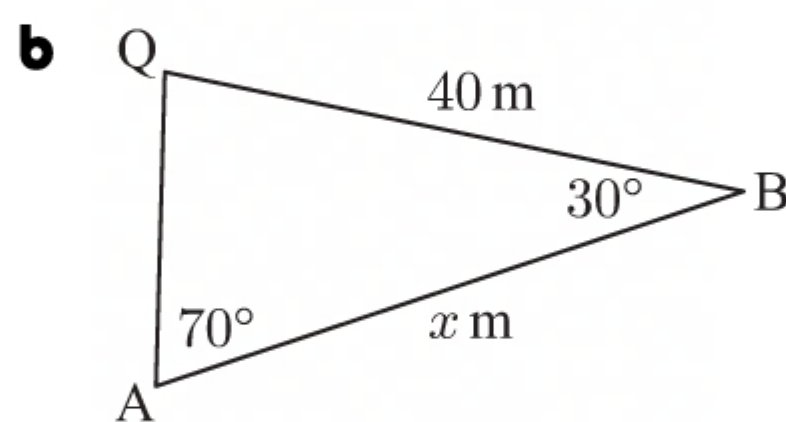


Let PQ be h m.

$$\therefore \tan 36^\circ = \frac{h}{40}$$

$$\therefore h = 40 \tan 36^\circ \approx 29.1$$

The height of the pole is about 29.1 m.



Let AB be x m.

$$\begin{aligned} \widehat{AQB} &= 180^\circ - 70^\circ - 30^\circ \quad \{\text{angles in a triangle}\} \\ &= 80^\circ \end{aligned}$$

$$\therefore \frac{x}{\sin 80^\circ} = \frac{40}{\sin 70^\circ} \quad \{\text{sine rule}\}$$

$$\therefore x = \frac{40 \sin 80^\circ}{\sin 70^\circ}$$

$$\therefore x \approx 41.9$$

The distance between A and B is about 41.9 m.

- 9 a** Let μ be the population mean flight time of Robin's arrows with the longer bow.

The hypotheses to be tested are:

$$H_0: \mu = 1.523 \quad \{\text{the mean flight time is 1.523 seconds}\}$$

$$H_1: \mu < 1.523 \quad \{\text{the mean flight time is less than 1.523 seconds}\}$$

- b i** $\bar{x} = 1.5094$ seconds, $s \approx 0.0203$ seconds, $n = 10$

$$\text{The value of the test statistic is } t \approx \frac{1.5094 - 1.523}{\frac{0.0203}{\sqrt{10}}} \approx -2.12$$

- ii** Since $H_1: \mu < 1.523$, $p\text{-value} \approx P(T < -2.12)$ where $T \sim t_9$
 ≈ 0.0315 {using technology}

- c** Since $p\text{-value} > 0.01 = \alpha$, we do not have enough evidence to reject H_0 in favour of H_1 at the 1% significance level. We therefore accept H_0 .

Since we have accepted H_0 , we cannot conclude that the longer bow has decreased the flight time of Robin's arrows.

- 10 a** The bin has capacity 500 litres $\equiv 500\,000 \text{ cm}^3$

$$\therefore \pi r^2 h = 500\,000$$

$$\therefore h = \frac{500\,000}{\pi r^2}$$

- b** Surface area $A = 2\pi r h + \pi r^2$

$$= 2\pi r \left(\frac{500\,000}{\pi r^2} \right) + \pi r^2$$

$$= 1\,000\,000 r^{-1} + \pi r^2$$

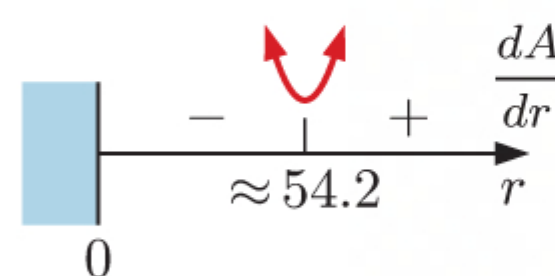
c $\frac{dA}{dr} = -\frac{1\,000\,000}{r^2} + 2\pi r$

Now $\frac{dA}{dr} = 0$ when $2\pi r = \frac{1\,000\,000}{r^2}$

$$\therefore 2\pi r^3 = 1\,000\,000$$

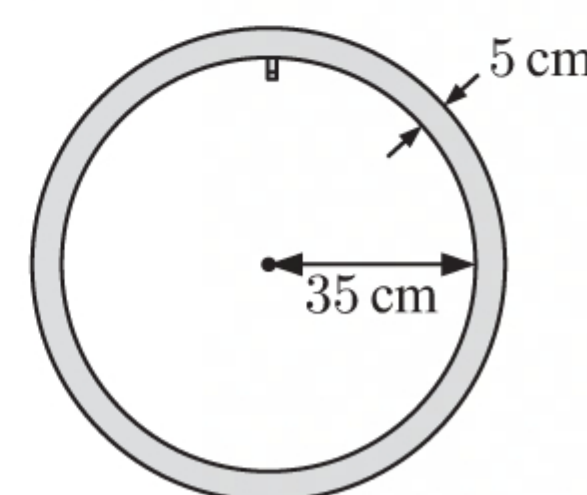
$$\therefore r = \sqrt[3]{\frac{1\,000\,000}{2\pi}} \approx 54.2$$

$$\text{and } h \approx \frac{500\,000}{\pi(54.2)^2} \approx 54.2$$



So, the surface area of the bin is minimised when the bin has a base radius and height of about 54.2 cm.

- 11 a** At 0 seconds, the valve is at its highest position, so the height of the valve above the road is $5 + 35 \times 2 = 75$ cm.



- b** $H(t) = a \cos(bt)^\circ + d$ cm

- i** Amplitude = radius of the wheel
 $= 35$ cm

$$\therefore a = 35$$

- ii** The centre of the wheel is 40 cm above the ground, so the principal axis is at $H = 40$ cm.

$$\therefore d = 40$$

- iii** There are 0.4 revolutions per second, so the period is $\frac{5}{2}$ second.

$$\text{The period} = \frac{360^\circ}{b}$$

$$\therefore \frac{5}{2} = \frac{360^\circ}{b}$$

$$\therefore b = 144^\circ$$

c From **b**, $H(t) = 35 \cos(144t)^\circ + 40$

Now $H(t) = 60$ when $35 \cos(144t)^\circ + 40 = 60$

$$\therefore t \approx 0.383 \quad \{\text{using technology}\}$$

It takes approximately 0.383 seconds for the valve to fall to 60 cm above the road.

12 a i If $A \propto \theta$ then $A = k\theta$ where k is a constant.

Using the first point, $10.48 = 3 \times k$

$$\therefore k = \frac{10.48}{3}$$

\therefore the model is $A = \frac{10.48}{3} \theta$

ii Substituting $\theta = 22^\circ$, $A = \frac{10.48}{3} \times 22 \approx 76.85$

Substituting $\theta = 37^\circ$, $A = \frac{10.48}{3} \times 37 \approx 129.25$

These values are significantly different from the areas observed, so we conclude that Robert's suggestion is incorrect.

b i

θ	3°	22°	37°	52°	74°
$\tan \theta$	0.0524	0.4040	0.7536	1.2799	3.4874
$A \text{ (m}^2\text{)}$	10.48	80.81	150.71	255.99	697.48

ii The correlation coefficient is very close to 1, so the fit is excellent.

The power is very close to 1, so it is reasonable to conclude that A is directly proportional to $\tan \theta$.

The model is $A \approx 200 \tan \theta$.

iii The power model suggests $A \propto \tan \theta$, and so supports David's claim.

iv When $\theta = 43^\circ$, $A \approx 200 \times \tan 43^\circ$
 $\approx 187 \text{ m}^2$

Des	Norm1	d/c	Real
PowerReg			
a	=199.999884		
b	=1.00003395		
r	=0.99999999		
r ²	=0.99999999		
MSe	=4.1844×10 ⁻⁹		
y=a·x^b			
[COPY] [DRAW]			

MIXED QUESTIONS SET 9

1 $f(x) = ax^2 + bx + 7$

a We have $f(2) = 7$ and $f(4) = 23$

$$\therefore 4a + 2b + 7 = 7$$

$$16a + 4b + 7 = 23$$

$$\therefore 4a + 2b = 0$$

$$16a + 4b = 16$$

$$\therefore 2a + b = 0 \quad \dots (1)$$

$$\therefore 4a + b = 4 \quad \dots (2)$$

b We solve equations (1) and (2) simultaneously using technology.

$\therefore a = 2$ and $b = -4$.

Math	Des	Norm1	d/c	Real
a _n X + b _n Y = C _n				
	a	b	c	
1	2	1	0	
2	4	1	4	
[SOLVE] [DELETE] [CLEAR] [EDIT]				

Math	Des	Norm1	d/c	Real
a _n X + b _n Y = C _n				
	a	b	c	
1	2	1	0	
2	4	1	4	
[SOLVE] [DELETE] [CLEAR] [EDIT]				

c Using **b**, $f(x) = 2x^2 - 4x + 7$

$$\therefore f(-1) = 2(-1)^2 - 4(-1) + 7$$

$$= 2 + 4 + 7$$

$$= 13$$

2 a Let u_n km be the distance Hayley cycled in the n th week, and v_n km be the distance Patrick cycled in the n th week. Hayley cycled an additional 20 km each week.

$$\therefore u_n = 60 + 20(n - 1)$$

$$\therefore u_5 = 60 + 20 \times 4 = 140$$

So, Hayley cycled 140 km in the 5th week of training.

Patrick increased his distance by 20% each week.

$$\therefore v_n = 60(1 + 0.2)^{n-1} \quad \{20\% = 0.2\}$$

$$= 60(1.2)^{n-1}$$

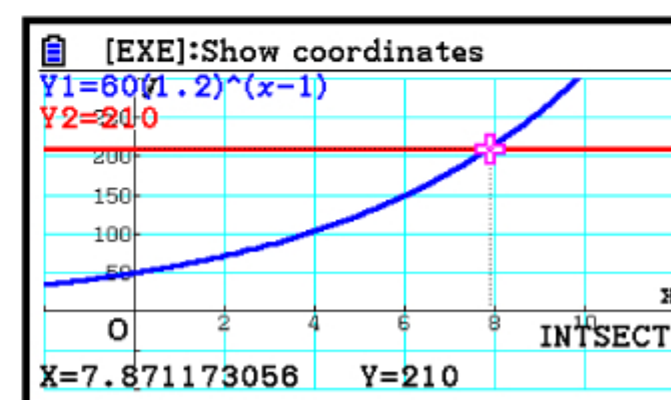
$$\therefore v_5 = 60(1.2)^4 \approx 124$$

So, Patrick cycled about 124 km in the 5th week of training.

$$\begin{aligned}
 \text{b } u_n &= 210 \text{ where } 60 + 20(n-1) = 210 \\
 &\therefore 20(n-1) = 150 \\
 &\therefore n-1 = 7.5 \\
 &\therefore n = 8.5
 \end{aligned}$$

$$v_n = 210 \text{ where } 60(1.2)^{n-1} = 210$$

Using technology, $n \approx 7.87$.



So, Hayley first cycled 210 km in the 9th week, and Patrick first cycled 210 km in the 8th week.

\therefore Patrick was the first to cycle 210 km in one week.

c u_n is an arithmetic sequence with $u_1 = 60$ and $d = 20$.

$$\begin{aligned}
 \therefore \text{the total distance Hayley cycled in the first 12 weeks} &= \frac{n}{2}(2u_1 + (n-1)d) \\
 &= \frac{12}{2}(2 \times 60 + 11 \times 20) \\
 &= 2040 \text{ km}
 \end{aligned}$$

v_n is a geometric sequence with $v_1 = 60$ and $r = 1.2$.

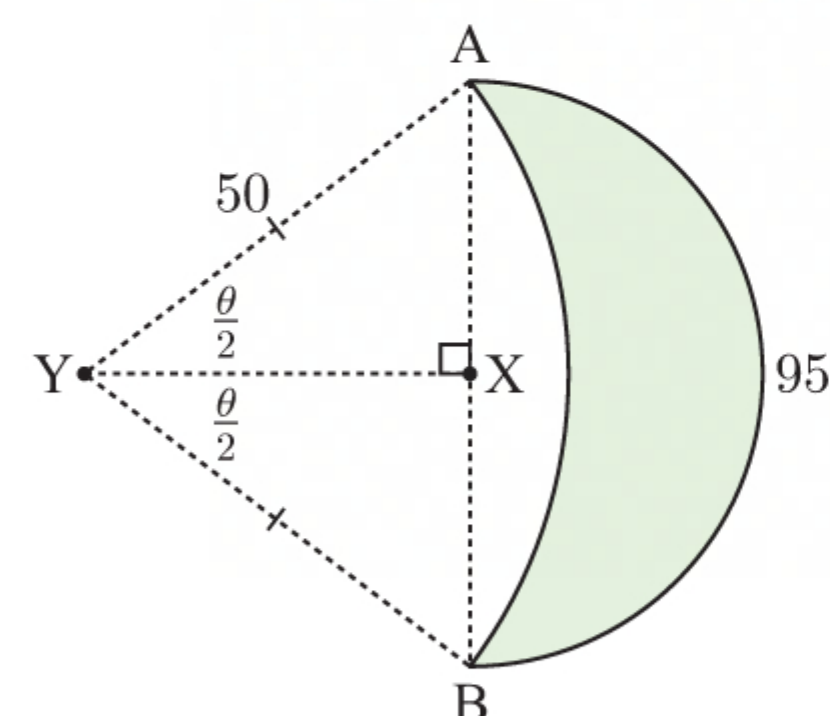
$$\begin{aligned}
 \therefore \text{the total distance Patrick cycled in the first 12 weeks} &= \frac{v_1(1-r^n)}{1-r} \\
 &= \frac{60(1-(1.2)^{12})}{1-1.2} \\
 &\approx 2375 \text{ km}
 \end{aligned}$$

So, Patrick cycled a greater total distance in the first 12 weeks.

3 a The circle centred at X has radius [AX].

The length of the large arc [AB] is half of the circumference.

$$\begin{aligned}
 \therefore \frac{1}{2}(2 \times \pi \times AX) &= 95 \\
 \therefore \pi \times AX &= 95 \\
 \therefore AX &= \frac{95}{\pi} \text{ units} \\
 &\approx 30.2 \text{ units}
 \end{aligned}$$



$$\begin{aligned}
 \text{b In } \triangle AXY, \sin \frac{\theta}{2} &= \frac{AX}{AY} \\
 &= \frac{\frac{95}{\pi}}{50} \\
 &= \frac{19}{10\pi} \\
 \therefore \frac{\theta}{2} &= \sin^{-1}\left(\frac{19}{10\pi}\right) \\
 \therefore \theta &= 2 \sin^{-1}\left(\frac{19}{10\pi}\right) \\
 &\approx 74.4^\circ
 \end{aligned}$$

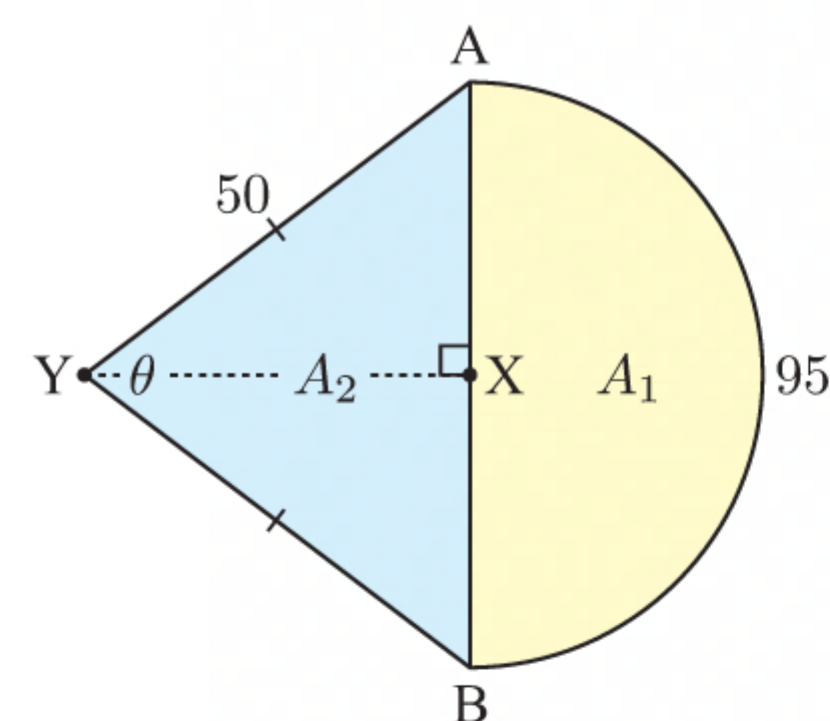
c We first divide the figure into areas A_1 and A_2 .

Now A_1 = area of semi-circle centred at X

$$\begin{aligned}
 &= \frac{1}{2} \times \pi \times \left(\frac{95}{\pi}\right)^2 \quad \{\text{from a}\} \\
 &\approx 1436 \text{ units}^2
 \end{aligned}$$

and A_2 = area of $\triangle AXY$

$$\begin{aligned}
 &= \frac{1}{2} \times 50 \times 50 \times \sin \theta \\
 &\approx 1250 \times \sin 74.4^\circ \quad \{\text{from b}\} \\
 &\approx 1204 \text{ units}^2
 \end{aligned}$$



So, total area of figure = $A_1 + A_2$

$$\begin{aligned}
 &\approx 1436 + 1204 \\
 &\approx 2640 \text{ units}^2
 \end{aligned}$$

Now, shaded area = total area of figure – area of sector ABY

$$\begin{aligned} &\approx 2640 - \frac{\theta}{360^\circ} \pi r^2 \\ &\approx 2640 - \frac{74.4}{360} \pi \times 50^2 \\ &\approx 1020 \text{ units}^2 \quad \{\text{to 3 significant figures}\} \end{aligned}$$

4 a $f(x) = xe^x$

i $n = 2, a = 0, b = 1$

$$h = \frac{b-a}{n} = \frac{1}{2}$$

$$x_i = \frac{1}{2}i$$

i	x_i	$f(x_i)$
0	0	0
1	$\frac{1}{2}$	0.824 361
2	1	2.718 282

Using the trapezoidal rule, the area $\approx \frac{h}{2}(f(x_0) + 2f(x_1) + f(x_2))$
 $\approx 1.0918 \text{ units}^2$

ii $n = 6, a = 0, b = 1$

$$h = \frac{b-a}{n} = \frac{1}{6}$$

$$x_i = \frac{1}{6}i$$

i	x_i	$f(x_i)$
0	0	0
1	$\frac{1}{6}$	0.196 893
2	$\frac{1}{3}$	0.465 204
3	$\frac{1}{2}$	0.824 361
4	$\frac{2}{3}$	1.298 489
5	$\frac{5}{6}$	1.917 480
6	1	2.718 282

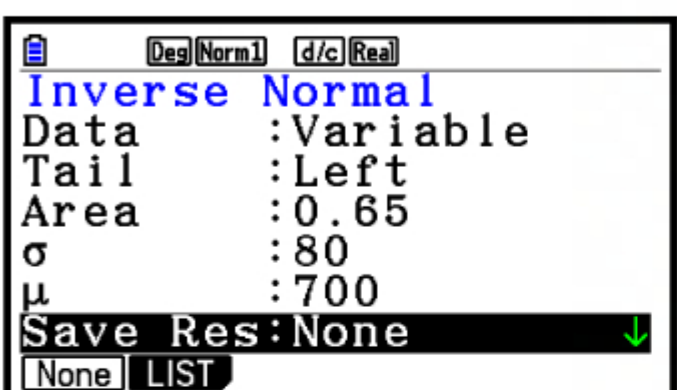
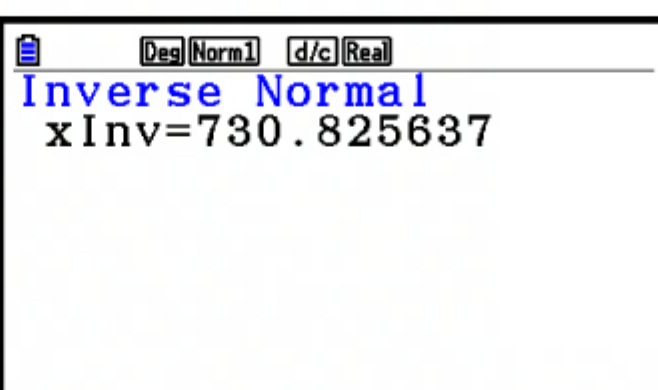
Using the trapezoidal rule, the area $\approx \frac{h}{2}(f(x_0) + 2f(x_1) + \dots + 2f(x_5) + f(x_6))$
 $\approx 1.0103 \text{ units}^2$

b Given the exact area is 1 unit^2 , the estimate in **a ii** is more accurate than our estimate in **a i**.

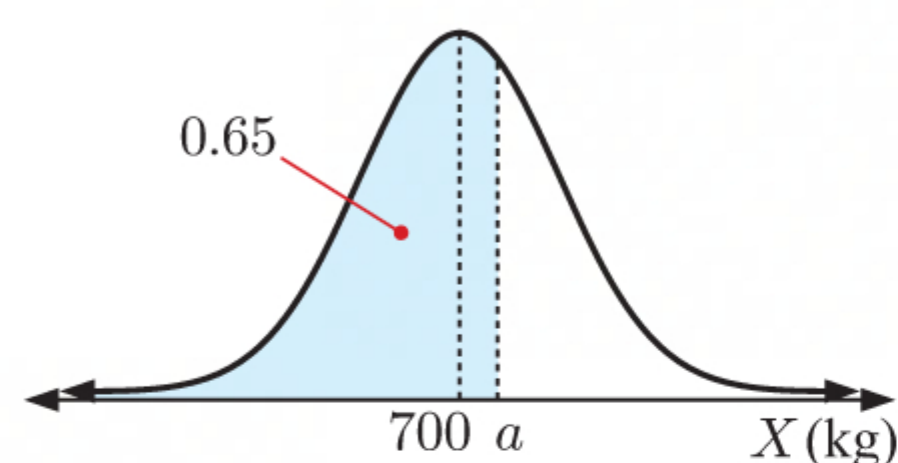
Increasing the number of subintervals increases the accuracy of our estimate.

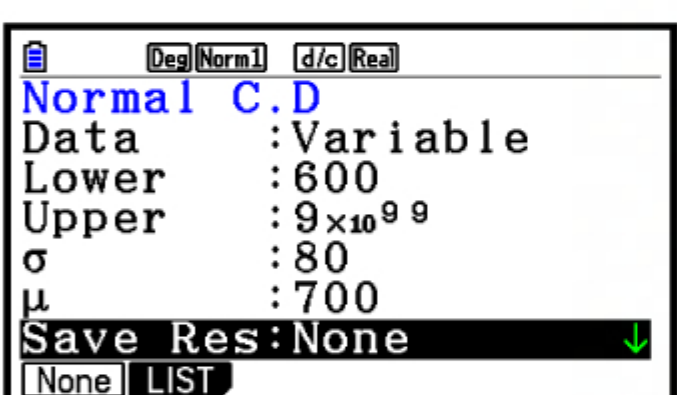
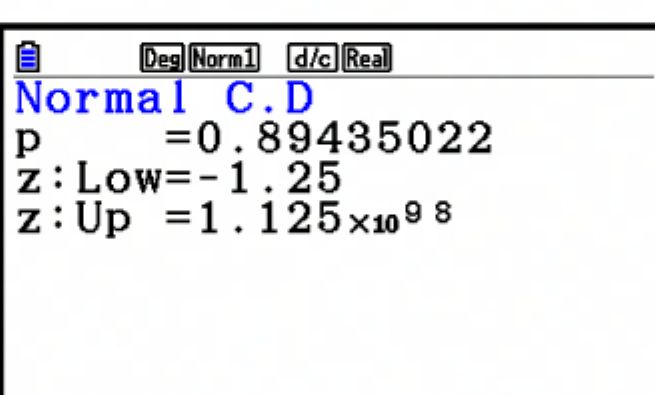
5 Let X kg be the mass of a randomly selected sea lion.

$$\therefore X \sim N(700, 80^2)$$

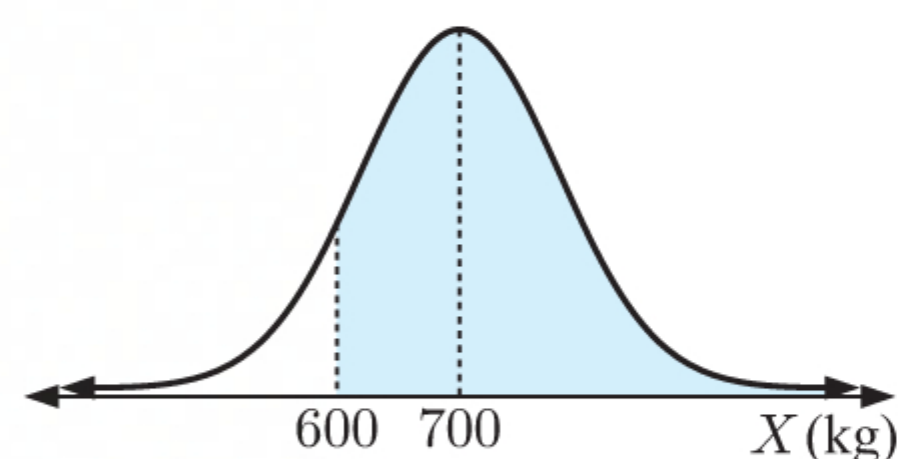
a		
----------	---	--

If $P(X < a) = 0.65$
 then $a \approx 731$



b		
----------	---	--

$P(X > 600) \approx 0.894 350$
 ≈ 0.894

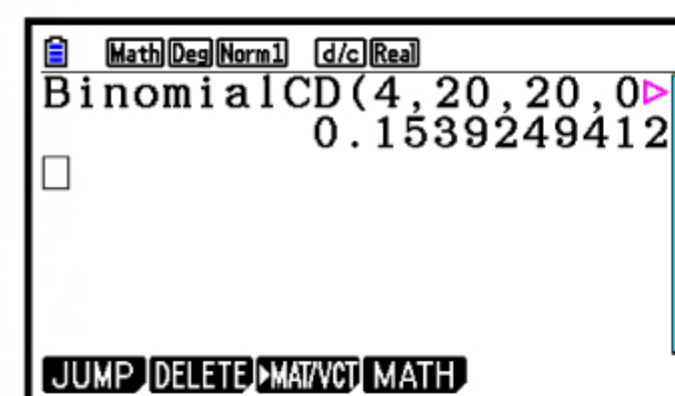


$$\begin{aligned} \text{c } P(X < 600) &= 1 - P(X \geq 600) \\ &\approx 1 - 0.894\,350 \quad \{\text{using b}\} \\ &\approx 0.105\,650 \end{aligned}$$

$$\therefore Y \sim B(20, 0.105\,650)$$

$$\begin{aligned} \text{i } \mu_Y &= np & \sigma_Y &= \sqrt{np(1-p)} \\ &\approx 20 \times 0.105\,650 & &\approx \sqrt{20 \times 0.105\,650 \times 0.894\,350} \\ &\approx 2.11 & &\approx 1.37 \end{aligned}$$

$$\begin{aligned} \text{ii Using technology, } P(Y > 3) &= P(Y \geq 4) \\ &\approx 0.154 \end{aligned}$$



6 The ordered data set is:

132 140 149 155 159 160 161 163 164 165 (20 data values)
169 171 173 181 185 191 200 207 212 303

a Since $n = 20$, $\frac{n+1}{2} = 10.5$ \therefore the median is the average of the 10th and 11th value.

~~132 140 149 155 159 160 161 163 164~~ 165
169 ~~171 173 181 185 191 200 207 212 303~~

$$\begin{aligned} \therefore \text{median} &= \frac{10\text{th value} + 11\text{th value}}{2} \\ &= \frac{\$165 + \$169}{2} \\ &= \$167 \end{aligned}$$

We have an even number of data values, so we include all data values when we split the data set in two.

lower half

132	140	149	155	159	160	161	163	164	165
169	171	173	181	185	191	200	207	212	303

upper half

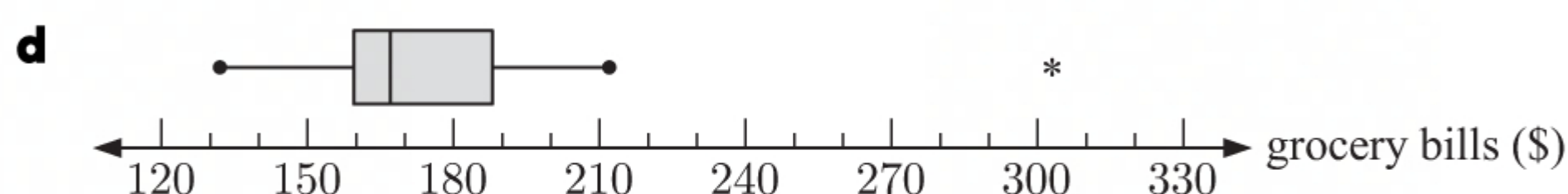
$$Q_1 = \text{median of lower half} = \frac{\$159 + \$160}{2} = \$159.50$$

$$Q_3 = \text{median of upper half} = \frac{\$185 + \$191}{2} = \$188$$

$$\begin{aligned} \text{b } \text{IQR} &= Q_3 - Q_1 \\ &= \$188 - \$159.50 \\ &= \$28.50 \end{aligned}$$

$$\begin{aligned} \text{c Test for outliers:} \quad & \text{upper boundary} & \text{and} & \text{lower boundary} \\ & = \text{upper quartile} + 1.5 \times \text{IQR} & & = \text{lower quartile} - 1.5 \times \text{IQR} \\ & = \$188 + 1.5 \times 28.50 & & = \$159.50 - 1.5 \times 28.50 \\ & = \$230.75 & & = \$116.75 \end{aligned}$$

\$303 is above the upper boundary, so it is an outlier.



7 $W(t) = 5 \times (0.965)^t$ grams, $t \geq 0$

- a** The weight of the radioactive substance at the *end* of each year forms a geometric sequence with common ratio $r = 0.965$.

$$\begin{aligned}\text{So, percentage decrease} &= (1 - r) \times 100\% \\ &= 0.035 \times 100\% \\ &= 3.5\%\end{aligned}$$

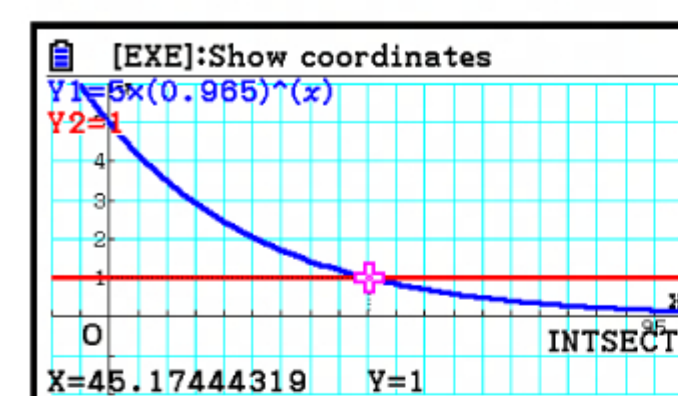
b $W(300) = 5 \times (0.965)^{300}$
 ≈ 0.000114
 $\approx 1.14 \times 10^{-4}$

The weight of the substance after 300 years is about 1.14×10^{-4} grams.

c We need to solve $W(t) = 1$
 $\therefore 5 \times (0.965)^t = 1$

Using technology, $t \approx 45.2$

\therefore it will take about 45.2 years for the weight of the substance to fall below 1 g.



- 8 a i** $(2, 4)$ is closest to Q, so we estimate a lead concentration of 47 ppm at $(2, 4)$.
ii $(-2, -3)$ is closest to R, so we estimate a lead concentration of 62 ppm at $(-2, -3)$.
iii $(-4, -1)$ is equally closest to P and R, so we estimate a lead concentration of $\frac{28 + 62}{2} = 45$ ppm at $(-4, -1)$.

- b i** S lies in the original cell R, so we construct the perpendicular bisector of [RS] within this cell.

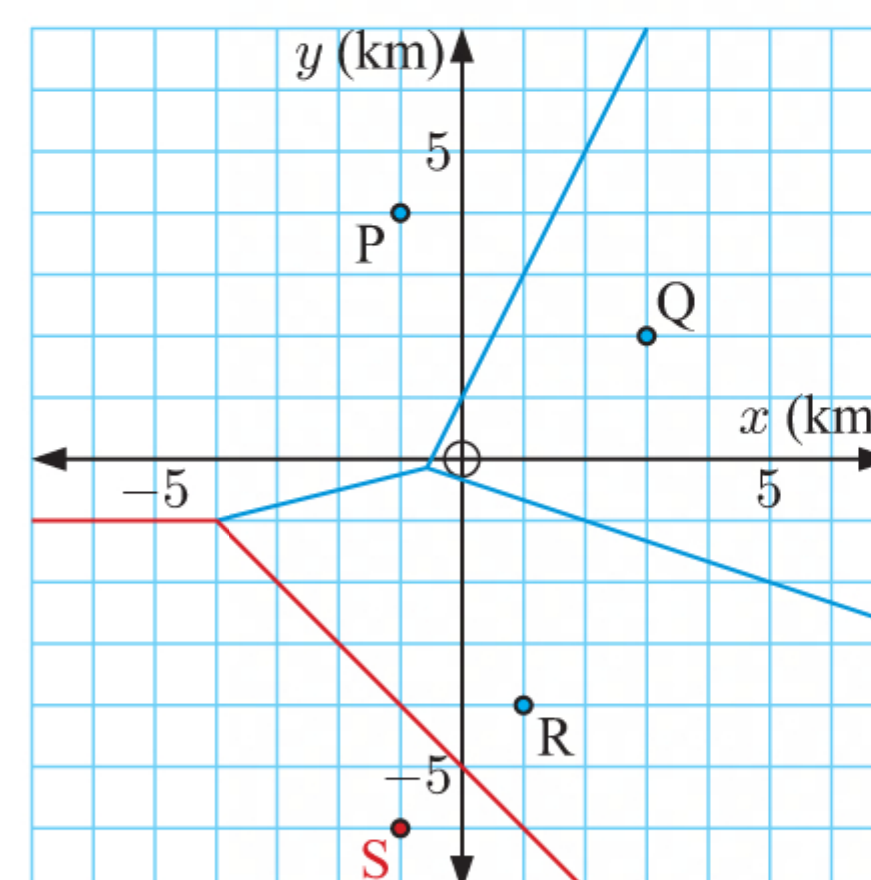
We then construct the perpendicular bisector of [PS] within the original cell P.

Finally, we remove the segment of the perpendicular bisector of [PR] which now lies within cell S.

- ii** $(2, 4)$ is still closest to Q, so its estimate is unchanged.

$(-2, -3)$ is now equally closest to R and S, so we estimate a lead concentration of $\frac{62 + 55}{2} = 58.5$ ppm at $(-2, -3)$.

$(-4, -1)$ is now equally closest to P, R, and S, so we estimate a lead concentration of $\frac{28 + 62 + 55}{3} \approx 48.3$ ppm at $(-4, -1)$.



9 a $y = x^3 + ax^2 + bx + 3$

The function passes through $(1, 8)$, so $(1)^3 + a(1)^2 + b(1) + 3 = 8$
 $\therefore 1 + a + b + 3 = 8$
 $\therefore a + b + 4 = 8$
 $\therefore a + b = 4 \quad \dots (1)$

Now $\frac{dy}{dx} = 3x^2 + 2ax + b$.

At $(1, 8)$ the tangent has equation $y = 2x + 6$ which has gradient 2.

$\therefore 3(1)^2 + 2a(1) + b = 2$
 $\therefore 3 + 2a + b = 2$
 $\therefore 2a + b = -1 \quad \dots (2)$

We solve equations (1) and (2) simultaneously using technology.

$\therefore a = -5$ and $b = 9$

- b** From **a**, $y = x^3 - 5x^2 + 9x + 3$ and

$$\frac{dy}{dx} = 3x^2 - 10x + 9$$

$$\begin{aligned} \text{Now when } x = -1, \quad y &= (-1)^3 - 5(-1)^2 + 9(-1) + 3 \quad \text{and} \quad \frac{dy}{dx} = 3(-1)^2 - 10(-1) + 9 \\ &= -1 - 5 - 9 + 3 \quad \quad \quad = 3 + 10 + 9 \\ &= -12 \quad \quad \quad = 22 \end{aligned}$$

So the point of contact is $(-1, -12)$ and the gradient of the normal is $-\frac{1}{22}$.

The equation of the normal is $y = -\frac{1}{22}(x - (-1)) - 12$

$$\therefore y = -\frac{1}{22}x - \frac{265}{22}$$

- 10 a** We need to find how long it will take for the future value to fall to €0.

$$I\% = 4.6, \quad PV = -500\,000, \quad PMT = 3000, \quad FV = 0, \quad P/Y = 12, \quad C/Y = 12$$

$$\therefore N \approx 266$$

Raman will be able to withdraw €3000 for 266 months, and then less in the 267th month.

- b** $N = 4 \times 12 = 48, \quad I\% = 4.6, \quad PV = -500\,000, \quad PMT = 3000, \quad P/Y = 12, \quad C/Y = 12$

$$\therefore FV \approx 443\,028.05$$

After 4 years, the balance is €443 028.05.

- c** $N = 15 \times 12 = 180, \quad I\% = 4.6, \quad PV = -443\,028.05, \quad FV = 0, \quad P/Y = 12, \quad C/Y = 12$

$$\therefore PMT \approx 3411.82$$

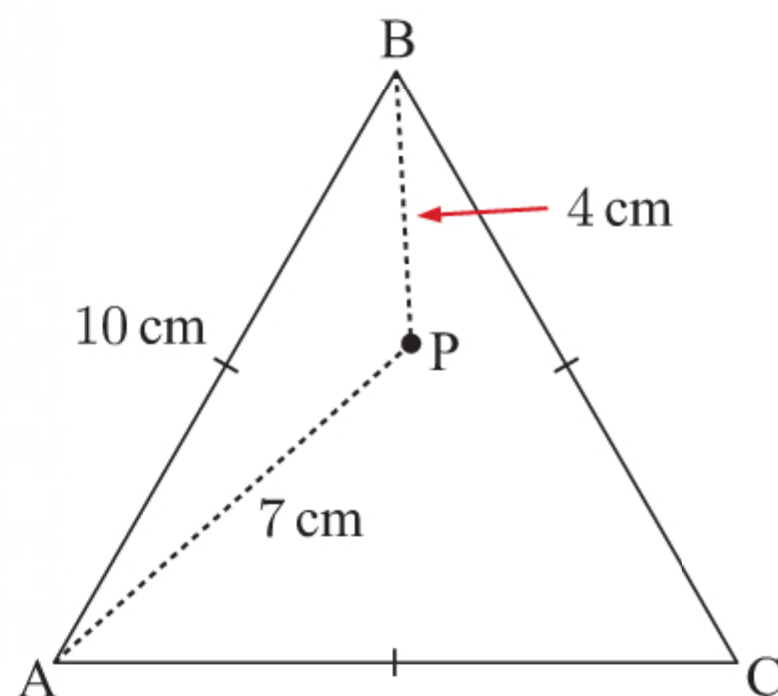
Raman can afford to withdraw €3411.82 each month.

Norm1	
Compound Interest	
n	=266.2227638
I%	=4.6
PV	=-500000
PMT	=3000
FV	=0
P/Y	=12
n	I% PV PMT FV AMORTZ

Norm1	
Compound Interest	
n	=48
I%	=4.6
PV	=-500000
PMT	=3000
FV	=443028.0463
P/Y	=12
n	I% PV PMT FV AMORTZ

Norm1	
Compound Interest	
n	=180
I%	=4.6
PV	=-443028.05
PMT	=3411.820714
FV	=0
P/Y	=12
n	I% PV PMT FV AMORTZ

- 11 a**



- b i** By the cosine rule in $\triangle BAP$:

$$\cos \hat{BAP} = \frac{10^2 + 7^2 - 4^2}{2 \times 7 \times 10}$$

$$\therefore \cos \hat{BAP} = \frac{133}{140}$$

$$\therefore \hat{BAP} = \cos^{-1}\left(\frac{133}{140}\right) \approx 18.2^\circ$$

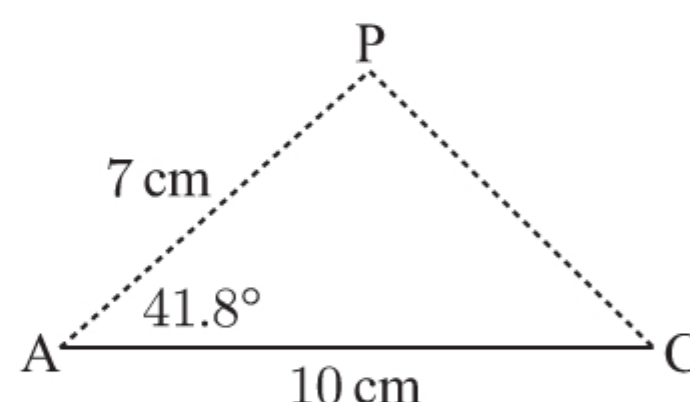
- ii** $\hat{BAC} = 60^\circ$ {angles in an equilateral triangle}

$$\therefore \hat{CAP} = 60^\circ - \hat{BAP}$$

$$\approx 60^\circ - 18.2^\circ$$

$$\approx 41.8^\circ$$

- c**



By the cosine rule in $\triangle APC$:

$$CP^2 \approx 10^2 + 7^2 - 2(10)(7) \cos 41.8^\circ$$

$$\therefore CP \approx \sqrt{10^2 + 7^2 - 2(10)(7) \cos 41.8^\circ}$$

$$\therefore CP \approx 6.68 \text{ cm}$$

- 12 a**

	Painting	Sketching	Sculpting	Sum
Male	30	35	15	80
Female	20	15	25	60
Sum	50	50	40	140

The expected number of male sculptors is $\frac{80 \times 40}{140} \approx 23$.

- b** Using technology, $\chi_{\text{calc}}^2 \approx 9.84$.

	Des Norm1	d/c Real	
A	1	2	3
1	30	35	15
2	20	15	25

25

ROW-OP ROW COLUMN EDIT

χ^2 Test

$\chi^2 = 9.84375$

$p = 7.2855 \times 10^{-3}$

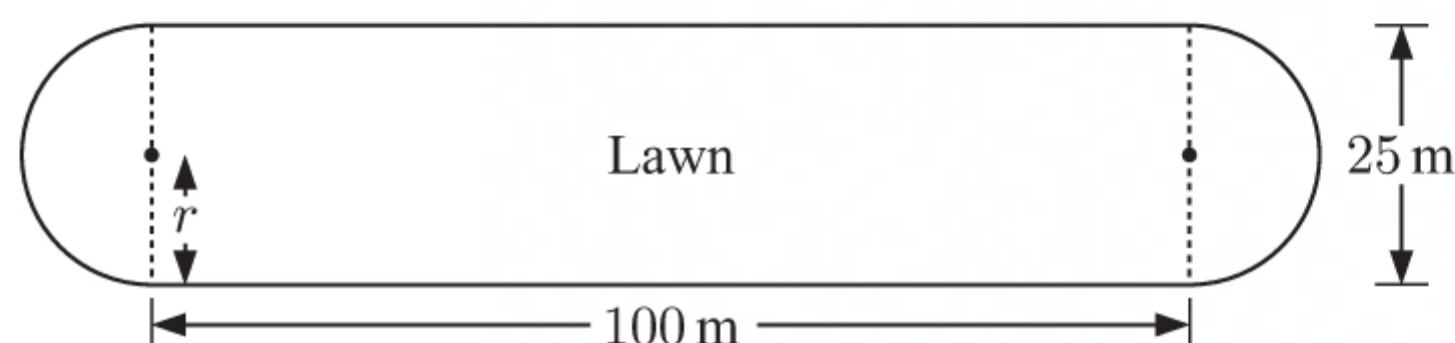
$df = 2$

▶MAT

- c** Since $\chi_{\text{calc}}^2 > \chi_{\text{crit}}^2$, we reject the null hypothesis that *gender* and *choice of art speciality* are independent. We therefore conclude at the 1% significance level that *gender* and *choice of art speciality* are dependent.

MIXED QUESTIONS SET 10

- 1 a** Area = area of two semi-circles + area of rectangle
 $= \pi r^2 + 100 \times 2r$
 $= \pi \times 12.5^2 + 100 \times 25$
 $= 156.25\pi + 2500 \text{ m}^2$

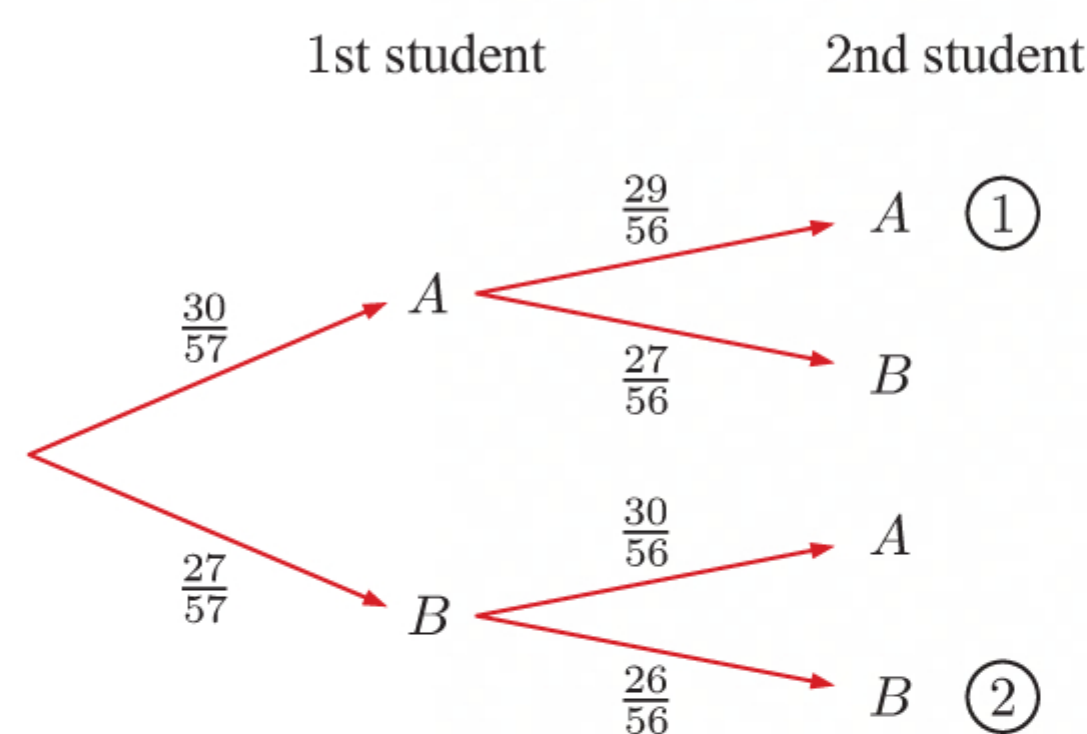


- b** Using $\pi \approx 3$, area $\approx 156.25 \times 3 + 2500$
 $\approx 2969 \text{ m}^2$

- c** Percentage error
 $= \frac{|V_A - V_E|}{V_E} \times 100\%$
 $= \frac{|2969 - (156.25\pi + 2500)|}{156.25\pi + 2500} \times 100\%$
 $\approx 0.73\% \quad \{2 \text{ significant figures}\}$

- 2 a** Total number of Year 7 students = $30 + 27 = 57$
 Let A represent a student selected from class A and
 B represent a student selected from class B.

$$\begin{aligned} P(\text{same class}) &= P(AA \text{ or } BB) \\ &= \underbrace{\frac{30}{57} \times \frac{29}{56}}_{(1)} + \underbrace{\frac{27}{57} \times \frac{26}{56}}_{(2)} \\ &= \frac{131}{266} \\ &\approx 0.492 \end{aligned}$$



\therefore the probability that in any given week the two selected students are in the same class is $\frac{131}{266} \approx 0.492$.

- b** Let X be the number of weeks out of 20 that the two selected students are in the same class.

$$\therefore X \sim B\left(20, \frac{131}{266}\right) \quad \{\text{using a}\}$$

$$\begin{aligned} E(X) &= np \\ &= 20 \times \frac{131}{266} \\ &\approx 9.85 \end{aligned}$$

\therefore we expect that the two selected students are in the same class about 9.85 times out of 20.

- 3** $V(t) = 10t^2 - \frac{1}{3}t^3, \quad 0 \leq t \leq 30$

- a** $V(5) = 10(5)^2 - \frac{1}{3}(5)^3$
 $= 250 - \frac{1}{3}(125)$
 $= 208\frac{1}{3} \text{ litres}$

After 5 minutes there is $208\frac{1}{3}$ litres of water in the tank.

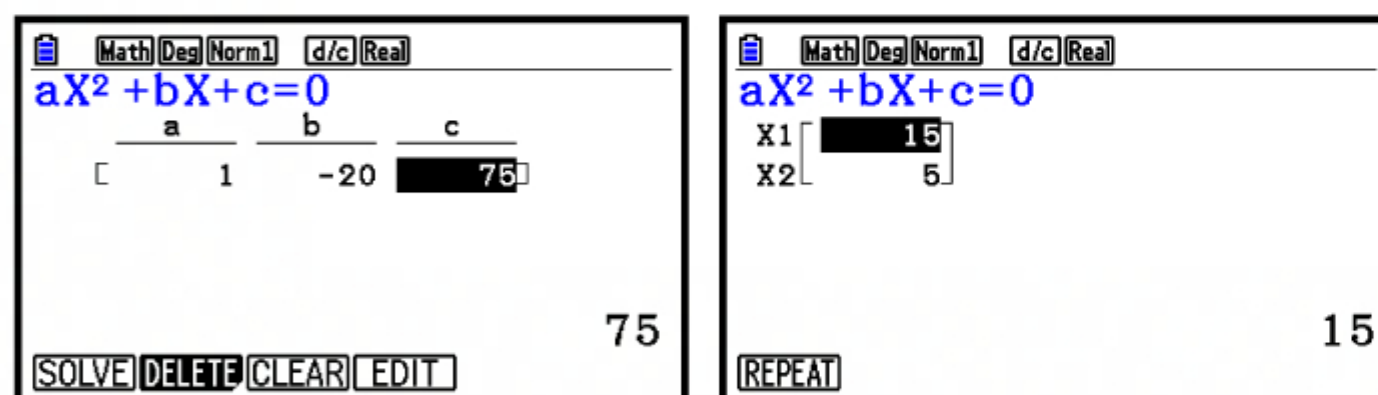
- c** $V'(t) = 0$
 $\therefore 20t - t^2 = 0$
 $\therefore t(20 - t) = 0$
 $\therefore t = 0 \text{ or } 20$

- b** $V'(t) = 20t - t^2$ litres per minute

- d** $V'(5) = 5(20 - 5)$
 $= 5(15)$
 $= 75 \text{ litres per minute}$

$$\begin{aligned} V'(25) &= 25(20 - 25) \\ &= 25(-5) \\ &= -125 \text{ litres per minute} \end{aligned}$$

e $V'(t) = 75$
 $\therefore 20t - t^2 = 75$
 $\therefore t^2 - 20t + 75 = 0$



Using technology, $t = 5$ or 15 .

So, the volume is increasing by 75 litres per minute at times 5 minutes and 15 minutes.

- 4 a** Let Maggie's eye level be at M, the car be at C, and the base of the building be at B.

$$\begin{aligned} MB &= \text{Maggie's height} + \text{building height} \\ &= 51.55 \text{ m} \end{aligned}$$

$$\text{Now } \widehat{MCB} = 67^\circ \quad \{\text{alternate angles}\}$$

$$\begin{aligned} \therefore \tan 67^\circ &= \frac{51.55}{d} \\ \therefore d &= \frac{51.55}{\tan 67^\circ} \approx 21.9 \end{aligned}$$

So the car is about 21.9 m away from the base of the building.

- b** Let S be Sven's location.

$$\begin{aligned} \text{i In } \triangle MBC, \quad \sin 67^\circ &= \frac{51.55}{MC} \\ \therefore MC &= \frac{51.55}{\sin 67^\circ} \\ &\approx 56.0 \text{ m} \end{aligned}$$

Since the car is directly opposite to Maggie, $\triangle MCS$ is right angled at C.

$$\begin{aligned} \therefore MS^2 &\approx 10^2 + 56.0^2 \quad \{\text{Pythagoras}\} \\ \therefore MS &\approx \sqrt{3236} \approx 56.9 \text{ m} \end{aligned}$$

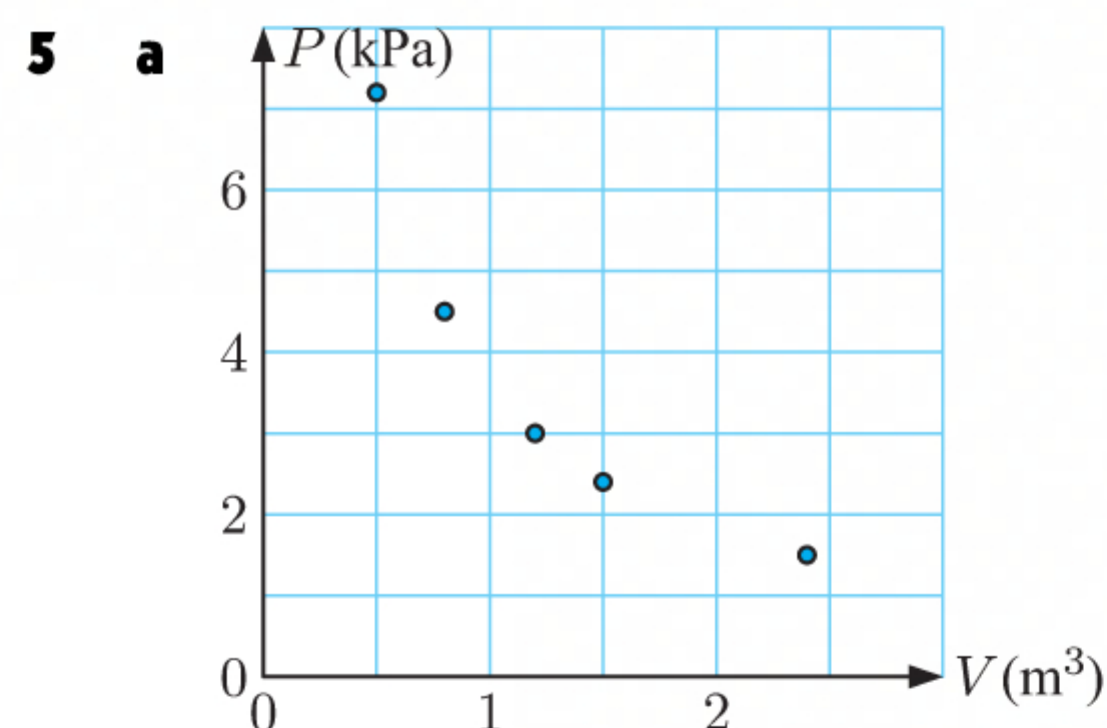
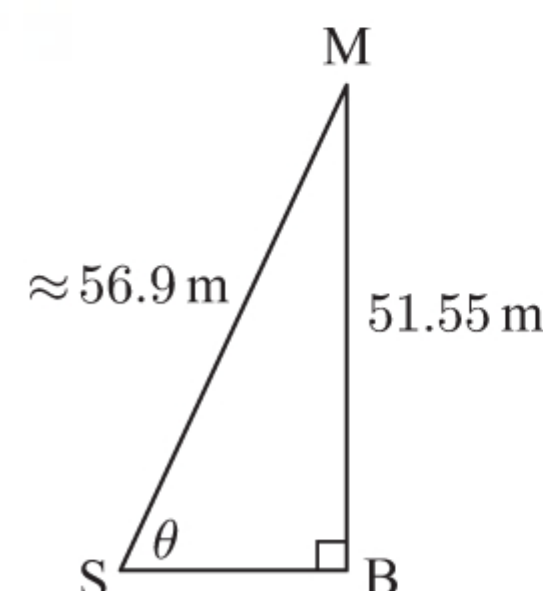
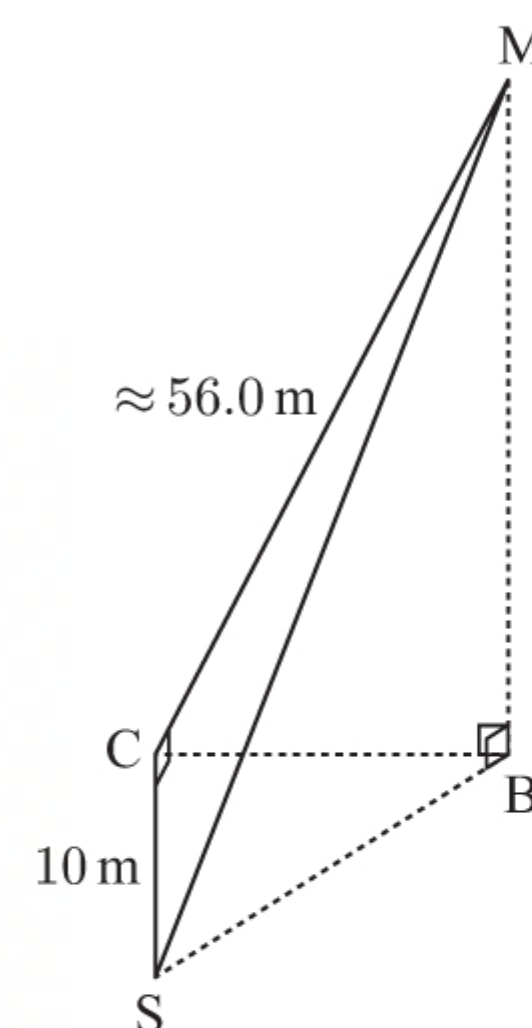
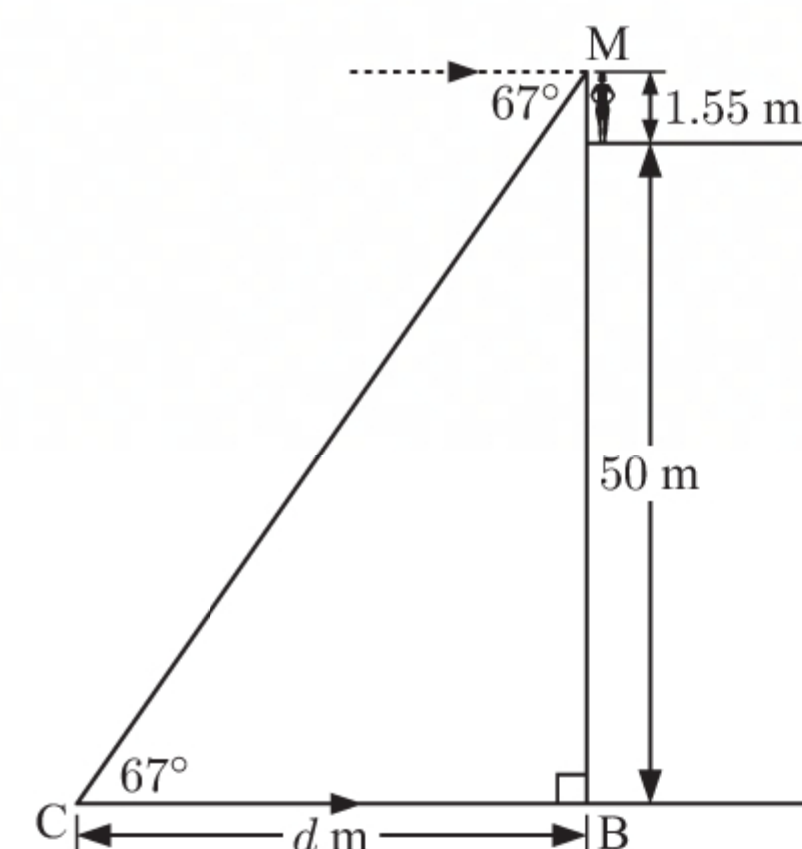
The distance between Maggie and Sven is about 56.9 m.

- ii** Let θ be the angle of elevation from Sven to Maggie.

Now $\triangle MBS$ is right angled at B.

$$\begin{aligned} \therefore \sin \theta &\approx \frac{51.55}{56.9} \\ \therefore \theta &\approx \sin^{-1}\left(\frac{51.55}{56.9}\right) \approx 65.0^\circ \end{aligned}$$

Sven needs to look up at an angle of about 65.0° to see Maggie.



The points appear to lie on a curve for which the axes are both asymptotes. This suggests that an inverse variation model is appropriate.

- b** If V and P are inversely proportional, then $P = \frac{k}{V}$ for some constant k .

When $V = 0.5$, $P = 7.2$, so $7.2 = \frac{k}{0.5}$

$$\therefore k = 7.2 \times 0.5 = 3.6$$

Check: When $V = 0.8$, $P = \frac{3.6}{0.8} = 4.5$ ✓

$$V = 1.2, \quad P = \frac{3.6}{1.2} = 3 \quad \checkmark$$

$$V = 1.5, \quad P = \frac{3.6}{1.5} = 2.4 \quad \checkmark$$

$$V = 2.4, \quad P = \frac{3.6}{2.4} = 1.5 \quad \checkmark$$

\therefore the model connecting V and P is $P = \frac{3.6}{V}$.

- c** When $V = 3$, $P = \frac{3.6}{3} = 1.2$ kPa

- 6 a** The interest is calculated annually, so $n = 7$ time periods.

$$\begin{aligned} u_7 &= u_0 \times (1 + i)^7 \\ &= 2000 \times (1.0825)^7 \quad \{8.25\% = 0.0825\} \\ &\approx 3484 \end{aligned}$$

The total value of Kapil's investment on January 1st 2019 is 3484 rupees.

- b** There are $n = 7 \times 12 = 84$ time periods.

Each time period the investment increases by $i = \frac{8\%}{12} \approx 0.6667\%$

$$\begin{aligned} \therefore \text{the value after 7 years is } u_{84} &= u_0 \times (1 + i)^{84} \\ &\approx 2000 \times (1.006667)^{84} \quad \{0.6667\% = 0.006667\} \\ &\approx 3495 \end{aligned}$$

The total value of Kapil's investment on January 1st 2019 would be 3495 rupees.

\therefore investing in the account paying 8% per annum interest compounded monthly is the better option.

- 7 a i** $P = 1000 + ae^{kn}$

The initial population was 2000.

So, when $n = 0$, $P = 2000$

$$\therefore 2000 = 1000 + ae^0$$

$$\therefore a = 1000$$

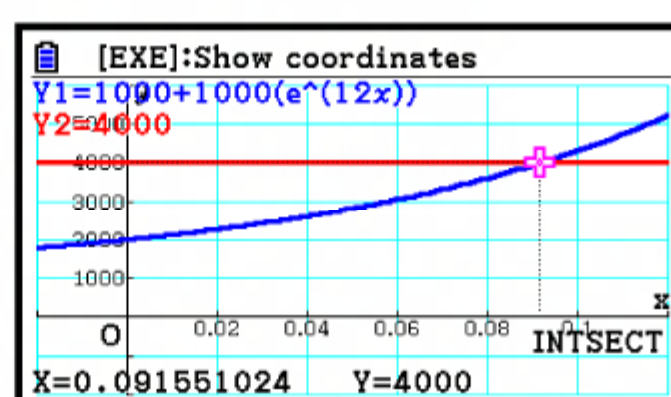
- ii** $P = 1000 + 1000e^{kn}$

After 1 year, the population was 4000.

So, when $n = 12$, $P = 4000$

$$\therefore 4000 = 1000 + 1000e^{k \times 12}$$

Using technology, $k \approx 0.0916$.

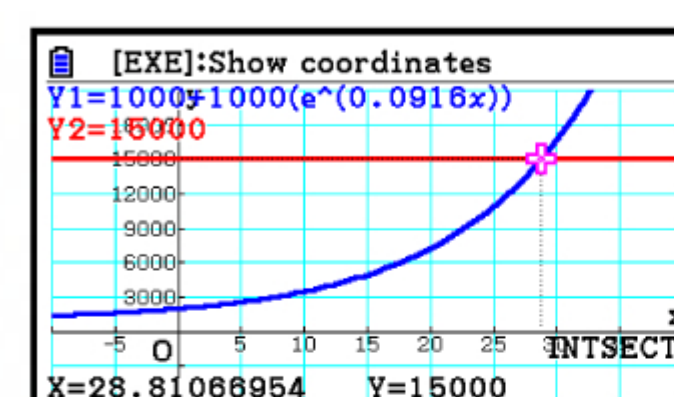


- b** $P \approx 1000 + 1000e^{0.0916n}$

Now $P = 15000$ when $15000 \approx 1000 + 1000e^{0.0916n}$.

Using technology, $n \approx 28.8$.

\therefore it will take about 28.8 months for the population to reach 15 000.



8 a

		Die 2			
		1	2	3	4
Die 1	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

b i $P(S = 5) = \frac{4}{16} = \frac{1}{4}$ ii $P(S > 5) = \frac{6}{16} = \frac{3}{8}$ iii $P(S = 5 | S > 3) = \frac{P(S = 5 \cap S > 3)}{P(S > 3)}$
 $= \frac{P(S = 5)}{P(S > 3)}$
 $= \frac{4}{13}$

c

Value of S	$S = 2$	$3 \leq S \leq 5$	$S > 5$
Number of points won or lost	32	16	-8
Probability	$\frac{1}{16}$	$\frac{9}{16}$	$\frac{3}{8}$

i The expected number of points
 $= 32 \times \frac{1}{16} + 16 \times \frac{9}{16} + (-8) \times \frac{3}{8}$
 $= 2 + 9 + (-3)$
 $= 8$

ii Let k be the number of points for $S = 2$.

For the expectation to be zero,

$$k \times \frac{1}{16} + 16 \times \frac{9}{16} + (-8) \times \frac{3}{8} = 0$$

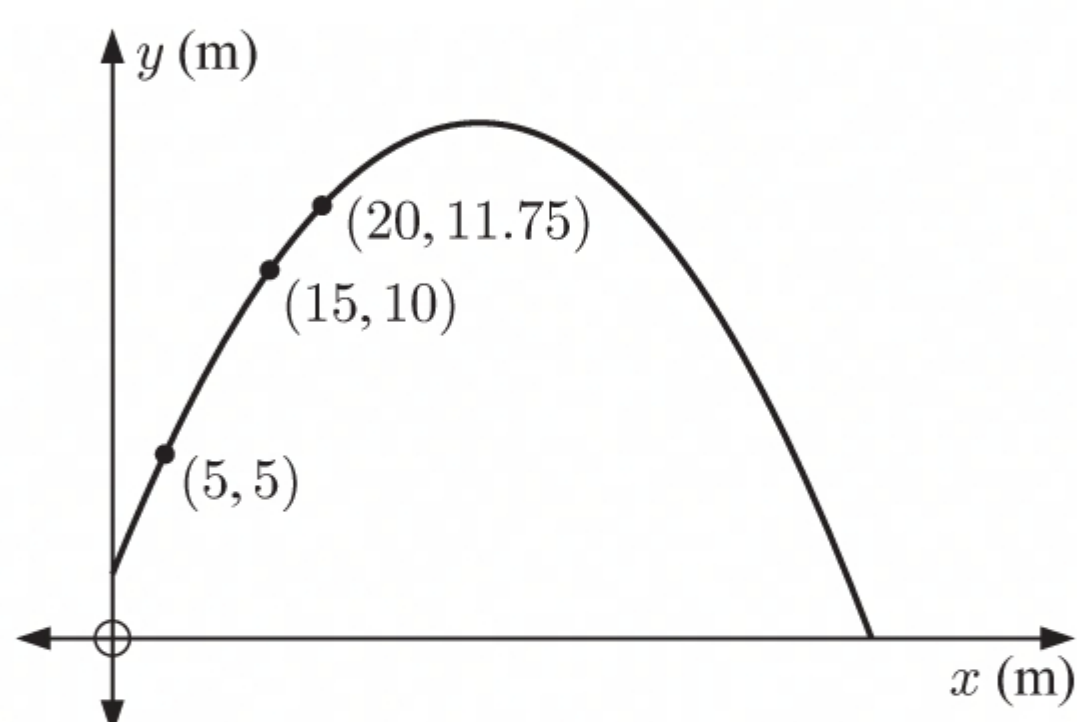
$$\therefore \frac{1}{16}k + 9 - 3 = 0$$

$$\therefore \frac{1}{16}k = -6$$

$$\therefore k = -96$$

So, $S = 2$ should correspond to a 96 point loss.

9



a $y = ax^2 + bx + c$

When $x = 5$, $y = 5$ $\therefore 5 = a(5)^2 + b(5) + c$ or $25a + 5b + c = 5$

When $x = 15$, $y = 10$ $\therefore 10 = a(15)^2 + b(15) + c$ or $225a + 15b + c = 10$

When $x = 20$, $y = 11.75$ $\therefore 11.75 = a(20)^2 + b(20) + c$ or $400a + 20b + c = 11.75$

b We solve the system of equations

$$\begin{cases} 25a + 5b + c = 5 \\ 225a + 15b + c = 10 \\ 400a + 20b + c = 11.75 \end{cases}$$

simultaneously using technology.

We find that $a = -0.01$, $b = 0.7$, and $c = 1.75$.

c $y = -0.01x^2 + 0.7x + 1.75$ {using b}

Since $a = -0.01 < 0$, the shape is .

The maximum height occurs when $x = -\frac{b}{2a} = -\frac{0.7}{2 \times (-0.01)} = 35$

When $x = 35$, $y = -0.01(35)^2 + 0.7(35) + 1.75$
 $= 14$

So, the discus reached a maximum height of 14 m.

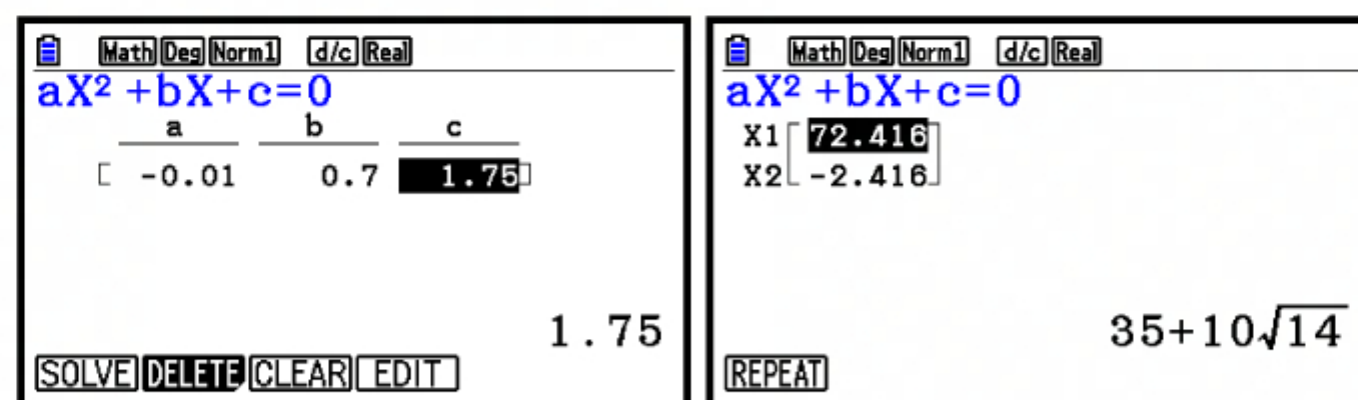
$\begin{matrix} \text{Math} & \text{Deg} & \text{Norm} & \text{d/c} & \text{Real} \\ \text{a} & \text{X} & \text{b} & \text{Y} & \text{c} & \text{Z} & \text{d} & \text{n} \\ 1 & 25 & 5 & 1 & 5 \\ 2 & 225 & 15 & 1 & 10 \\ 3 & 400 & 20 & 1 & 11.75 \end{matrix}$	$\begin{matrix} \text{Math} & \text{Deg} & \text{Norm} & \text{d/c} & \text{Real} \\ \text{a} & \text{X} & \text{b} & \text{Y} & \text{c} & \text{Z} & \text{d} & \text{n} \\ \text{X} & -0.01 \\ \text{Y} & 0.7 \\ \text{Z} & 1.75 \end{matrix}$
$\begin{matrix} \text{SOLVE} & \text{DELETE} & \text{CLEAR} & \text{EDIT} \end{matrix}$	REPEAT

d When $y = 0$, $-0.01x^2 + 0.7x + 1.75 = 0$

Using technology, $x \approx -2.42$ or ≈ 72.4

$$\therefore x \approx 72.4 \quad \{x \geq 0\}$$

\therefore the disc travelled about 72.4 metres before it hit the ground.



10 a i The blue edge is the perpendicular bisector of $[AB]$.

The midpoint of $[AB]$ is $\left(\frac{-10+6}{2}, \frac{21+13}{2}\right)$ or $(-2, 17)$.

The gradient of $[AB]$ is $\frac{13-21}{6-(-10)} = \frac{-8}{16} = -\frac{1}{2}$.

So, the blue edge has gradient 2.

$$\begin{aligned} \therefore \text{its equation is } y &= 2(x - (-2)) + 17 \\ &= 2(x + 2) + 17 \\ &= 2x + 4 + 17 \\ &= 2x + 21 \end{aligned}$$

ii The green edge is the perpendicular bisector of $[AD]$.

The midpoint of $[AD]$ is $\left(\frac{-10+(-16)}{2}, \frac{21+(-9)}{2}\right)$ or $(-13, 6)$.

The gradient of $[AD]$ is $\frac{-9-21}{-16-(-10)} = \frac{-30}{-6} = 5$.

So, the green edge has gradient $-\frac{1}{5}$.

$$\begin{aligned} \therefore \text{its equation is } y &= -\frac{1}{5}(x - (-13)) + 6 \\ &= -\frac{1}{5}(x + 13) + 6 \\ &= -\frac{1}{5}x - \frac{13}{5} + 6 \\ &= -\frac{1}{5}x + \frac{17}{5} \end{aligned}$$

b The blue edge and green edge intersect where $2x + 21 = -\frac{1}{5}x + \frac{17}{5}$

$$\begin{aligned} \therefore \frac{11}{5}x &= -\frac{88}{5} \\ \therefore 11x &= -88 \\ \therefore x &= -8 \end{aligned}$$

$$\begin{aligned} \text{When } x = -8, \quad y &= 2(-8) + 21 \quad \{\text{using the blue line}\} \\ &= -16 + 21 \\ &= 5 \end{aligned}$$

$\therefore V_1$ has coordinates $(-8, 5)$.

c i V_1 is equidistant from A, B, and D.

$$\begin{aligned} V_1A &= \sqrt{(-10 - (-8))^2 + (21 - 5)^2} \\ &= \sqrt{(-2)^2 + 16^2} \\ &= \sqrt{260} \text{ km} \end{aligned}$$

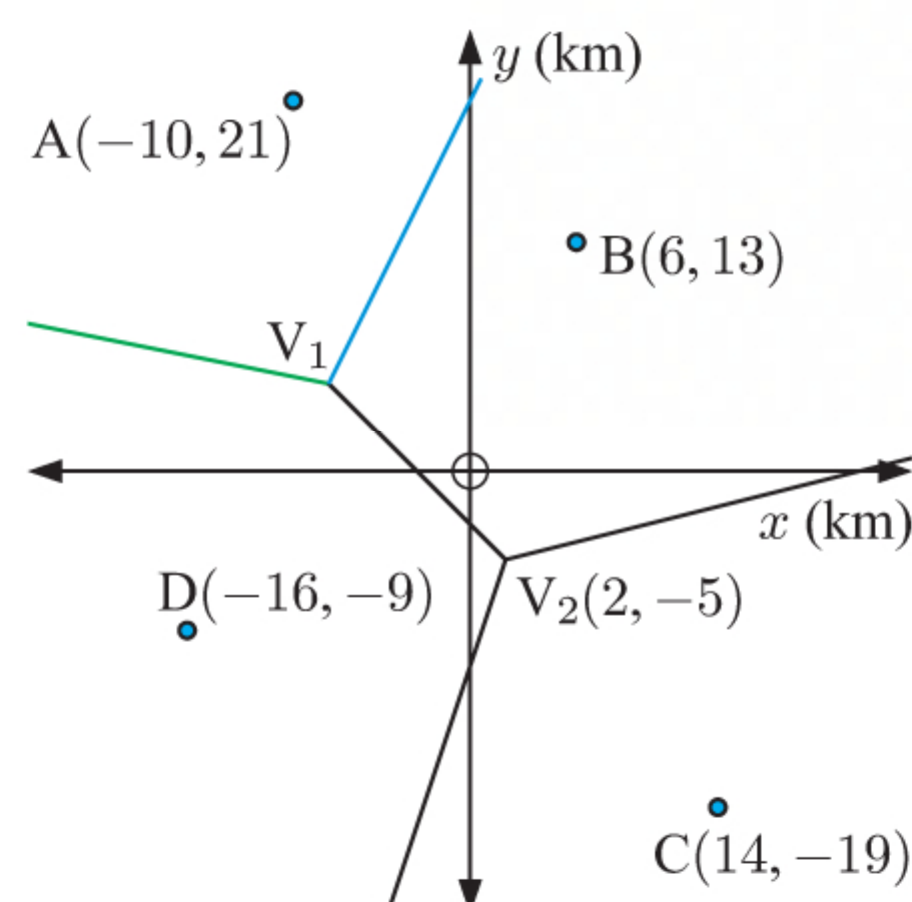
V_2 is equidistant from B, C, and D.

$$\begin{aligned} V_2B &= \sqrt{(6 - 2)^2 + (13 - (-5))^2} \\ &= \sqrt{4^2 + 18^2} \\ &= \sqrt{340} \text{ km} \end{aligned}$$

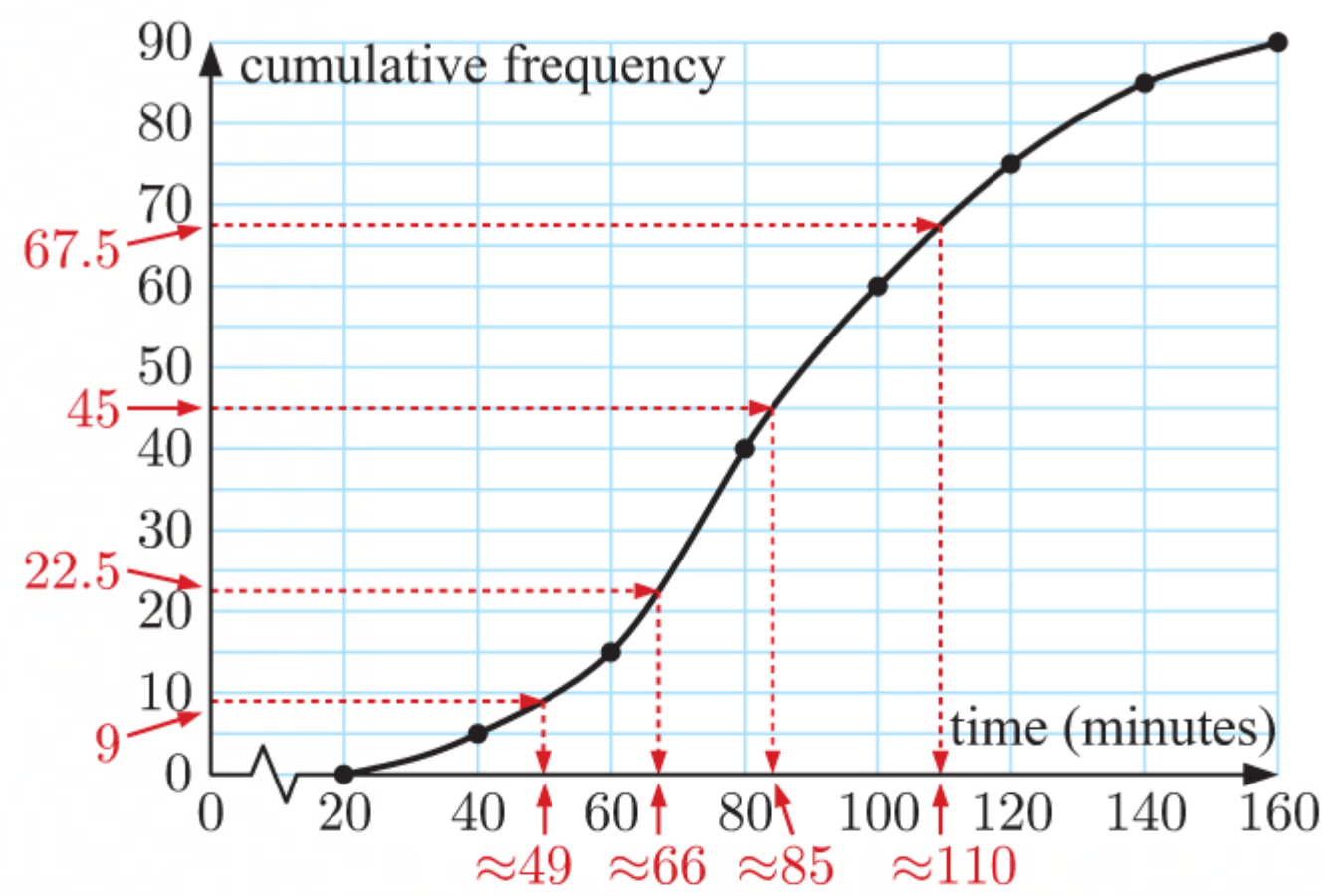
So, the largest empty circle has centre $V_2(2, -5)$ and radius $\sqrt{340}$ km.

\therefore Ivan should open the weekend retreat at $V_2(2, -5)$.

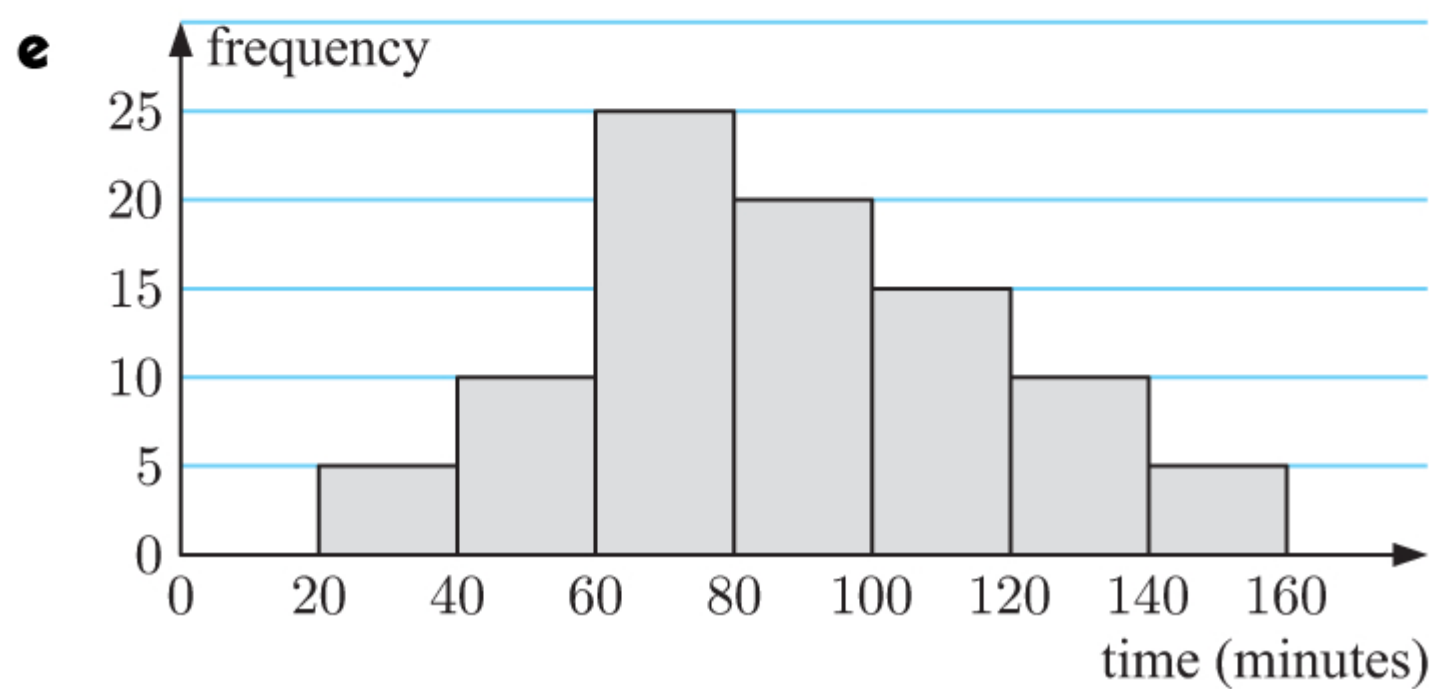
ii Towns B, C, and D are closest to the retreat.



11



- a** From the graph, 90 games were played.
- b** The median corresponds to cumulative frequency = 45.
Hence, the median game length is about 85 minutes.
- c** $IQR = Q_3 - Q_1$
 Q_3 corresponds to cumulative frequency = 67.5 which is ≈ 110 minutes.
 Q_1 corresponds to cumulative frequency = 22.5 which is ≈ 66 minutes.
 $\therefore IQR \approx 110 - 66 = 44$ minutes.
- d** The 10th percentile corresponds to cumulative frequency = 9 which is ≈ 49 minutes.



- 12 a** From the graph, $f(-2) = -1$

$$\therefore -2 + \frac{k}{(-2)^2} = -1$$

$$\therefore \frac{k}{4} = 1$$

$$\therefore k = 4$$

- c** P is a stationary point of $f(x)$.

$$\text{Now } f'(x) = 0 \text{ where } 1 - \frac{8}{x^3} = 0$$

$$\therefore x^3 = 8$$

$$\therefore x = 2$$

$$f(2) = 2 + \frac{4}{2^2}$$

$$= 2 + 1$$

$$= 3$$

So, P has coordinates (2, 3).

e Shaded area = $\int_2^6 f(x) dx$

$$= \left[\frac{1}{2}x^2 - \frac{4}{x} \right]_2^6 \quad \{\text{using d}\}$$

$$= \left(\frac{1}{2}(6)^2 - \frac{4}{6} \right) - \left(\frac{1}{2}(2)^2 - \frac{4}{2} \right)$$

$$= \frac{52}{3} - 0$$

$$= \frac{52}{3} \text{ units}^2$$

b $f(x) = x + \frac{4}{x^2} = x + 4x^{-2}$

$$\therefore f'(x) = 1 - 8x^{-3} = 1 - \frac{8}{x^3}$$

d $\int f(x) dx = \int (x + 4x^{-2}) dx$

$$= \frac{1}{2}x^2 + \frac{4}{-1}x^{-1} + c$$

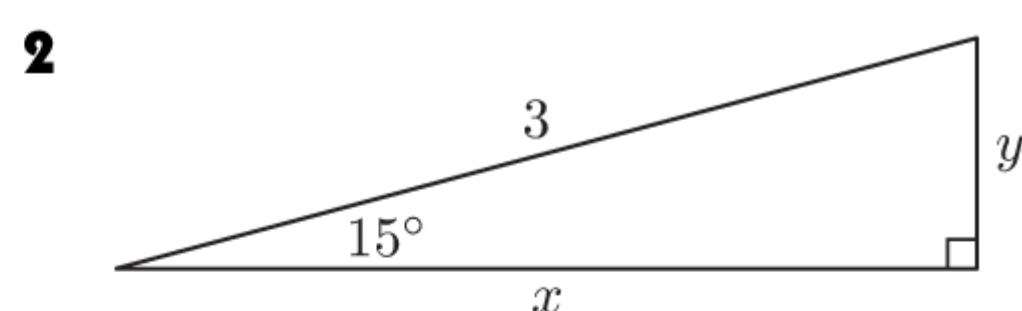
$$= \frac{1}{2}x^2 - \frac{4}{x} + c$$

TRIAL EXAMINATION 1

PAPER 1

- 1 a** $90 + 6 \times 15 = 180 \text{ cm}$ A1
- b** $H(t) = 6t + 90$ A1
- c** $H = 6t + 90$
 $\therefore 6t = H - 90$
 $\therefore t = \frac{H - 90}{6}$ A1
- d** When $H = 300$, $t = \frac{300 - 90}{6} = 35 \text{ weeks}$ M1
 \therefore range of t is $\{t \mid 0 \leq t \leq 35\}$ A1

Total [5 marks]



$$\sin 15^\circ = \frac{y}{3}$$

$$\therefore y = 3 \sin 15^\circ$$

$$\cos 15^\circ = \frac{x}{3}$$

$$\therefore x = 3 \cos 15^\circ$$

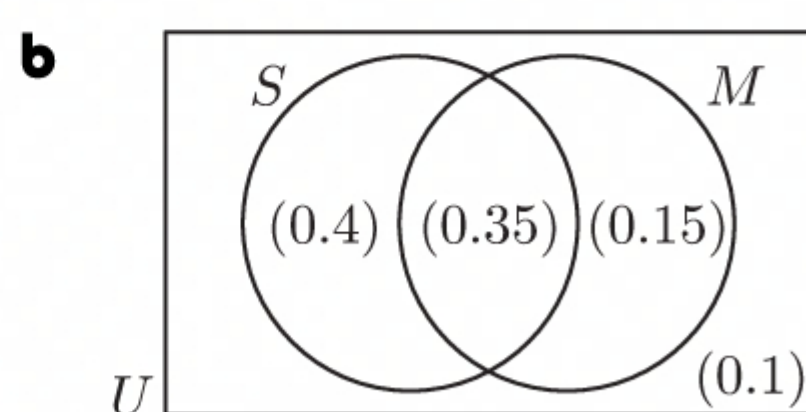
$$\therefore \text{area} = \frac{1}{2} \times (3 \cos 15^\circ) \times (3 \sin 15^\circ)$$

$$= 1.125 \text{ m}^2$$

A1
A1
(M1)
A1

Total [4 marks]

- 3 a** $P(S \cup M) = P(S) + P(M) - P(S \cap M)$
 $\therefore 0.9 = 0.75 + 0.5 - P(S \cap M)$ (M1)
 $\therefore P(S \cap M) = 0.35$ A1



$$P(S \mid M') = \frac{P(S \cap M')}{P(M')}$$

$$= \frac{0.4}{0.5}$$

$$= 0.8$$

M1
A1

Total [4 marks]

- 4 a** $X \sim B(7, 0.6)$ A1
 Assuming shots are independent events. R1
- b** $P(X < 3) \approx 0.0963$ A1
- c** $E(X) = 7 \times 0.6 = 4.2$ A1
 $\text{Var}(X) = 7 \times 0.6 \times 0.4 = 1.68$ A1

Total [5 marks]

- 5 a** $N = 36$, $I\% = 4.1$, $PV = -5000$, $FV = 0$, $P/Y = 12$, $C/Y = 12$ M1
 $\therefore PMT = \$147.85$ A1
- b** $N = 12$, $I\% = 4.1$, $PV = -5000$, $PMT = 147.85$, $P/Y = 12$, $C/Y = 12$ M1
 $\therefore FV = \$3400.97$ A1

$$\mathbf{c} \quad N = 24, \quad I\% = 5.3, \quad PV = -3400.97, \quad FV = 0, \quad P/Y = 12, \quad C/Y = 12$$

M1

$$\therefore PMT = \$149.67$$

A1

Total [6 marks]

$$\begin{aligned} \mathbf{6} \quad \mathbf{a} \quad d &= \frac{4.9 + 2.5}{2} \\ &= \frac{7.4}{2} \\ &= 3.7 \end{aligned}$$

M1

AG

$$\begin{aligned} \mathbf{b} \quad a &= \frac{4.9 - 2.5}{2} \\ &= \frac{2.4}{2} \\ &= 1.2 \end{aligned}$$

A1

$$\mathbf{c} \quad \frac{1}{2} \text{ of a period is } 09:18 \text{ to } 15:30 \text{ which is } 6 \text{ h } 12 \text{ min}$$

$$\therefore \text{period} = 12 \text{ h } 24 \text{ min} = 12.4 \text{ h}$$

A1

$$\therefore \text{period} = \frac{360}{b}$$

$$\therefore b = \frac{360}{12.4} \approx 29.03$$

A1

$$\mathbf{d} \quad w(t) \approx 1.2 \sin(29.03t) + 3.7, \quad 0 \leq t \leq 24$$

$$\text{Using technology, } w(t) > 3.3 \text{ when } 0 < t < 6.87 \text{ and } 11.73 < t < 19.27$$

M1

$$\therefore \text{latest time is } t = 19.27 \approx 19:16$$

A1

Total [6 marks]

$$\mathbf{7} \quad y = x^2 + \frac{4}{x}$$

$$\mathbf{a} \quad \text{When } x = 2, \quad k = 2^2 + \frac{4}{2} = 6$$

A1

$$\begin{aligned} \mathbf{b} \quad \frac{dy}{dx} &= 2x - 4x^{-2} \\ &= 2x - \frac{4}{x^2} \end{aligned}$$

M1A1

$$\mathbf{c} \quad \text{Gradient at A} = 2(2) - \frac{4}{2^2} = 3$$

M1

$$\therefore \text{the tangent at A has equation } y = 3x + c$$

M1

$$\text{When } x = 2, \quad y = 6, \text{ so } 6 = 3(2) + c$$

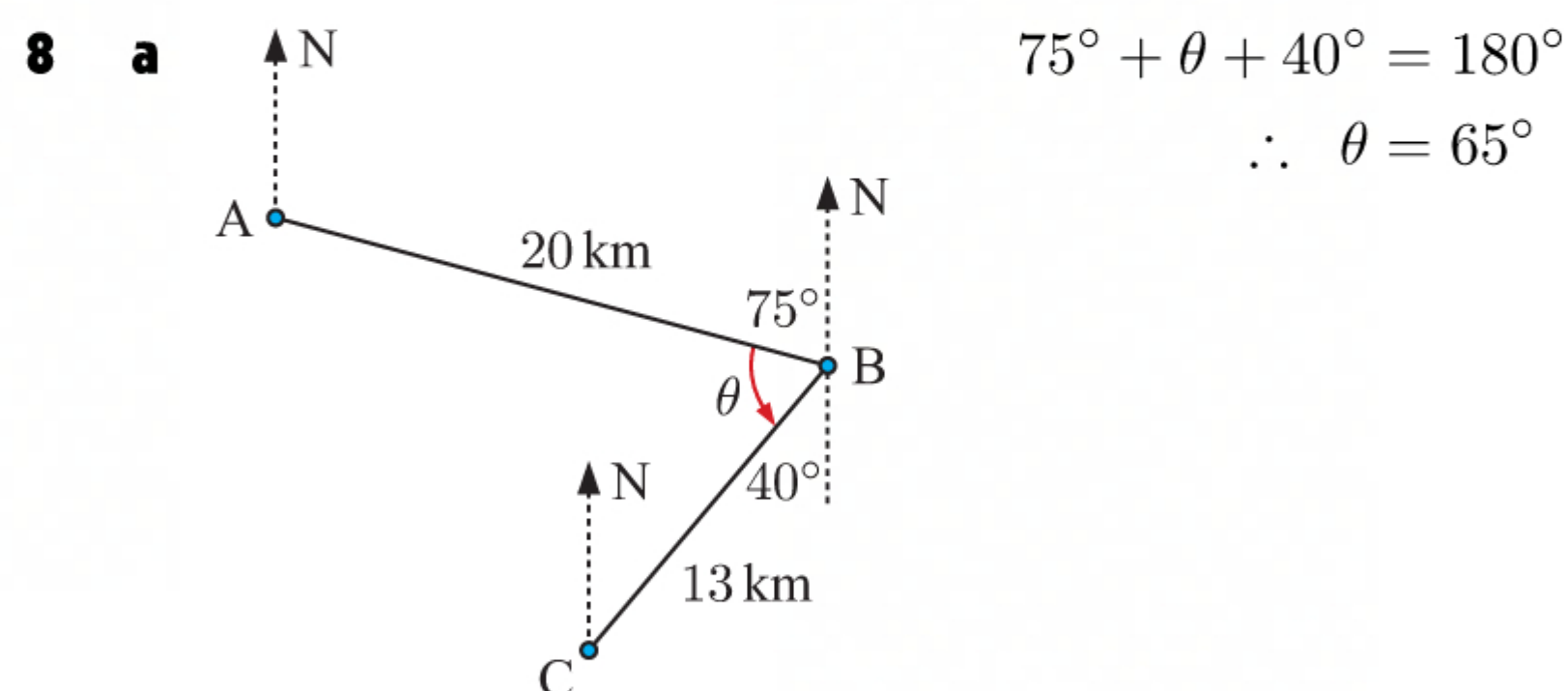
$$\therefore c = 0$$

R1

So, the tangent at A passes through the origin.

AG

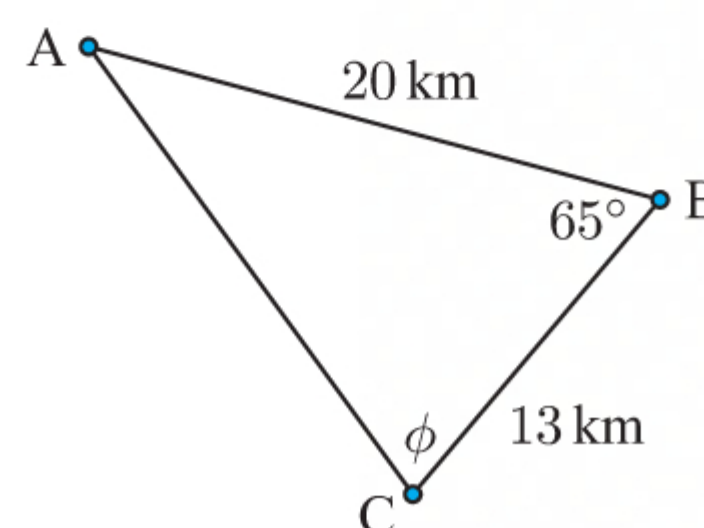
Total [6 marks]



R1

AG

$$\begin{aligned} \mathbf{b} \quad AC^2 &= 20^2 + 13^2 - 2 \times 20 \times 13 \times \cos 65^\circ \\ \therefore AC &\approx 18.7 \text{ km} \end{aligned}$$

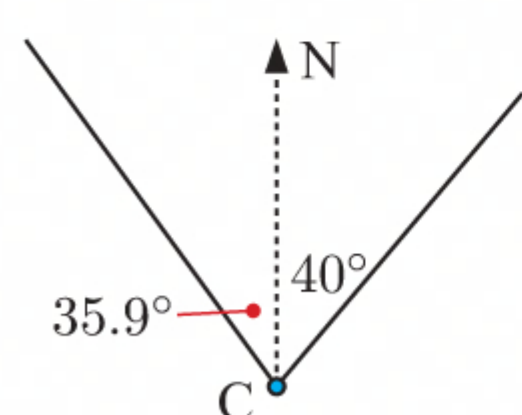


(M1)A1

A1

$$\text{c } \cos \phi \approx \frac{13^2 + 18.7^2 - 20^2}{2 \times 13 \times 18.7} \approx 0.243 \quad (\text{M1})$$

$$\therefore \phi \approx 75.9^\circ \quad \text{A1}$$



Bearing from C to A

$$\approx 360^\circ - 35.9^\circ$$

$$\approx 324^\circ$$

A1

Total [7 marks]

9 H_0 : Age independent of feedback

A1

H_1 : Age not independent of feedback

Degrees of freedom = 4

A1

$$\chi^2 \approx 7.04$$

A1

$$p\text{-value} \approx 0.134$$

A1

$$0.134 > 0.1 \quad \{10\% \equiv 0.1\}$$

R1

\therefore accept H_0

A1

\therefore age is independent of feedback.

Total [6 marks]

10 a 22°C

A1

$$\text{b } T = 22 + 78 \times (0.85)^5$$

(M1)

$$\approx 56.6^\circ\text{C}$$

A1

$$\text{c } \text{Using technology, } 22 + 78(0.85)^t < 30 \text{ when } t > 14.0$$

(M1)

\therefore it takes ≈ 14.0 minutes for the temperature of the soup to drop below 30°C .

A1

Total [5 marks]

$$\text{11 a } \text{Area} = \frac{60}{360} \times \pi \times 20^2$$

(M1)

$$\approx 209.44 \text{ cm}^2$$

A1

$$\text{b } h \approx \frac{1250}{209.44} \approx 5.97$$

A1

$$\text{c } l = \frac{60}{360} \times 2\pi \times 20 \approx 20.94$$

A1

$$\text{d } \text{Total surface area} \approx 2(209.44) + 2(20 \times 5.97) + (20.94 \times 5.97)$$

(M1)(M1)

$$\approx 783 \text{ cm}^2$$

A1

Total [7 marks]

12 a H_0 : $\mu_o = \mu_n$ no difference between the old and new models.

A1

H_1 : $\mu_o > \mu_n$ the new model is faster than the old model.

$$\text{b } t \approx 1.89$$

A1

$$p\text{-value} \approx 0.0385$$

A1

$$\text{c } 0.0385 < 0.05 \quad \{5\% \equiv 0.05\}$$

R1

\therefore we reject H_0 , and accept H_1

A1

\therefore the times for the new model are lower.

Total [5 marks]

$$\text{13 a } X \sim N(4.7, 0.9^2)$$

A1

$$P(X < 3) \approx 0.0295$$

A1

$$\text{b } P(X > a) = 0.1 \Rightarrow a \approx 5.85 \text{ kg}$$

M1A1

$$\text{c } P(\text{one too light and one XL}) \approx 2 \times 0.0295 \times 0.1$$

M1

$$\approx 0.00589$$

A1

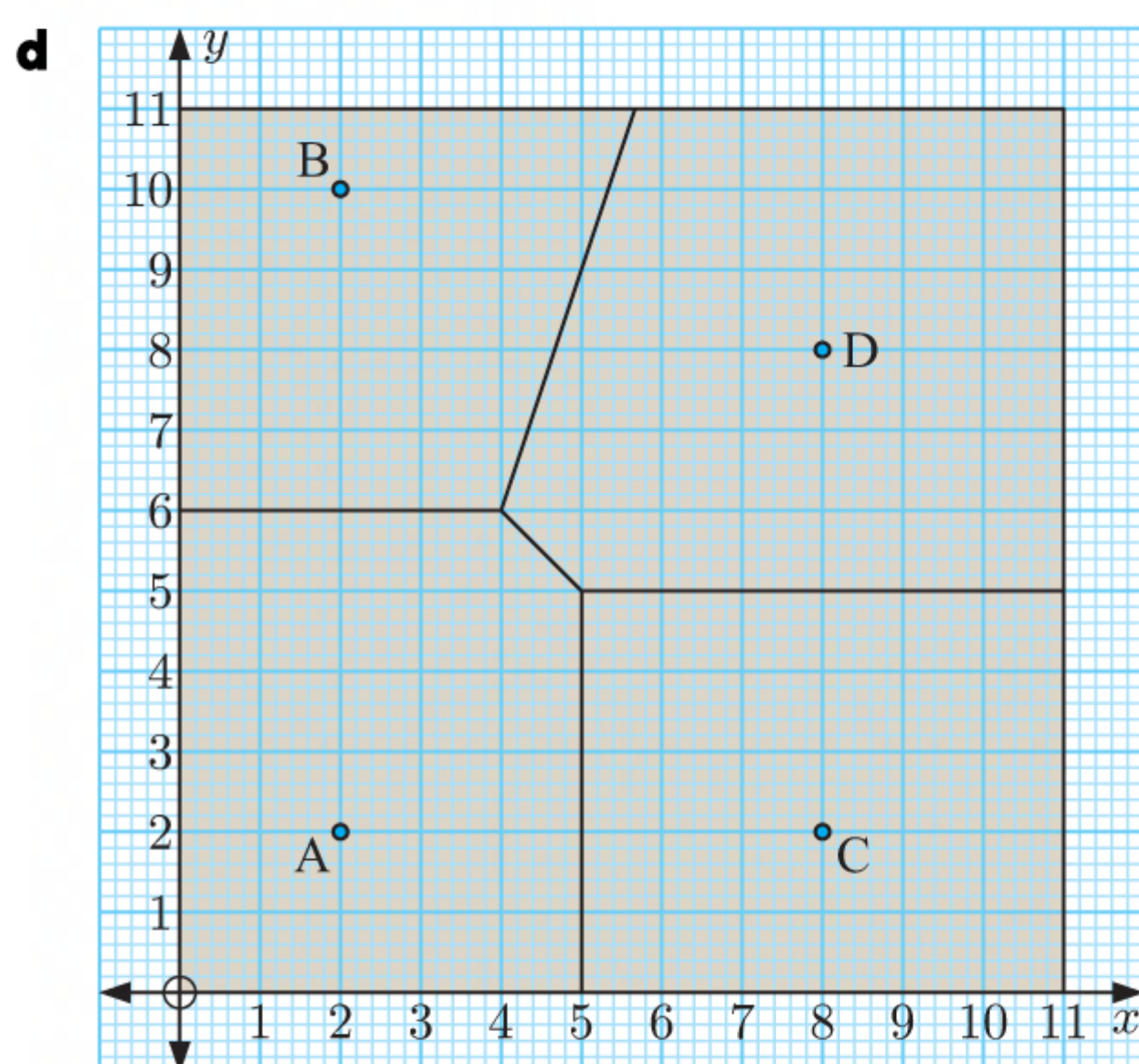
Total [6 marks]

- 14 a** Geometric $\Rightarrow u_{14} = 600 \times (1.10)^{13}$ (M1)
 $= \$2071.36$ A1
- b** Arithmetic $\Rightarrow S_{12} = \frac{12}{2}(2 \times 1000 + 11 \times 50)$ (M1)
 $= \$15\,300$ A1
- c** Total from Ann's contract after n months $= \frac{n}{2}[2000 + 50(n - 1)]$ M1
 $= 975n + 25n^2$
- Total from Greg's contract after n months $= \frac{600(1.1^n - 1)}{1.1 - 1}$ M1
 $= 6000(1.1^n - 1)$
- Using technology, $6000(1.1^n - 1) > 975n + 25n^2$ when $n > 16.48$ A1
- \therefore total amount from Greg's contract first exceeds the total amount from Ann's contract when $n = 17$, A1
 which is May 2021.

Total [8 marks]

PAPER 2

- 1 a** Midpoint $= \left(\frac{2+8}{2}, \frac{10+8}{2} \right) = (5, 9)$ A1
- b** Gradient $= \frac{8-10}{8-2}$ M1
 $= \frac{-2}{6}$
 $= -\frac{1}{3}$ A1
- c** The perpendicular bisector has gradient $-\frac{1}{-\frac{1}{3}} = 3$ M1
 \therefore its equation is $y = 3x + c$ M1
 $(5, 9)$ lies on the line, so $9 = 3(5) + c$
 $\therefore c = -6$ M1
- \therefore the perpendicular bisector of [BD] has equation $y = 3x - 6$
 $\therefore y - 3x + 6 = 0$ AG



- e** The optimum position for the new station is at the vertex with the circle of largest radius to adjacent sites.
- For vertex $(4, 6)$, radius $= \sqrt{2^2 + 4^2} = \sqrt{20}$ M1
- For vertex $(5, 5)$, radius $= \sqrt{3^2 + 3^2} = \sqrt{18}$ M1
- \therefore optimal position is $(4, 6)$. A1

Total [14 marks]

(M1) : 2 correct PB's
 (M1) : 4 correct PB's
 (M1) : 5 or 6 correct PB's
 A1 : 2 correct Voronoi regions
 A1 : 4 correct Voronoi regions

2 a $y = \frac{1}{50}x(x^2 - 26x + 160)$

$\therefore y = \frac{1}{50}x^3 - \frac{26}{50}x^2 + \frac{160}{50}x$ (M1)

$\Rightarrow \frac{dy}{dx} = \frac{3}{50}x^2 - \frac{52}{50}x + \frac{160}{50}$ A2

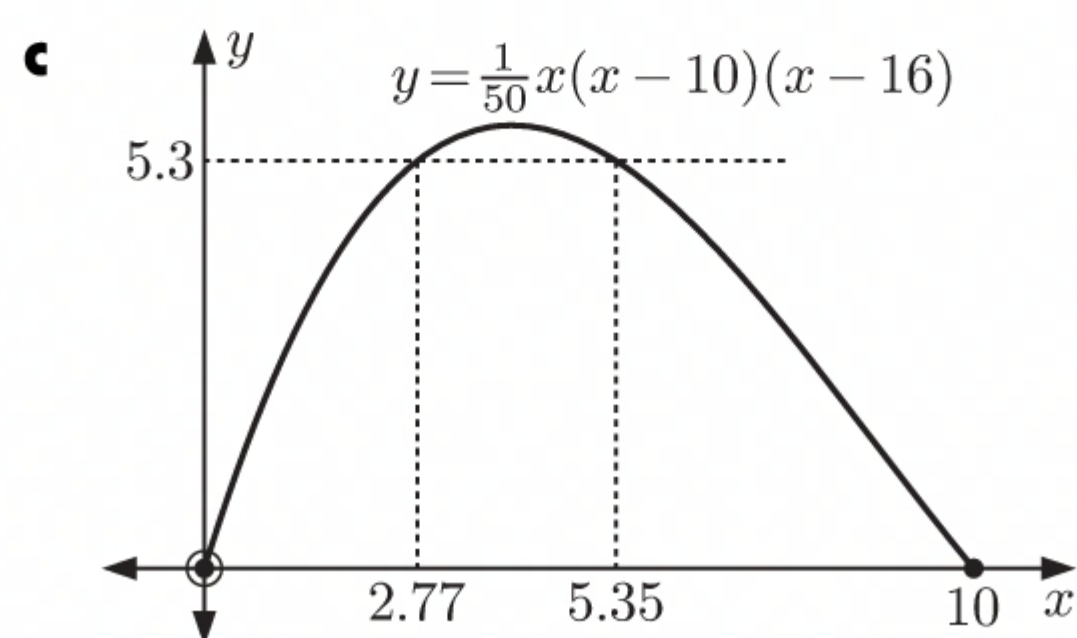
b $\frac{dy}{dx} = 0$ when $3x^2 - 52x + 160 = 0$

$\therefore (3x - 40)(x - 4) = 0$

$\therefore x = 4 \quad \{0 \leq x \leq 10\}$ A1

$\therefore y = 5.76 \text{ m}$ A1

or using technology, maximum occurs at (4, 5.76).



When $y = 5.3$, $x \approx 2.77$ or 5.35

M1

\therefore maximum width $\approx 5.35 - 2.77$

$\approx 2.58 \text{ m}$ M1

$2.8 > 2.58$, so the truck cannot fit in the tunnel.

R1

d Area $= \int_0^{10} \frac{1}{50}x(x-10)(x-16) dx$

M1

$= 36\frac{2}{3} \text{ m}^2$ (A1)

Volume $= 100 \times 36\frac{2}{3}$

$= 3666\frac{2}{3}$

$\approx 3667 \text{ m}^3$

A1

e Area $\approx \frac{2}{2}[0 + 0 + 2(4.48 + 5.76 + 4.8 + 2.56)]$

(M1)

$\approx 35.2 \text{ m}^2$

(A1)

Volume $\approx 100 \times 35.2$

$\approx 3520 \text{ m}^3$

A1

f % error $= \frac{3666\frac{2}{3} - 3520}{3666\frac{2}{3}} \times 100\%$

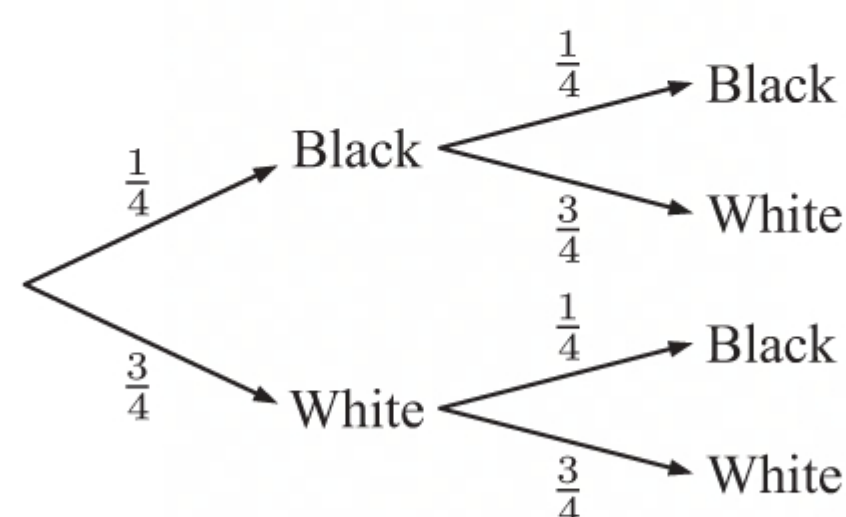
M1

$= 4\%$

A1

Total [16 marks]

3 a



A1A1

b $P(X = 0) = P(WW)$

$= \frac{3}{4} \times \frac{3}{4}$

$= \frac{9}{16}$

$\therefore P(X = 1) = 1 - \frac{9}{16} - \frac{1}{16}$

$= \frac{6}{16}$

A1

A1

c $E(X) = 0 \times \frac{9}{16} + 1 \times \frac{6}{16} + 2 \times \frac{1}{16}$

$= \frac{8}{16}$

$= 0.5$

M1

A1

x	0	1	2
$P(X = x)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

$$\begin{aligned} \text{d } E(\text{Winnings}) &= 0 \times \frac{9}{16} + 5 \times \frac{6}{16} + 10 \times \frac{1}{16} \\ &= \frac{40}{16} \\ &= \$2.50 \end{aligned}$$

A1

$$\therefore \text{expected profit per game} = \$2.50 - \$3 = -\$0.50$$

A1

$$\therefore \text{expected loss from 10 games is } 10 \times \$0.50 = \$5$$

A1

e

0 blacks	1 black	2 blacks
56.25	37.5	6.25

A2

f Degrees of freedom = $3 - 1 = 2$

A1

g $\chi^2 \approx 4.107$

A1

$$p\text{-value} \approx 0.128$$

A2

h $0.128 > 0.05$

R1

There is insufficient evidence to conclude that the spinner is biased.

A1

Total [17 marks]

4 a For example, use the 12 students in her class as the sample.

A1

b For example, choose every 10th student from an alphabetised list of the students in her grade.

A1

c 9 33 56 | 58 61 69 | 72 72 72 | 79 80 85

$$\text{Median} = 70.5$$

A1

$$\text{Lower quartile} = 57, \quad \text{Upper quartile} = 75.5$$

(M1)

$$\therefore \text{IQR} = 75.5 - 57 = 18.5$$

A1

d $1.5 \times \text{IQR} = 1.5 \times 18.5 = 27.75$

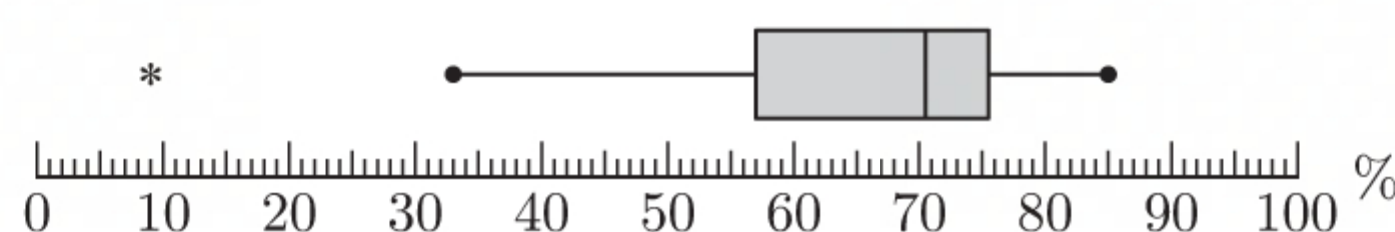
$$\text{lower boundary} = 57 - 27.75 = 29.25$$

$$\text{upper boundary} = 75.5 + 27.75 = 103.25$$

M1

$\therefore 9$ is an outlier.

A1



A1

e $r \approx 0.862$

A2

f The regression line of p against m is $p \approx 0.7857m + 17.64$

M1A1

$$\therefore \text{when } m = 68, p \approx 71\%$$

A1

g Ranks are

rank of p	1	2	3	4	5	8	6	8	10	8	11	12
rank of m	1	2	3	4	5.5	5.5	7	8	9	10	11	12

(M1)

(M1)

$$\therefore r_s \approx 0.956$$

A1

h r_s less sensitive to outliers than r .

R1

Total [17 marks]

5 a $N(0) = 99$

$$\therefore a \times b^0 = 99$$

$$\therefore a = 99$$

A1

$$N(5) = 184$$

$$\therefore 99 \times b^5 = 184$$

$$\therefore b = \sqrt[5]{\frac{184}{99}} \approx 1.132$$

A1

b 13% increase.

A1

- c** $N(20) = 99 \times 1.132^{20}$ (M1)
 ≈ 1180 birds A1
- d** $N(30) = 1180 \times (0.83)^{10}$ (M1)
 ≈ 183 birds A1
- e** $N(t) < 30$ when $t > 39.7$ (M1)A1
 \therefore the population will first drop below 30 in the 40th year after 1980 which is the year 2020. A1
- f** $0 < t < 20$, $N(t) > 500$ when $t > 13.1$ (M1)
 $t > 20$, $N(t) > 500$ when $t < 24.6$ (M1)
 $\therefore N(t) > 500$ when $13.1 < t < 24.6$ which is ≈ 11.5 years. A1
- g** $N(40) = 1180 \times (0.83)^{20} \approx 28$ birds. (M1)
 \therefore Population in 2020 = $28 + 100 = 128$ birds.
 \therefore in 2030, population = $128 \times (1.08)^{10}$ (M1)
 ≈ 276 birds A1

Total [16 marks]

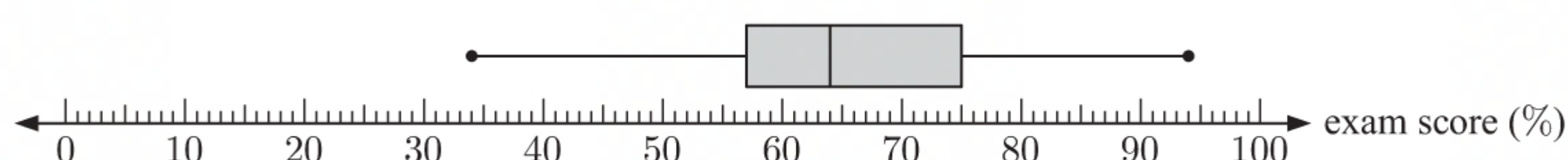
TRIAL EXAMINATION 2

PAPER 1

- 1 a** $129 - 13 = 116$ out of 129, which is about 89.9% of students, obtained at least 50% for their examination. **A1**

- b** The 5-number summary for the results is:
- $$\begin{aligned} \min &= 34 \\ Q_1 &= 57 \\ \text{median} &= 64 \\ Q_3 &= 75 \\ \max &= 94 \end{aligned}$$
- (M1)**

The box and whisker diagram is:



A1A1

- c** The interquartile range $= Q_3 - Q_1$
 $= 75 - 57$
 $= 18$ **A1**

The middle half of results are spread across 18%. **A1**

Total [6 marks]

- 2 a** Each year, the rent Georgio pays is multiplied by the same constant 1.025.
 \therefore the rent follows a geometric sequence with common ratio $r = 1.025$. **R1**
A1

- b** The rent in 2020 is $u_1 = \text{€}12\,000$
 \therefore the rent in 2025 is $u_6 = \text{€}12\,000 \times 1.025^5$ **M1**
 $= \text{€}13\,576.90$ **A1**

- c** The total rent Georgio pays from 2020 to 2025 inclusive is $S_6 = \frac{u_1(r^6 - 1)}{r - 1}$ **M1**
 $= \text{€} \frac{12\,000(1.025^6 - 1)}{0.025}$
 $= \text{€}76\,652.84$ **A1**

Total [6 marks]

- 3 a** In a stratified sample, each age group is sampled according to its proportion of the population.
 Now 28.3% of $250 = 0.283 \times 250$ **M1**
 $= 70.75$

\therefore 71 customers in the age range 51 - 70 were sampled. **A1**

- b** The total number of customers surveyed $= 250$
 $\therefore 55 + x + 56 + 42 + y + 17 + 6 = 250$
 $\therefore x + y = 74 \dots (1)$ **A1**

The mean number of visits per week was 3.08.

$$\begin{aligned} \therefore \frac{1 \times 55 + 2x + 3 \times 56 + 4 \times 42 + 5y + 6 \times 17 + 7 \times 6}{250} &= 3.08 \\ \therefore 2x + 5y + 535 &= 770 \\ \therefore 2x + 5y &= 235 \dots (2) \end{aligned}$$

A1

- c** Using (1), $2x + 2y = 148 \dots (3)$
 $(2) - (3)$ gives $3y = 87$
 $\therefore y = 29$ **A1**
 $\therefore x = 74 - 29 = 45$ **A1**

Total [6 marks]

- 4 a** Since $V \propto r^3$, if the radius of a sphere is doubled, its volume is multiplied by $2^3 = 8$. A1

b $V = \frac{4}{3}\pi r^3$

$$\therefore r^3 = \frac{3V}{4\pi}$$

$$\therefore r = \sqrt[3]{\frac{3V}{4\pi}}$$

A1

This function tells us the radius of a sphere with volume V .

A1

Total [3 marks]

5 a $P(0) = 24$

$$\therefore A \times 2^0 = 24$$

$$\therefore A = 24$$

A1

b $P(2) = 2 \times 24 = 48$

$$\therefore 24 \times 2^{2k} = 48$$

(M1)

$$\therefore 2^{2k} = 2$$

$$\therefore k = \frac{1}{2}$$

A1

c $P(t) = 24 \times 2^{\frac{1}{2}t}$

$$\therefore P(11) = 24 \times 2^{\frac{11}{2}}$$

M1

$$\approx 1086$$

A1

After 11 months there will be about 1086 guinea pigs.

Total [5 marks]

- 6 a** H_0 : Teacher type and pet preference are independent. A1

- b** The 3×4 contingency table is:

	Cat	Dog	Bird	Other	Sum
Arts & Humanities	12	17	16	11	56
Sciences	18	9	5	8	40
Sports	5	8	3	2	18
Sum	35	34	24	21	114

The expected frequency table is:

	Cat	Dog	Bird	Other
Arts & Humanities	17.19	16.70	11.79	10.32
Sciences	12.28	11.93	8.42	7.37
Sports	5.53	5.37	3.79	3.32

Using technology, $\chi_{\text{calc}}^2 \approx 9.98$

A1A1

c $\chi_{\text{crit}}^2 = 12.59$

Since $\chi_{\text{calc}}^2 < \chi_{\text{crit}}^2$, we do not have enough evidence to reject H_0 in favour of H_1 on a 5% significance level. We therefore accept H_0 .

R1A1

Total [5 marks]

- 7 a** The missing edge is the perpendicular bisector of CD.

$$\text{The gradient of CD} = \frac{3-1}{2-(-4)} = \frac{2}{6} = \frac{1}{3}$$

A1

$$\text{The midpoint of CD is } \left(\frac{-4+2}{2}, \frac{1+3}{2} \right) \text{ which is } (-1, 2).$$

A1

$$\therefore \text{the missing edge has equation } y - 2 = -3(x + 1)$$

$$\text{Rearranging, } y - 2 = -3x - 3$$

$$\therefore 3x + y + 1 = 0$$

A1

- b** The missing edge must intersect OP at P.

$$\begin{aligned} \text{Substituting } y = -\frac{2}{3}x \text{ into the equation of the missing edge, } 3x + \left(-\frac{2}{3}x\right) + 1 &= 0 \\ \therefore \frac{7}{3}x &= -1 \\ \therefore x &= -\frac{3}{7} \\ \therefore y &= \frac{2}{7} \end{aligned}$$

A1

$$\therefore P \text{ is } \left(-\frac{3}{7}, \frac{2}{7}\right)$$

A1

- c** For Hamsika's restaurant to be as far as possible from the other restaurants, it should be located at one of the Voronoi vertices $\left(-\frac{3}{7}, \frac{2}{7}\right)$, $(0, 0)$, or $(1, 0)$. **(M1)**

$$\text{Now } PD = \sqrt{\left(2 + \frac{3}{7}\right)^2 + \left(3 - \frac{2}{7}\right)^2} \approx 3.64$$

$$OD = \sqrt{2^2 + 3^2} \approx 3.61$$

$$\text{and } QD = \sqrt{(2-1)^2 + 3^2} \approx 3.16$$

M1

P is the most appropriate location, as it is furthest from the existing restaurants.

A1

Total [8 marks]

- 8 a i** a is the amplitude of the function, which is the length of the minute hand.

$$\therefore a = 4.3$$

A1

- ii** d is the mean height of the tip of the minute hand above the ground.

$$\therefore d = 53$$

A1

- iii** The period of the function is the time it takes for the minute hand to complete one revolution.

$$\therefore \text{the period is 60 minutes.}$$

M1

$$\therefore \frac{360}{b} = 60$$

$$\therefore b = 6$$

A1

$$\mathbf{b} \quad h(t) = 4.3 \cos(6t)^\circ + 53 \text{ m}$$

$$\therefore h(23) = 4.3 \cos 138^\circ + 53$$

M1

$$\approx 49.8 \text{ m}$$

A1

23 minutes past the hour, the tip of the minute hand is about 49.8 m above the ground.

Total [6 marks]

- 9 a** $H_0: \mu_m = \mu_f$ {males and females have the same mean weight}

A1

$$H_1: \mu_m \neq \mu_f \text{ {males and females have different mean weights}}$$

A1

- b** Using technology to conduct a two-sample t -test, p -value ≈ 0.954 .

A2

- c** Since p -value > 0.05 , we do not have enough evidence to reject H_0 in favour of H_1 on a 5% significance level. We therefore accept H_0 . **R1A1**

Total [6 marks]

- 10 a** $f(x) = -(x+2)(x-1)$ is zero when $x = -2$ or 1 .

$$\therefore \text{the } x\text{-intercepts are } -2 \text{ and } 1.$$

A1

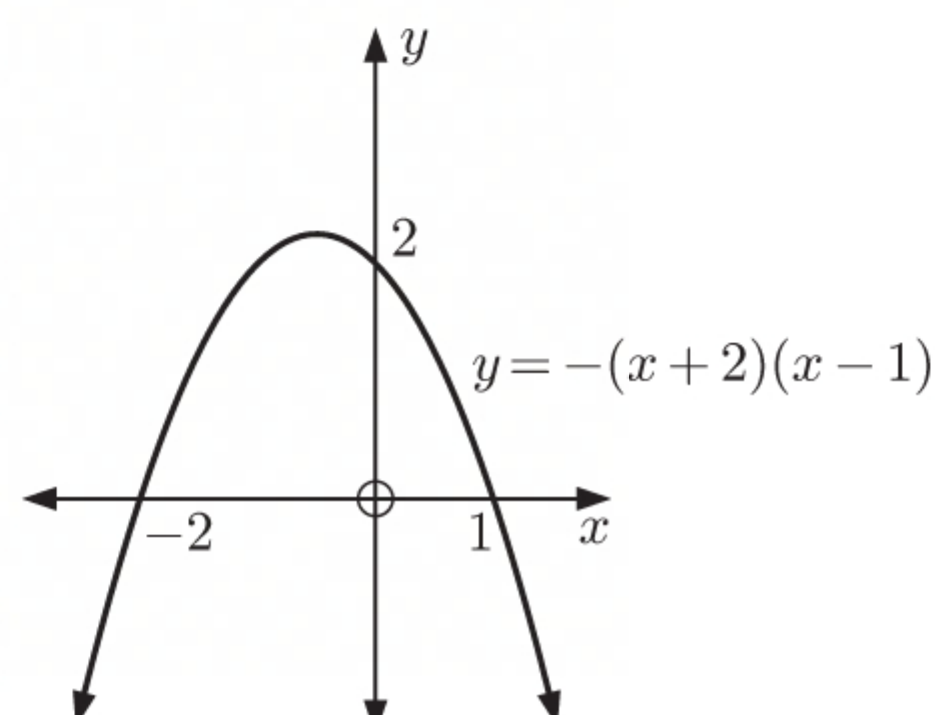
$$f(0) = -(2)(-1) = 2$$

$$\therefore \text{the } y\text{-intercept is } 2.$$

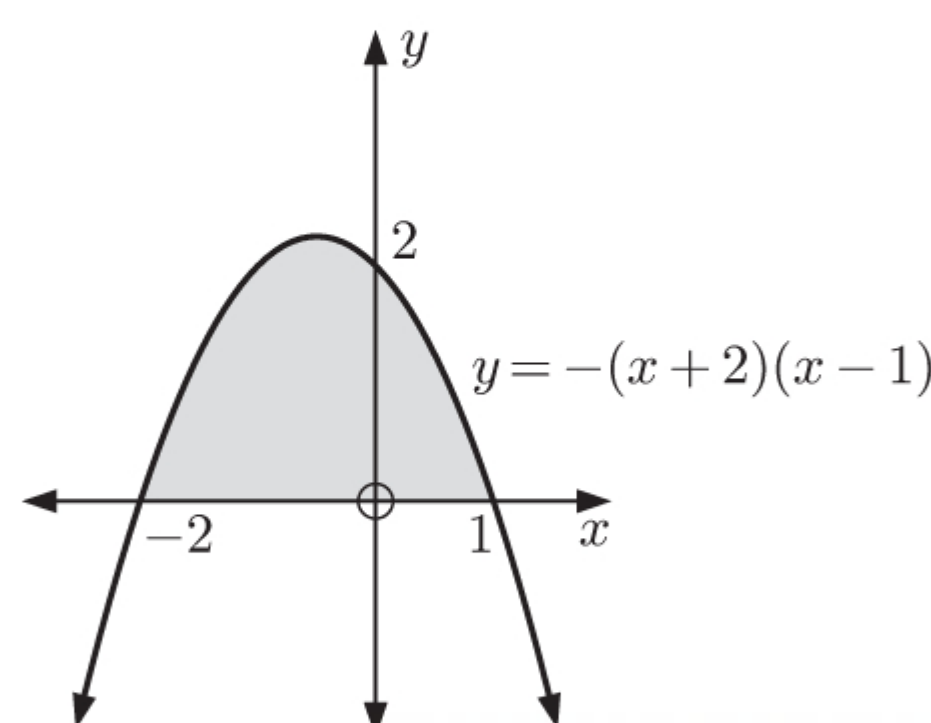
A1

b

A2



$$\begin{aligned}
 \text{c Area} &= \int_{-2}^1 -(x+2)(x-1) dx \\
 &= - \int_{-2}^1 (x^2 + x - 2) dx \\
 &= - \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x \right]_{-2}^1 \\
 &= - \left(\frac{1}{3} + \frac{1}{2} - 2 \right) + \left(\frac{-8}{3} + \frac{4}{2} + 4 \right) \\
 &= \frac{7}{6} + \frac{10}{3} \\
 &= 4\frac{1}{2} \text{ units}^2
 \end{aligned}$$



M1

A1

Total [6 marks]

$$\begin{aligned}
 11 \text{ a Area of triangle OAB} &= \frac{1}{2}r^2 \sin \theta \\
 &= \frac{1}{2} \times 0.4^2 \times \sin 70^\circ \\
 &\approx 0.0752 \text{ m}^2
 \end{aligned}$$

M1

A1

$$\begin{aligned}
 \text{b Volume of log} &= \text{cross-sectional area} \times \text{length} \\
 &= \left(\frac{290}{360} \times \pi r^2 + \text{area of triangle OAB} \right) \times 3 \\
 &\approx \left(\frac{29}{36} \times \pi \times 0.4^2 + 0.0752 \right) \times 3 \\
 &\approx 1.44 \text{ m}^3
 \end{aligned}$$

(M1)

M1

A1

Total [5 marks]

$$\begin{aligned}
 12 \text{ a } k &= 1 - 0.46 - 0.15 - 0.06 - 0.03 - 0.01 \\
 &= 0.29
 \end{aligned}$$

\therefore 29% of people leave without buying any fish.

A1

$$\begin{aligned}
 \text{b Expected number of fish types bought} \\
 &= 0 \times 0.29 + 0.46 \times 1 + 0.15 \times 2 + 0.06 \times 3 + 0.03 \times 4 + 0.01 \times 5 \\
 &= 1.11
 \end{aligned}$$

M1

A1

$$\begin{aligned}
 \text{c } P(\text{a person buys more than one type of fish}) &= 1 - 0.29 - 0.46 \\
 &= 0.25
 \end{aligned}$$

(M1)

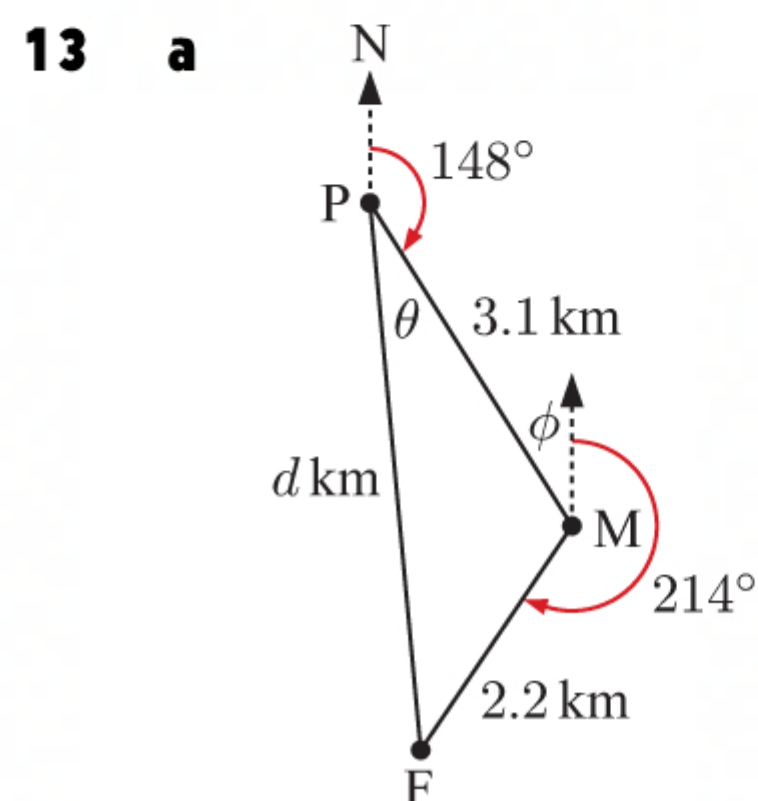
Let X be the number of people in the sample of 3 who buy more than one type of fish.

$$\therefore X \sim B\left(3, \frac{1}{4}\right)$$

$$\begin{aligned}
 P(X \geq 1) &= 1 - P(X = 0) \\
 &= 1 - \binom{3}{0} \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^3 \\
 &\approx 0.578
 \end{aligned}$$

A1

Total [5 marks]



Suppose Eleanor runs d km.

Let θ and ϕ be the angles marked.

$$\begin{aligned}
 \phi &= 180^\circ - 148^\circ \quad \{\text{co-interior angles}\} \\
 &= 32^\circ
 \end{aligned}$$

(M1)

$$\begin{aligned}
 \therefore \widehat{\text{FMP}} &= 360^\circ - 32^\circ - 214^\circ \\
 &= 114^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{Using the cosine rule, } d^2 &= 3.1^2 + 2.2^2 - 2 \times 3.1 \times 2.2 \times \cos 114^\circ \\
 \therefore d &\approx 4.47
 \end{aligned}$$

M1

A1

Eleanor runs about 4.47 km.

$$\begin{aligned}
 \text{b Using the sine rule } \frac{\sin \theta}{2.2} &= \frac{\sin 114^\circ}{d} \\
 \therefore \sin \theta &= \frac{2.2 \sin 114^\circ}{d} \\
 \therefore \theta &\approx 26.7^\circ
 \end{aligned}$$

M1

A1

\therefore Eleanor runs on the bearing $\approx 148^\circ + 026.7^\circ$ which is about 174.7° .

A1

c Morris runs 5.3 km in $\frac{5.3}{8.4} \approx 0.631$ hours
 ≈ 37.86 minutes A1

Eleanor runs about 4.47 km in $\frac{4.47}{6.9} \approx 0.648$ hours
 ≈ 38.87 minutes A1

Morris arrives first and about 1 minute sooner than Eleanor. A1

Total [9 marks]

14 a Let X be the donations received by the animal welfare foundation in one week.
 $\therefore X \sim N(13\,600, 3100^2)$ (M1)

$P(X > 10\,000) \approx 0.877$ {using technology} A1

\therefore the probability that the foundation receives more than €10 000 in a random selected week is approximately 0.877.

b $52 \times 0.877 \approx 45.6$ M1A1

We expect the foundation to receive more than €10 000 in about 45.6 weeks of a 52 week year.

Total [4 marks]

PAPER 2

1 a $N = 5 \times 12 = 60$
 $I\% = 10.1$
 $PMT = 297.57$
 $FV = 0$
 $P/Y = 12$
 $C/Y = 4$ (M1)(A1)
 $\therefore PV = -14\,000$ A1

Corina borrowed \$14 000.

b Interest paid $= 60 \times \$297.57 - \$14\,000$ (M1)
 $= \$3854.20$ A1

c $N = 30 \times 12 = 360$
 $I\% = 5.2$
 $PV = -865\,400$
 $FV = 0$
 $P/Y = 12$
 $C/Y = 12$ (M1)(A1)
 $\therefore PMT = 4752.00$ A1

Corina can afford to withdraw \$4752.00 each month.

d i $N = 6 \times 12 = 72$
 $I\% = 5.2$
 $PV = -865\,400$
 $PMT = 4752.00$
 $P/Y = 12$
 $C/Y = 12$ (M1)(A1)
 $\therefore FV = -780\,952.25$ A1

There is \$780 952.25 in the account when Corina decides to increase her withdrawals.

ii $I\% = 5.2$

$$PV = -780\,952.25$$

$$PMT = 5400$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

$$\therefore N \approx 228$$

(M1)(A1)

A1

Corina's money will run out in the 228th month after her decision to increase her withdrawals.

At this time, she will be $66 + \frac{228}{12} = 85$ years old.

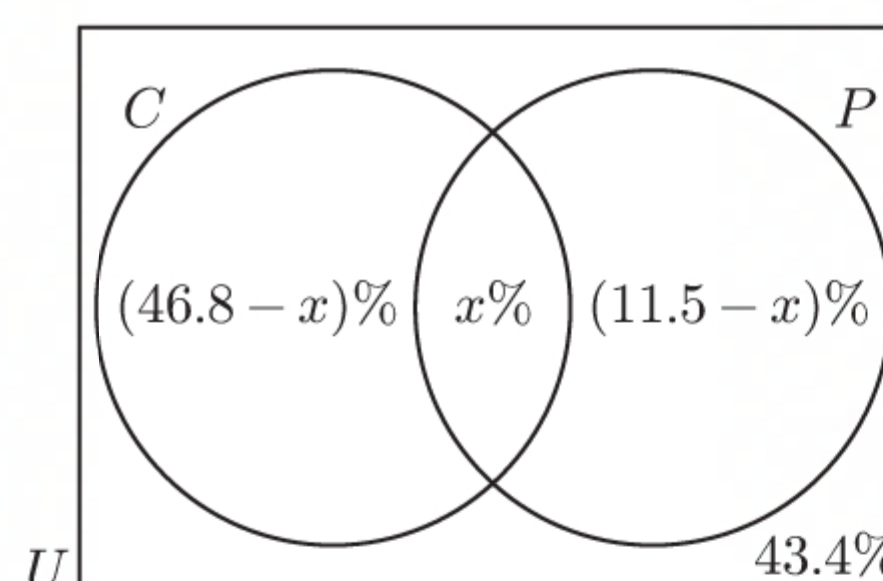
Total [14 marks]

2 a Let the percentage in $C \cap P$ be x .

\therefore the percentage in $C \cap P'$ is $46.8 - x$ and the percentage in $C' \cap P$ is $11.5 - x$.

$$(46.8 - x) + x + (11.5 - x) + 43.4 = 100$$

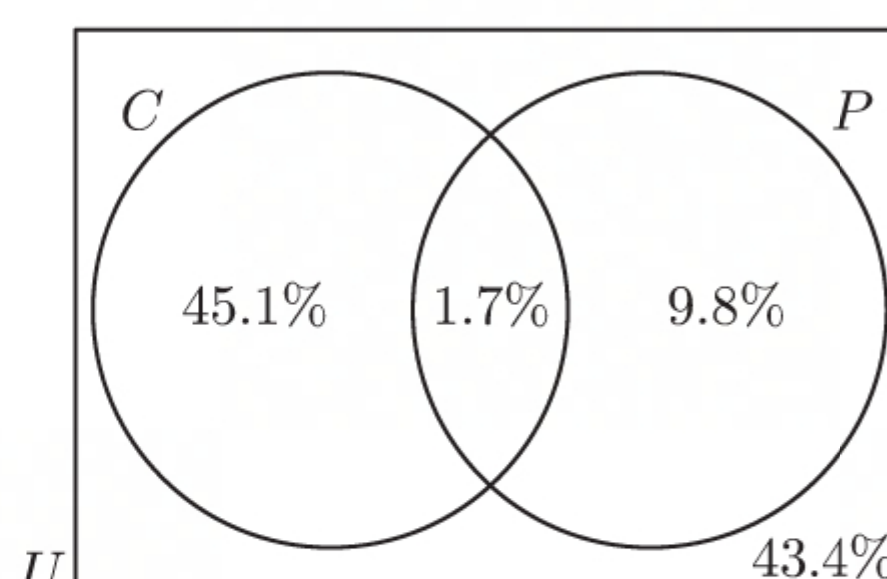
$$\therefore x = 1.7$$



(M1)

(A1)

The completed Venn diagram is:



A2

b $P(C \cap P) = 1.7\% = 0.017$

A1

c
$$P(P' | C') = \frac{P(P' \cap C')}{P(C')}$$

$$= \frac{0.434}{0.098 + 0.434}$$

$$\approx 0.816$$

M1

A1

A1

d The expected number of staff in $C \cap P'$ is $0.451 \times 58\,463 \approx 26\,367$.

M1A1

e i
$$n(C' \cap P) = 500 - 9 - 236 - 205$$

$$= 50$$

A1

ii Let $p_1 = P(C \cap P)$, $p_2 = P(C \cap P')$, $p_3 = P(C' \cap P)$, and $p_4 = P(C' \cap P')$.

The hypotheses that should be tested are:

$$H_0: p_1 = 0.017, p_2 = 0.451, p_3 = 0.098, p_4 = 0.434$$

A1

$$H_1: \text{at least one of } p_1 \neq 0.017, p_2 \neq 0.451, p_3 \neq 0.098, p_4 \neq 0.434$$

A1

iii $df = 4 - 1 = 3$

A1

iv

Group	Expected frequency
$C \cap P$	$0.017 \times 500 = 8.5$
$C \cap P'$	$0.451 \times 500 = 225.5$
$C' \cap P$	$0.098 \times 500 = 49$
$C' \cap P'$	$0.434 \times 500 = 217$

A2

v
$$\chi^2_{\text{calc}} = \sum \frac{(f_{\text{obs}} - f_{\text{exp}})^2}{f_{\text{exp}}}$$

$$= \frac{(9 - 8.5)^2}{8.5} + \frac{(236 - 225.5)^2}{225.5} + \frac{(50 - 49)^2}{49} + \frac{(205 - 217)^2}{217}$$

$$\approx 1.20$$

A2

- vi Since $\chi^2_{\text{calc}} < \chi^2_{\text{crit}}$, we do not have enough evidence to reject H_0 in favour of H_1 on a 5% level of significance. We therefore accept H_0 .

R1A1

\therefore this sample is suitable to answer the extensive questionnaire.

Total [20 marks]

3 a i Height of the model $= 32\,400 \times \frac{8}{12\,500}$
 ≈ 20.736
 $\approx 20.7 \text{ cm}$

M1

- ii The case has base with sides $8 \times 1.02 = 8.16 \text{ cm}$ and height $\approx 20.736 \times 1.02$
 $\approx 21.15 \text{ cm}$

A1

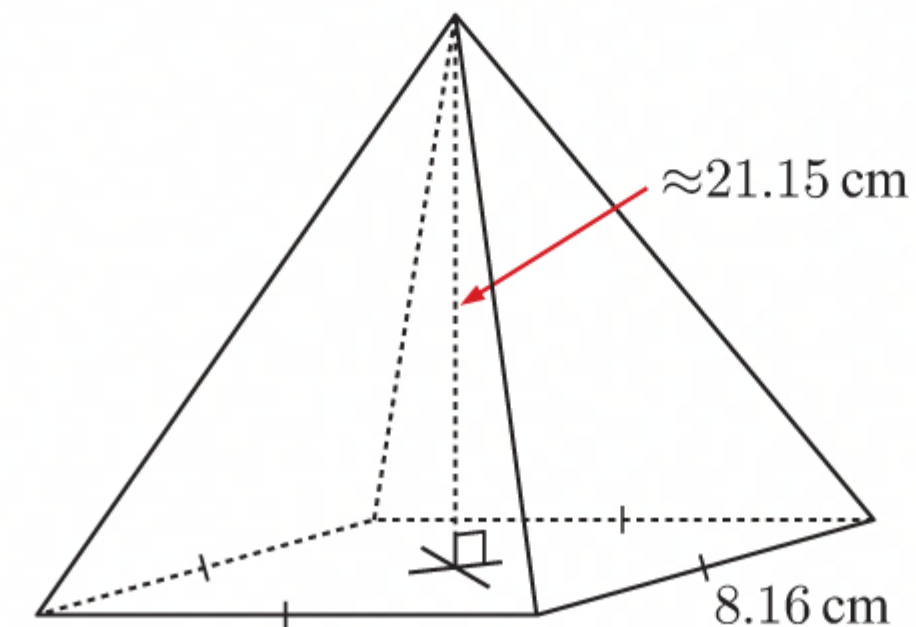
A1

\therefore the volume of the glass pyramid $\approx \frac{1}{3} \times 8.16^2 \times 21.15$
 $\approx 469 \text{ cm}^3$

A1

M1

A1



- b i If the shop owner does not buy any models, he cannot sell any models, so he makes neither profit nor loss.

R1

$\therefore P(0) = 0$

$\therefore d = 0$

- ii Using the table of values,

$$P(4) = 4^3a + 4^2b + 4c = 32\,000$$

$$\therefore 64a + 16b + 4c = 32\,000 \quad \dots (1)$$

A1

$$P(9) = 9^3a + 9^2b + 9c = 85\,500$$

$$\therefore 729a + 81b + 9c = 85\,500 \quad \dots (2)$$

A1

$$P(12) = 12^3a + 12^2b + 12c = 115\,200$$

$$\therefore 1728a + 144b + 12c = 115\,200 \quad \dots (3)$$

A1

Solving these simultaneously, using technology, $a = -\frac{100}{3}$, $b = \frac{2200}{3}$, $c = 5600$

$$\therefore P(x) = -\frac{100}{3}x^3 + \frac{2200}{3}x^2 + 5600x \text{ euros}$$

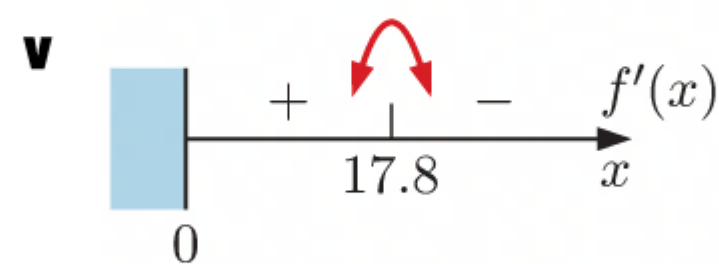
A1

iii $P'(x) = -100x^2 + \frac{4400}{3}x + 5600$

A2

- iv Using technology, $P'(x) = 0$ when $x \approx -3.14$ or 17.8

A2



The maximum profit will occur when the shop owner buys about 17 800 models.

A1

The profit in this case is $P(17.8) \approx \text{€}144\,000$

A1

Total [17 marks]

- 4 a Using technology, $r_p \approx 0.605$.

A1

There is a weak positive correlation between a student's mass and their BMD.

A1

- b Using technology, the least squares regression line is $y \approx 0.0319x - 2.43$

A2

- c When $x = 92$, $y \approx 0.0319 \times 92 - 2.43$

M1

$$\approx 0.497$$

A1

We estimate this student will have $\text{BMD} \approx 0.497$. However, we cannot expect this to be a reliable estimate because, although it is an interpolation, the correlation between the variables is weak.

A1

d

Student	A	B	C	D	E	F	G	H	I	J
rank of mass x	7	2	5	6	9	1	10	4	3	8
rank of BMD y	5	6	4	3	10	1	7.5	2	7.5	9

A1A1

e Using technology, $r_s \approx 0.620$. A2

This confirms there is a weak positive correlation between a student's mass and their BMD.

Since $r_p \approx r_s$, there is no suggestion that a non-linear model would be a better fit. A1

Total [12 marks]

5 a Distance fallen $= \int_0^{\tau} v(t) dt$ metres. A1

b i $v \propto t$

$\therefore v = \beta t$ for some constant β

Now $v(1) = 9.8$

$\therefore \beta = 9.8$

(M1)

So, the proportionality constant is 9.8, and $v = 9.8t \text{ m s}^{-1}$.

A1A1

ii Distance fallen in the first 2 seconds $= \int_0^2 9.8t dt$ M1

$$= [4.9t^2]_0^2$$

$$= 4.9 \times 4 - 0$$

$$= 19.6 \text{ m}$$

A1

c i $V(0) = 53(1 - e^0) = 53(1 - 1) = 0$ A1

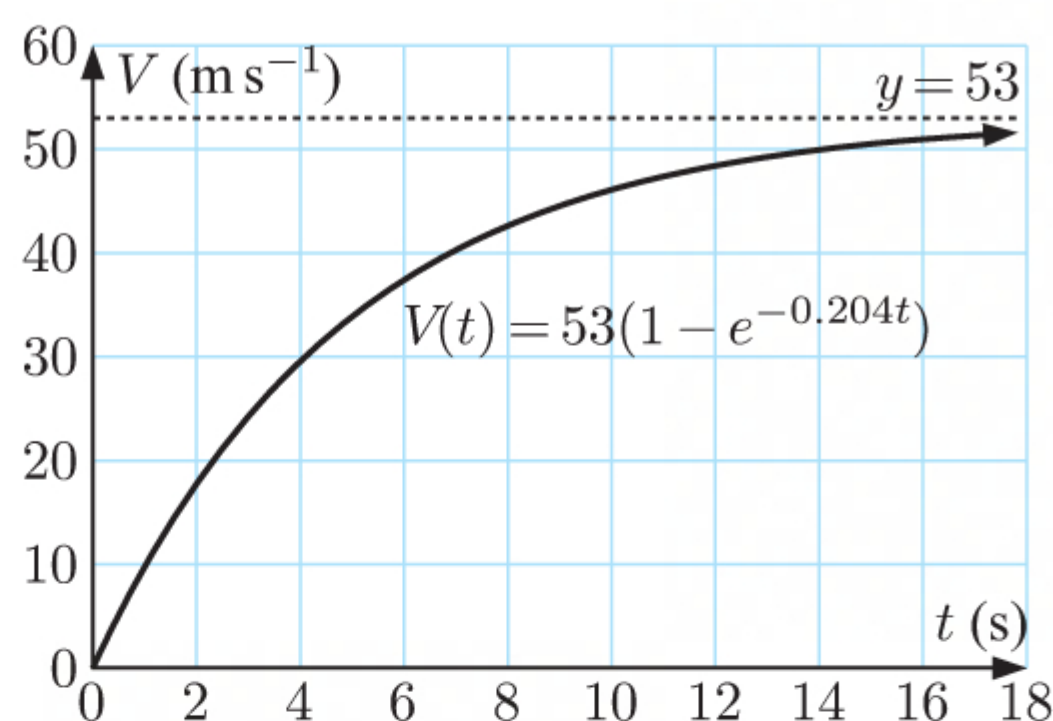
This matches $v(0) = 0$ and the fact that at time 0 seconds, the skydiver jumps and has therefore not yet fallen. R1

ii $V(1) = 9.8$

$\therefore 53(1 - e^{-k}) = 9.8$ (M1)

Solving using technology, $k \approx 0.204$ A1

iii A2



Over time, the speed of the skydiver approaches 53 m s^{-1} . A1

iv Distance fallen in the first 5 seconds $= \int_0^5 53(1 - e^{-0.204t}) dt$ M1A1

$\approx 99 \text{ m}$ {using technology} A1

Total [17 marks]

TRIAL EXAMINATION 3

PAPER 1

- 1 a** Volume of Saturn $\approx \frac{4}{3}\pi r^3$
 $\approx \frac{4}{3}\pi(5.8232 \times 10^4)^3$
 $\approx 8.2713 \times 10^{14} \text{ km}^3$ which agrees with the NASA data. M1A1

- b** Radius of outer ring $R \approx 5.8232 \times 10^4 + 2.82 \times 10^5 \approx 3.40232 \times 10^5 \text{ km}$ A1
 Circumference of orbit $\approx 2\pi R$
 $\approx 2\pi \times (3.40232 \times 10^5)$ M1
 $\approx 2.14 \times 10^6 \text{ km}$ A1

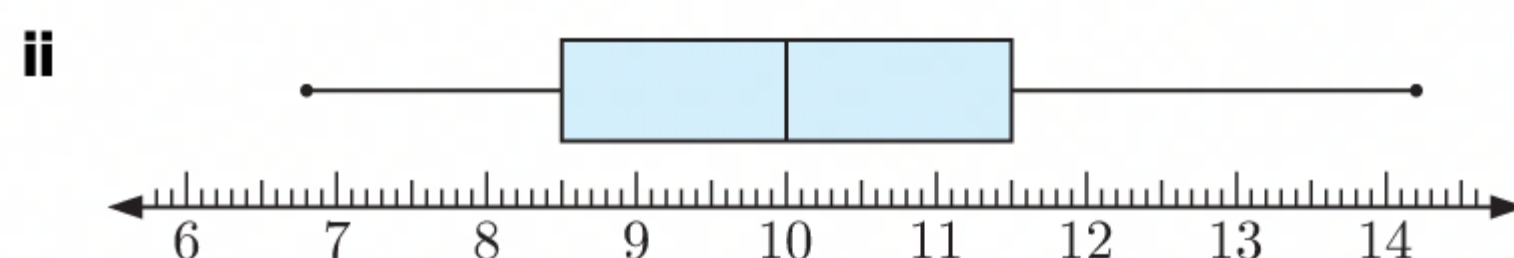
Total [5 marks]

- 2 a** There are $6 \times 6 = 36$ possible outcomes when rolling the dice.
 Of these, the outcomes with sum 5 are: (1, 4), (2, 3), (3, 2), (4, 1). (A1)
 So, the probability that the sum is 5 is $\frac{4}{36} = \frac{1}{9}$. A1

- b** When the pair of dice is rolled 10 times, let X be the number of times that the sum of the dice is 5.
 $\therefore X \sim B(10, \frac{1}{9})$ M1A1
 Using technology, $P(X \geq 2) \approx 0.307$ A1

Total [5 marks]

- 3 a i** minimum = 6.8
 $Q_1 = 8.5$ A1
 median = 10.0
 $Q_3 = 11.5$ A1
 maximum = 14.2 A1



A1 : scale
 A1 : box
 A1 : whiskers

- b i** Range = $13.1 - 3 = 10.1$ A1
 IQR = $11.3 - 8 = 3.3$ A1
ii Each statistic of the 5-number summary for sample B is lower than the corresponding statistic for sample A. R1
 The IQR for sample A is also less than the IQR for sample B. R1
 \therefore sample A had better and more reliable growing conditions. A1

Total [11 marks]

- 4 a** Using technology, $r \approx 0.974$ A1
b Using technology, $y \approx 2.72x + 8.21$ A1A1
c Letting $y = 50$, $2.72x + 8.21 \approx 50$ M1
 $\therefore 2.72x \approx 41.79$
 $\therefore x \approx 15.4$ A1

Jody would expect the worker to travel about 15.4 km to the office.

Total [5 marks]

- 5 a** As t becomes very large $(\frac{1}{2})^{\frac{t}{400}}$ approaches zero.
 $\therefore M$ approaches A . R1
 $A = 97.8\%$ of 18.61 g
 $\therefore A \approx 18.2 \text{ g}$ A1

$$\begin{aligned} \mathbf{b} \quad M(0) &= 18.61 & \mathbf{M1} \\ \therefore 18.61 &\approx 18.2 + B\left(\frac{1}{2}\right)^0 & \\ \therefore B &\approx 0.41 & \mathbf{A1} \end{aligned}$$

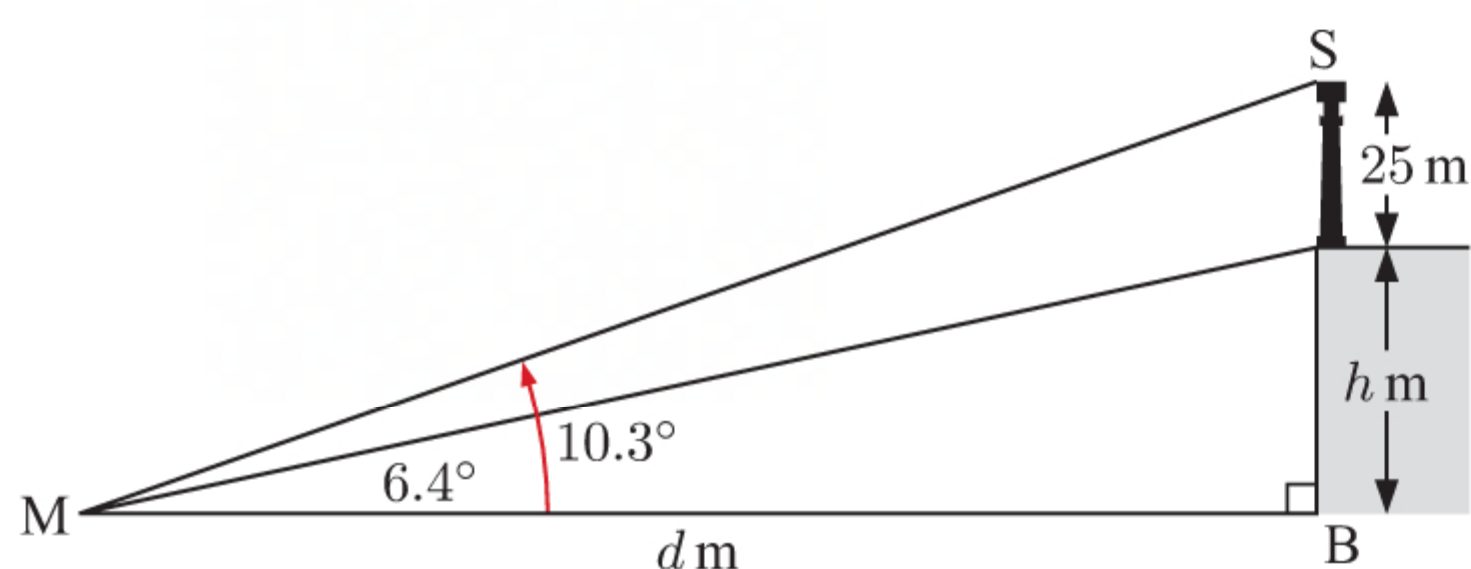
\mathbf{c} We assume $2 \text{ years} = 2 \times 365.25 \text{ days}$
 $= 730.5 \text{ days}$

$$\begin{aligned} M(730.5) &\approx 18.2 + 0.41\left(\frac{1}{2}\right)^{\frac{730.5}{400}} & \mathbf{M1} \\ &\approx 18.3 & \mathbf{A1} \end{aligned}$$

\therefore after 2 years, the mass is about 18.3 g.

Total [6 marks]

6 a



Suppose the cliff is h m high and that Markus is d m from the base of the cliff.

$$\therefore \tan 6.4^\circ = \frac{h}{d} \text{ and } \tan 10.3^\circ = \frac{h + 25}{d} \quad (\mathbf{M1})$$

$$\therefore \tan 10.3^\circ = \tan 6.4^\circ + \frac{25}{d} \quad (\mathbf{M1})$$

$$\therefore \frac{25}{d} = \tan 10.3^\circ - \tan 6.4^\circ$$

$$\therefore d = \frac{25}{\tan 10.3^\circ - \tan 6.4^\circ}$$

$$\therefore d \approx 359.39 \quad \mathbf{A1}$$

\therefore Markus is about 359 m from the base of the cliff.

$$\begin{aligned} \mathbf{b} \quad h &= d \tan 6.4^\circ & (\mathbf{M1}) \\ &\approx 40.3 & \mathbf{A1} \end{aligned}$$

\therefore the cliff is about 40.3 m high.

Total [5 marks]

$$\begin{aligned} \mathbf{7} \quad \mathbf{a} \quad M &= \frac{2}{3} \log_{10}(7.8 \times 10^{13}) - 3.6 & (\mathbf{M1}) \\ &\approx 5.66 & \mathbf{A1} \end{aligned}$$

The magnitude of the earthquake is about 5.66.

$$\mathbf{b} \quad \text{When } M = 2.6, \quad 2.6 = \frac{2}{3} \log_{10} E - 3.6 \quad (\mathbf{M1})$$

$$\text{Using technology, } E \approx 2.00 \times 10^9 \quad \mathbf{A1}$$

The earthquake releases about 2.00×10^9 J of energy.

Total [4 marks]

$$\begin{aligned} \mathbf{8} \quad \mathbf{a} \quad &\text{If } P(X) \text{ is the probability of finding a card featuring Professor } X, \text{ then} & \mathbf{A1} \\ &H_0: P(D) = 0.6, P(M) = 0.2, P(T) = 0.1, P(F) = 0.07, \text{ and } P(S) = 0.03. \end{aligned}$$

\mathbf{b}	<i>Professor</i>	D	M	T	F	S	$\mathbf{A2}$
	<i>Expected frequency</i>	$150 \times 0.6 = 90$	$150 \times 0.2 = 30$	$150 \times 0.1 = 15$	$150 \times 0.07 = 10.5$	$150 \times 0.03 = 4.5$	

$$\mathbf{c} \quad df = 5 - 1 = 4 \quad \mathbf{A1}$$

$$\mathbf{d} \quad \text{Using technology, } p\text{-value} \approx 0.595. \quad \mathbf{A2}$$

$$\mathbf{e} \quad \text{Since } p\text{-value} > 0.05, \text{ we do not have enough evidence to reject } H_0 \text{ at a 5\% significance level, so we accept that the cards are distributed as claimed.} \quad \mathbf{R1A1}$$

Total [8 marks]

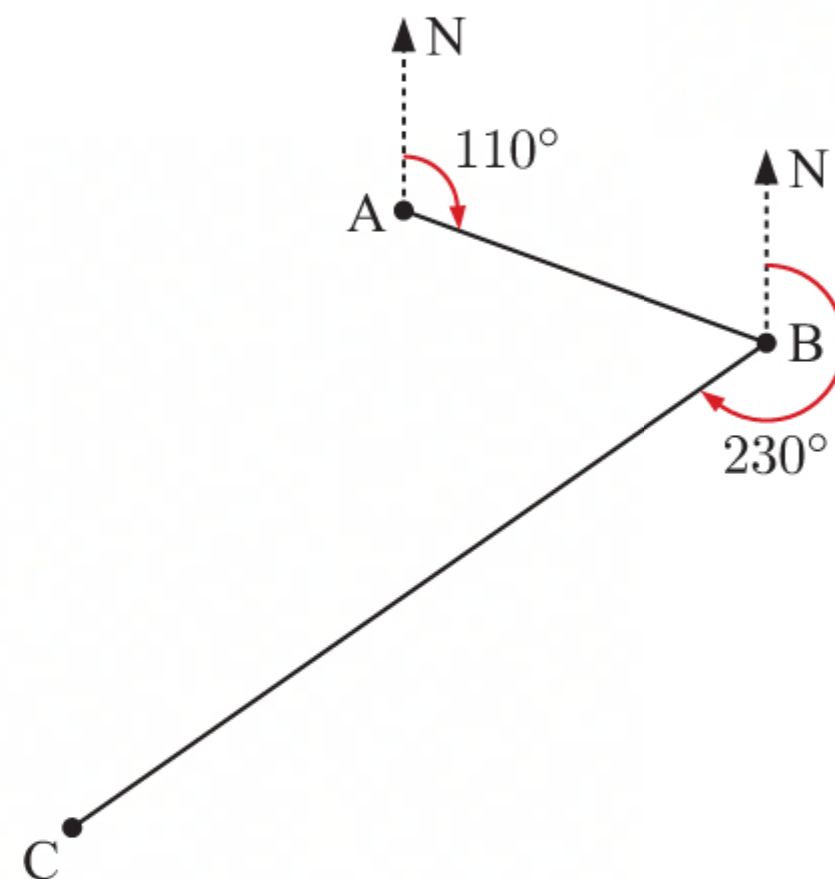
$$\mathbf{9} \quad \mathbf{a} \quad a = 1 \quad \mathbf{A1}$$

$$\begin{aligned} k(-1)(3) &= 6 \\ \therefore k &= -2 & \mathbf{A1} \end{aligned}$$

$$\begin{aligned}
 \text{b Area} &= \int_{-3}^0 f(x) dx \\
 &= \int_{-3}^0 -2(x-1)(x+3) dx && \text{(M1)} \\
 &= \int_{-3}^0 (-2x^2 - 4x + 6) dx \\
 &= \left[-\frac{2}{3}x^3 - 2x^2 + 6x \right]_{-3}^0 && \text{(M1)} \\
 &= 0 - \left(-\frac{2}{3}(-3)^3 - 2(-3)^2 + 6(-3) \right) \\
 &= -(18 - 18 - 18) \\
 &= 18 \text{ units}^2 && \text{A1}
 \end{aligned}$$

Total [5 marks]

10 a



$$\begin{aligned}
 \widehat{ABC} &= 360^\circ - 230^\circ - (180 - 110)^\circ \\
 &= 60^\circ
 \end{aligned}$$

M1

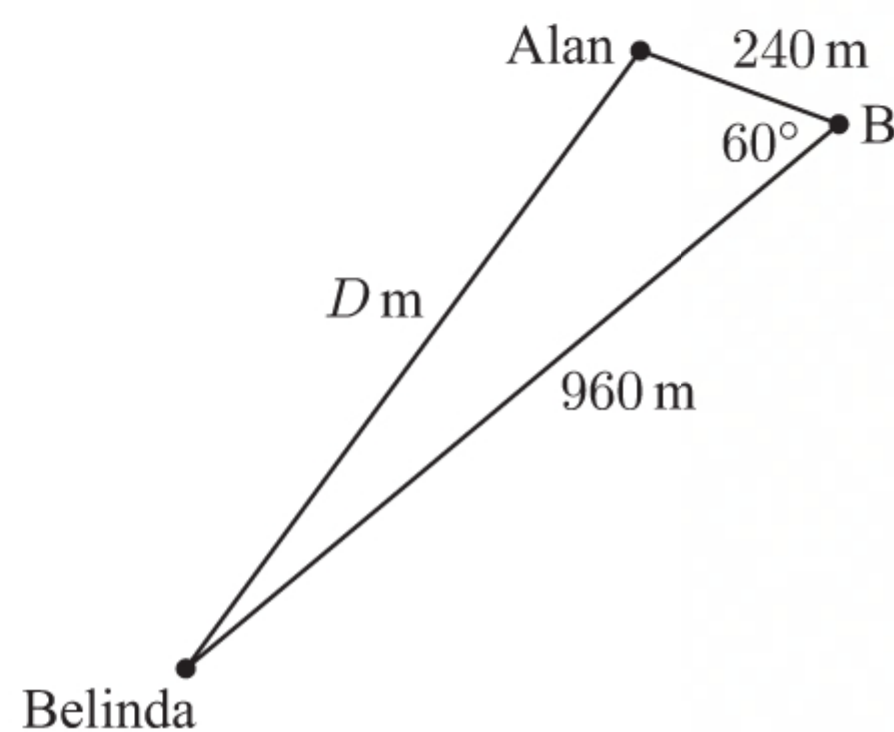
A1

 b After 2 minutes, Alan is $(600 - 3 \times 120) = 240$ m from B.

M1

 In the same time, Belinda cycles $8 \times 120 = 960$ m.

M1


 Let the distance between Alan and Belinda be D m.

Using the cosine rule,

$$\begin{aligned}
 D^2 &= 240^2 + 960^2 - 2 \times 240 \times 960 \times \cos 60^\circ \\
 &= 748\,800
 \end{aligned}$$

M1

$$\therefore D \approx 865$$

A1

After 2 minutes, the distance between Alan and Belinda is about 865 m.

Total [6 marks]

11 a

$$\begin{aligned}
 N &= 3 \\
 PV &= -36\,995 \\
 PMT &= 0 \\
 FV &= 21\,600 \\
 P/Y &= 1 \\
 C/Y &= 1 \\
 \therefore I\% &\approx -16.420
 \end{aligned}$$

M1A1

A1

The annual rate of depreciation was about 16.4%.

b

$$\begin{aligned}
 N &= 10 \\
 I\% &\approx -16.420 \\
 PV &= -36\,995 \\
 PMT &= 0 \\
 P/Y &= 1 \\
 C/Y &= 1 \\
 \therefore FV &\approx 6154.30
 \end{aligned}$$

M1A1

A1

After 10 years, we would expect the forklift to be worth \$6154.30.

Total [6 marks]

- 12

a

H_0 : An adult’s *age group* is independent of their *opinion* about their government’s handling of the COVID-19 pandemic.
- A1
- b

Using technology, $p\text{-value} \approx 0.495$.
- A2
- c

$p\text{-value} > 0.1$, so we do not have enough evidence to reject the null hypothesis, and therefore should accept it.
- R1A1
- We conclude that at a 10% level of significance, *age group* and *opinion* are independent.

Total [5 marks]

- 13

a

Using technology, $\text{volume} \approx 0.0160 \times (\text{height})^{3.00}$
where $r \approx 1.00$.
- A1
- A1
- b

Since $r \approx 1$ for the model in **a**, and the power 3.00 is consistent with volume being proportional to the cube of a length, it is reasonable to believe that the models are mathematically similar.
- R1A1
- c

Using the model from **a**, if $\text{height} = 5.17\text{ m}$ then $\text{volume} \approx 0.016 \times 5.17^3$
 $\approx 2.21\text{ m}^3$
- M1
- This is about 5.3% more than the actual volume of David, so we conclude the replicas are not to scale.
- R1

Total [6 marks]

- 14

a

$k = 1 - 0.21 - 0.36 - 0.16 - 0.04 - 0.01$
 $= 0.22$
- A1
- b

Expected number of errors $= \sum_{i=0}^5 x_i p_i$
 $= 0 \times 0.21 + 1 \times 0.36 + 2 \times 0.22 + 3 \times 0.16 + 4 \times 0.04 + 5 \times 0.01$
 $= 1.49\text{ errors per 100 words.}$
- M1
- A1

Total [3 marks]

PAPER 2

- 1

a

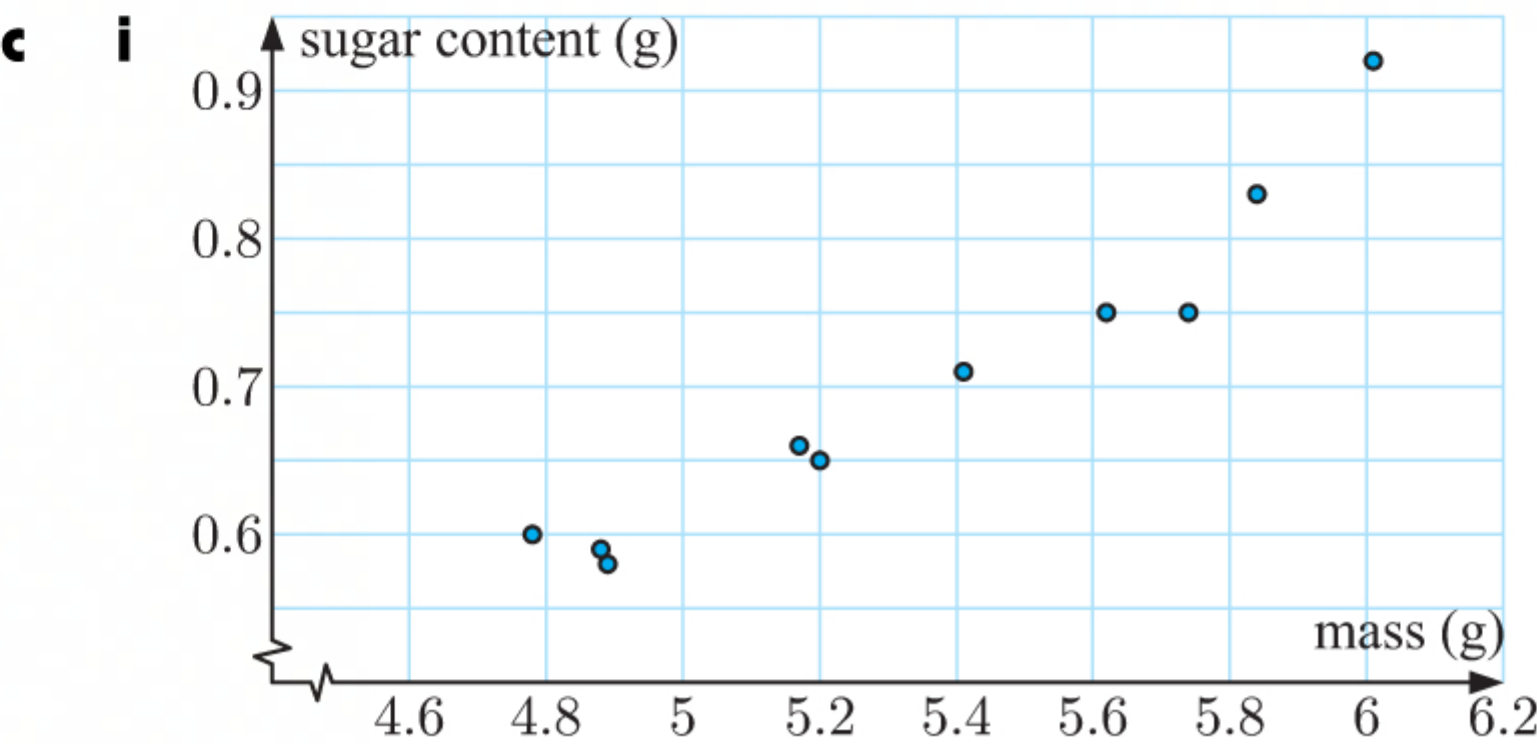
$X \sim N(5.38, 0.62^2)$
 $P(X < k) = 0.4$
Using technology, $k \approx 5.22$
This means that 40% of the cherries have mass less than about 5.22 g.
- M1
- A1
- A1
- b

i

Using technology, $P(X > 6) \approx 0.1587 \approx 0.159$
- A1
- ii

Christina would expect $\approx 50 \times 0.1587 \approx 8$ cherries to have mass greater than 6 g.
- M1A1
- iii

Let Y be the number of cherries in Christina’s sample with mass greater than 6 g.
 $\therefore Y \sim B(50, 0.1587)$
Using technology, $P(Y \geq 4) \approx 0.967$
- M1
- A1



- ii

From the scatter diagram, the data appears non-linear. We therefore favour Spearman’s rank correlation coefficient rather than applying Pearson’s correlation coefficient directly.
- R1

iii

Mass (g)	5.17	5.84	6.01	5.74	4.88	5.41	5.62	4.78	4.89	5.20
rank of mass	4	9	10	8	2	6	7	1	3	5
Sugar content (g)	0.66	0.83	0.92	0.75	0.59	0.71	0.75	0.60	0.58	0.65
rank of sugar content	5	9	10	7.5	2	6	7.5	3	1	4

A1

A1

iv Using technology, $r_s \approx 0.936$

A2

v There is a strong positive correlation between the variables.

A1

Total [16 marks]

- 2 a The data begins with high tide at $t = 0$.

The cosine model is more appropriate, since it starts at its maximum value.

R1A1

$$\begin{aligned} \text{b Amplitude} &= \frac{\text{maximum} - \text{minimum}}{2} \\ &\approx \frac{3.3 - 0.9}{2} \\ &\approx 1.2 \end{aligned}$$

A1

$$\therefore a \approx 1.2$$

A1

$$\begin{aligned} \text{c Mean water depth} &= \frac{\text{maximum} + \text{minimum}}{2} \\ &\approx \frac{3.3 + 0.9}{2} \\ &\approx 2.1 \end{aligned}$$

A1

$$\therefore d \approx 2.1$$

A1

- d Period = time between high tides
 ≈ 12.4 hours

A1

$$\therefore \frac{360}{b} \approx 12.4$$

$$\therefore b \approx \frac{360}{12.4} \approx 29.0$$

A1

e $h(t) \approx 1.2 \cos(29.0t)^\circ + 2.1$

M1

$$\begin{aligned} \therefore h(14) &\approx 1.2 \cos(29.0 \times 14) + 2.1 \\ &\approx 2.93 \end{aligned}$$

A1

\therefore the depth of the water is about 2.93 m.

$$\begin{aligned} \text{f } h(t) = 1.4 \text{ when } 1.2 \cos(29.0t)^\circ + 2.1 &= 1.4 \\ \therefore 1.2 \cos(29.0t)^\circ &= -0.7 \\ \therefore \cos(29.0t)^\circ &= -\frac{0.7}{1.2} = -\frac{7}{12} \end{aligned}$$

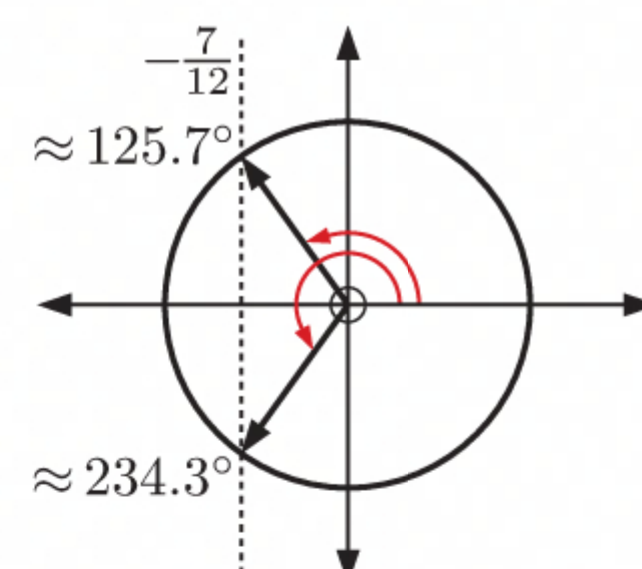
(M1)

For $0 \leq \theta \leq 360^\circ$,

$$\cos \theta = -\frac{7}{12}$$

when $\theta \approx 125.7^\circ$

and $\theta \approx (360 - 125.7)^\circ \approx 234.3^\circ$



$$\therefore \cos \theta < -\frac{7}{12} \text{ for } 125.7^\circ < \theta < 234.3^\circ.$$

(M1)

\therefore the fishing boats can pass over the reef for about $\frac{360 - (234.3 - 125.7)}{360} \times 100\% \approx 69.8\%$ of the time, which is, on average, about $0.698 \times 24 \approx 16.8$ hours per day.

M1A1

- g The top of the reef is 55 cm higher than the sea floor beside the pier, so the depth of water at the reef is 55 cm less.

$$\therefore r(t) \approx 1.2 \cos(29.0t)^\circ + 1.55 \text{ m}$$

A1

Total [15 marks]

- 3 a Investment A is simple interest, so it results in an arithmetic sequence.

A1

Investment B is compound interest, so it results in a geometric sequence.

A1

b Each quarter, the interest paid is $\text{€}10\,000 \times \frac{0.05}{4}$

A1

$$\begin{aligned} \therefore \text{after } n \text{ quarters, the investment is worth } &\text{€}10\,000 + n \times \text{€}10\,000 \times \frac{0.05}{4} \\ &= \text{€}10\,000(1 + 0.0125n) \end{aligned}$$

A1

- c i** Each quarter, the value of the investment is multiplied by $1 + \frac{0.044}{4} = 1.011$ A1
 \therefore after n quarters, the investment is worth $\text{€}10\,000 \times 1.011^n$ A1
ii After 7 quarters, the compound interest would be worth $\text{€}10\,000 \times 1.011^7 = \text{€}10\,795.88$ A1

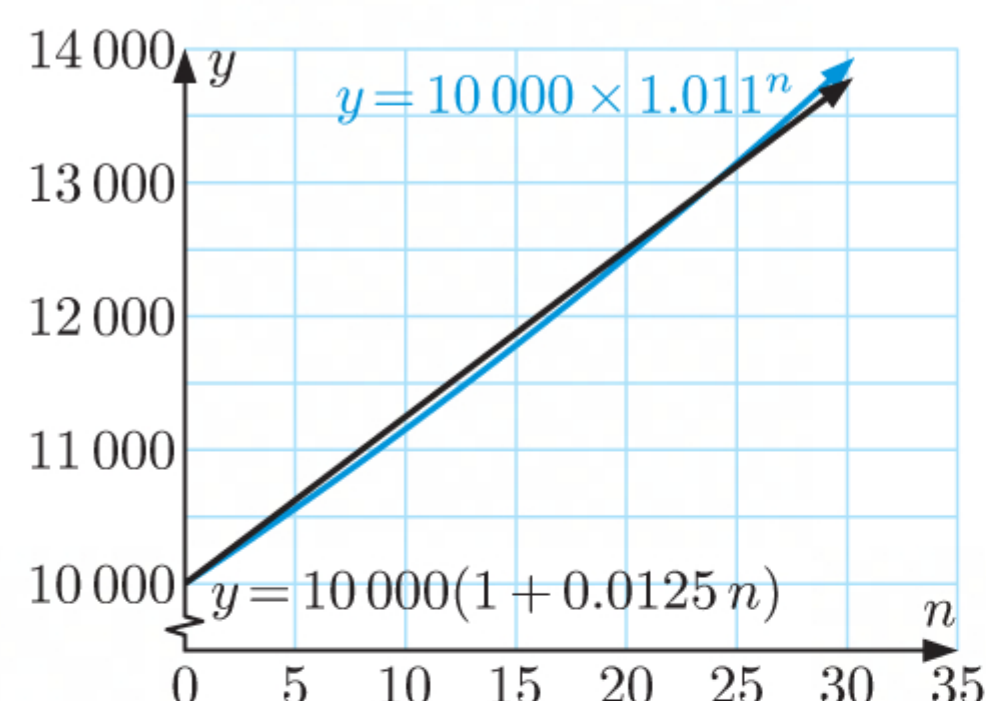
- d** We use technology to compare the graphs of

$$y = 10\,000(1 + 0.0125n)$$

and $y = 10\,000 \times 1.011^n$

The graphs intersect when $n \approx 23.85$

\therefore it will take 24 quarters, which is 6 years, for the compound interest investment to be the better option.



- e i** 15 years = 15×4 quarters = 60 quarters M1
The starting balance is $\text{€}10\,000 \times 1.011^{60}$ A1
 $= \text{€}19\,278.33$ A1
ii $N = 5 \times 12 = 60$
 $I\% = 2.8$
 $PV = -19\,278.33$
 $FV = 0$
 $P/Y = 12$
 $C/Y = 12$ M1A1
 $\therefore PMT \approx 334.69$ A1
Portia can afford to withdraw $\text{€}334.69$ each month.

Total [15 marks]

- 4 a** Systematic sampling A1
b i $\bar{x} \approx 162.5$ (162.4643) A1
 $s \approx 5.01$ (5.008 85) A1
ii A one-tailed t -test for a population mean, as James needs to be confident that the mean weight is *more than* 160 kg, and he only knows the *sample* standard deviation. R1A1
iii $H_0: \mu = 160$ A1
 $H_1: \mu > 160$ A1
iv Using technology, p -value ≈ 0.0443 A1
Since p -value < 0.05 , there is sufficient evidence to reject H_0 in favour of H_1 at the 5% significance level. R1A1
James can therefore confidently sell the bales he is receiving.
c i A two-sample t -test for comparing population means. A1
ii James will need to assume that the standard deviation of his sample is the same as Susan's. A1
iii If the mean of James' bales and Susan's bales are μ_J and μ_S respectively, then
 $H_0: \mu_J = \mu_S$ A1
 $H_1: \mu_J \neq \mu_S$ A1
iv Using technology, p -value ≈ 0.312 A1
Since p -value > 0.05 , there is not sufficient evidence to reject H_0 in favour of H_1 at the 5% significance level. R1A1
James therefore concludes that the mean weight of his bales is not significantly different from the mean weight of Susan's bales.

Total [17 marks]

5 a The midpoint of $[AB]$ is $\left(\frac{-5+1}{2}, \frac{3+5}{2}\right)$ which is $(-2, 4)$. **M1**

The gradient of $[AB]$ is $\frac{5-3}{1-(-5)} = \frac{2}{6} = \frac{1}{3}$

\therefore the gradient of the perpendicular bisector is -3 . **M1**

\therefore the equation of the perpendicular bisector is $y - 4 = -3(x + 2)$
which is $y = -3x - 2$ **A1**

b i The gradient of the perpendicular bisector of $[AC]$ is $\frac{2}{3}$.

\therefore the gradient of $[AC]$ is $-\frac{3}{2}$. **M1**

$$\therefore \frac{k-3}{-1-(-5)} = -\frac{3}{2}$$

$$\therefore k-3 = -6$$

$$\therefore k = -3$$
 A1

ii P lies at the intersection of $y = -3x - 2$ and $y = \frac{2}{3}x + 2$. **M1**

Equating values of y , $-3x - 2 = \frac{2}{3}x + 2$

$$\therefore -\frac{11}{3}x = 4$$

$$\therefore x = -\frac{12}{11}$$
 A1

$$\therefore y = -3\left(-\frac{12}{11}\right) - 2 = \frac{36-22}{11} = \frac{14}{11}$$
 A1

\therefore P is at $\left(-\frac{12}{11}, \frac{14}{11}\right)$.

c i M lies in the region closest to A, so its altitude is about 2635 m. **A1**

ii N lies on the boundary between C and D so its altitude $\approx \frac{2307 + 2683}{2}$
 ≈ 2495 m **A1**

d Q lies at the intersection between B, C, and D, so $\frac{z + 2307 + 2683}{3} = 2602$ **M1**

$$\therefore z + 4990 = 7806$$

$$\therefore z = 2816$$
 A1

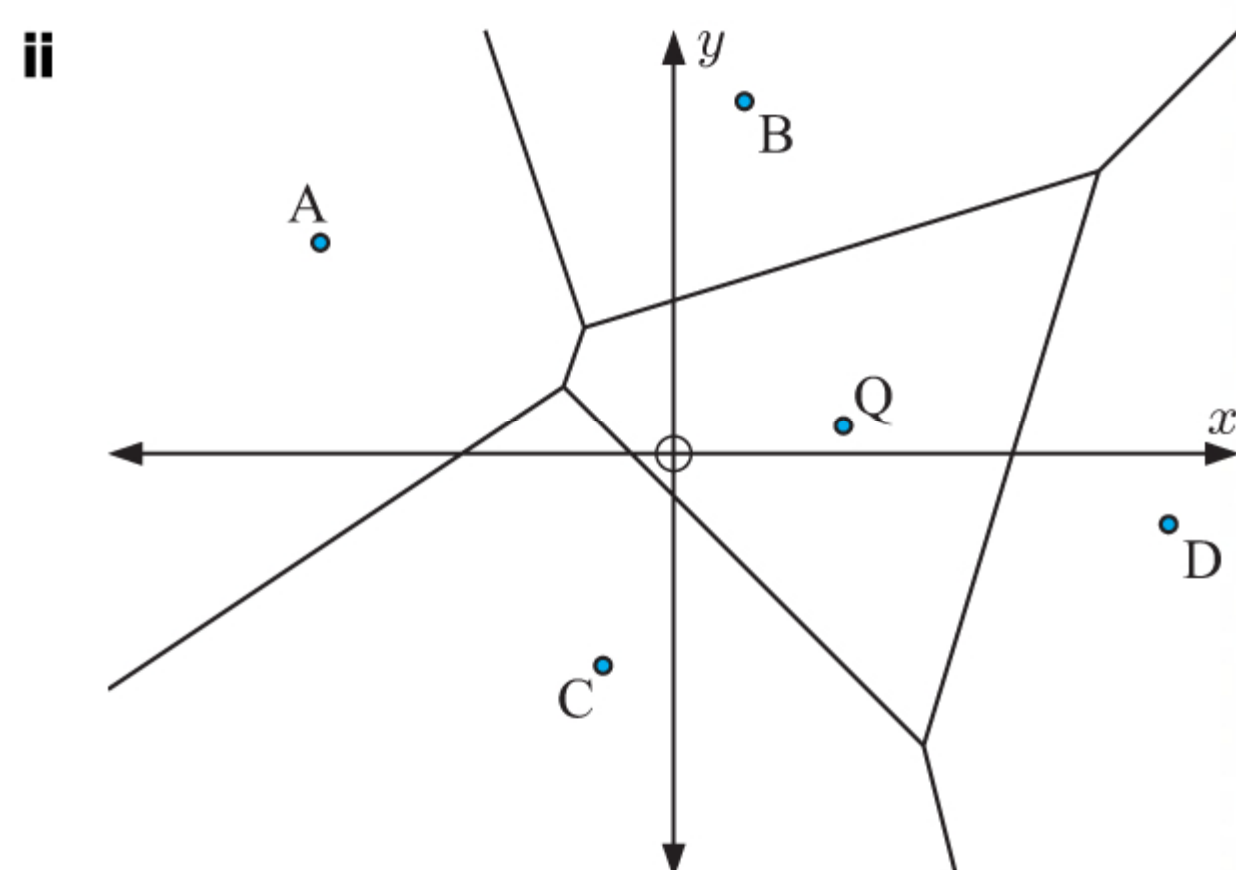
e i $CP = \sqrt{\left(-\frac{12}{11} - (-1)\right)^2 + \left(\frac{14}{11} - (-3)\right)^2}$
 $= \sqrt{\left(-\frac{1}{11}\right)^2 + \left(\frac{47}{11}\right)^2}$
 ≈ 4.27 km **M1**

$$CQ = \sqrt{\left(\frac{12}{5} - (-1)\right)^2 + \left(\frac{2}{5} - (-3)\right)^2}$$

$$= \sqrt{\left(\frac{17}{5}\right)^2 + \left(\frac{17}{5}\right)^2}$$

$$\approx 4.81$$
 km **M1**

\therefore point Q is farthest from the existing weather stations, so it is the location to choose. **A1**



Total [17 marks]

TRIAL EXAMINATION 4

PAPER 1

$$\begin{aligned} \mathbf{1} \quad \mathbf{a} \quad \text{Arc length} &= \frac{128}{360} \times 2\pi(5) && \text{(M1)} \\ &= 11.170 \dots \text{ cm} && \text{A1} \end{aligned}$$

$$\begin{aligned} \text{Perimeter} &= 5 + 5 + 11.170 \dots \\ &\approx 21.2 \text{ cm} && \text{A1} \end{aligned}$$

b Method 1

$$\text{Angle subtended by major sector} = 360^\circ - 128^\circ = 232^\circ \quad \text{(A1)}$$

$$\begin{aligned} \text{Area of major sector} &= \frac{232}{360} \times \pi(5)^2 && \text{(M1)} \\ &\approx 50.6 \text{ cm}^2 && \text{A1} \end{aligned}$$

Method 2

$$\begin{aligned} \text{Area of minor sector} &= \frac{128}{360} \times \pi(5)^2 \\ &= 27.925 \dots \text{ cm}^2 && \text{(A1)} \end{aligned}$$

$$\begin{aligned} \text{Area of major sector} &= \pi(5)^2 - 27.925 \dots && \text{(M1)} \\ &\approx 50.6 \text{ cm}^2 && \text{A1} \end{aligned}$$

Total [6 marks]

$$\mathbf{2} \quad \mathbf{a} \quad r \approx 0.977 \quad \text{A2}$$

$$\begin{aligned} \mathbf{b} \quad a &\approx 0.636 && \text{A1} \\ b &\approx 0.545 && \text{A1} \end{aligned}$$

$$\text{Accept } y \approx 0.636x + 0.545$$

$$\begin{aligned} \mathbf{c} \quad y &\approx 0.636(10) + 0.545 && \text{(M1)} \\ y &\approx 6.91 \end{aligned}$$

$$\text{They would have scored 7 in the Physics test.} \quad \text{A1}$$

Total [6 marks]



$$\begin{aligned} \mathbf{b} \quad P(\text{Julie wins 2nd game}) &= \left(\frac{1}{3} \times 0.5\right) + \left(\frac{2}{3} \times 0.2\right) && \text{(M1)} \\ &= 0.3 && \text{A1} \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad P(\text{Julie wins 1st game} \mid \text{Julie wins 2nd game}) &= \frac{\frac{1}{3} \times 0.5}{0.3} && \text{M1} \\ &= \frac{5}{9} && \text{A1} \end{aligned}$$

Total [6 marks]

$$\mathbf{4} \quad \mathbf{a} \quad X \sim B(30, 0.2) \quad \text{(M1)A1}$$

$$P(X = 12) \approx 0.00638 \quad \text{A1}$$

$$\mathbf{b} \quad P(X \leq 9) \approx 0.939 \quad \text{(M1)A1}$$

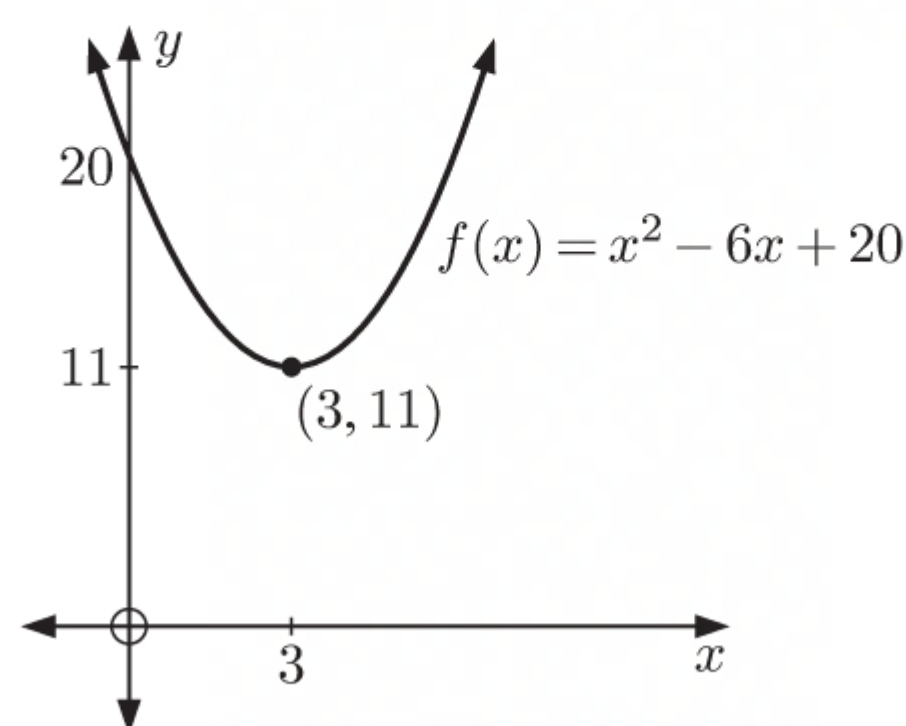
Total [5 marks]

5 a $f(5) = 15$

A1

b Sketch from GDC

(M1)



Range is $\{y \mid y \geq 11\}$.

A1

c Minimum point of $y = x^2 - 6x + 20$ is $(3, 11)$.

(M1)

$\therefore a = 3$

A1

Total [5 marks]

6 a C

A1

b The vertices are $V_1(2, 1)$, $V_2(1, 0)$, and $V_3(2, 3)$.

$$V_1A = \sqrt{(2 - (-1))^2 + (1 - 3)^2} = \sqrt{9 + 4} = \sqrt{13} \approx 3.6055 \dots$$

$$V_2A = \sqrt{(1 - (-1))^2 + (0 - 3)^2} = \sqrt{4 + 9} = \sqrt{13} \approx 3.6055 \dots$$

$$V_3A = 3$$

M1M1

The new recycling centre should be placed at $(2, 1)$ or $(1, 0)$.

A1

Total [4 marks]

7 a Using GDC:

$$N = 5, \quad I\% = 2.5, \quad PV = 0, \quad PMT = -500, \quad P/Y = 1, \quad C/Y = 2$$

(M1)(A1)

$$FV \approx 2628.99, \text{ so she will have saved } \pounds 2628.99.$$

A1

b $I\% = 2.5, \quad PV = 0, \quad PMT = -500, \quad FV = 10\,000, \quad P/Y = 1, \quad C/Y = 2$

(M1)(A1)

$$N \approx 16.4, \text{ so it will take 17 years.}$$

A1

Total [6 marks]

8 a When $t = 75$, $L = 10.5 + 13.9 \ln 85$

(M1)

$$\approx 72.3 \text{ years}$$

A1

b When $L = 60$, $60 = 10.5 + 13.9 \ln(t + 10)$

(M1)

$$\therefore t \approx 25.2$$

\therefore in the year 1925.

A1

Total [4 marks]

9 a $\frac{dV}{dr} = 250 - 3\pi r^2$

A1

b $250 - 3\pi r^2 = 0$

$$\therefore 3\pi r^2 = 250$$

$$\therefore r^2 = \frac{250}{3\pi}$$

M1

$$\therefore r = \sqrt{\frac{250}{3\pi}} \quad \{r > 0\} \quad (\approx 5.15)$$

A1

c When $r \approx 5.15$, $V \approx 250 \times 5.15 - \pi \times (5.15)^3$

M1

$$\approx 858$$

A1

Total [5 marks]

10 a Mean = $\frac{12 + 17 + 12 + 4 + \dots + 10}{20}$

M1

$$= 11$$

A1

- b** $\text{IQR} = 14 - 7$ M1
 $= 7$ A1
- c** Upper boundary $= 14 + (1.5 \times 7)$ M1
 $= 24.5$ A1
- \therefore the largest value that is not an outlier is 18. A1

Total [7 marks]

- 11 a** The sequence is geometric with $u_1 = 12$ and $r = 1.1$. A1
- Solving $2000 = \frac{12((1.1)^n - 1)}{1.1 - 1}$ (M1)
gives $n = 30.129 \dots$ {GDC} (A1)
It will take 31 days in total. A1
- b** She will cycle furthest on the 30th day. (M1)
 $u_{30} = 12(1.1)^{29}$ (M1)
 ≈ 190 miles A1

Total [7 marks]

- 12 a** $H_0: p_1 = \frac{1}{6}, p_2 = \frac{1}{6}, p_3 = \frac{2}{3}$
 $H_1: p_1 \neq \frac{1}{6}, \text{ or } p_2 \neq \frac{1}{6}, \text{ or } p_3 \neq \frac{2}{3}$
where p_1, p_2 , and p_3 represent the proportions of sweet peas with red, white, and pink flowers respectively. A2
- OR
- H_0 : the proportions are in line with the theory.
 H_1 : the proportions are not in line with the theory. A2
- b** 2 A1
- c** Expected values: red = 20
white = 20
pink = 80 (M1)
 $p\text{-value} \approx 0.000\,659$ A1
- d** $0.000\,659 < 0.01 \therefore$ reject H_0 R1
At a 1% significance level the flower colours do not follow the proportions in the theory. A1

Total [7 marks]

- 13** $\tan 15^\circ = \frac{30}{\text{AT}}$ (M1)
 $\therefore \text{AT} = \frac{30}{\tan 15^\circ} = 111.96 \dots \text{ m}$ A1
- $\tan 13^\circ = \frac{30}{\text{BT}}$ (M1)
 $\therefore \text{BT} = \frac{30}{\tan 13^\circ} = 129.94 \dots \text{ m}$ A1
- $\text{AB}^2 = (111.96 \dots)^2 + (129.94 \dots)^2 - 2(111.96 \dots)(129.94 \dots) \cos 60^\circ$ M1A1
 $\therefore \text{AB} \approx \sqrt{14\,872.14}$ {AB > 0} (M1)
 $\therefore \text{AB} \approx 122 \text{ m}$ A1

Total [8 marks]

14 a

	A	B	C	D	E	F	G	H
True rank	1	2	3	4	5	6	7	8
Butcher's rank	1	2.5	4	2.5	7	7	5	7

M1 : tied ranks
A1 : correct values in both rows

- b** Using GDC, $r_s \approx 0.835$. M1A1

Total [4 marks]

PAPER 2

- 1 a** 1.4 cm A1
- b** Volume of the crayon = volume of the cylinder + volume of the cone (M1)

$$= \pi(0.45)^2 \times 7.8 + \frac{1}{3}\pi(0.45)^2(1.4)$$
 M1A1

$$= 5.2590 \dots \approx 5.26 \text{ cm}^3$$
 A1
- c** $s^2 = 1.4^2 + 0.45^2$ {Pythagoras theorem} (M1)

$$\therefore s = \sqrt{1.4^2 + 0.45^2}$$

$$\therefore s = 1.4705 \dots \approx 1.47 \text{ cm}$$
 A1
- d** Total surface area = $\pi(0.45)(1.4705 \dots) + 2\pi(0.45)(7.8) + \pi(0.45)^2$ M1A1

$$= 2.0789 \dots + 22.0539 \dots + 0.6361 \dots$$

$$= 24.769 \dots \text{ cm}^2$$
 A1

$$\therefore \text{percentage covered by label} = \frac{22.0539 \dots}{24.769 \dots} \times 100\%$$
 M1A1

$$\approx 89.0\%$$
 A1
- e** Amount which has been taken from the top = $\frac{1}{3}\pi(0.36)^2(1.12) = 0.1520 \dots \text{ cm}^3$ (M1)A1

$$\therefore \text{volume remaining} = 5.2590 \dots - 0.1520 \dots$$
 (M1)

$$\approx 5.11 \text{ cm}^3$$
 A1

Total [17 marks]

- 2 a** Systematic A1
- b** $P(> 3 \text{ hours screen time}) = \frac{9 + 7 + 7 + 9}{4 + 6 + 8 + \dots + 7 + 7 + 9}$ M1

$$= \frac{32}{95}$$
 A1
- c** $P(\text{Year 11} \cap < 1 \text{ hour screen time}) = \frac{8}{95}$ A2
- d** $P(> 3 \text{ hours} \mid \text{Year 12}) = \frac{9}{11 + 14 + 9}$ (M1)A1

$$= \frac{9}{34}$$
 A1
- e** H_0 : Year group and the amount of screen time are independent A1
- f** 6 A1
- g** $p\text{-value} \approx 0.770$ (0.769 58 ...) A2
- h** $0.770 > 0.1$, so accept H_0 R1
 Year group and the amount of screen time are independent. A1

Total [14 marks]

- 3 a** 35° A1
- b** $\widehat{FBC} = 180^\circ - 35^\circ = 145^\circ$
 $\widehat{BFC} = 180^\circ - 145^\circ - 29^\circ = 6^\circ$ (A1)(A1)
- In $\triangle BCF$, $\frac{FC}{\sin 145^\circ} = \frac{7.5}{\sin 6^\circ}$ M1A1

$$\therefore FC = \frac{7.5 \sin 145^\circ}{\sin 6^\circ}$$

$$= 41.154 \dots$$

$$\approx 41.2 \text{ m}$$
 A1
- c** In $\triangle ACF$, $\sin 29^\circ = \frac{FA}{41.154 \dots}$ (M1)A1

$$\therefore FA = 19.952 \dots$$
 A1
 The height of the flagpole is about 20.0 m.

- d** In $\triangle ACF$, $\tan 29^\circ = \frac{19.952 \dots}{AC}$ (M1)
 $\therefore AC = \frac{19.952 \dots}{\tan 29^\circ}$ (A1)
 $= 35.994 \dots$ (M1)
 $\therefore \text{total distance} = 35.994 \dots \times 4$ (M1)
 $\approx 144 \text{ m}$ A1

Total [13 marks]

4 a

x	2	2.5	3	3.5	4	4.5	5	5.5	6
$y = \sqrt{5x-1}$	3	3.3912	3.7417	4.0620	4.3589	4.6368	4.8990	5.1478	5.3852

A2

- b** $h = \frac{1}{2}$ (A1)
 $\int_2^6 \sqrt{5x-1} dx$
 $\approx \frac{1}{2} \left(\frac{1}{2} \right) [(3 + 5.3852) + 2(3.3912 + 3.7417 + 4.0620 + 4.3589 + 4.6368 + 4.8990 + 5.1478)]$ (M1)A1
 ≈ 17.215 A1

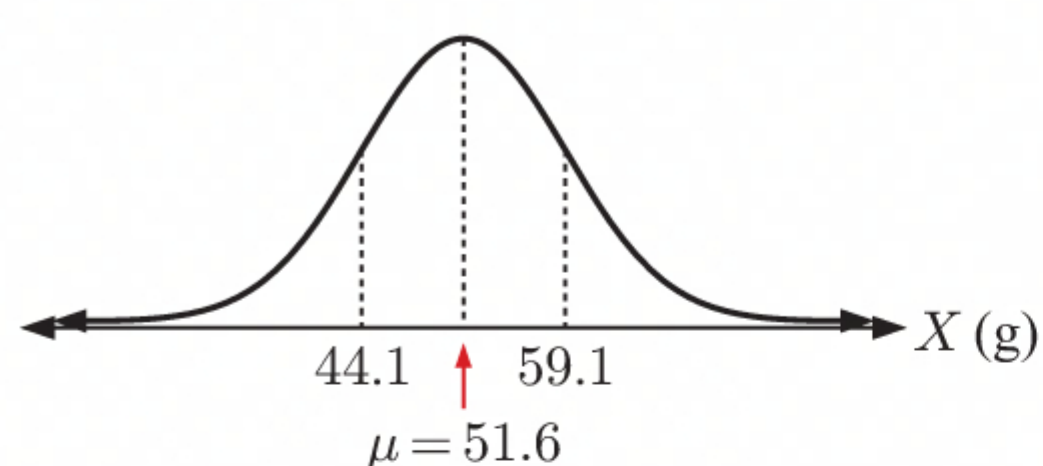
- c** $\int_2^6 \sqrt{5x-1} dx = 17.222\ 637 \dots$
 ≈ 17.223 A2

- d** $\frac{|17.215 - 17.223|}{17.223} \times 100\%$ (M1)A1
 $\approx 0.0464\%$ A1

- e** Area = $\int_2^6 \sqrt{5x-1} dx - \text{area of triangle}$ (A1)
 $\approx 17.223 - \frac{1}{2} \times 4 \times 3$ M1A1
 $\approx 11.223 \text{ m}^2$ A1

Total [15 marks]

- 5 a** $X \sim N(51.6, 7.5^2)$



A1 : “bell curve” shape
 A1 : mean correctly labelled
 A1 : one standard deviation labelled

- b** $P(X < 49) \approx 0.364$ (M1)
 $\therefore \approx 36.4\%$ of eggs weighed less than 49 g. A1
- c** $P(X > k) = 0.2787$ (M1)
 $\therefore k \approx 56.0$ A1
- d** Let Y be the number of large eggs selected.
 $Y \sim B(40, 0.2787)$ M1
 $E(Y) = 40 \times 0.2787$ (A1)
 $= 11.148 \text{ eggs}$ (accept 11 or 11.1) A1
- e** $\frac{2}{5}$ of 40 = 16 (M1)
 $P(Y = 16) \approx 0.0328$ (M1)A1
- f** $\bar{x} = 53.1$
 $s \approx 13.0$ A3
- g i** $H_0: \mu_0 = 51.6$
 $H_1: \mu_0 > 51.6$ A1

ii $p\text{-value} \approx 0.267$

$t \approx 0.630$

A2

h $0.267 > 0.1$, so we accept H_0

R1

The eggs are not significantly heavier than last year, so the farmer is not correct.

A1

Total [21 marks]